Pulsewidth Modulation for Power Electronic Converters
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Lecture - 36
Analysis of overmodulation in sine-triangle PWM from space vector perspective

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters.

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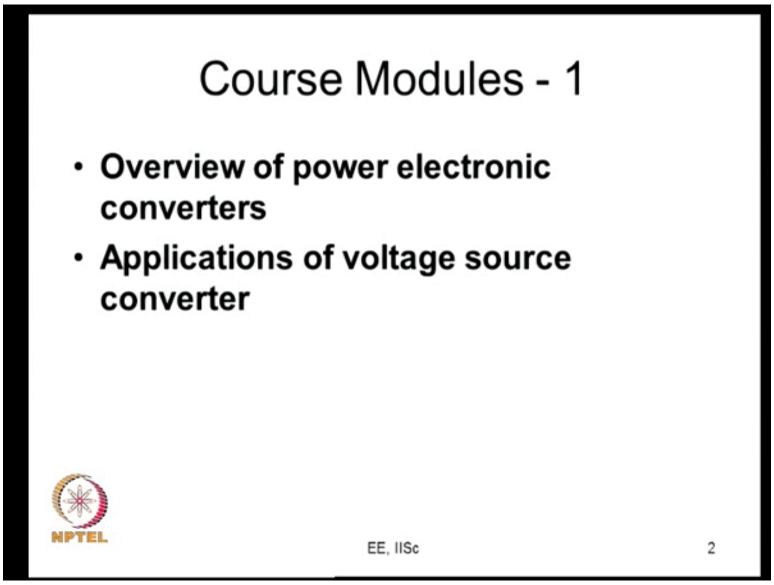

So, we have been looking through various course modules. These are first couple of I am trying to organise the course modules for your understanding. So, you have this overview of power electronic converters as one of the first module, but we saw where we looked at different topologies of power electronic converters. And then we looked at different applications of voltage source converters, such as motor drive, active front end converter, and reactive power compensator. So, these are somewhat like a typical power electronics course. These 2 modules fall under somewhat typical power electronic courses, and you which very could have some more lap with other things. But we just did this so that you know we would not do pulsewidth modulation of power converters we need to be clear what the power converters are, and what they are going to be used.

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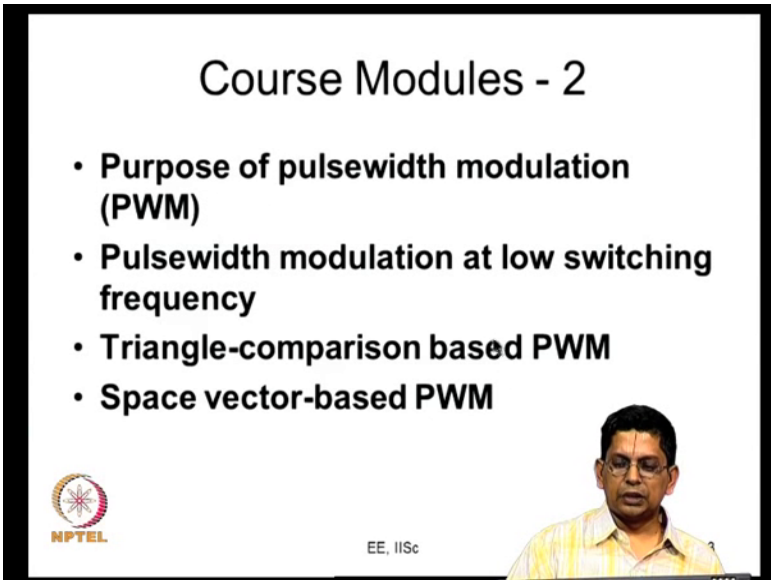

So, then the second set of modules, this is actually would be second set of modules, they are more about pulsewidth modulation. So, about Fourier series and how you would switch them various symmetries, how you would generate the pulsewidth modulation wave forms at low switching frequency, and how you would do it with the high switching frequencies. So, the last 2 are more to do with high switching frequencies, this is based on the classical way of comparing 3 phase modulation signals again it is triangular carrier. And this is relatively more modern of getting through space vector based PWM.

So, this comes from the modulation theory that they have used in communication and so on. This basically originates from electric machines. So, though they are origins are different, there is a lot of similarity between the 2. Which has been established in the literature, and we also saw that now. So, everything that is done by the triangle comparison PWM can also be done by space vector. The only difference what we say in this course is; certain things done by space vector cannot be done by triangle comparison approach.

