Pulsewidth Modulation for Power Electronic Converters
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Lecture - 09
Purpose of PWM - I

Good morning. Welcome back to this lecture series on a Pulsewidth Modulation for Power Electronic Converters. We have been you know first in this course we looked at various kinds of power electronic converters such as dc to dc converters and dc to ac converters etcetera.

Now, we are looking at this question of pulsewidth modulation which is beginning to look at this issue of pulsewidth modulation. In fact, in today's lecture we just going to see what is the purpose of pulsewidth modulation. Let us what we are going to see in this in the coming couple of lectures, why do we need pulsewidth modulation that is basic question that we are going to address here.

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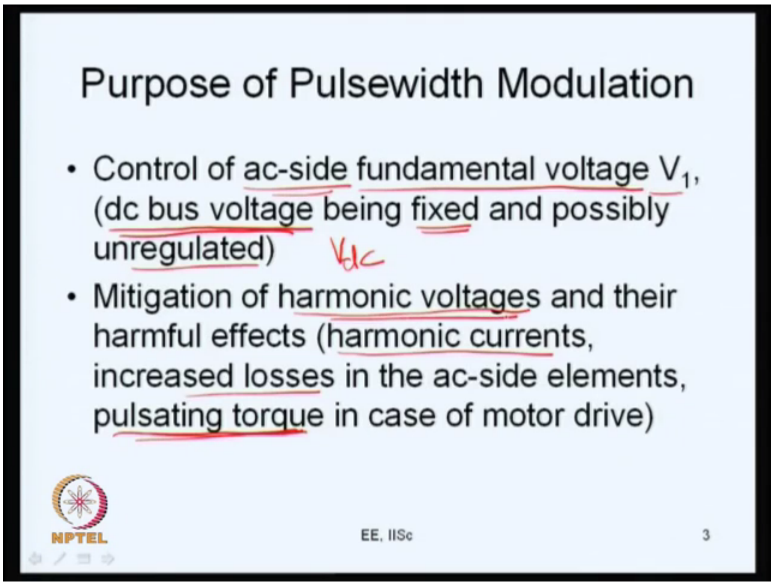

Now, if you look at this what you need is you have a voltage source inverter and that voltage source inverter has a dc bus voltage, and the dc bus voltage is being large fixed, you know it is not vary it is not vary remain it is fixed and it could possibly be unregulated you mean you can have a dc source this inverter may be feed from a dc source and that could be a dc source such as an active PWM rectifier or in some kind of controlled rectifier in which case you know that could be well regulated. On the other hand it could be simply a diode bridge rectifier with some L filter I mean with the C filter or an LC filter in which case the dc bus voltage could also be unregulated.

So, what we have is let us you know ignore this regulation or unregulation for the time being now let us just say it is fixed at a particular value , we want to realize whatever fundamental voltage we need on the ac side we need certain fundamental voltage let us call it V1, given certain dc bus voltage which we can call it as v dc, we want to realize this V1 and who should do this it is the PWM converter should do this now and the inverter as can produce a range of fundamental voltage for this given v dc.

Now, you want a specific voltage let us call it 60 percent or 70 percent or whatever, now if the first goal of this pulsewidth modulation is to ensure that the fundamental voltage has the desired amplitude, that it should have the desired amplitude is one of the first goals that it you need to control the ac side fundamental voltage. There are other cases you know you may also have to control the frequency of the fundamental voltage and there are also both the phase of the fundamental voltage etcetera, controlling the ac side fundamental voltage given a fixed dc bus is one of the goals of this you know pulsewidth modulation. Beyond that what happens is what you produce on the ac side is not a sinusoidal waveform we are trying to synthesize some sinusoid you know of some 50 hertz or whatever modulating frequency, but it is essentially a non sinusoidal waveform.

We are trying to produce ac using dc, it is you know what you are playing basically is not a sinusoidal waveform we apply various pulses, you may apply positive pulses during the positive of cycle and negative pulses during the negative of cycle and so on, and it is not a sinusoidal waveform it is it has some non sinusoidal I mean it has certain harmonics it is non sinusoidal, when you say non sinusoidal it has certain harmonic voltages.

These harmonic voltages are going to result in certain undesirable you know going to have certain undesirable effects this harmonic voltages will cause certain harmonic currents to flow and these harmonic currents for example, can go about increase the losses this harmonic currents will now flow on top of the fundamental current through all the line side components, it could be a line side inductor are could be an induction motor whatever it could be. These are going to increase the losses and sometimes the core losses in those reactive elements and the motor can also increase now.

What happens as an effect of you know the harmonics is that the losses are increased now and then you in case of motor drive you may also have a pulsating torque what you normally have in a motor in an induction motor fed from a sinusoidal voltage sources the torque is steady you get a steady torque because it is a result for interaction of 2 different fluxes are one flux and a current or whatever and both are sinusoidal and you know if you look at them through phasors we will deal with all this at a later in a greater detail, but just to help you grab this. You have 2 fields one pertaining to the stator and another pertaining to the rotor which both revolve at the same frequency and they have a fixed phase angle because of the interaction between the 2 you produce a steady torque.

But what could happen in a motor drive is that you have not only a sinusoidal voltage, but you also have harmonic voltages getting applied them, because of these harmonic voltages you can have a harmonic fluxes and harmonic currents. The result of interaction between a fundamental current and harmonic flux are between steady flux and harmonic currents you get pulsating torque as we will see later now. These are some examples of the harmful effects of harmonic voltages, pulsewidth modulation what it tries to do is, it first tries to give the desired fundamental voltage V 1, now this voltage V 1 is achievable through a number of means.

Now you can you can use this and you select a particular method of producing this V 1which results in certain amount of reduction in the harmful effects of the harmonic voltages now, the pulsewidth modulation has it is second goal this mitigation of the harmful effects of the harmonics. Sometimes you may say I do not want specific harmonics you can say I do not want 5th harmonic, I do not want 7th harmonic etcetera that is what is known as harmonic elimination. Sometimes you might want to reduce certain harmonic currents in certain range and set that sometimes you might want to reduce the pulsating torque. So, your goal could be different, but overall you know what can be said is you know it aims at pulsewidth modulation aims at reducing the harmonics and they are harmful side effects.

This is what I would you call as the basic purpose of pulsewidth modulation now and we go further to see how we control fundamental voltage and how to calculate harmonics and how to calculate you know the harmonic currents and things like that now.

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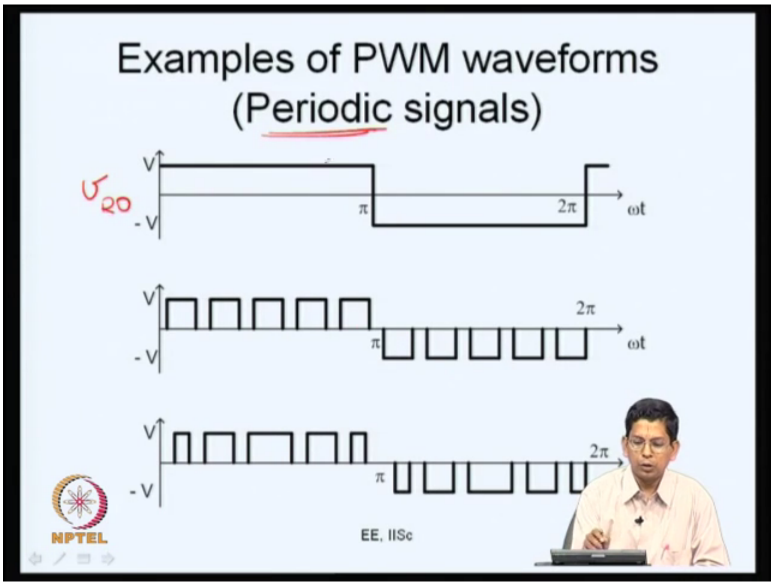

So, moment I said you know there is a PWM waveform what these are some examples of waveforms that we will see in dc to ac converters. The first is basically a square wave now you can regard this as your pole voltage V RO, that is we look at the pole I mean the midpoint of the load terminal of a leg and measured it with respect to the dc bus neutral or the midpoint of the dc bus, this is the kind of waveform and the if the inverter (Refer Time: 06:58), you know switched in a square wave fashion this is the kind of voltage waveform you will get this is your pole voltage waveform.

This is a periodic waveform, similarly you can modulate in several ways some examples which we have already seen now you can modulate in all these ways all these are periodic signals now, many of such simple PWM waveforms are periodic signals. So, what we can do is, we can use Fourier series we want to know calculate how much harmonic it has, I mean how much fundamental component it has, how much whether it has a specific harmonic or not and what is the amplitude of that harmonic and so, on, what we can use Fourier series to do that now.

