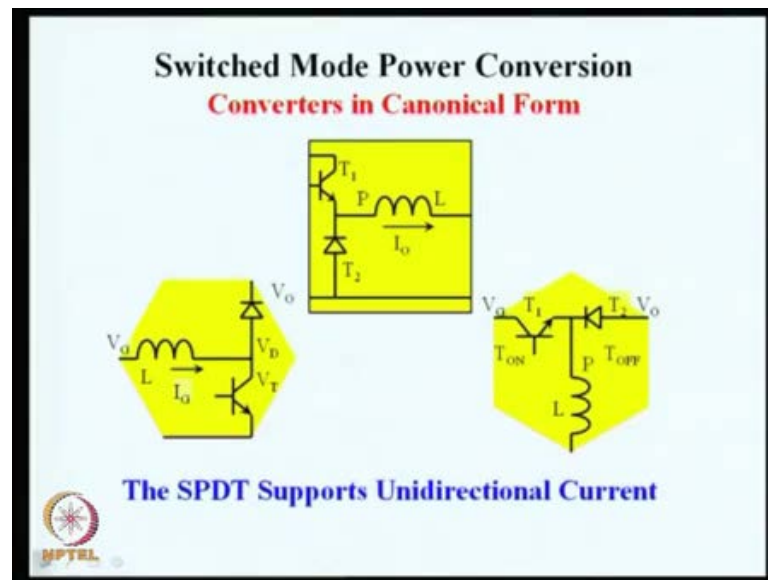


Switched Mode Power Conversion
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Lecture - 16
Conduction Mode

Good day to all of you. In today's lecture we will go through the basic converter circuit and identify their different mode of operation, which is common among all kinds of switched mode power converters.

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So, what you see here in this slide or the 3 basic canonical form converters. The first converter, which is the buck converter, you see here which steps down the DC voltage from a source voltage of V_G to an output voltage of V_{naught} , which is less than the source voltage. Then we also look at the second converter which is the boost converter. This is a canonical model where the source voltage V_G that you see here is boosted up to voltage of V_{naught} , which is higher than the input voltage.

The third canonical circuit which is called the fly back converter or the buck boost converter where the source voltage V_G is converted to the V_{naught} and V_{naught} can be anywhere below V_G or above V_G . Now, in all these converters if you notice the circuit diagram, the prevailing path is through a way diode. The prevailing path in each one of them is through a diode right.

In this and on account of this, when we have a current flowing through the inductor, what do you see here when you see a current flowing through this inductor? I naught can flow only in one direction because the diode in all the circuits are present in that part of the circuit and it will not allow the current to change direction. So, the SPDT, that is the single pole double throw switch that we use in all this canonical circuits consist of an active switch, which is the consistor and a passive switch which is the diode.

In this kind of realization, they do not allow the current in the inductor to be bidirectional. It supports only unidirectional current. So, on account of this the inductor current cannot change direction and this gives rise to 2 different operating modes in these converters. These operating modes are the continuous current mode of operation and the discontinues mode of operations. Now, in this lecture today we will try to go through what is the special features of these 2 operating modes.

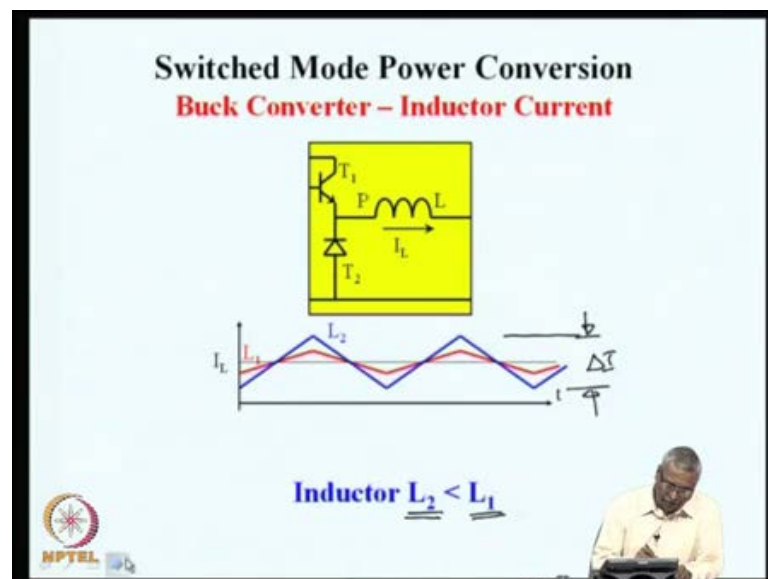
How does one get the converter gain in operating mode? That is in the continuous current mode of operation or in the discontinuous current mode of operation? So, we will take first the buck converter, the boost converter and then the fly back converter. We will go through the process of the prevailing duration and try to see if the current does not change direction and what are the consequences of that depending on the different operating conditions in the converter.

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The slide is titled "Switched Mode Power Conversion" and "Buck Converter - Inductor Current". It features a circuit diagram of a buck converter with a MOSFET (T₁), a diode (T₂), an inductor (L), and a load resistor (P). Below the circuit is a graph of inductor current (I_L) versus time (t). The graph shows a sawtooth waveform that is always positive, indicating continuous current. The rising slope is labeled $di/dt > 0$ and the falling slope is labeled $di/dt < 0$. At the bottom of the slide, the text "Inductor Current is Positive and Contin..." is visible. The MPTEL logo is in the bottom left corner, and a small image of a person is in the bottom right corner.

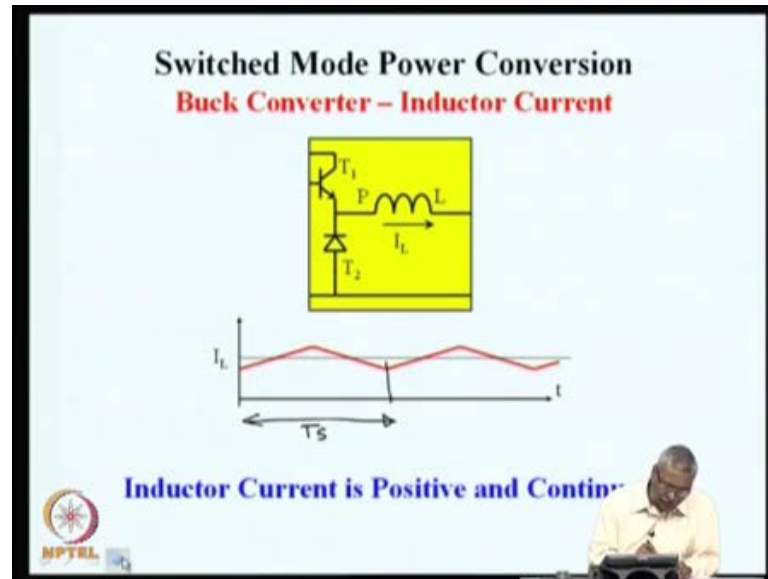
For example, the inductor current in these converters is positive. It has 2 regions. The first one is the raising region of current $\frac{di}{dt} > 0$ and another is the falling region where $\frac{di}{dt} < 0$. Under the steady state of operation, the current is periodic. So, the fall in current in the off period and the raise in current in the on period are equal and give we get an inductor current which is positive all the time and continuous. So, let us look at what happens. So, this inductor current on certain, changes in the operating condition.

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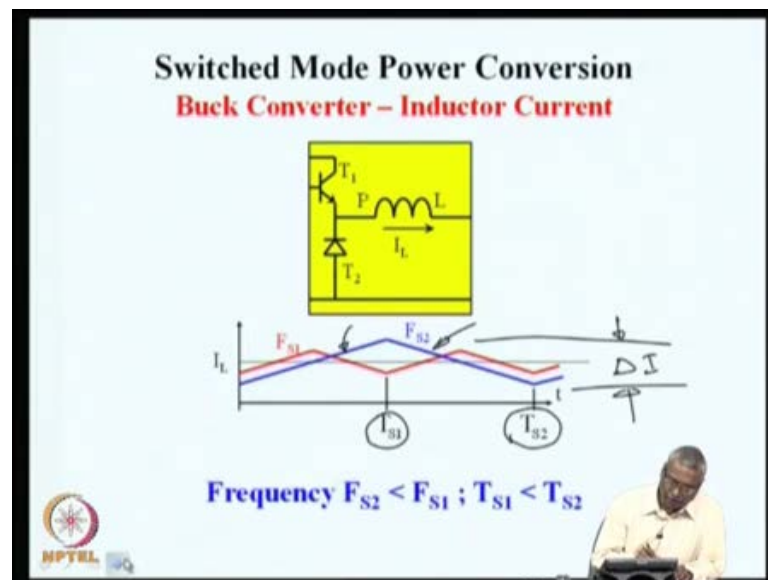
We can consider the condition where we have the inductance value being reduced. For example, the inductor L is reduced from the previous value L_1 to L_2 . L_2 is less than L_1 . The consequence of this is that the current in the inductor is now swinging between larger values of ΔI . This is a consequence of the lower value of inductance.

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In a similar way, it is possible to look at the inductor current when the switching period is changed or switching frequency is changed.

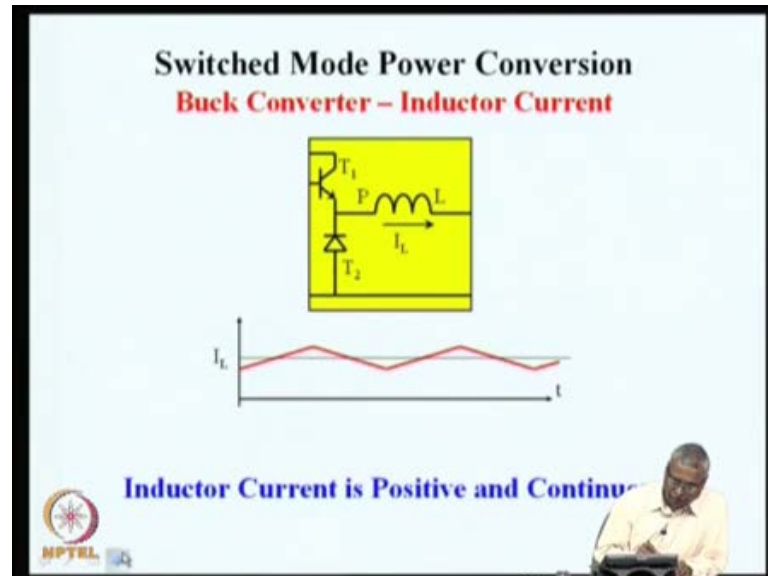
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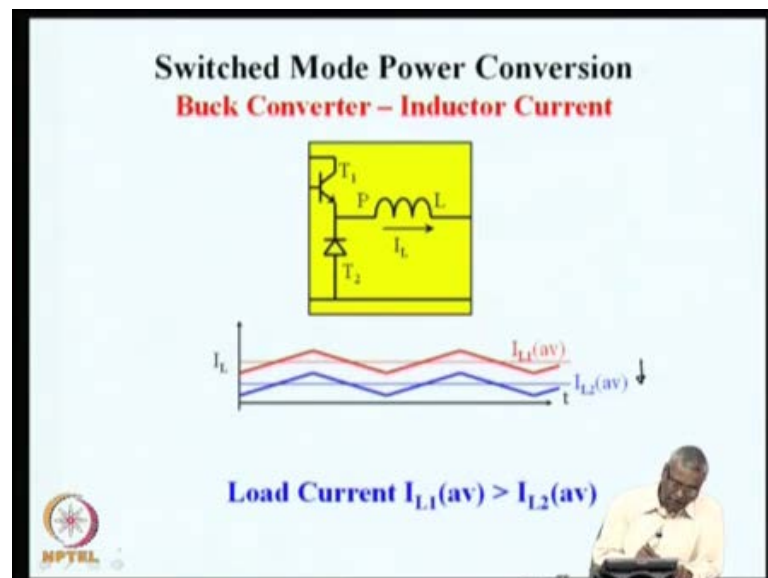
For example, if the switching frequency is changed from T_{s1} to T_{s2} , you can see that now the raising region of the inductor current is prevailing for a larger duration and the falling region of the current is also prevailing over a larger duration. On account of that, this swing ΔI or the ripple current has increased compare to the previous condition. If the switching frequency is decreased, current ripple increases. In the earlier case we had