Also, there are methods called continuous PWM methods, which can be implemented either using the former approach or the latter approach. There are discontinuous PWM methods, or bus clamping PWM methods, which can be implemented either using a triangle comparison approach or space vector approach, but there are those advanced bus clamping PWM methods, which normally cannot be produced by comparing 3 phase modulating waves against triangular carrier they need this space vector. Say in some sense space vector based PWM is more general than the triangle comparison base PWM.

So, in the triangle comparison base PWM, what you do is you have 3 phase sinusoids, you also add common mode components addition of common mode component to the 3 phase sine waves is equivalent to dividing the null vector time. So, when you in every sub cycle we operate to active vectors and a null vector, and the null vector is applied through 2 different 0 states in the inverter, where the 1 0 state is when all the top switches are on other one is when all the bottom switches are on.

So, it is the division of active vector null vector time that we have seen. That is really the degree of freedom available. But in space vector based PWM we not only do that, we also divide the active vector time. It is also possible to apply in active state more than once; multiple application of active state and you can you know. So, so called division of active vector time could be done. So, the advance base clamping PWM methods, they do that they apply only one active I mean only 1 0 state, but they apply one of the active states twice. And they produce some interesting waveforms and they are capable of improving the performance particularly at high modulation indices and so on and so forth at some level now.

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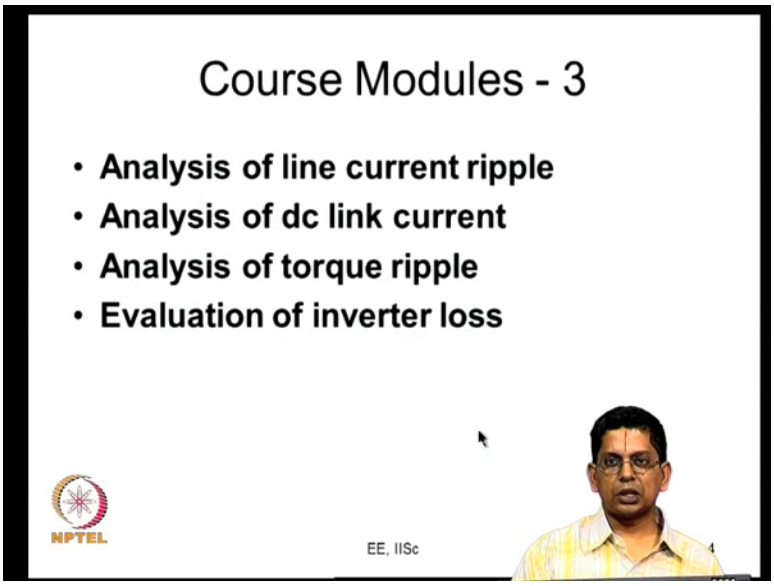

So, let us go on from that is the third second set of module. Now the third set of modules we have been more into analysis; that is, you know, earlier the focus was more on generating the given amount of PWM wave form. And may be enhancing the dc base utilization for a given amount of dc voltage how much ac voltage you can produce. So, that is one of the reason why we went in for common mode injection and so on. Now the emphasis shifts more to the harmonic components, not just the fundamental component. The harmonic components are in the voltage lead to harmonic currents. And so, these are at various frequencies.

And so, the sum of all the harmonic currents we can turn them as the current ripple. And we try to analyse the current ripple. So, we try to come up with you know methods of calculating the hormones current ripple particularly considering the high switching frequency case. So, we consider the error between the applied voltage vector and the reference voltage vector we integrate that, and try to evaluate that now. Then we did the dc link current analysis. So, the ac side you may have sinusoidal currents with little ripple flowing there.

Now the dc link current is basically the sum of the top switched currents, and the every top switched current is the product of that switching function and the current that is flowing through that leg. So, we try to analyse the dc link current; so which has a dc component and has harmonic components. And the dc component flows from the dc side, and most of the harmonic the dc side mean the power supply, let us say that is a rectifier or battery or some active front end converters so on. And the harmonic components and the dc link current largely flow through the capacitors dc electrolytic capacitors.

The electrolyte capacitors are the main sources for the ripple component and this dc link current. And therefore, we evaluated that so as to be able to evaluate how much ripple current the dc capacitors should give us. So, we looked at these kinds of analysis. One is on the line current ripple and then the dc link current. Then we looked at torque ripple. Because you know we are considering induction motor or drive as our main application here. And so, in the induction motor drive when you apply harmonic voltages, it causes harmonic currents, harmonic fluxes, and there are also harmonic torques. And we found that the harmonic torques are normally caused by you know, interaction of fundamental flux with some harmonic current are fundamental current with harmonic flux.