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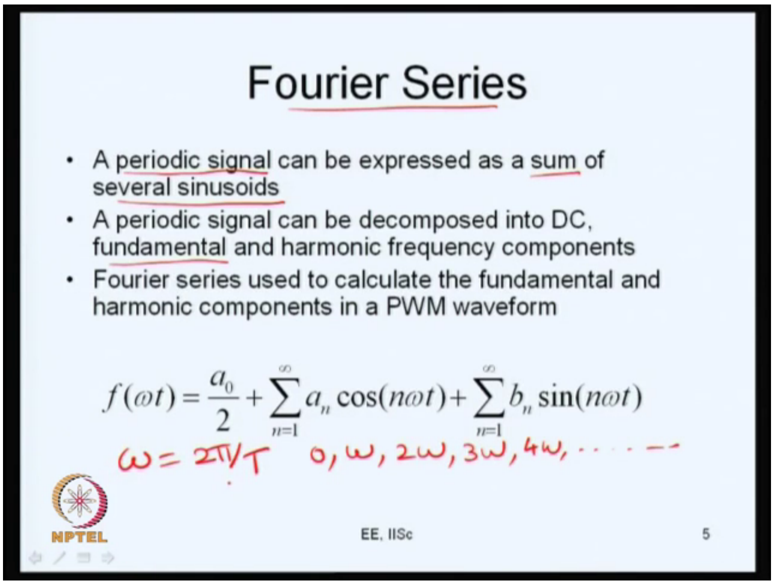

What is Fourier series, you may have any periodic signal, you may have a periodic signal and this periodic signal may have any shape does not matter, but it can be expressed as a sum of several sinusoids. Let us define the sinusoids a little more closely, these sinusoids may have certain frequencies. Firstly, it could be 0 frequency or there can be dc now, next is it can have what is called as the fundamental frequency, that is the same periodicity as the actual waveform and it can also have harmonic frequencies which are integral multiples of the fundamental frequencies.

Periodic signal can be decomposed into a dc component, a fundamental component and a set of harmonic components as indicated here now. So, here what I have given is sum f of omega t it has been you know the variable instead of time I have taken angle at the fundamental frequency, here omega is equal to 2 pi upon t where t is the time period of the waveform in question. So, I we have expressed this as a function of the angel at the fundamental frequency or what we will call as the fundamental angle. Now, this f is f of omega t has could have a dc component which is given by the a naught by 2 and it could have several follow frequency components one of the frequency component could be omega itself.

If you look at the frequencies 0 is one that is dc, omega is another one 2 omega is 3 omega, 4 omega and so, on, this can have all these various frequencies now. So, Fourier series helps us expand any given f of omega t into such a series, it helps us calculate the coefficients of various terms, I mean once we are able to calculate the coefficients of various terms we know how much harmonic is there. For example, in this term if we can calculate a naught we know what is the dc value which is a naught by 2, if we can calculate a 1 and b 1, we know what is the sinusoidal component, it will be a 1 squared plus b 1 squared and the root would be the amplitude of the fundamental component here if you want the nth harmonic component we must try and get this a n b n here.

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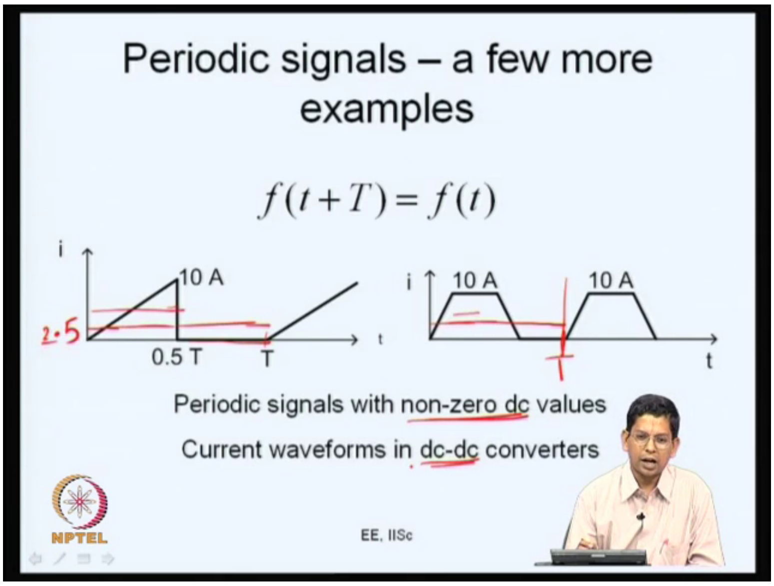

Let us move on to see a few more periodic signals now, this periodic signal now these are all periodic because they repeat over this with this periodicity t. So, now, they are defined over f of T plus capital T is equal to f of t and t is this period now, one example that has been given as a wave form ramps up and then falls back and then it is 0 and once again the same thing begins and this is your time period now this is one example of a periodic signal. Another example is given here current trices stays flat falls back and then it is 0 for certain amount of time and this is your time period t and the same thing repeats now, these are also certain other examples of periodic signals.

In this case you know there is a difference between the previous one and here the previous ones were all by enlarge ac waveforms, the ones these were all by enlarge ac waveforms and they did not contain any dc component at all. They have a 0 average value if you look at this it as 0 average, that is if you look at the area under this and the area enclosed by this they are equal and they have just have an opposite sign, if you look at the total area within over a cycle it is 0 it has no average value.

But these are certain examples of signals which have some average value, if you look at the average here this average is in the average it over the 50 percent the average is somewhere like this, if you average it over the entire cycle then it is something like this, in this case actually 2.5 is the average value of this current that you have now. If you look at the other case the average value here if you look at you know over this it is also possible for you to calculate certain average like this.

So, these are examples of some periodic signals which have non 0 dc values they have some average values and you know these are actually taken from some dc - dc convertors these are some current waveforms in certain dc - dc converters there, they always have certain dc component flowing whereas, in dc to ac converters you know on the when you are looking at the ac side there are I mean there is no dc component here they are all ac waveforms, these waveforms would typically have you know fundamental and other harmonic frequencies, but not the dc.

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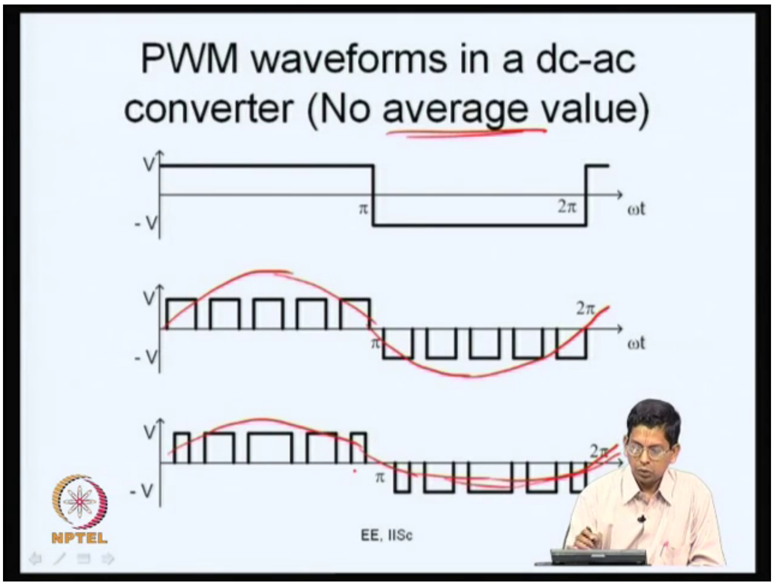

So, that is what I said here, these are examples where they have no average value you integrate this whole waveform we can consider the first waveform integrated over the end with respect to d omega t start from 0 to 2 pi and you find that the value is 0 you integrate this waveform you will also find that it is 0, you can take the third example also you do that integration you will find that it is 0, that is because these are waveforms on the ac side of dc to ac converter now, these are all essentially ac waveforms we ideally we wish that they were sinusoidal waveforms.

So, like you know what we want to see here is some sinusoidal waveform like this , but then it has several other non sinusoidal components which we cannot help you know because we are producing ac using dc we are applying basically positive dc pulses and negative dc pulses to get something like this. So, what we are trying to do is now the width of these pulses you can see controls the fundamental voltage which is one of the purpose that we were talking about earlier and now if you look at here this could also have some ac waveform, but in such you know there is some fundamental component such cases this is one example where the harmonics could be a little lower, then let us say fall the pulses were of equal bits, you need to do a complete Fourier analysis to you know expand them as Fourier series to get the exact magnitudes of how much etcetera here.