seen that if the inductance value reduces the ripple current is increasing. In the same way, we can also look at the change in the load current.

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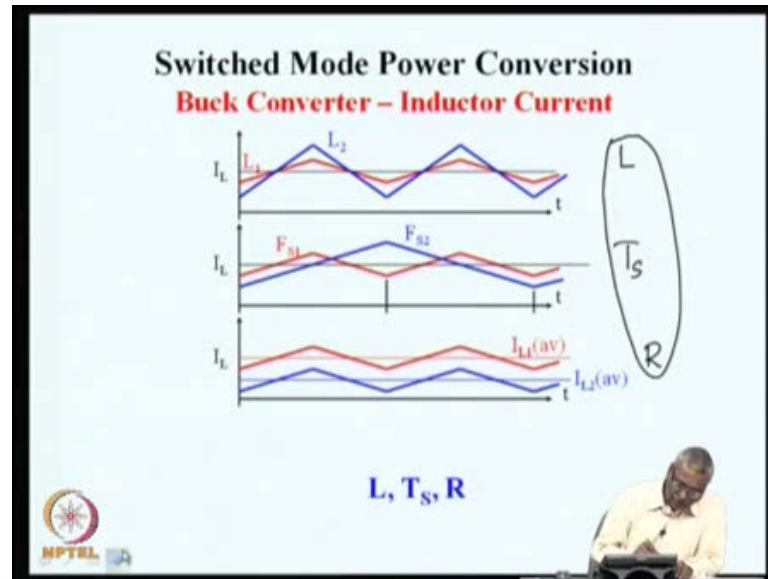


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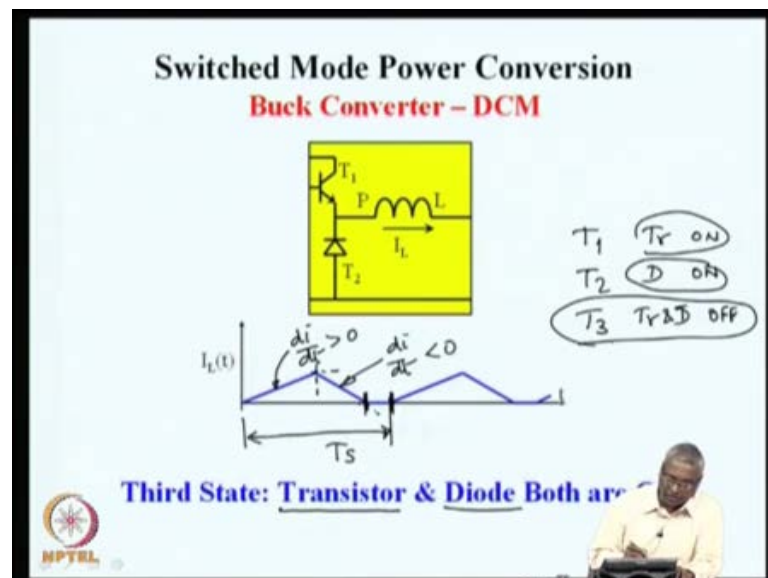
If the load current changes, the net average current in the inductor has dropped down, but the ripple current remains the same or if you see a ripple factor which is ΔI expressed over the average current, that increases. So, we find that there are 3 possible directions in which the ripple current is affected.

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These three can be summarized as change in L . If inductance becomes smaller, the ripple current increases. If the inductance is higher, the ripple current will decrease. If the switching frequency drops, then the ripple current increases. The third one is the load. If the load current is reducing, then also the ripple factor or the ripple current is increasing. So, these are the three parameters, which are governing the ripple current or the ripple factor. Now, the consequence of these changes can be seen in a particular mode of operation.

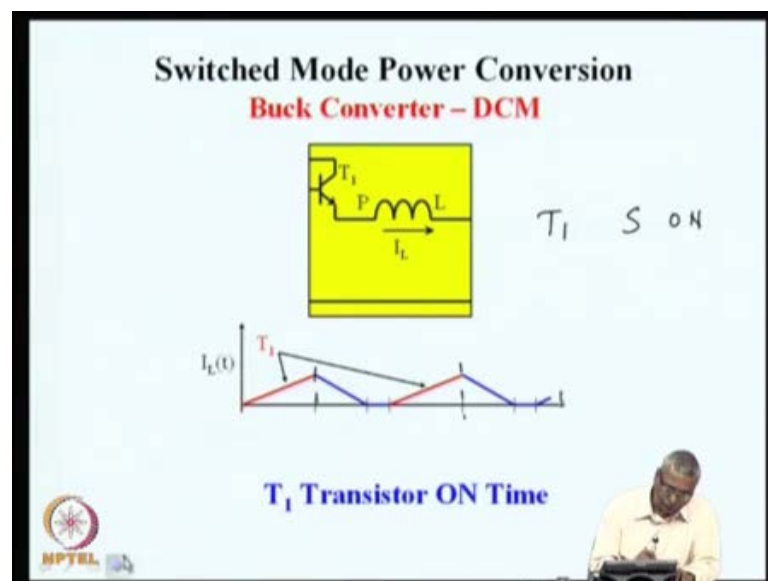
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For example, if the inductor current increases during on of switch to some value and decreases during the off of the switch and if the inductor current comes to 0, even before the end of the cycle, at this point T_s , we see that the current cannot reverse in direction on account of the diode in the circuit. So, during the short interval between the time, diode current comes to 0 and in the time when the next cycle starts the diode and the transistor both are off.

So, we say that there is a third state when the diode and transistor are off, So, we have an interval T_1 when transistor is on, an interval T_2 when diode is on and an interval T_3 when the transistor and diode both are off now. Unlike the pervious mode of operation where current in inductor continuous, we had only 2 mode and 2 sub intervals in 1 cycle where transistor of this is on and diode is on in the next cycle slots. Now, in this mode of operation there is a third interval when both transistor and the diode are off. Now, this particular mode of operation is called discontinues conduction mode of operation or the DCM mode of operation

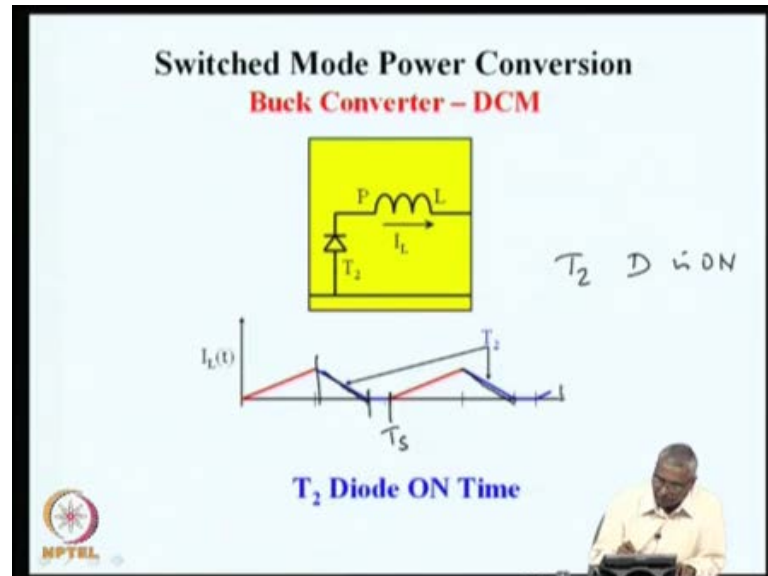
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You can see that it as an interval T_1 when switch is on, which is what is shown here as the first interval where the current is increasing. Then, we have an interval T_2 when the diode is on. So, during this interval whatever stored energy in the inductor is lost, eventually it comes backs to 0, before the end of the cycle. If the cycle ends, T_s even

before the end of the cycle, the energy in the inductor is completely lost. The diode in the circuit ensures that the current cannot reverse in the inductor.

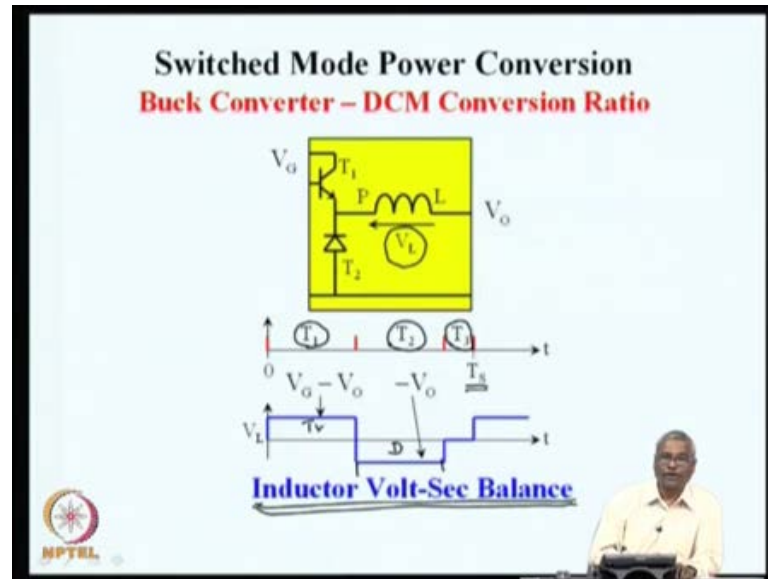
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So, that being the case we had a third interval which is, when both the transistor and the diode are off. No current is flowing through the inductor and the inductor is floating on one end. The other end which is not connected anywhere and current through the inductor is 0. This is the third interval, that is the T_1 interval, T_2 interval and T_3 interval. T_1 is when the switch is on, T_2 is when the diode is on and T_3 is when none of them are on or all of them are off.