So, we tried to analyse this torque ripple considering 2 cases. When you had low switching frequency, and when you have high switching frequency. Then we looked at evaluating the inverter loss. So, up to this point we regarded the devices ideal. Here we started considering the forward drop in the device, and tried to calculate the conduction loss. Again, we tried to look at the transitions in the devices turn on turn off transitions and the energy that is lost, and calculate the switching energy loss. And we also worked out some PWM methods which is which are capable of reducing the inverter switching loss compared to conventional space vector PWM.

Interestingly bus clamping PWM methods and some advanced bus clamping PWM methods are capable of doing that. So, like our you know, in this module we had 3 lectures. The first one focusing on conduction loss, second focusing on switching loss and the third one exclusively focused on this reduction. And we looked at this advanced bus clamping PWM methods, we saw how they could probably be used you know, there are some at least some directions in that regard. How they could be used to reduce the inverter switching frequency? So, this is I mean there is inverter switching loss, and also the power conversion loss all right.

So, this is now kind of little bit of analytical learned this issues now. And now you have the other set of modules which I would call as the last set of modules. So, the overall there are 13 modules, I am now putting them as 4 sets for your easy understanding. So, here this is a little I would call them as slightly complex in the sense like, you we were not considering the inverter switching transitions at all, and then they you know at some stage we started considering them, that is in the previous stage we started considering; for example, the switching loss and so on.

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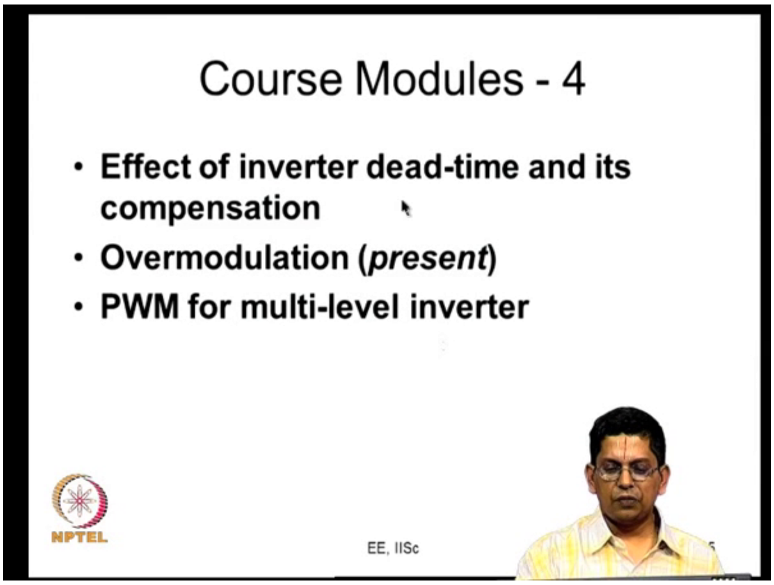

Here we are considering the effect of dead time. That is, you know the dead time is like the top and the bottom devices are not exactly complementary in an actual inverter. Not switched complimentarily. You switch off the outgoing device first, and then you switch on the incoming device which sounds in dead time. And we looked at what are the effects of the dead time we found that you know that there are how we found that the dead time you know analysed it is effect for continuous PWM methods. We found it for all continuous PWM methods as the effect of dead time is quite similar, and it is dependent on power factor. And we also looked at the effect of dead time for bus clamping PWM methods. We found that the dead time effect under given condition changes from one bus clamping PWM to another bus clamping PWM. And also for the same bus clamping PWM, the dead time effect varies from one power factor to another power factor.

So, these are certain things we found, and the dead time effect and the compensation is just the other side of the coin. If you know what is the effect of the dead time you can compensate for that. We found that it basically introduces certain kind of error voltage, you can compensate for that error voltage by adding the negative of it, I mean equivalent what is equivalent to the negative of the error voltage to the modulating signal. So, we though your emphasis was entirely on the inverter dead time, I mean the we the compensation is just one step away. So, we could do that. So, here we are going to look at something what is non-linear here.

Firstly, what we found as the effect of that is; the fundamental voltage is no longer proportional to the reference, because that is the ideal fundamental voltage. On top of the ideal fundamental voltage there is a small component that was getting added, which is the fundamental error voltage caused by dead time. And whatever you are getting is the phasorial sum of these 2, the ideal fundamental voltage, and the error voltage fundamental voltage where you know because of that. So, you have the phasorial sum.