Once again going back to Fourier series these are examples that we have (Refer Time: 13:52) no average value now.

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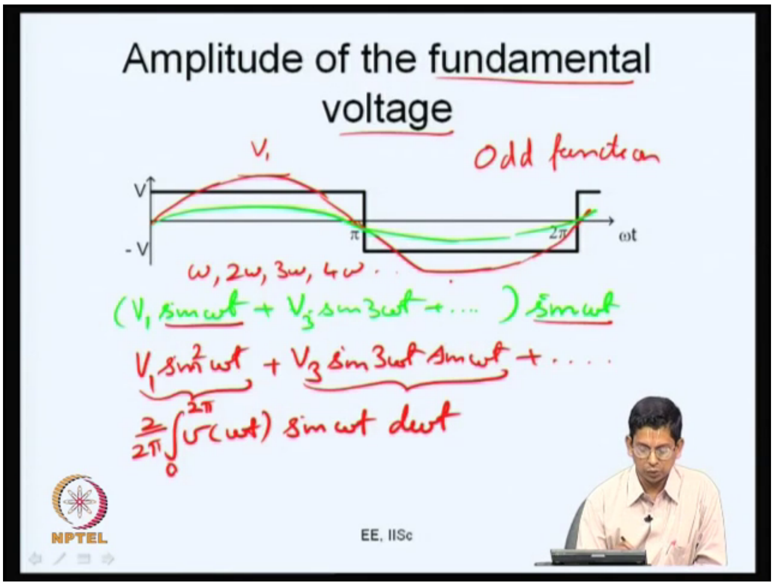

The average is taken off there is there is no dc component then let us look at the fundamental component now. We want to understand how much, what is the amplitude of the fundamental voltage. So, if you look at this waveform the waveform certainly has a fundamental component like this. The fundamental component is a sinusoid of the same freak periodicity as the original parent waveform right and we are also able to judge the phase this should be the phase of this sinusoidal waveform, what we do not know is this amplitude let me call this as V1, we do not know what is that amplitude, how can you calculate that.

What we need to do is going by Fourier series this waveform has several frequencies you know like 0 omega, 2 omega etcetera. We know that for sure that it has no 0, it could have all these frequencies omega 2, omega 3, omega etcetera it could have it could have all these frequencies. So, we still do not know, now we are interested in finding out the amplitude of the fundamental frequency, how do we do that, what we need to do is if you multiply this waveform by a sine let us say we multiply this waveform by a sine waveform let me just choose a different color here maybe this let me multiply this by a sinusoid, sinusoid of the same frequency as the fundamental component, why do I do that if I do that, why do I choose the same frequency as the fundamental component.

Now, this waveform in question has several components let me write the mass V1 sine omega t for example, plus some other you know let me say certain v n or you know some v let us say v 3 sin 3 omega t plus so on, it has all these various components. If I multiply this by sin omega t what happens, you first have this product let me once again change the color to something a little brighter. Now, let us say this sin omega t and sin omega t you have, this product gives you sin squared omega t it gives you a sin squared omega t term.

What you will have is basically this will become V1 sin squared omega t and the second term will become V3 sin 3 omega t sin omega t and so on. So, you will get some waveforms, if you look at this waveform and in a if you find out it is dc value it is average of this term over cycle it will have some dc value.

This average value is a measure of what is your V1 if you consider this waveform for example sin 3 omega t into sin omega t you look at the product waveform and you integrated over a cycle it is average value will be 0. So, all the other components also you know their averages all the product terms that you will have here their averages over cycle will be 0 except for this term, how to get this what you need to do is you given wave form you multiplied by sin of the same frequency and the sin can be unit amplitude.

Now, what is the phase of the sin in this case the sin is defined as sin omega t, now what we are trying to do in this case is we are trying to do v of omega t multiplied by sin omega t d omega t and we are integrating this from 0 to 2 pi and this is 2 upon 2 pi this is what we do to get our amplitude of the fundamental component this is what you need to do to get this. Now we are multiplying it by sin omega t and because you know this is what is called as one way to put this is this is an example of what is called as an odd function. If you extend the wave form to the left side whatever is it is value at some theta at this minus theta it will have the negative of it, in these kind of cases the wave form has only the sin components in does not have the cosine components.

So, for example, if you go through the previous ones, it will have only this b and coefficients listed in it will not have this, a and coefficients listed here now, this is one, you know that you have to multiply this by a sin function here now, let us look at it more.

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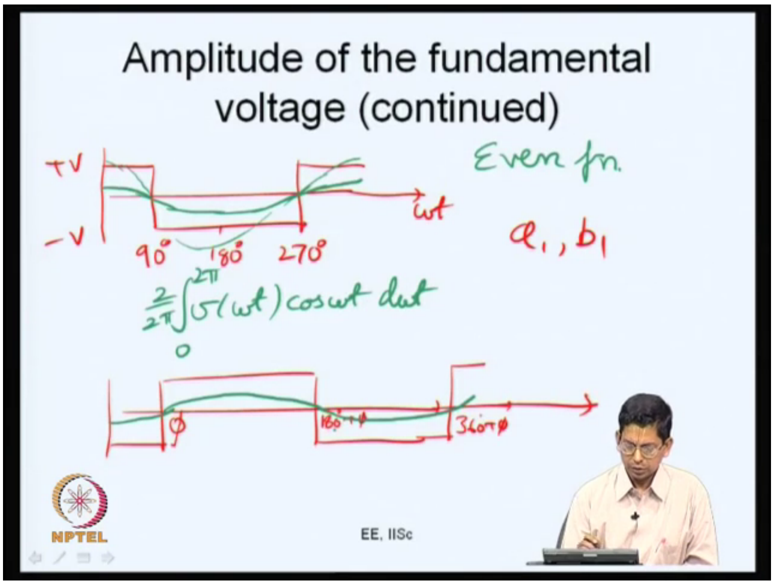

Let us say the wave form is considered differently you know there are you know the same voltage wave form let us take it like that, this is the omega t the same plus v and the same minus v. Now what we have is this is 90 degrees, this is 270 and 180 comes in between now this is the natural wave form now. So, in this wave form I am trying to see what is the fundamental component, what should I do now the same way, but what I have to see is it is fundamental I can see that it is fundamentally is something like that is fundamental has it is phase like this it is fundamental is something like that, I need to multiply this by a unit sine as shown here.

What I do here is I have v of omega t as defined here I multiply this by cos omega t and I integrate this with respect to omega t from 0 to 2 pi and 2 upon 2 pi, I do this to get the fundamental amplitude. So, here this is the case of what is called as an even function, this is a case example of what is called as an even function if you look at the negative values go along the negative axis I extend this to the negative horizontal axis also you will see that this waveform is symmetric about the vertical axis. So, vf minus omega t will be basically equal to vf omega t, that is the kind of symmetry and that is what is called as an even function. In such cases the b terms do not exist, but the a terms a coefficients in the Fourier series v exist, you go about doing it now.

Let us take it more generally, now let us say my waveform is something like that this starts at some angle phi and it goes on till some angle 180 plus phi r pi plus phi and this is 360 plus phi. In such cases if we do not know what this phi is are you know if r we are unable to see I mean are in any kind of a general waveform, what we need to be doing is we need to multiply this waveform by sin component by sin omega t and do the integration over the cycle and also similarly we must multiply the waveform by cos omega t also and get this. So, this multiplication by sin and averaging over a cycle gives one component of that and by doing it you know the other component is available what are called as a one and b one both these components have to be evaluated and then from a 1 square plus b 1 square under root will give you this amplitude that is your actual way of doing it now.

So, what I can see is we can kind of generalize now in the earlier case you multiply it by a unit sine as shown here, in the second case you are multiplying it by a unit sine as shown here. Now here what you should be doing you must actually be multiplying it by a unit sine like this, if you can multiply it by unit sine such as shown here you do your integration over a cycle or take the average of the product then we will get the amplitude I mean the average value is proportional to the amplitude of the fundamental voltage. So, what is being then if the phase is known that is what we are trying to do is you know we are trying to multiply by a sine wave what is the frequency of the sine wave it is the fundamental frequency that is the same periodicity as the parent waveform in question right.

Now, what should be the amplitude of the sin it is unit amplitude, what should be the phase the sin actually should have the same phase as the fundamental component of the original waveform. In this case we are able to judge that in certain cases we may not be able to judge that if we are able to judge that or if we know that then we can multiply it by a sin who which is in phase with the fundamental component of the original waveform and we can do that integration of the product and get the amplitude out of that when we are not aware of the exact phase what we should be doing is we must multiplied this by sin omega t and also by cos omega t you know get those both those a 1 and b 1 terms and from there get the overall amplitude from doing it here now.