So, in 1 interval 1 is in switching period T_s , there are 3 sub interval T_1 , T_2 and T_3 . T_1 is the interval which is the duty ratio of the switch called D . T_2 is the interval, we might call as the diode conduction ratio which is d_2 and the rest of them is $1 - d - d_2$, so that all of them together add up a total normalized switching period of 1. So, we have a duty cycle d , a diode conduction ratio d_2 . Then the rest of the cycle time, that is $1 - d - d_2$ and the total switching time which is normalized to be 1.

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Now, in this particular slide you see that these 3 intervals in 1 cycle T_s , there is an interval T_1 when the diode is a transistor is on. Then there is an next interval T_2 when the diode is on and then the last interval T_3 when none of them are on. So, the current through this is 0. This third interval is 0. Now, the conversion ratio can be found out just as we did for the converters by taking the V_L , the inductor voltage in 1 cycle and finding out the volts. On this volt, the second balance as we see over 1 period now consist of the interval T_1 when the voltage is $V_G - V_O$ and interval T_2 when the voltage is $-V_O$ and the interval T_3 when the voltage across the inductor is 0 or the inductor is just floating.

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$
 $\frac{T_2}{T_s} = d_2$

$(V_g - V_o)T_1 - V_o T_2 = 0$

$\frac{V_o}{V_g} = M = \frac{d}{d + d_2}$

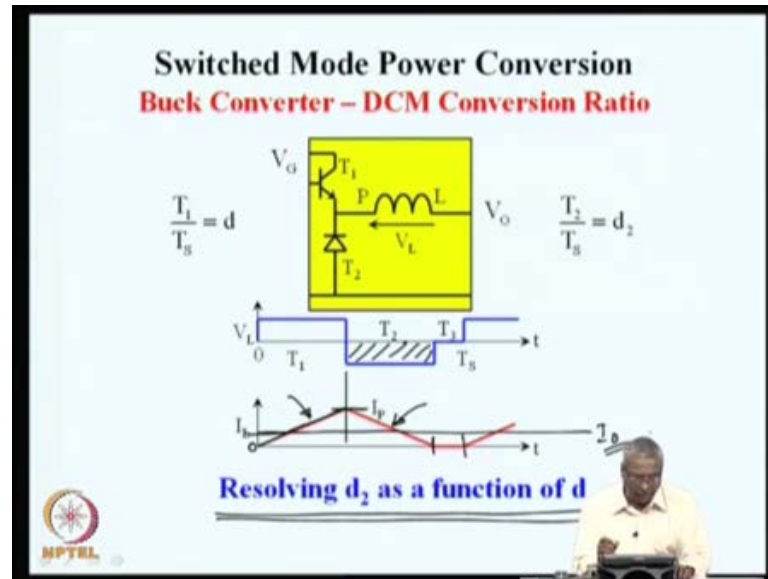
$dV_g = (d + d_2)V_o$
 $M = \frac{d}{d + d_2}$

Voltage Conversion Ratio

So, if we now write down the Volts second balance equation for the inductor, it consist of totally 2 sections. During T 1 your voltage of V G minus V naught is applied across the inductor. During T 2 a voltage of minus V naught is a applied across the inductor. So, we defined this ratio T 1 by T s as the duty ratio d. T 2 by T s is the duty ratio d 2. This is a duty ratio of diode conduction time. This is the duty ratio of switch conduction time and if we write the Volts second balance in terms of voltage multiplied by time, it can be expressed in terms of these ratios as d times V G.

The main switch duty ratio times V G, is now d plus d 2 times V naught. This equation, if it is written in terms of duty ratio you can divide it everywhere by T s and write in this form. From this, it is possible to find out the voltage conversion ratio and it is d by d plus d 2. Now, V in continuous conduction mode, you will find d 2 will be V entire 1 minus d and the denominator will become 1 and conversion ratio is just d. In the discontinuous conduction mode, there is a duration in any cycle when the inductor is left open. On account of that, the duty ratio M is now d divided by d plus d 2.

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What we see here? Now, this d and d_2 are related to each other. Normally when we operate the converter we control only d and d_2 based on the stored energy in the inductor. At the start of the first interval T_1 , the inductor charges up starting from 0 current. It charges up to a current of I_p . Depending on what is this peak current, in the next interval T_2 , the inductor loses all its energy. So, there is a certain rate of rise of current in this interval which is decided by the voltage across the inductor.

During the on period, there is a certain d_i by $d t$ during this diode conduction interval which is decided by the voltage across the inductor during the diode conduction period. There is a time when the voltage is 0 across the inductor or the inductor is left floating. So, it is possible to resolve this lead to as a function of d by realizing that the net dc current flowing out of the inductor is the output load current. The inductor current is shown here in this red wave form. The average value of that is same as the output current.

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$$\frac{T_1}{T_s} = d$$

$$\frac{T_2}{T_s} = d_2$$

$$I_p = d_2 V_o T_s / L$$

$$I_o = \frac{V_o}{R} = \frac{1}{2} I_p (d + d_2) = \frac{1}{2} (d + d_2) \frac{d_2 T_s V_o}{L}$$

Resolving d_2 as a function of d

If you relate this, it is possible for us to find out what is d_2 in terms of d . For example, the peak current is decided by the half interval $d_2 T_s$ multiplied by the d_i by $d t$ during the off interval. That is $d_2 T_s$ into V naught by L . This tells us what is peak current reached in the first interval. Then during the second interval d_2 , this energy is completely lost and we can save at the average value of the wave form that we saw.

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$$\frac{T_1}{T_s} = d$$

$$\frac{T_2}{T_s} = d_2$$

Resolving d_2 as a function of d

The previous average value of this is I_p by 2, on its own base and because this total duration is total ratio is d plus d_2 .

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$$\frac{T_1}{T_s} = d$$

$$\frac{T_2}{T_s} = d_2$$

$$I_p = d_2 V_o T_s / L$$

$$I_o = \frac{V_o}{R} = \frac{1}{2} I_p (d + d_2) = \frac{1}{2} (d + d_2) \frac{d_2 T_s V_o}{L}$$

Resolving d_2 as a function of d

The average current is 1 half I_p into d plus d_2 . This is the average current which is V_o naught by R i naught. It is same as V_o naught by R . This I_p , we can replace from the first equation so that now we get a result that V_o naught by R is same as 1 half of d plus d_2 times $d_2 T_s V_o$ naught by L . Here V_o naught is on both sides.

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$$\frac{T_1}{T_s} = d$$

$$\frac{T_2}{T_s} = d_2$$

$$I_o = \frac{V_o}{R} = \frac{1}{2} I_p (d + d_2) = \frac{1}{2} (d + d_2) \frac{d_2 T_s V_o}{L}$$

$$d_2 (d + d_2) = \frac{2L}{R T_s} \sqrt{K} \quad \underline{d_2^2 + d d_2 - K = 0}$$

d and d_2 are related through Conduction Parameter K

If we cancel that, this $2L$ by $R T_s$ can be taken to the other side. So you see a relationship d_2 into d plus d_2 is equal to K . This can be written as a second order equation in d_2 , namely d_2^2 plus d times d_2 minus K equals to 0. So, for the buck

converter, the relationship between d and d_2 is related through this quadratic equation in d_2 . So, from this equation it is possible to evaluate d_2 in terms of d . Naturally, it will also be a function of K . This K is defined as the conduction parameter. It is related to $2L$. It is related to L .

It is related to R and it is related to T_s . We had seen graphically right in the beginning that the conduction, whether it is continuous or discontinuous has 3 significant causes. 1 of them is the value of inductance, the other is frequency and the third is the load. We saw 3 different causes which can force the inductor current to come closer to 0 and get into discontinuous mode of operation. We see that this effect is consolidated by a single parameter called K and this K is equal to $2L$ divided by $R T_s$.

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

The slide features a circuit diagram of a buck converter in discontinuous conduction mode (DCM). The input voltage is V_o . The switching transistor is labeled T_1 and the diode is T_2 . The inductor current is I_L and the output voltage is V_o . The duty cycle of the transistor is $\frac{T_1}{T_s} = d$ and the duty cycle of the diode is $\frac{T_2}{T_s} = d_2$.

The conduction parameter K is defined as:

$$d_2(d + d_2) = \frac{2L}{RT_s} = K$$

The duty cycle d_2 is given by the quadratic equation:

$$d_2 = \frac{-d + d\sqrt{1 + (4K/d^2)}}{2}$$

d and d_2 are related through Conduction Parameter K

The slide also includes the NPTEL logo and a small image of a person in the bottom right corner.

Now, this quadratic equation can be solved so that we can write d_2 in terms of d and K . So, this is the relationship between d and d_2 .

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$$d_2 = \frac{-d + d\sqrt{1 + (4K/d^2)}}{2}$$

$$M = \frac{V_o}{V_g} = \frac{d}{1 + \sqrt{1 + (4K/d^2)}}$$

Conversion Factor is a Function of d and K

$f(d, K)$
 $\frac{V_o}{V_g} = \frac{d}{1 + d_2}$

If we now substitute back in the first equation, it is possible to find out the conversion gain M as a function of d , which is duty ratio and conduction parameter K , which is this $2L$ by $R T_s$. This is also the same as d by d plus by d_2 . So, V_o by V_g can be expressed in terms of d and d_2 . It can also be expressed as V_o by V_g or can be expressed in terms of d and K or alternately in terms of K and d_2 or in terms of any 2 of the other variables.

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Switched Mode Power Conversion
Buck Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$$M = \frac{d}{d + d_2} > \underline{d} \quad (\because (d + d_2) < 1)$$

Conversion Factor in DCM is More

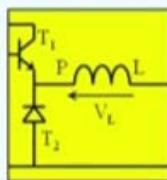
d
 $\frac{d}{d + d_2} < 1$

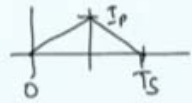
Now, we see that in comparison with the continuous mode of operation, this function in d has become d by d plus d^2 . In the discontinuous mode of operation, d plus d^2 is always less than 1 because the total switching period T_s has T_1 plus T_2 plus T_3 and d plus d^2 is ratio only for 2 of the intervals. The third interval is not counted. So, we see that this addition d plus d^2 will always be less than 1.