Now, we are going to look at something where the ideal fundamental voltage itself is not proportional to your voltage reference. We are going to go into to the non-linear region of operation of PWM rectifiers; that is, where your reference might not really be you know, the inverter voltage may not be proportional to that. So, why do you have to do that? So, we can trace it back to sine triangle PWM. With sine triangle PWM you know very simple way of doing things. So, you can calculate you know you can I mean modulate the ac side voltage reasonably good harmonic spectrum. But we wanting for common mode injection.

Why did we going for common mode injection? Among the various reasons, one reason why we needed to do that to ask to increase the dc bus utilization. So, we increase the dc bus utilization, now with sine triangle PWM the maximum fundamental whatever you could get was 0.5 V dc. The peak phase fundamental voltage is V dc by 2. When we went for common mode injection or certain kinds of appropriate common mode injection, we could increase V dc by 2 to V dc by root 3 that is 0.577 times V dc.

Now, by adopting to over modulation, you can take your inverter all the way till square wave mode, which can give you 2 upon pi times V dc. Or 0.644 times V dc, that is what we are trying to do. So, why do we need over modulation? In one sense it is to increase the dc bus volt utilization further. That is, if you reserved to over modulation, now you can produce higher ac side voltage. At what cost? The same dc bus voltage. Since the dc bus voyage is same the device voltage ratings are also same, no problem. So, with the same dc bus voltage and same device rating, you are able to have an inverter whose voltage rating is higher, whose ac voltage rating is higher. But there is a problem. It introduces low frequency distortion in your output. Therefore, there are going to be low frequency currents, and that is going to lead to several problems in closed loop control it is going to be much more difficult to handle that. So, these are some things that we will look at now.

So, today's and the next lectures would exclusively focus on over modulation now. Today's lecture would focus on the over modulation for triangle comparison based PWM. We tried to get an understanding of that problem. And in the next class we would do some space vector algorithms for over modulation and so on. And then the last one we would be for this PWM for multi-level inverter, all that we have been able to study for this 2-level inverter we should be able to extend them to 3 level inverter, and we will do a few things some triangle comparison based PWM, and some space vector based PWM their analysis for multi-level inverter now.

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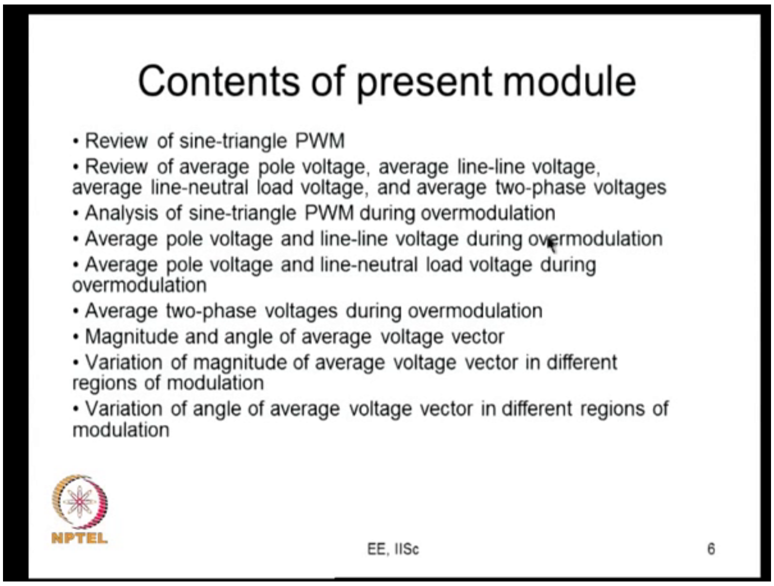

So, after this rather long introduction to this let me take you to this; present module in greater detail. So, I said I am going to deal with over modulation. So, what we are going to first do is to review sine triangle PWM. It is better that we pick up our threats from there. And then let us review these quantities what we have called as average pole voltage, average line to line voltage, and average line to neutral voltage and average 2 phase voltages. So, then what we would essentially do today is this, analysis of sine triangle PWM during over modulation.

So, when there is there is an inverter, it is feeding a load. Let us take it as an induction mode load for example. And that inverter is having a dc bus voltage it is operating in over modulation; that is, it is peak value of sine is greater than the peak value of carrier. So, what happens? That is what we are essentially going to see now. So, how would we look at that what do you mean by what happens? So, what are the quantities you look at? We will look at the average pole voltage. We will look at the average line to line voltage have the inverter. We will also look at the average pole voltage and the average line to neutral of voltage applied on the load during this over modulation. We will we will just see what happens to these wave forms they will not be sinusoidal. That is that is certainly you can understand. And then once you have this average 3 phase voltages, you can transform them into average 2 phase voltages this is space vector transformation.