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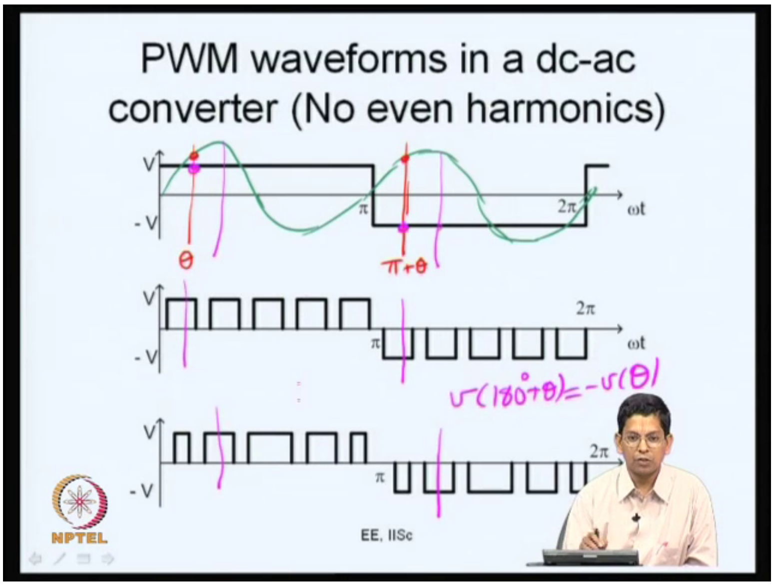

This is how you determine the fundamental compute of the fundamental component now let us say how you determine the amplitude of second harmonic. Here we multiplied as we saw a little earlier we multiplied this whole thing what is within the bracket represents the waveform we multiplied by sin omega t because we were interested to see whether there is any omega t you know sin omega t term exist we wanted to find out the amplitude of the fundamental frequency.

If you see that you know there is a second harmonic could be present here, what we must be doing is we should be multiplying this by a second harmonic sine wave. So, let me say that I am multiplying it by a second harmonic sine wave like this you know this is a second harmonic sine wave. Now I want to get the product of these 2 what would be the product of these 2 like now let me consider some point let me consider certain instant here let me call this instant as theta and let me consider another instant which is 180 degrees away. So, what do I find these 2 instance, I find that the even harmonic has the same value at these 2 instance I consider theta and I consider pi plus theta are 180 plus theta at both this instances second harmonic has the same value because this pi is measured at the fundamental angle 180 degrees for the fundamental is one complete cycle for I mean what is one half cycle for the fundamental is one full cycle for a second harmonic, it has the same value.

Now, how about the waveform in question that is the square wave here that we have considered let us let me take this, the square wave has this value and what is the value here, here it is plus V here it is minus V. The value at 180 plus theta is negative of whatever it is at theta. If you take this product, this product is something at theta and what is the product at 180 plus theta it also has the same value, but it has a negative sign when you add these 2 these 2 add up to 0. In such a wave this it is not only at this instance you take any other instance, you take here and you take the corresponding instance there you will always find this is some other theta dash in 180 plus theta dash in even we looking at some arbitrary angles.

So, you will find that at both these instants the even harmonic I mean the second harmonic or any even harmonic for that matter would have the same value, but now the waveform in question is such that you know it is value is the negative of the other at 180 plus theta is the negative of whatever it is at theta. So, such a waveform as no even harmonic you consider the same thing here also you consider this angle and this is at 180 plus theta. This waveform also has no even harmonic you consider this waveform you consider this angle and you consider this about 180 degrees later the waveform here whatever value it has at V of theta at 180 plus theta it has a different value.

Whatever the waveform is at 180 plus theta it is negative of it is value at theta this is what we observe in all these now and whenever this condition is satisfied there are no even harmonics.

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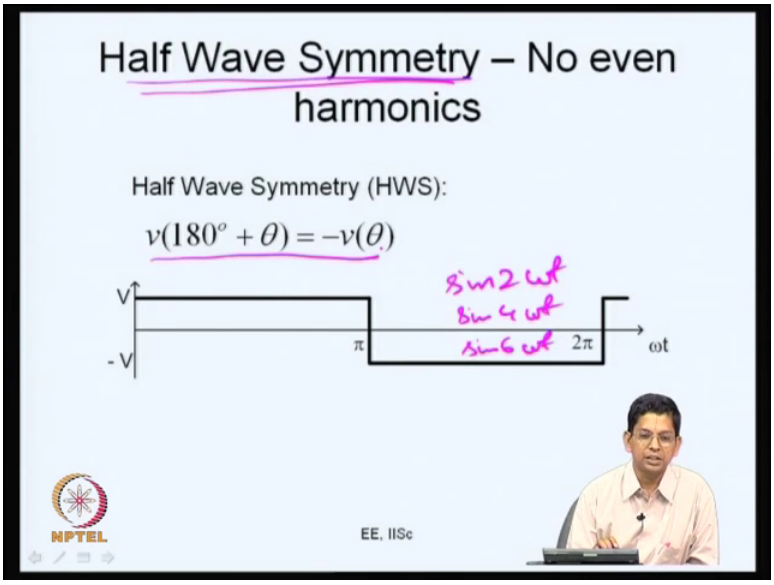

This is what we call as Half Wave Symmetry I should caution new that you know symmetries are defined in slightly different terms by various authors and all that. We are trying to define it for the purpose of course, here when we say half wave symmetry what we mean is that the wave form in question 4 satisfies this property. If we consider the wave form at theta and at one eighty plus theta where this angle is a fundamental angle at the fundamental frequency then whatever is it is value at 180 plus theta will be the negative of it is value at theta that is. So, one example of this is the square wave and all the other wave PWM wave forms there are a couple of other PWM wave forms we saw earlier also satisfy this particular property.

This half wave symmetry basically means no even harmonics, if you multiply this 180 plus theta here at sin to omega t and sin omega t which is what we just saw right now you know if you do that integration over a half a cycle or a complete cycle you will see that the product I mean you multiply this waveform by sin 2 omega t or sin 4 omega t or sin 6 omega t or anything like that, you will always integrate it over a cycle you will find that the product has 0 average, it has no even harmonics. So, significance of half wave symmetry whenever waveform satisfy this condition 180 plus theta is equal to minus v of theta you have no even harmonics present there.

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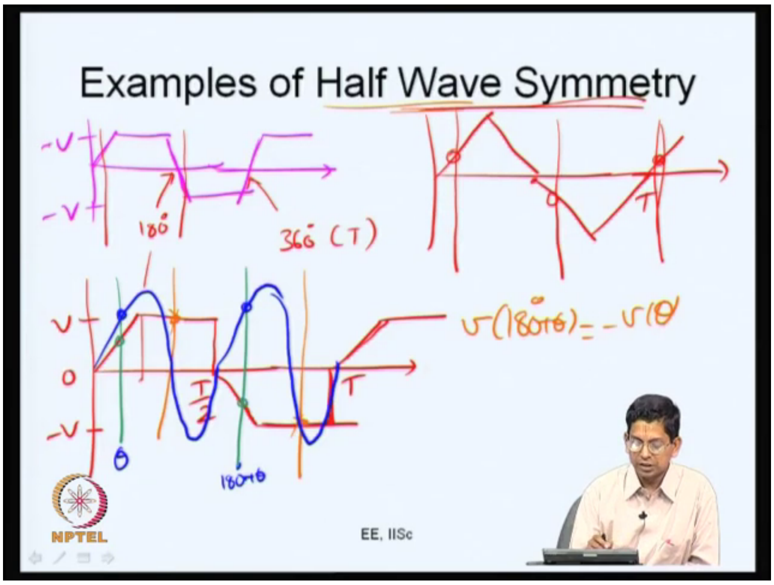

Let us look at certain examples of half wave symmetry I mean examples of wave forms that exhibit half wave symmetry sine wave of course, has half wave symmetry because sin of 180 plus theta let us say if you want to take sin of 90 degrees is basically minus sin 10 degrees. So, it has that, all the sinusoidal kind of waveforms whatever phase shifts you consider they all have this cosine whatever you might call, certain other examples that you can think of or I am just giving some arbitrary examples let us say a trapezoidal wave this waveform also has half wave symmetry I mean the slopes are equal here there are certain drawing inaccuracies, but the slopes are actually equal it is sum plus V and it is sum minus V here.