So, the conversion gain d by d plus d^2 will be always greater than d , which is the continuous mode of operation gain. So, we can say that the conversion factor in discontinuous mode of operation is always more compared to the conversion factor in the continuous mode of operation.

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Switched Mode Power Conversion
Buck Converter – Border Between DCM & CCM

$\frac{T_1}{T_s} = d$

 $\frac{T_2}{T_s} = d_2$

$d_2(d + d_2) = (1-d) = \frac{2L}{RT_s} = K$


$\frac{2L}{RT_s} > (1-d) \Rightarrow \text{Op. Mode is CCM}$
 $\frac{2L}{RT_s} < (1-d) \Rightarrow \text{Op. Mode is DCM}$

Border of DCM and CCM

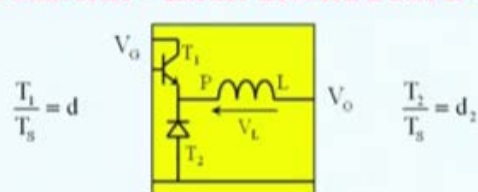
Now, we can also find out when does the converter operate on the border, that is we can say that it is both in our continuous mode of operation or in discontinuous mode of operation. For this to be true, the current has to start from 0, go to the peak and when exactly the cycle ends, it has to come back to 0, so that T_3 is 0 or d_2 is 1 minus d . We can also say that when d_2 is equal to 1 minus d , the operation will be on the border. When d_2 calculated from this is greater than 1 minus d , the converter will be in continuous conduction and when d_2 is calculated as for this relationship, if it is less than 1 minus d , then the converter will be in discontinuous mode of operation.

So, if you substitute that particular relationship, we find that if this $2L$ by RT_s , which is called the conduction parameter, if this on and is equal to 1 minus d , then the converter

will be in the operating in the continuous mode. If it is more than 1 minus d, it will be in the continuous mode of operation. If it will be less than 1 minus d, it will be in the discontinuous mode. When $2L$ by $r T_s$ is greater than 1 minus d, then the operating mode is in continuous mode of operation and when $2L$ by $R T_s$ is less than 1 minus d, the operating mode will be in discontinues mode of operation. When $2L$ by $R T_s$ is equal to 1 minus d, the operation will be on the border between continuous and discontinuous mode.

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Switched Mode Power Conversion
Buck Converter – Border Between DCM & CCM



Define $K_{cri} = (1-d)$

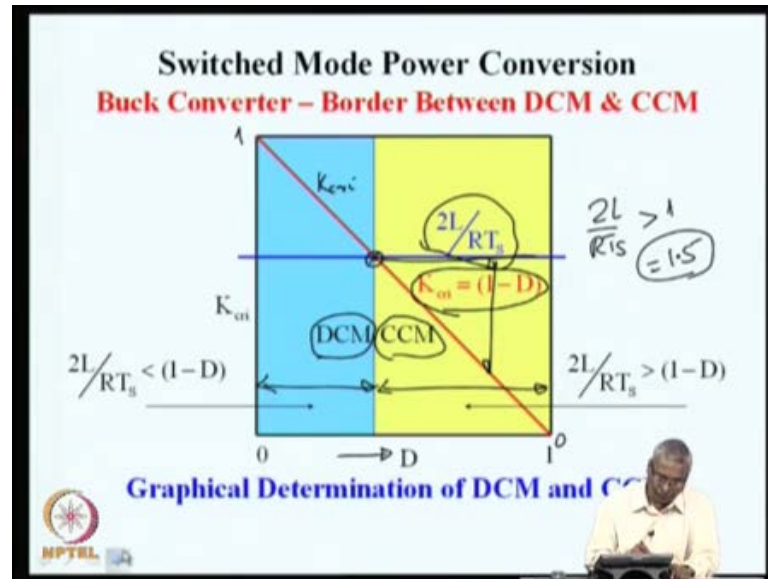
$K > K_{cri}$: CCM Operation $K < K_{cri}$: DCM Operation

Border of DCM and CCM

Now, we can define this function 1 minus d as a critical value of K and if K is greater than that critical value, then the operation in CCM operation mode. If K is less than that critical value operation is in DCM, this is the border.

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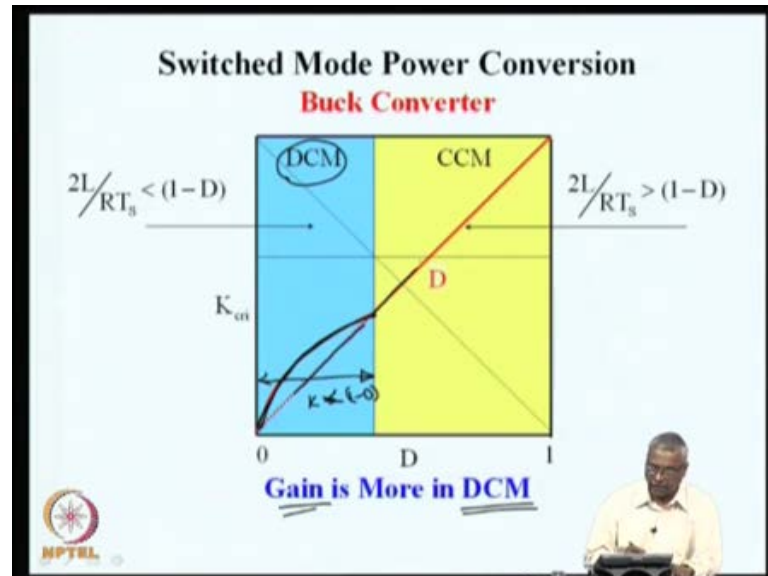
It can also be seen in my graphical form, we can plot the critical value of K which is the conduction parameter, as a function of d . This d is duty ratio and K critical is $1 - d$. Let it be a function starting from 1 and ending up at 0 because when d equal to 0 , K critical is 1 . When d equal to 1 , K critical is 0 and the conduction parameter of converter is $2L / RT_s$ that will be a fixed number for a converter. So, when you compare both of them you can see that at some point both may be equal and on the left hand side of that the blue region is where this K critical which is $1 - d$ is more than $2L / RT_s$ or $2L / RT_s$ is less.

Then, K critical operation is in discontinuous mode of operation. On the right hand side, $2L / RT_s$ is more than $1 - d$, so the operation is in continuous mode of operation. So, graphically it is possible for one to find out what is the region in which the conduction will be in continuous or discontinuous mode. There is an important observation here. The critical conduction parameter is the one at d . At all over points it is less than 1 .

Supposing, if this $2L / RT_s$ greater than 1 is equal to 1.5 , then such a converter will have this L always above 1 . So, it will never be operating in discontinuous mode of operation. Such a buck converter were the inductance, resistance and switching time or such that this quantity is more than 1 , will never operate in discontinuous mode of operation. Only if this is less than 1 , following this picture it is possible to find out what

is a region when conduction is discontinuous and what is the region 1 conduction is continuous.

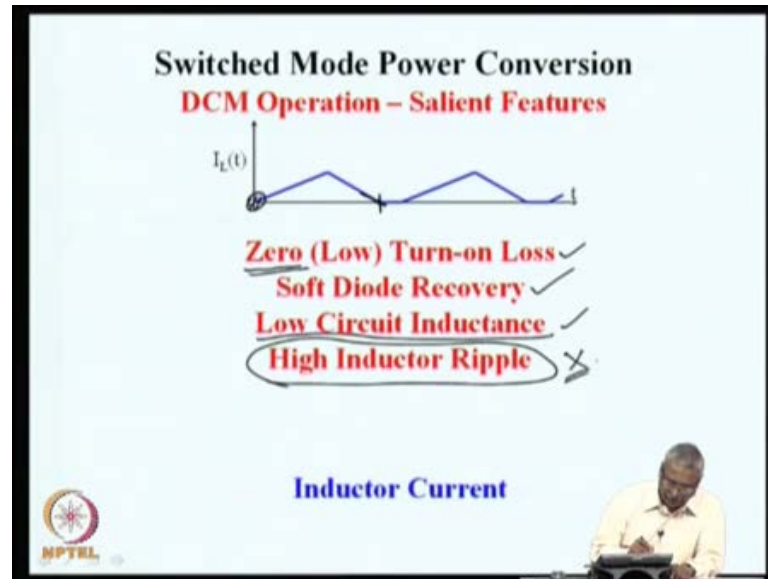
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We had seen that in the discontinuous mode of operation, the gain is always more than the gain for continuous mode of operation. So, from this picture you can see that this is the duration of d where K is greater than K_r . K is less than $1 - d$. So, that blue region is when the operation is in the discontinuous mode. So, in this region the function d which is our gain in continuous mode of operation is shown by the dotted lines here, but because it is operating in discontinuous mode of operation, the gain is more than what we would have got by this bottom line.

So, in the blue region the converter produces voltage which is higher than what it would produce in continuous mode, but after you have move to the yellow region the gain is given by d which is a duty ratio. So, it is possible to see graphically, how the conversion of weak operating condition will be DCM or CCM and if it is a DCM, how the gain will be more than the normal continuous conduction gain, and so on?

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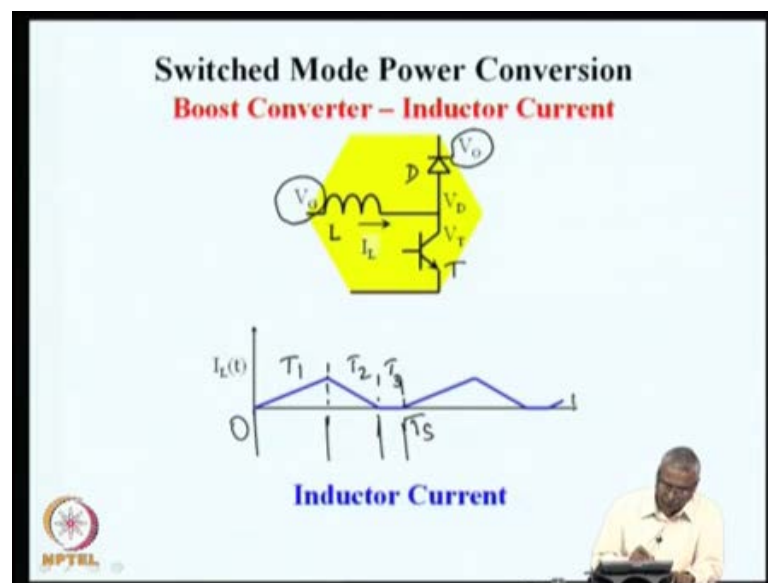
Now, the salient features of discontinuous mode of conduction are summarized here. You can see that during every cycle, the current in the inductor starts from 0. So, because it starts from 0 current the turn on loss in the transistor or in the switch is 0. This is a very important advantage, because now it is possible to switch the converter at higher and higher frequencies because the switch on loss has come to 0. During the off period the diode recovers smoothly with finite $\frac{di}{dt}$.