Now, we will look at how these 2 phase voltages vary during over modulation. Now you have come to the 2 phase voltages, which are basically the orthogonal components of the voltage phase vector or the average voltage vector. Now we will see how the magnitude and angle of the average voltage vector are. They will not be for example; the magnitude of the average voltage vector cannot be expected to be constant throughout the cycle. Well that is the case in linear modulation; in linear modulation at the study operating condition the average voltage vector has a constant magnitude which will not be in the case of for example, in over modulation right. So, you have some things like. So, we will look at how that magnitude varies.

And that itself would might vary differently in different regions of modulation. From sine triangle PWM you slightly go into over modulation it may be different, when you go into you know deeper into over modulation closer to 6 top more it can different. So, you look at that we will also look at how does the angle of the average voltage vector vary in different regions of that. This would essentially give us an idea on how to deal with over modulation, what does over modulation how to deal with over modulation. And this analysis and the space vector domain will be giving us enough inputs, enough parts, on how we can handle over modulation when we are dealing with space vector modulated inverters.

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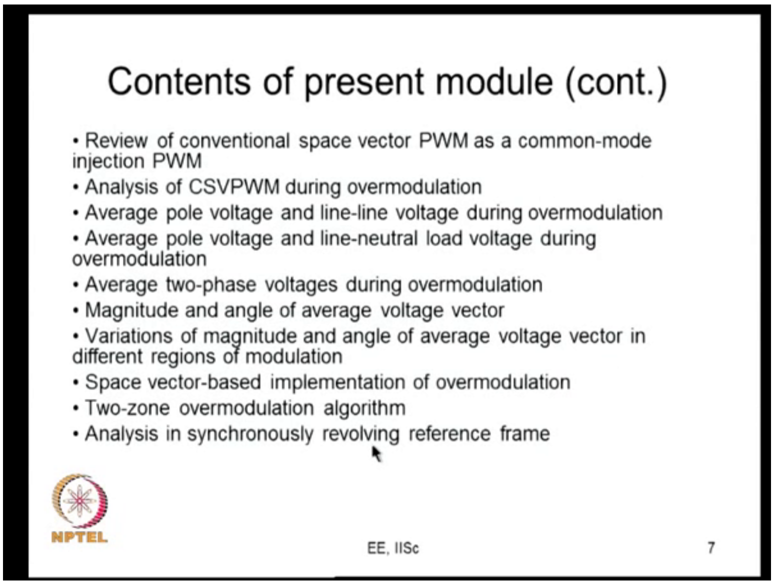

So, in the next lecture what we will probably do is we will take it up from that. We will review conventional space vector PWM as a common mode injection PWM. And we will do a singular analysis of what we are going to do today for sine triangle PWM, we will do the same thing for conventional space vector PWM. And we will go look at the average pole voltage average, line to line voltage average, line to neutral voltage, and we will we will transform them into the 2 phases. And then we will see that we will come very close to certain space vector based methods of doing them now.

At as the result of all this we will we will be able to understand you know how to implement over modulation in space vector based PWM. We will discuss a few algorithms. Over modulation algorithms for space vector based inverters, and do some analysis in the synchronously revolving reference frame. This is what we would probably do in the next lecture. So now, coming to the today's purpose we are going to deal with voltage source inverter. So, we are going to deal with the voltage source inverter which has a fixed dc bus voltage V dc. And which is modulated using sine triangle PWM. And it is modulated such that it is the inverters are in the over modulation; that is the peak of this sine sinusoidal wave voltage is you know they are greater than the peak of the carrier.

So, it is that is over modulation. So, we are going to analyse this over modulation, and how are you going to analyse we are actually going to analyse it from this space vector perspective. So, that is part we are going to do today.