If you see here whatever is the value at theta the wave form has the same thing at 180 degrees, this is 360 degrees are what corresponds to t in terms of time and this is 180 degrees or what corresponds to t by 2, this is 180 degrees. So, whatever it is at theta if you take 180 plus theta the wave form has this property that at 180 plus theta it is value is the negative of whatever it was a (Refer Time: 29:46). This is one example of half wave symmetry like this you can go about forming several such examples that you can go on giving. Let me say shall I draw let us consider a triangular wave this is a complete triangular wave this is the time t, this is the time t by 2 here also if I see is and I consider another instant is 180 degrees away whatever is the value here and the value here they are equal in magnitude, but opposite in sin they just have you know one is negative of the other the same way if I go another 180 degrees later I will have the same thing the value will be the same just the sin goes on changing.

This is what we have a half wave symmetry you can go about constructing certain other examples also which anyway we will revisit see and let me just give you one example of half wave symmetry you know let me say there is a wave form of signal rises like this and then it is flat now it falls down, now let us say from here it takes the shape now this is t, this is t by 2 the signal rises from 0 to v in some you know some duration over some duration and then it is flat at v till t by 2 then it comes down to 0 and 0 it goes to minus v with some rate which is same as here except for the sin the slope here and here are equal, but just for the sin now.

It goes down like this and it is here now is this wave form half wave symmetric is the half wave symmetric let us try and apply the condition now. So, let us say let me just change the ink color for some clarity let me consider certain angle theta let me consider one eighty plus theta. So, whatever is the value here it is the same here, but for a opposite sin let me take consider certain other instant let me consider this instant if I go 180 degrees later I have this. So, what you see the values here and there they are the same except for the sin that is this value and this value are the same except for the sin.

This waveform also has half wave symmetry there is something not so symmetric it does not appeal to the I there is something not so symmetric, we there are some additional symmetries which will come to a little later, but this waveform nevertheless has half wave symmetric because it satisfies the property that V of 180 plus theta is equal to minus V of theta and this waveform you can be very sure that it has no even harmonic you can try doing the multiplication check it around yourself now maybe if you want you can try that you multiply this by a second harmonic sine, it would be like this is half cycle this is one cycle of a second harmonic and this is like this now.

If you take this green instant whatever is the value here the same value the second harmonic has here, but if you look at the original waveform they are opposite in sin. If you add the product at this instant to the product at this instant that is add the product at theta to the product at 180 plus theta the sum is 0 this is valid for any theta, on the whole this reduces to 0. This is an example of a wave form which has half wave symmetry you know it is could be little deceptive you may the first side many students generally tend to say well it does not have a half wave symmetry, but it has and you know that is the property and that is how we define half wave symmetry here and it has the meaning of having no even harmonic.

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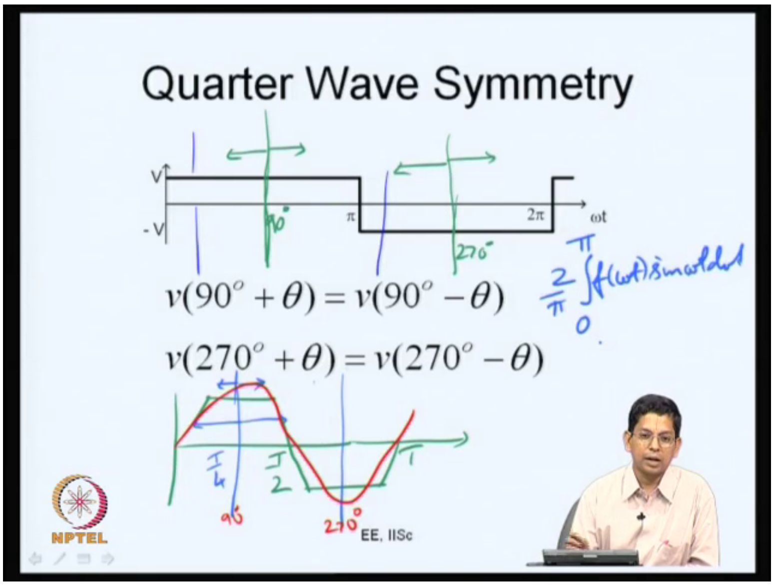

Let us go further now within half is this wave form half wave symmetric yes because you take this angle at theta and you take the corresponding angle at 180 plus theta you know one is the reverse of the other. Now what else does it have it also has an interesting property that you know let me consider the middle of one half cycle of it is you know fundamental of it is you know let us say it is positive cycle. If I travel some distance from the middle and travel the same distance from this side the wave form has the same value is on top of quarter wave symmetry half wave symmetry it has this property remember this wave form the last wave form we do here did not have this property. Whereas, if you look at here you travel to the left or you travel to the right by the same distance long as you have travelled the same distance it will have you know the same value, that is what we call as quarter symmetry let me call this as 90 degrees or pi by 2.

So, whatever it is 90 plus theta is the same at 90 minus theta. So, whatever it is at 270 plus theta the value is same as 270 minus theta you also have the symmetry observed around 270 degrees, from 270 degrees you go some distance to the right or you go to same distance go the same distance to the left the wave form will have same values.

You can look at certain other examples also before we go into that now from the previous examples we had constructed we had constructed a trapezoidal waveform some of this you know this was one example we considered this is a trapezoidal waveform one time period t, this is t by 2 . The wave form has half wave symmetry as we already seen and now it also has quarter wave symmetry it also has quarter wave symmetry let us this is the middle point this is t by 4 and you go some distance to the right goes the equal distance to the left the value is the same.

Let us say you go further the same it is equal, it is symmetric about this t by 4 or 90 degrees similarly it is also symmetric about if you can just consider one half cycle and you take the central instant at the half cycle what is that now. So, what is it that we are saying we are saying that you know these are symmetric about 90 degrees and this is symmetric about 270 degrees what is special about 90, what is special about 270 degrees. If you look at the wave form let us say here, where is the fundamental component line, the fundamental component is something like this, the fundamental component is something like that.

So, what we are saying as 90 degrees here and what we are saying as 270 degrees here they are nothing, but the instants at which the fundamental component is at it is peak the instant at which the fundamental components at peak.

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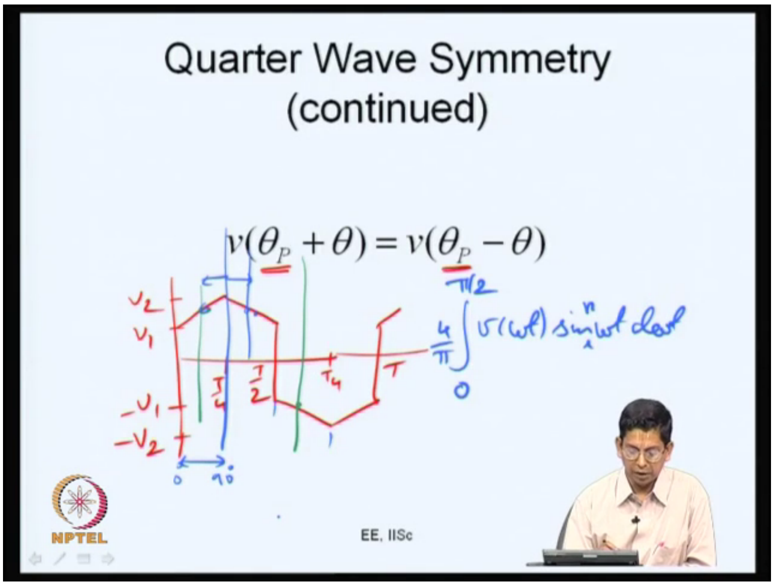

Let us say theta p is the instant at which the fundamental component is at peak any instant at which the fundamental component is at it is peak now. If this property is satisfied in addition to half wave symmetry we call this quarter wave symmetry, let me just construct one other waveform here let us say the wave form like this is t this is t by 2 and this is t by 4 t by 4 and these values are let us say if it is V1, V 2 and these are the same values with negative sign minus V1 and minus V2.

This has half wave symmetry and also has quarter wave symmetry how do we say that, you consider this instant theta you consider 180 degrees away or t by 2 away. So, what you have it is the function as the same value, but for opposite sign that is half wave symmetry it has no even harmonic, on top of that you look around this point you go some distance and you go the same distance on this side you find the values to be equal, this is quarter wave symmetry.

Someone can ask now well what is the significance of quarter wave symmetry in the significance of half wave symmetry is that you know there are no even harmonics what is the significance of quarter wave symmetry, one answer to this question is if you have half wave symmetry as in any of these cases and if you want to find out the fundamental amplitude what you normally do is you carry out an integration from 0 to 2 pi instead you can only do an integration over 0 to pi and you can take this factor to be 2 upon pi, you can let us say f of omega t you multiply this by sin omega t d omega t and you do an integration to get the fundamental component you do not have to do that you know if wave form as half wave symmetric you do not have to consider the wave form over 0 to 2 pi you can consider it from 0 to pi or any from any theta to pi plus theta you must consider one half cycle of the waveform you can calculate an average over that, that is one of the advantages now.