So, at the end of this diode conduction period, the recovering current through the diode is limited and the recovery charge is also limited and the diode recovers very softly. This is a second advantage. This has significant influence on the e_{mi} generated by the converter. The electromagnetic interference because at all the instances, you find that the current in the conductor has a very well defined $\frac{di}{dt}$. There are no sharp rise in current and if there are no sharp rise in current there will be no sharp rise in magnetic field around the inductors.

So, e_{mi} effect will be very good. This will not have too high frequency or very high frequency electromagnetic interference effects. Another significant feature is that, because the ripple current is large we can manage with low inductance in the circuit. The filtered inductance L is now smaller compared to continuous current mode. If the inductor is smaller, the size of the inductor also will be small, the weight of the inductor will be smaller and the weight of the overall converter can also be small.

Therefore, it is possible to achieve a higher power from lower weight of the convertor. Because, of these advantages there are some situations where the discontinuous mode of conduction is preferred. The disadvantage is that the inductor ripple is large. So, the inductor has to be designed for a higher value of peak current. So, this is a compromise which one has to accept. There are several advantages at low power levels. These advantages are significant compare to the disadvantage of high inductor current ripple. So, there are so many converter, which are consciously selected to operate in DCM operation if the power level is low.

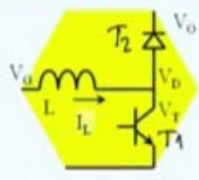
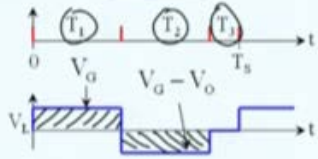
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Now, we will go on to see the same effect of the operating modes of discontinuous mode of conduction and the continuous mode of conduction on the second converter, which is a boost converter. What you see is the boost converter which converts the source voltage V_G to L , load voltage V through an inductor L and active switch transistor T or s and passive switch which is a diode, are very similar. If we draw the inductor current in any 1 of this converter when you are operating in discontinuous mode of conduction, it is very easy to identify the operating mode. If we look at the inductor current, it will have now the 3 sub intervals. In 1 circuit you call 0 to T_s as the full circuit. You have now 3 sub intervals T_1 , T_2 and T_3 .

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Switched Mode Power Conversion
Boost Converter – DCM Conversion Ratio

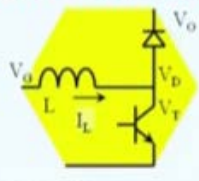
Inductor Volt-Sec Balance

Just we had seen before that T 1 is when the active switch transistor is conducting, T 2 is when the passive switch diode is conducting, and T 3 is when our 1 of them are conducting. The conversion factors is obtained just as before finding out the volts second balance in the inductor. What is the voltage and duration during the on time? What is the voltage and duration during the off time on the inductor? It is possible to write down simple volts second balance on the inductor. As you see here, we defined the active switch duty ratio is d, the passive switch duty ratio is d 2 and write the volts second balance is V G.

(Refer Slide Time: 32:06)

Switched Mode Power Conversion
Boost Converter – DCM Conversion Ratio



$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$



$$V_g T_1 + (V_g - V_o) T_2 = 0$$

$$\frac{V_o}{V_g} = M = \frac{d + d_2}{d_2}$$

$$(d + d_2) V_g = d_2 V_o$$

$$\frac{d + d_2}{d_2}$$

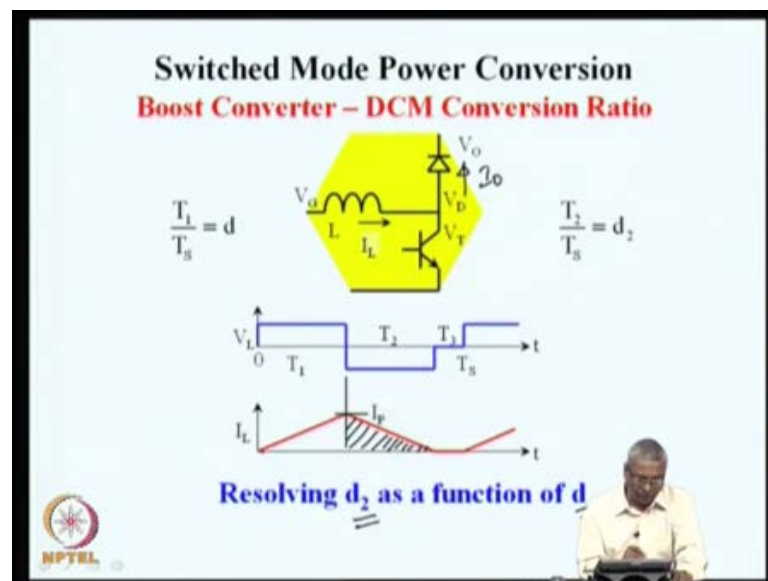
Voltage Conversion Ratio

When the switch s is on, the voltage across the inductor is V_G into T_1 . When the switch is off, the voltage across the inductor is V_G minus V_{naught} into T_2 and during the third interval the inductor is floating. So, if we add up this volts seconds what you get is V_G into T_1 plus V_G minus V_{naught} into T_2 and this total has to add up to 0 for steady state. This can be written in terms of duty ratios as d plus d_2 times V_G is d_2 times V_{naught} .

This results in the conversion ratio which is d plus d_2 . The ratio of the active switch on time passive switch on time divided by the passive switch on is d plus d_2 divided by d_2 . In the continuous conduction mode d_2 is same as 1 minus d . So, the numerator becomes 1 and the denominator becomes 1 minus d . The difference compared to the continuous mode of conduction is that you have now in the numerator instead of 1 d plus d_2 and in the denominator instead of 1 minus d , you have d_2 .

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So, this voltage conversion ratio also requires resolution of d_2 and d as before. It is possible to evaluate the d_2 in terms of d by finding out what is the average current flowing out of this. Average current flowing out of this is whatever highest area I am showing here. If that is averaged over 1 cycle, that will give the average current, which is I_{naught} .

(Refer Slide Time: 34:01)

Switched Mode Power Conversion
Boost Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$$I_p = \frac{d V_g T_s}{L}$$

$$I_o = \frac{V_o}{R} = \frac{1}{2} I_p d_2 = \frac{1}{2} d_2 \frac{d T_s V_g}{L}$$

Resolving d_2 as a function of d

This is shown as I_p naught, which is average over d_2 . Thus d_2 only the current is going to the output and this can be written in terms of this peak current. The peak current is the slope during the V_g into d times T_s is T_1 . V_g by L is the slope during the on time. So the peak current is given by this expression.

(Refer Slide Time: 34:45)

Switched Mode Power Conversion
Boost Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$$I_o = \frac{V_o}{R} = \frac{1}{2} I_p d_2 = \frac{1}{2} d_2 \frac{d T_s V_g}{L}$$

$$K(d + d_2) = d d_2^2 \quad \left(K = \frac{2L}{R T_s} \right)$$

d and d_2 are related through Conduction Parameter K

If you express the average current in terms of all this quantities, you get now the relationship between d , d_2 and K as before, where k is the same conduction parameter for all these converters. It is the ratio of $2L$ by $R T_s$ and the interesting point is that this

conduction parameter has no dimensions. It is dimensionless parameter. So, whatever consistency units you use. This $2L$ by RT is if used, K will always be the same number irrespective of what units are used.

So, long as consistency units are used the K will be a parameter which has no dimension. So, it is constant irrespective of the units that are used. This second-order quadratic equation which relates d , d^2 square minus Kd^2 minus Kd equal to 0. So, this is the quadratic equation relating d and d^2 and it is quite easy to find out what is d^2 in terms of d and K .

(Refer Slide Time: 35:51)

The slide displays the following content:

- Title:** Switched Mode Power Conversion Boost Converter – DCM Conversion Ratio
- Circuit Diagram:** A boost converter in Discontinuous Conduction Mode (DCM). It consists of an input voltage source V_d , an inductor L with current I_L , a MOSFET switch with voltage V_T , a diode with voltage V_D , and an output voltage V_o .
- Equations:**
 - $\frac{T_1}{T_s} = d$ (on the left)
 - $\frac{T_2}{T_s} = d_2$ (on the right)
 - $K(d + d_2) = dd_2^2$ (in the center)
 - $d_2 = f(K, d)$ (handwritten note on the right)
 - $d_2 = \frac{K}{d} \left\{ 1 + \sqrt{1 + (4d^2/K)} \right\} / 2$ (boxed equation)
- Text at the bottom:** d and d_2 are related through Conduction Parameter K .
- Logos:** NPTEL logo in the bottom left corner and a small image of a person in the bottom right corner.

So, this is that relationship. We can resolve d^2 as a function of the conduction parameter K and the switch on time d function of K and d . As I said before, it is also possible to write d as a function of K and d^2 or K as a function of d and d^2 through all these relationships.

(Refer Slide Time: 36:24)

Switched Mode Power Conversion
Boost Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$$\frac{V_o}{V_g} = \frac{d+d_2}{d_2} = \frac{d}{K} d_2$$

$$\frac{V_o}{V_g} = \frac{\{1 + \sqrt{1 + (4d^2/K)}\}}{2}$$

(d, K)

d and d₂ are related through Conduction Parameter K

We can substitute that value of d_2 back in the original conversion gain equation to find out what is the conversion gain V_o/V_g in terms of d and K . Now, V_o/V_g and V_g is not just a function of d in continuous conduction mode. V_o/V_g is just $1/(1-d)$. It is a function of only 1 variable d , where it is a function of load. Also, d and K do not load K , the conduction parameter which is a function of L , T_s and R .

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Switched Mode Power Conversion
Boost Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$$M = \frac{d+d_2}{d_2} > \frac{1}{(1-d)} \quad (\because (d+d_2) < 1)$$

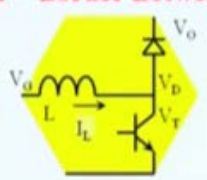
Conversion Factor in DCM is More

Just as before, this discontinuous mode of conduction resulted in a gain. During that time the ratio of V_o by V_g or the conversion factor in the discontinuous mode of conduction is always more than conversion factor in the continuous mode of operation.

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Switched Mode Power Conversion
Boost Converter – Border Between DCM & CCM

$\frac{T_1}{T_s} = d$





$\frac{T_2}{T_s} = d_2$

$K(d + d_2) = dd_2^2$ $d_2 \approx (1-d)$

$K_{crit} = d(1-d)^2$

Border of DCM and CCM

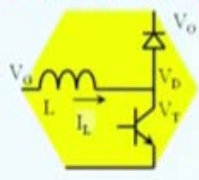



These things can also be seen in the form of our graphical form. We can find out the value of K for which this d_2 is $1 - d$. In this relationship if we put that in here, K_{crit} is that value of K when d_2 is $1 - d$ or that value of K when the converter is operating on the border between DCM and CCM. In this particular boost converter, the critical value of K is $d(1 - d)^2$.