It reduces your calculation if you have quarter wave symmetry what you can further say is you need to consider only one quarter for example, you can only consider this quarter and you have this V of omega t defined only over this quarter that is 0 to 90 degrees and you perform an integration that is V of omega t multiplied by sin of omega t this is integrated with respect to d omega t starting from 0 to pi by 2 and then it is 2 upon pi by 2 or 4 by pi this will give you the value.

So, this spares you the trouble of defining V of omega t from 0 to 19 or that what you need to do, but if you want to care you know use do the calculation or entire cycle you have to define V between 0 to 90 which is some equation, you have to define V between 90 to 180 it is another equation this is another straight line, if you have to define V between 180 to 270 it is yet another equation between 270 and 360 it is yet another equation.

If you have half wave symmetry you do not have to define the wave form over the entire cycle we can just consider one half cycle do that if you have quarter wave it is enough if you define the wave form just from 0 to 90 degrees and you perform this integration over 0 to 90 degree you can come up with your component.

The same thing is possible for any nth harmonic component also if for nth harmonic component you are going to multiply this by certain sin n omega t and you going to do this, your calculation burden reduces that is one of the advantages that you might have with quarter wave symmetry. Then what could be the other one like you know what are the significance is what we have been talking about now what you can say is in a wave form that lacks quarter wave symmetry.

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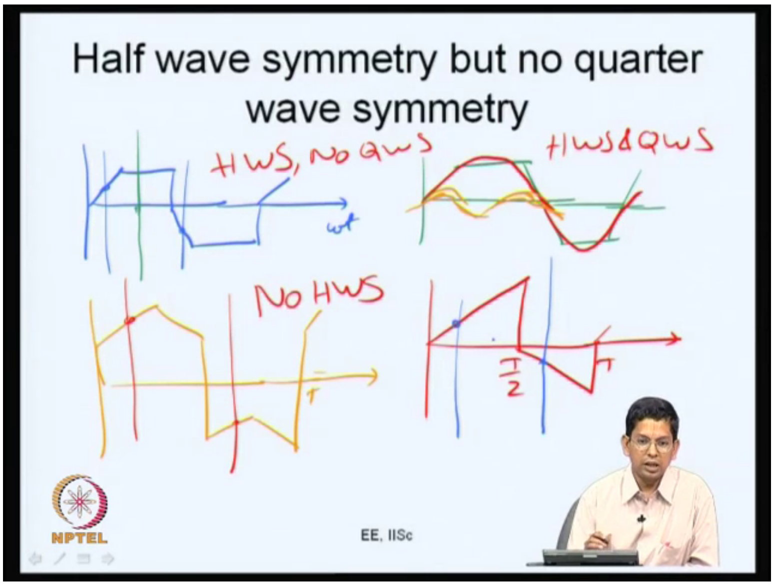

We may be we should first look at some examples where you have half wave symmetry, but no quarter wave symmetry of course, one of the examples I had already constructed, this is a wave form which has half wave symmetry, but no quarter wave symmetry. Once again to make my point clearer I would say you consider this theta and you consider this pi plus theta the values you look at the values, one is the negative of the other, it has half wave symmetry.

But if you look at quarter wave symmetry you look at 90 degrees you take t by 4 I do not know where it is 90 degree is you know strictly speaking, but you see anywhere from here the wave form is not really symmetric the waveform is not symmetric about a particular line here, this lacks quarter wave symmetry.

In such cases what happens is it is a little difficult for us to judge where the fundamental phases in a previous example let us say if the same thing happens to be a symmetric trapezoidal wave. We very easily say that the fundamental component is here, we say that this is the fundamental component the phase of the fundamental component is clear to us whereas, the phase of the fundamental component is not very clear to us and in cases where there is quarter wave symmetry wherever the fundamental has 0 crossing the harmonics will also have 0 crossings the harmonics will also have you may have another harmonic the harmonic will have a 0 crossing here like this whereas, where there is no quarter wave symmetry wherever the fundamental has a 0 crossing the harmonics need not have their 0 crossing that is another issue it gives you certain information about the phase of the harmonics. They are now whenever the fundamental crosses 0 the harmonic also crosses 0 maybe you know from negative to positive or positive negative, but they also cross there is yet another thing that you have when you have you know when you do not have quarter wave symmetry.

This is one example that I gave to show that you know waveform could have a half wave symmetry and thereby you know have no even harmonics, but still not have quarter wave symmetry. I would give another one other example this is a waveform this is time t this waveform somehow looks nice, but has no half wave symmetry if you consider a particular instant here and you consider the next instant here this is not the negative of the other this has no half wave symmetry. This is not basically an example of half wave symmetry, but no quarter wave symmetry this has no half wave symmetry this is an example of half wave symmetry, but no quarter wave symmetry this is an example of both half wave symmetry and quarter wave symmetry.

So, you can certainly construct various examples let me just take a triangular case let me say the waveform goes like this and then comes back like this is time t and it continues, this is t by 2. If you take any instant like here you consider some instant and a corresponding instant half period away the values are one is the negative of the other. So, it has even I mean it has half wave symmetry and has no even harmonic, but this waveform once again lacks quarter wave symmetry.

These are some examples that you can really think of and as I mentioned earlier this definition slightly vary I know some authors tend to use different definitions now that is why I reemphasize you know for the purpose of this course you will define quarter wave symmetry as here and half wave symmetry as here that is V of 180 plus theta as minus V of theta in the wave form satisfies that property that is half wave symmetry and if we have something like V of 90 plus theta is ninety minus theta or more generally V of theta plus theta is equal to V of theta P minus theta, where theta P is the instant at which the fundamental component is at it is peak maybe the positive peak or may be the negative peak theta P is the instant when the fundamental component is at it is peak this is theta p and this is also a candidate for theta P.

Then we call this as quarter wave symmetry if we have quarter wave symmetry over on top of half wave symmetry then what we can say is the wave form not only has you know you we can say that it has no even harmonics and you can say that you know you need to consider only one quarter of the waveform starting from one 0 crossing to the peak for example, or any of the 4 quarters like this of the basically the fundamental cycle to do your PWM calculations you do not have to define the entire waveform over the entire cycle it is enough if you define it mathematically over one quarter. So, your calculation burden simplifies and you can say something about the face of all the harmonics whenever the fundamental cross is 0 the harmonics also cross 0. These are a few things that you can say when you have quarter wave symmetry to now.

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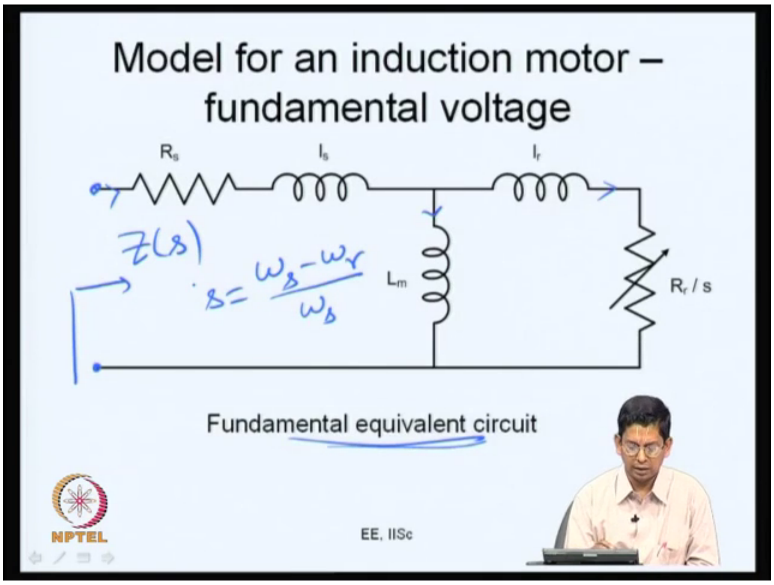

Now this is all about you know a kind of a review of Fourier series. So, that we can use Fourier series effectively to calculate the fundamental voltages and harmonic voltages of a several different you know PWM waveforms. The next issue is when the harmonic voltages are there what are their effects is what we are trying to see now what gets supplied to an induction motor for example, you know like a an induction motor fed from voltage source inverter is not only the sinusoidal fundamental component also harmonics get fed there.

Let us say we consider only the fundamental component if you see only the fundamental component it is a sinusoidal quantity, the fundamental component sees the induction motor as it is fundamental equivalent circuit. So, which as the standard things this is your you know rotor stator winding resistance and this is the stator leakage inductance and this is the magnetizing inductance and you have a rotor leakage and then this Rr is once again the Rotor resistance and s is the slip now.

This Rr by s can be divided into 2 parts as Rr and the remaining the Rr alone stands for the rotor resistance and the remaining part stands for the mechanical power developed the whatever power is dissipated through this resistor is equal to the mechanical power developed by the machine now. This is an equivalent circuit what do we mean by that, when applied you know when some fundamental voltage is applied the current drawn by the machine is equal to whatever the this fundamental circuit draws the same amount of current now that is there is a you know the terminal relationships are the same now for a certain amount of fundamental voltage whatever is the current drawn by the actual machine is the same as what is drawn by this circuit.

So, this circuit gives us a measure of what is the stator current and what is the magnetizing current and what is the rotor current all these measures can be obtained from here now and we can also see how much power gets dissipated here and from there you can also come up with how much mechanical power is developed and etcetera. So, this is the fundamental equivalent circuit the fundamental voltage the inverter voltage has fundamental as well as harmonic components the fundamental component c is the machine as some such equivalent circuit here where the slip s is given by whatever is your synchronous speed omega s minus omega r divided by omega s. So, omega s is the you know synchronous frequency that is if you apply the stator frequency like 50 hertz or, the revolving magnetic field revolves at it is synchronous frequency which is decided by you know the frequency of the applied voltage and the number of poles there is something like that.

This is the synchronous speed the machine will also run the synchronous speed if the machine is not loaded, but you know any machine as you know I even under the so called no load condition has some amount of loading on that and when you actually load it what happens is there is a slip the rotor speed is a little lower than the synchronous speed . So, that is what you get by the term omega s minus omega r that is normalized with respect to omega s gives you the slip. So, slip is 0 under no load condition and the slip has some rated value of a few percentages something like 5 percent or, when it is under rated condition many typical machines now.

So, what happens is the slip is really low and therefore, the term Rr by s is significantly high that is what we have right, we move on.

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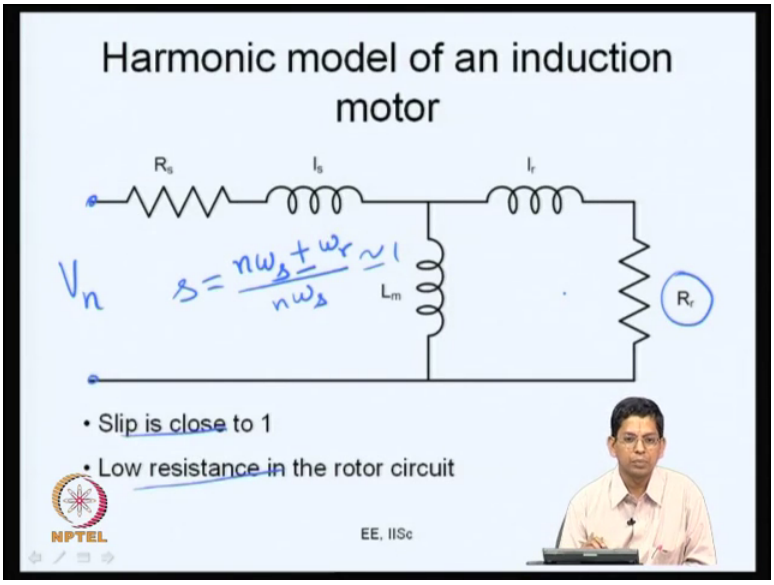

When we look at the harmonic voltage let us say some instead of the fundamental component some nth harmonic voltage may be the 5th harmonic or may be the 7 th harmonic is applied now. So, what happens now in this case you as you find the slip the synchronous speed is higher now the fundamental has 50 hertz, the 5th harmonic is 250 hertz, the 7 th harmonic is 350 hertz and 5 th harmonic also revolves in a direction opposite to that of the fundamental and you know the 7 th harmonic revolves in the same direction.

The synchronous speed to start with is not omega s, but it is n times omega s where n is the harmonic order and the relative speed between the synchronous speed and the rotor is now it could be n omega s plus omega Rr minus omega r, it is n omega s plus r minus omega r. If it were the 5th harmonic for example, this omega and the synchronous the revolving magnetic fields that the 5th harmonic produces are in the opposite directions. So, the relative speed is n omega s 5 omega s plus omega r if it were the 7 th harmonic then the 7 th harmonics revolving magnetic field and the rotor revolve in the same direction it will be 7 omega s minus omega r, if this is what you have n omega s plus or minus omega r and you normalize it with respect to n omega s.

If you do this is almost equal to one because n omega s is much higher than omega r, you have something which is almost close to one for high values of n. So, what do you have is Rr instead of Rr by s, now Rr by s is big because s is small therefore, Rr by s is big, if s is 0.05 Rr by s is some 20 times Rr now it is only Rr, Rr by itself is only the winding resistance of the rotor which is a very small number, what happens is it reduces slip is close to one the rotor side resistance is now very low.

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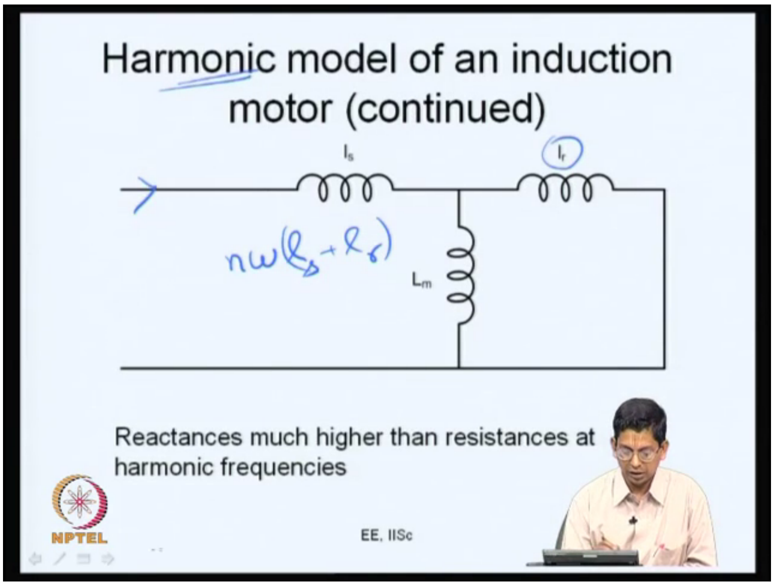

So, you move to the next point, what you have is you basically had only the stator winding resistance on the rotor winding resistance and when you look at harmonic frequencies this omega l this actually offers n times omega l where omega could be the fundamental frequency the harmonic frequencies n times omega n times omega l is the reactance seen by this and n times omega l r is the reactance seen by that, this is the total reactance. If you see these reactances this n omega ls for example, is much bigger than the rotor resistance rs and similarly this n omega lr the reactance pertaining to lr is much larger than rr at the harmonic frequencies therefore, you can ignore those 2 for the point of view of calculating how much harmonic current is being drawn you can ignore those two.

If you go further this inductance is much larger than this inductance, this inductance is practically like an open circuit of the parallel combination of this 2 almost dominated by this it is almost close to this value of inductance and that is what we get here.

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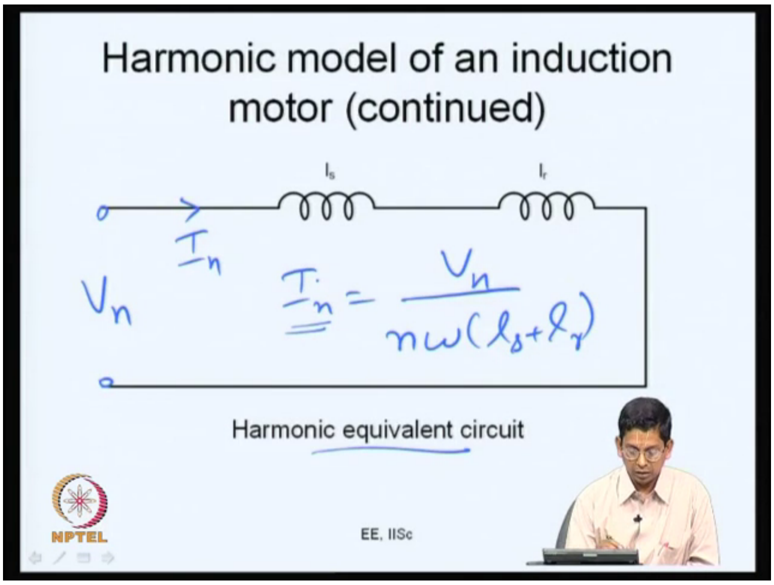

This is what is called as a harmonic equivalent circuit of an induction motor the harmonic voltage that is applied essentially sees an induction motor as an equal as it is total leakage inductance. If you want to calculate the I n, this I n is simply equal to your V n divided by n omega l s plus l r is that right. This is the reactance pertaining to the leakage reactance, you can very easily calculate that, now you must remember that this equivalent circuit is mainly to calculate I n, you can calculate I n considering the resistances for example, you can consider the various other things, but I n would not be significantly different even if you neglected those, but it you know and you use this I n it does not mean that you know there is no loss this is to calculate once you have calculated this I n you can use this to calculate your copper loss.