(Refer Slide Time: 38:02)

Switched Mode Power Conversion
Boost Converter – Border Between DCM & CCM

$\frac{T_1}{T_s} = d$





$\frac{T_2}{T_s} = d_2$

Define $K_{crit} = d(1-d)^2$

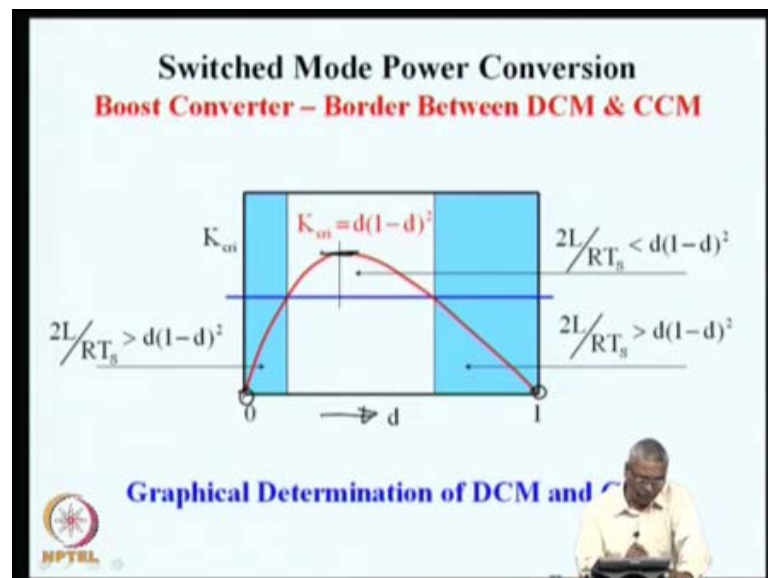
$K > K_{crit}$: CCM Operation $K < K_{crit}$: DCM Operation

Border of DCM and CCM



It is possible that once this K critical is defined, we can find out when the converter will be in CCM operation and when it will be in DCM operation.

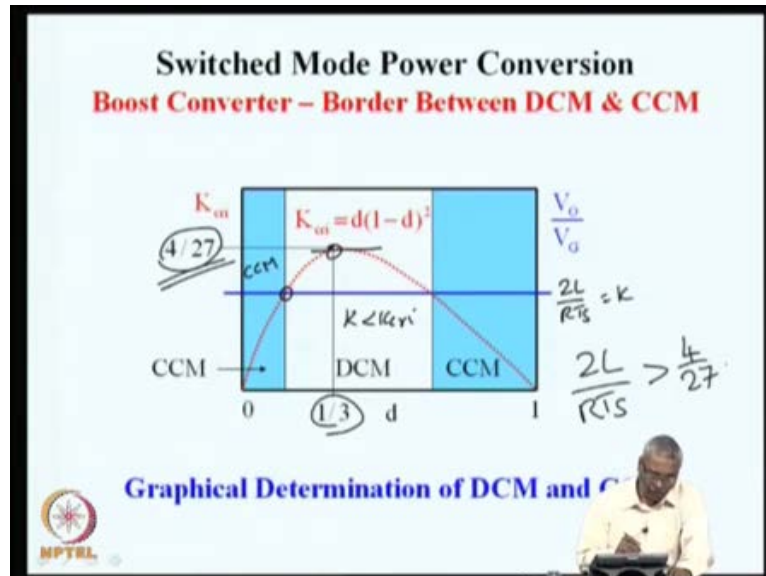
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This can be seen in the form of a graphical form. So, as d varies from 0 to 1, this K critical starting from 0 reaches a peak and comes back to 0. In the buck converter, we saw that this function was 1 minus d . So, it is started from 1 and came all the way to 0. Now, it starts from 0. K critical is d into 1 minus d the whole squared. So, when d is 0, K

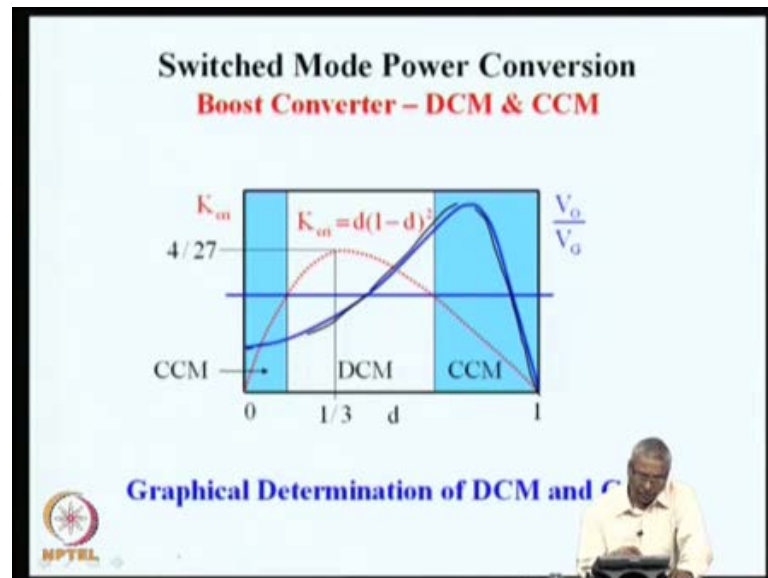
critical is 0 and when d is 1 also k critical is 0. Both values and this K critical has some maximum at some particular point in between.

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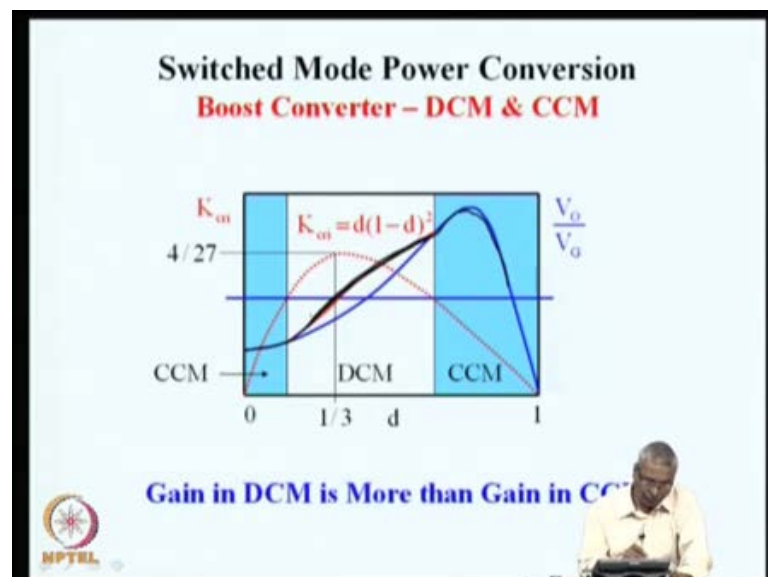
You can see that that maximum occurs when d is 1 by 3. This maximum value of K critical is 4 by 27. It is very easy to see if the value 2 L by R T s, if K is greater than K critical then the converter will be in CCM and if this K is greater than K is critical again it is in DCM and in between region where K is less than K critical. Then the converter is operating in discontinuous current mode of operation. Again, you can see that because this K critical has a maximum value of 4 by 27 in boost converter, if 2 L by R T s is greater than 4 by 27, the converter will never go into discontinuous mode of operation. It will always be in continuous mode of operation.

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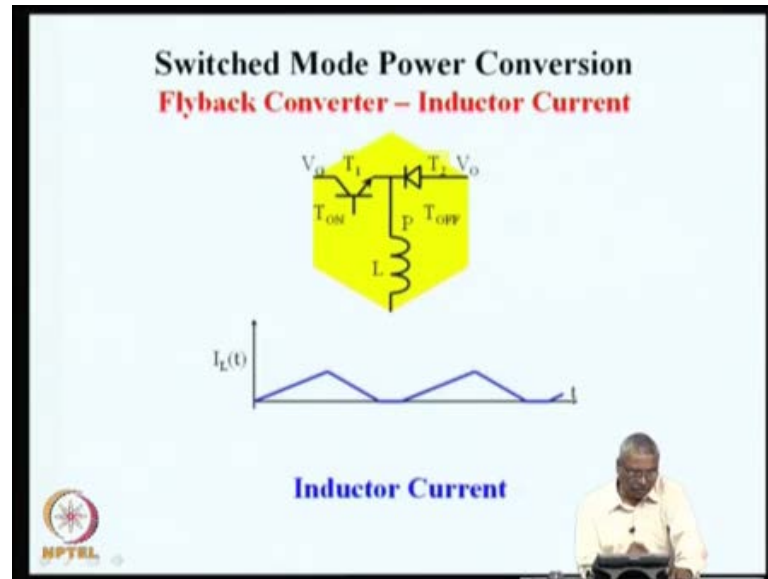
It is also possible to relate in the discontinuous mode of operation what happens to conversion gain. What you see here is the conversion gain as a function of d if the converter is operating in continuous mode of operation.

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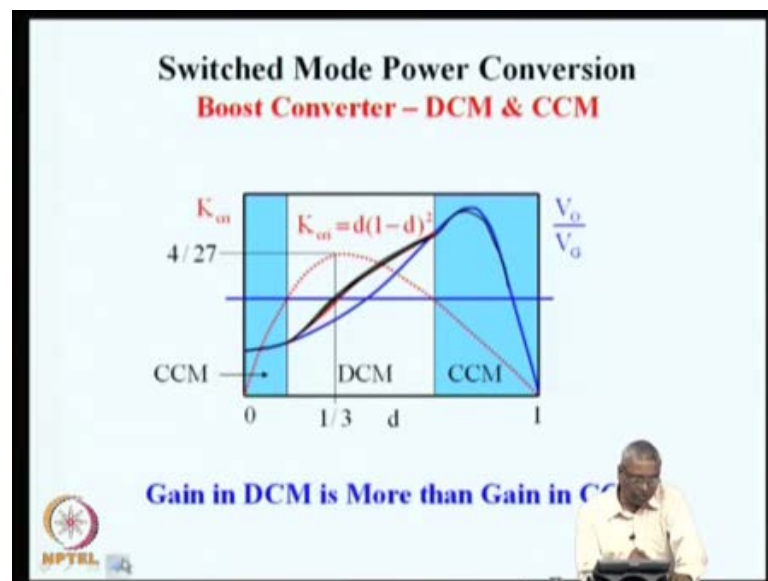


We know that in the discontinuous mode of operation, the gain is more than the continuous mode gain. So, actual gain follows this curve maximum of the two things. In this DCM mode of operation the gain is more than the continuous mode of operation.

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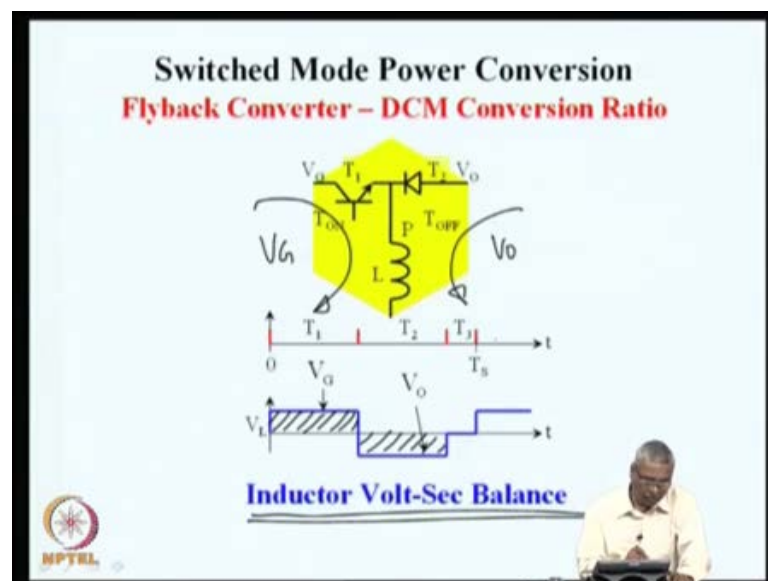


So, what we had seen in the boost converter also has the same features, that the turn on losses is 0, inductor value is low, the recovery of the diode is soft and the ripple current is large. These are the salient features of the converter. So, what is seen here is the third canonical form or the third basic converter which is the fly back converter. Now, fly back converter takes in the energy during the on time from the source and supplies the energy during the off time to the load.

You can see that the when polarity of power will also change, polarity of voltage is also change. Some times this fly back converter is also called inverting buck boost converter, that is output voltage can be in magnitude less or more then the input buck or boost. But, the output polarity is opposite to that of the input polarity. So, non isolated fly back converter in some literature as the inverting buck boost converter which is what is shown here. The output polarity is of this polarity.

The input polarity is plus and minus now. The continuous current mode and discontinuous current mode are characterized by 3 intervals. Very clearly, it can be seen on the inductor current wave form, the interval T 1, the interval T 2 and interval T 3. T 3 is when none of the devices are conducting. The inverter is floating and there is no energy drawn from the source. No energy is supplied to the load site from the inductor.

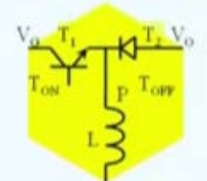
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The conversion gain for these converters follows the same procedure that we find out under steady state. What is the volts second balance on the inductor for that? Find out what is the voltage across the inductor during the switch on time, which is V_G in this particular case? During the on time this is V_G and during the off time this is V_{naught} . So, you can find out the volt second balance, by finding out V_G into T_1 plus V_{naught} into T_2 . The third interval does not contribute to the volt second.

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Switched Mode Power Conversion
Flyback Converter – DCM Conversion Ratio



$\frac{T_1}{T_s} = d$



$\frac{T_2}{T_s} = d_2$

$V_g T_1 + V_o T_2 = 0$

$\frac{V_o}{V_g} = M = -\frac{d}{d_2}$

$dV_o = -d_2 V_o$

Voltage Conversion Ratio

If you had all these, $V_g T_1 + V_o T_2 = 0$. That gives us the relationship in terms of the duty ratios d times V_g is minus d_2 times V_o or V_o by V_g is the ratio of d by d_2 with an inverting sign for the change in polarity. It is ratio of the duty ratio of the switch divided by the duty ratio of the diode that is d by d_2 . In a continuous current mode of operation, this d_2 will be equal to $1 - d$. This relationship we had seen earlier that ideally for the fly back converter, the conversion ratio is minus d divided by $1 - d$.

Now, instead of the $1 - d$ in the denominator, the inductor demagnetizes or inductor discharge time is now not $1 - d$. It is only d_2 and on the account of that the ratio is just minus d by d_2 . This voltage conversion ratio will be higher than what we had seen in continuous current mode as minus d by $1 - d$ because d_2 is less than $1 - d$.

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Switched Mode Power Conversion
Flyback Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$ $\frac{T_2}{T_s} = d_2$

Resolving d_2 as a function of d

As before d_2 and d have to be resolved, if we want to get output voltage purely as a function of the duty ratio and the parameters of converter we follow the same principle that we find out. What is this load current which is supplied here? I_{naught} is same as the average of the inductor current during the diode conduction time. Diode conduction time is d_2 . Conduction time ratio is d_2 . So, $\frac{1}{2} I_p$ into d_2 . Then d_2 will be the average current of what is shown in the hatch region and that is the same as I_{naught} .

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Switched Mode Power Conversion
Flyback Converter – DCM Conversion Ratio

$\frac{T_1}{T_s} = d$ $\frac{T_2}{T_s} = d_2$

$I_p = \frac{d V_g T_s}{L}$

$\frac{2L}{R T_s} : K = d_2^2$

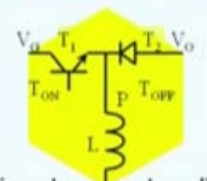
$I_o = \frac{V_o}{R} = \frac{1}{2} I_p d_2 = \frac{1}{2} d_2 \frac{d T_s V_g}{L V_o} \frac{d_2}{d}$

Resolving d_2 as a function of d

So, as before we can write I_{avg} as half of I_p into d . The way I had shown the hatch region and I_p itself is the on duration d and T_s multiplied by the slope in the on time. This is V_G by L . Now, if this I_p is replaced in this you get I_{avg} is V_{avg} by R . This is the same as $\frac{1}{2} d^2 d T_s V_G$ by L . If you bring this V_{avg} here and this V_{avg} by V_G can be replaced by d^2 by d , this d will cancel and you will find that $2L$ by $R T_s$ which is K is nothing but d^2 squared.

(Refer Slide Time: 46:02)

Switched Mode Power Conversion
Flyback Converter – DCM Conversion Ratio



$$\frac{T_1}{T_s} = d \qquad \frac{T_2}{T_s} = d_2$$

$$I_o = \frac{V_o}{R} = \frac{1}{2} I_p d_2 = \frac{1}{2} d_2 \frac{dT_s V_G}{L}$$

$$K = d^2 \qquad K = \frac{2L}{RT_s} \qquad d_2 = \sqrt{K}$$

d and d_2 are related through Conduction Parameter K

So, if this is simplified it turns out be K is equal to d^2 squared. K is what same conduction parameter, a dimension less parameter decided by the inductance in the circuit resistance of the load and the switching period T_s . All these quantities d and d_2 or related through the parameter K , you can simply now in this converter write d^2 as square root of K . It is not even related to d . It is simply a function of K , only in this particular converter.

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Switched Mode Power Conversion
Flyback Converter – DCM Conversion Ratio

$$\frac{T_1}{T_s} = d$$

$$\frac{V_o}{V_g} = \frac{d}{d_2}$$

$$\frac{T_2}{T_s} = d_2$$

$$d_2^2 = K$$

$$\frac{V_o}{V_g} = \frac{d}{\sqrt{K}} \leftarrow \frac{d}{d_2}$$

d and d₂ are related through Conduction Parameter K

So, we can say that d^2 is equal to K or d is square root of K . V_o/V_g had seen d/d_2 which turns out to be d/\sqrt{K} and d_2 being less than $1 - d$.

(Refer Slide Time: 47:00)

Switched Mode Power Conversion
Flyback Converter – DCM Conversion Ratio

$$\frac{T_1}{T_s} = d$$

$$\frac{T_2}{T_s} = d_2$$

$$|M| = \frac{d}{d_2} > \frac{d}{(1-d)} \quad (\because (d+d_2) < 1)$$

Conversion Factor in DCM is More

The conversion factor in discontinuous mode will always be more than the conversion factor in the continuous mode of operation. In this mode this conversion factor will be more.

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Switched Mode Power Conversion
Flyback Converter – Border Between DCM & CCM

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$K = d_2^2$

$K_{crit} = (1-d)^2$

$d_2 = (1-d)$

$K_{crit} = (1-d)^2$

Border of DCM and CCM

The critical conduction parameter is formed out by finding out what is d_2 in the border of discontinuous conduction and discontinuous conduction on the border d_2 . It is same as 1 minus d . So, if we substitute this in this relationship the critical value of key K is that value of K for which the converter is on the border between continuous and discontinuous mode of operation. So, that value is characterized by this relationship that $2L$ by $R T_s$ is equal to $(1-d)^2$. If $2L$ by $R T_s$ is more than this quantity, the converter will be in continuous mode of operation. If $2L$ by $R T_s$ is less than this critical value, the converter will be in discontinuous mode of operation.

(Refer Slide Time: 48:13)

Switched Mode Power Conversion
Flyback Converter – Border Between DCM & CCM

$\frac{T_1}{T_s} = d$

$\frac{T_2}{T_s} = d_2$

$\text{Define } K_{crit} = (1-d)^2$

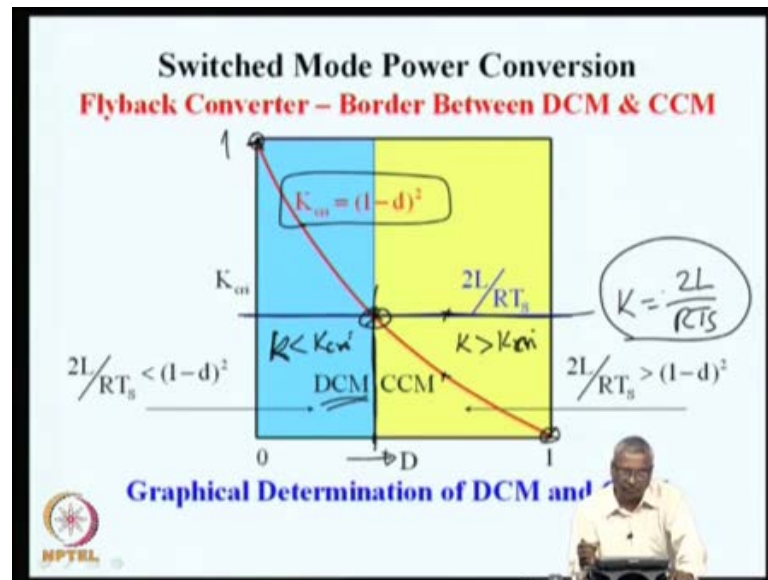
$K > K_{crit} : \text{CCM Operation}$

$K < K_{crit} : \text{DCM Operation}$

Border of DCM and CCM

So, we have seen this. We defined the critical value of conduction as 1 minus d the whole squared, that is the border value. If K is greater than that the border value, you are in continuous conduction and if K is less than that border value the operation is in discontinuous mode of operation.