Now the current harmonic what we see is if you take the stator winding resistance like what is indicated here what flows through the actual stator winding is not only the fundamental current also several harmonics flow. So, when I n flows that it also produces I n squared R s will also produce you know nth harmonic current also produces certain loss.

The harmonic currents also produce the stator copper loss is given by I squared r and in fact, the resistance corresponding to harmonics could be a little higher because of high frequency effects just the skin effect, the resistance could really be higher too, but we ignore those and you know what we want to do is we want to consider the same resistance and want to be able to calculate a value of I n and this is an approximation that is reasonably valid for calculating I n now this is simplified analysis you can you can add more and more details to it, but simplified analysis gives you a first cut figure which you can relay on and also it gives you a certain amount of insight, what is that insight we will use this extensively the harmonic you know well the fundamental equivalent circuit sees the machine as impedance and that is a function of slip, that is a function of slip the machine is seen as some impedance which is a function of slip that is fuzzy fundamentally.

But if you look at the harmonic the harmonic equivalent the harmonic voltage sees the machine simply as reactance. If you take a particular harmonic voltage and a particular harmonic current they are related as though you know their relationship is similar to that of the voltage and current through an inductor now this is of consequence as we will see later this will help us in analyzing the harmonic currents very effectively later on.

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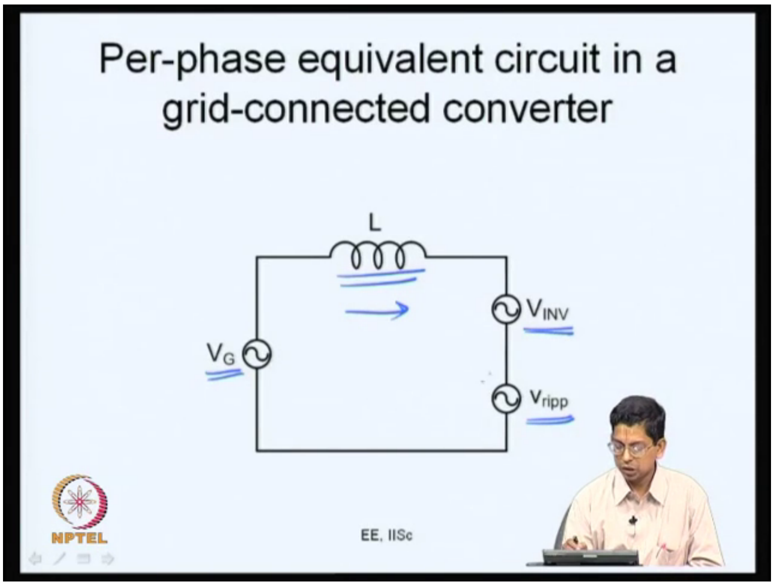

Now, instead of a motor drive application if you take it to be a grid connected converter you know the line side is basically connected to the ac side of the inverter is basically connected to the mains through line inductor l in that case what you have is VG is the mains voltage this is what we saw in you know one or 2 lectures back we were discussing front end converters and we were discussing statcom kind of applications this is the grid voltage and this is the fundamental component of the inverter voltage and this is the ripple part of the inverter voltage, the ripple has fundamental I mean the inverter output voltage the terminal voltage has fundamental which is that line frequency plus some ripple added to the ripple is the sum of all the harmonics.

If you want the fundamental equivalent circuit that is you want to calculate the fundamental current drawn through this then you can just ignore the ripple here.

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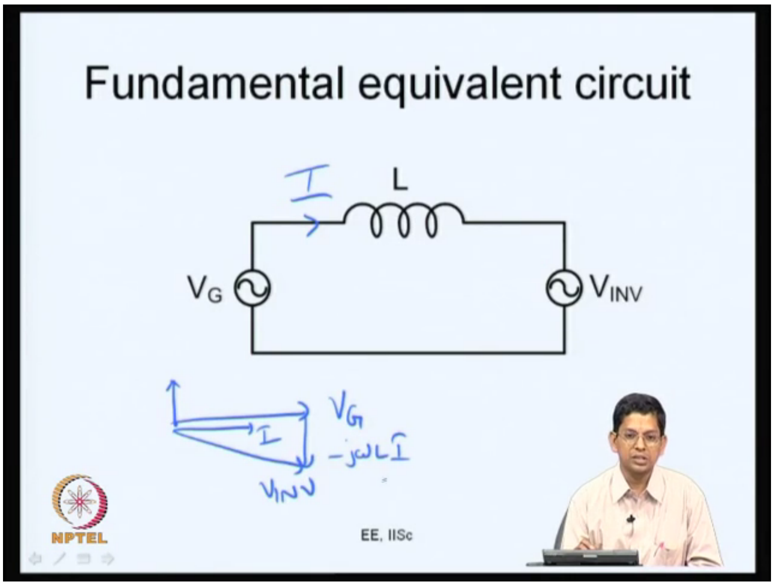

This is your equivalent circuit now and if you are drawing certain amount of current I you can even draw a phasor diagram this is VG and this is I that you want to draw and the oh the drop across the inductance due to I is going to be some j omega Lyou subtract that this is going to be minus j omega L I and this is what is going to be our V inverter voltage you can very easily draw when the phasor diagram and you can relate all the various quantities here, this is the fundamental equivalent circuit.

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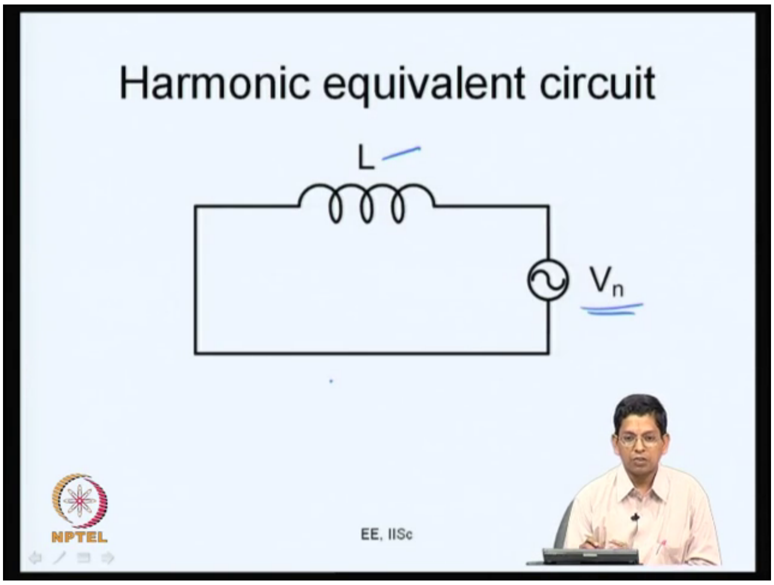

And you move over this is the harmonic equivalent circuit which also we saw, here if the fundamental components of the grid and the inverter do not make any effect here and you can calculate your nth harmonic current simply using your nth harmonic voltage and the inductance here.

These are the models which will help us calculate the harmonic currents in various applications when based on those harmonic currents we may be able to calculates you know these the copper losses and they now we may be able to calculate many other things. So, what we have been looking at is these is a measure for calculating harmonic current and from there we will be able to quantify some of the undesirable effects due to of harmonics that is such as the increased copper loss etcetera.

So, as I mentioned the purpose of pulsewidth modulation is to control the fundamental voltage and to mitigate the harmonics and their harmful side effects. We have reviewed Fourier series which tells us how to calculate the fundamental and the harmonic components and we have developed models. So, that you know given a fundamental voltage or a fundamental current you know or harmonic voltage we are able to calculate the corresponding fundamental current and harmonic currents.

Let us continue this in the next class and get into or business of understanding PWM effectively I thank you very much for your interest and for your time and your patience.

And I hope that you will you know have continued interest in this lecture series and you know see you again in the next lecture.

Thank you very much.