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This also can be plotted in the form of graph so that it is very easy to look in 1 place in a picture. What will be the operating mode? What is the gain? So, on we have seen for this fly back converter the critical value of K is given by 1 minus d whole squared. So we can plot that as a function of d when duty ratio varies from 0 to 1. Now, this critical value follows a second order curve, 1 minus d whole squared. Finally, when d is 1 critical value is 0, when d is 0 critical value is one. In the case of buck converter we saw that critical value of K was just 1 minus d.

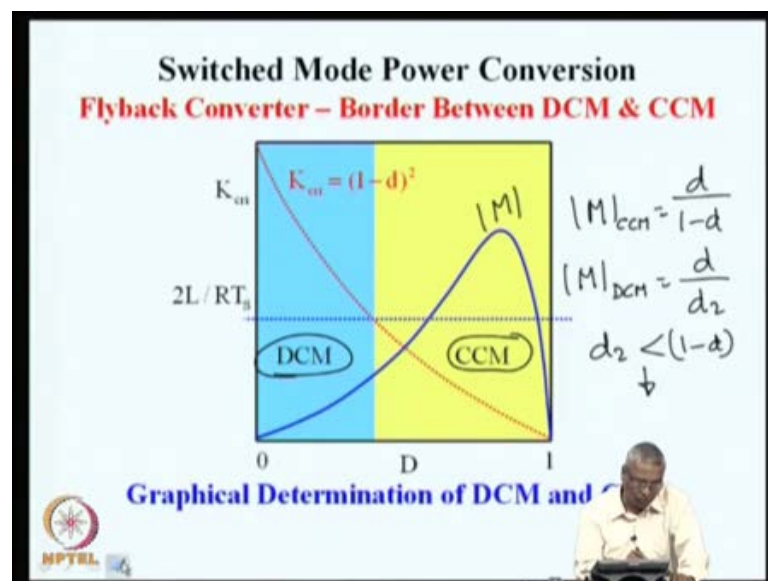
So, it followed straight lines connecting one at this end and 0 at this end, but in the fly back converter this critical value is a second order relationship. K critical is 1 minus d whole squared. So, this is a parabola. For example, if duty ratio is 0.5. At that time K critical will be 0.25. Then you can also arm the same graph. You can put the conduction parameter K equal to 2 L by R T s. So, if this line is drawn on the same picture then you can see that this meets the critical value of K, that is the point of duty ratio where the conduction is on the border between discontinuous and continuous mode of conduction.

On the left hand side, this blue region is where L is less than K critical. The conduction parameter is less than K critical. If that is so, the converter is in discontinuous mode of operation. On the right hand side, this is K and this is K critical. K is greater than K critical. On the yellow region or on the right hand side region, the operation is in continuous mode of conduction. Just as we had seen in the other converters, in the buck converter we saw that the maximum value of K critical is one in the boost converter.

The maximum value of K critical was 4 by 27 and in the fly back converter the maximum value of K critical is one that occurs when duty is ratio is 0. So, if we select our converter designs such that $2L$ by $R T_s$ is greater than 1. For example, any converter if you select a switching frequency high enough so that T_s is small enough, this K can be made greater than 1. The converter will always be in continuous mode of conduction. So, even for a converter which is designed with certain value of L and for certain load condition, it is possible to take it from discontinuous mode of conduction mode of conduction.

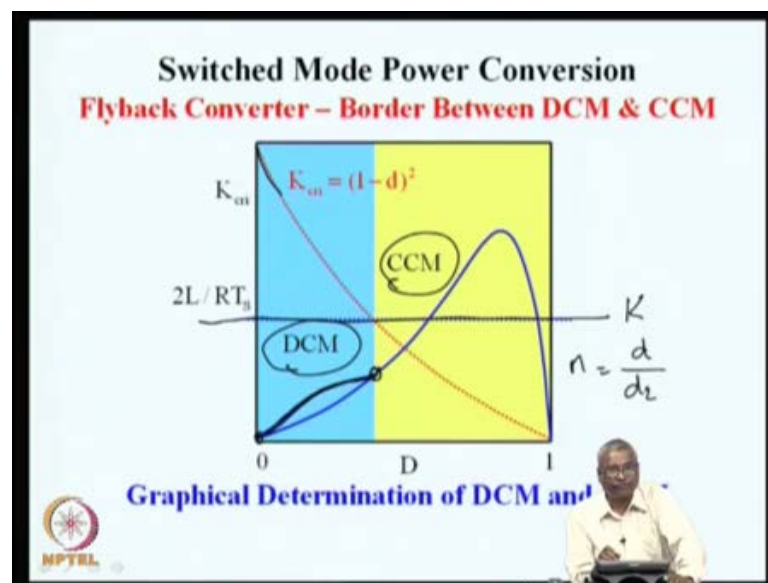
This is done just by changing the switching frequency following this relationship that if $2L$ by $R T_s$ is greater than $1 - d$ whole squared. Then the convertor will get into continuous mode of operation. So, many times if we prefer continuous mode of operation in a particular converter, we can always change the switching frequency, so that the converter can be brought into continuous mode of operation.

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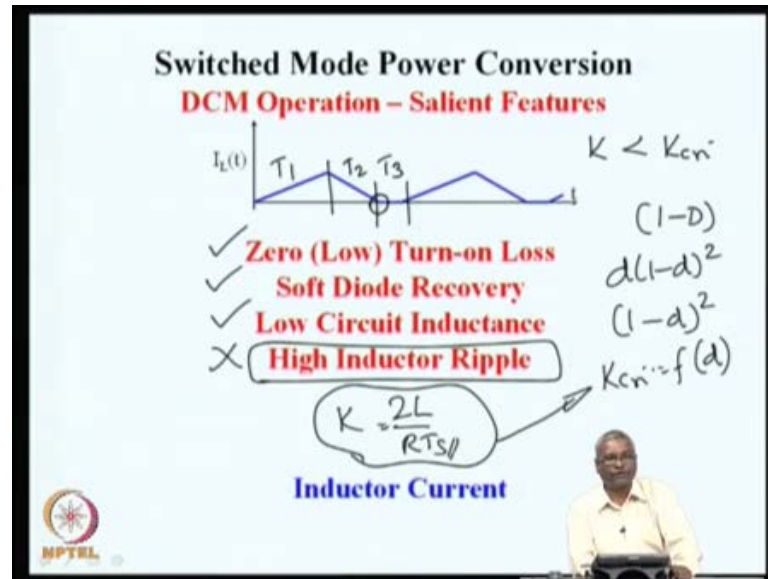
Now, in the continuous mode of operation in the region, where the operation is discontinuous and region where the operation is continuous this is the gain with continuous mode of operation. In the discontinuous mode of operation, the gain differently is going to be something more than this. It is because in the denominator now we have d by d^2 . Earlier it was just d $1 - d$. Now, in the denominator this M in continuous mode of conduction is d by $1 - d$. This M in discontinuous mode of conduction is d by d^2 and d^2 is always less than $1 - d$. So, from this you can see that the gain in continuous mode will be more than the gain in the discontinuous mode

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You can see that the discontinuous mode. The actual gain will be more than what you get in the continuous mode. On the border, both of them are equal. They follow the same curve as before. So, it is this type of graphical representation that can give as instantly if you over rely on this. What is the value of K ? What is the value of K critical? You can instantly find out the region of discontinuous mode. The region of continuous mode and then the additional gain which is d by d^2 in the discontinuous mode of operation, all this can be plotted nicely on a graph and then understood fully.

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Now, again finally, to summarize what we notice is that the discontinuous current mode of operation has 3 intervals in 1 cycle that is, T 1, T 2 and T 3. Recognizing that a converter is in continuous mode of operation or discontinuous mode of operation is quite simple. All that one has to do is to monitor the current in inductor. If the current in the inductor has 3 distinct sub intervals, switch on time, diode on time and then diode switch. When both of them are off T 1, T 2 and T 3, then you can conclude that the converter is operating in discontinuous mode of operation.

If you do not measure it, if you know the conduction parameter is less than K critical, then also one can say that it is in discontinuous mode of operation. This K critical is $1 - d$ in the case of buck converter. It is $d(1 - d)^2$ in the case of boost converter and $(1 - d)^2$ in the case of fly back converter. So, if we find out what is this conduction parameter $2L / (RTS)$ in any converter and try to compare it with the K critical value, it is possible to find out whether it is in DCM or CCM? If you have access to the current in the inductor it is directly possible to find out the intervals and do that.

Now, there are many positive advantages of discontinuous mode of operation. The first advantage is that because the current starts from 0, the converter turns on losses practically 0. So, if half the switching loss is eliminated it is possible to switch at higher switching frequency. Because the switching loss is now low. you can go to higher

switching frequency and because of switching frequency is higher, the inductor value, capacitor value and so on will get reduced.

The second advantage is that the diode recovery process is very soft. The current in the diode smoothly comes to 0 and stays at 0 with smooth recovery of the diode. This is very good from e m i, that is electromagnetic interference created by the converter on the surrounding environment. So, this is a very good advantage especially if you are switching at high switching frequencies. We also get an advantage because we have to use a small value of inductance.

The current ripple being high, the inductance value is smaller. So, smaller the size, the cost will be smaller and so on. The disadvantage is that the inductor current ripple is high. But if this compromise can be accepted mostly it is acceptable in low power converters where the currents are not very large. Many designers consciously choose discontinuous mode of operation if the power levels are small 10 hertz, 20 hertz, 30 hertz or 50 hertz.

At smaller power levels discontinuous mode of conduction can give you advantages in efficiency, in e m i performance and in the overall constants size. What we have seen up to now, we have summarized as 2 different modes of operation in all these converters. These are possible because the prevailing diode or the diode conduction period cannot support current in both directions. These 2 different modes of operation are named as discontinuous conduction mode and continuous conduction mode.

It is possible to find out the border between the 2 modes of operation by looking at the conduction parameter of the converter K . This is the ratio $2L$ by $R T_s$, this is a dimension less parameter and compare it with a critical value of K , which is a function of the duty ratio for different converters. For buck converter, it is $1 - d$. For boost converter it is $d / (1 - d)^2$ and so on.

So, by comparing the conduction parameter it is possible to find out the operation. You can consciously choose the conduction parameter by a high switching frequency so that the converter will be operating in discontinuous mode of operation. Learn all these advantages. So, with this we will stop on the different modes of operation.

In the following lectures, we will spent some time on solving a few problems related to the steady state operation of different types of power converters, both in continuous mode and in discontinuous mode, different circuit topologies and different power conversion levels.

Thank you very much.