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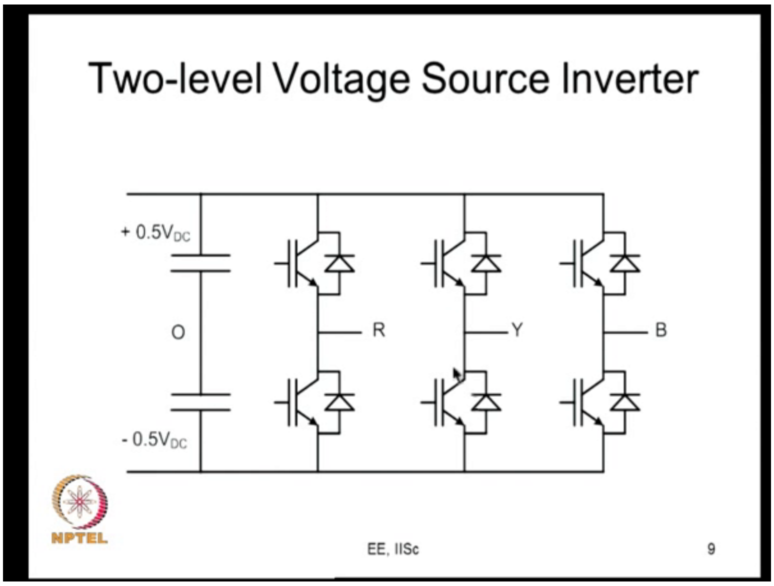

Let us get started. So, this is the inverter. Now you have 2 legs which are switched in the complementary fashion, and we are going to ignore dead time. Today once again like we did the analysis for the linear modulation, because our focus here is now understanding the difference between linear modulation and over modulation. And what is going to happen is now? So, there is a 3-phase load let us we can presume it to be may be an induction motor, and you have this now.

So, we are going to look at these basic quantities. So, let us it is it is better that we take a good I mean quick recap of that. So, this is R midpoint there is a low terminal midpoint as leg V R O is the pole voltage. So, R this midpoint of R phase leg measured with respect to O. Similarly, V Y O is the Y phase pole voltage. Voltage at this point measured with the respect to O V B O is the B phase pole voltage; that is measured at B with respect to O. So, these are pole voltages 3 phase pole voltages. V R Y would be the line to line voltage V R V R O minus V Y O will give you that.

Similarly, V Y O minus V B O will give you V Y B, and V B O minus V R O will give you V B R. So, these are the line to line voltages. And then there is a 3-phase load. We assume it to be a balanced star connected load and whose neutral is N. So, there is V R with respect to the load neutral N and Y with the respect to load neutral N and again B wave with respect to load neutral N. So, these are the various 3 phase voltages the same 3 phase voltages can actually be represented in terms of the pole voltages or the line to line voltages or the line to neutral voltage applied on that now.

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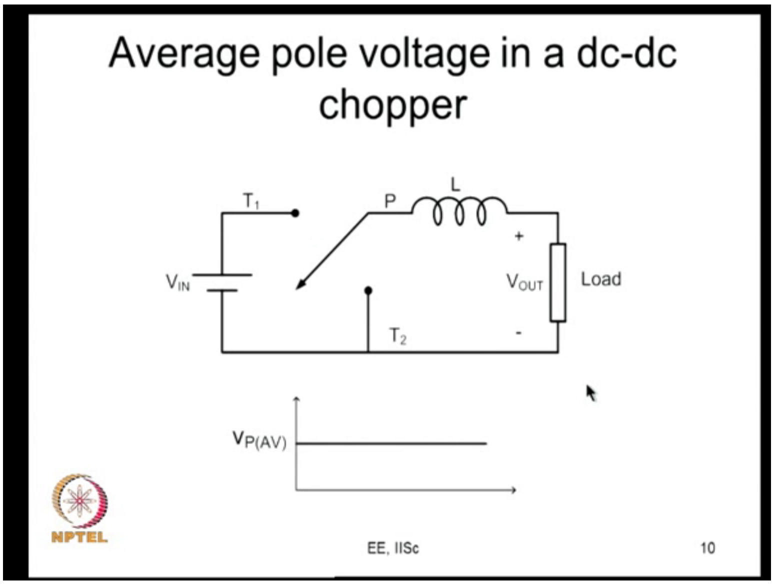

So, to recap you know let us go to a dc d c chopper. What do you want here? You want this voltage. So, you want certain output voltage, there is some average output the input is V in we want an output V out which is let us say 50 percent V in or 80 percent V in or whatever with something lower than that. Now this is an inductive filter. So, whatever is the average voltage you want you want the same average voltage here now. And how should that average voltage be? That is, we call as V p, and that V p average should be horizontal. And what does this V p average? If p is connected to throw out 2 it is 0. If p is connected to throw T 1 then the voltage is whatever comes between that that is V in comes here. So, it is sometimes equal to V in and sometimes equal to 0, and you might be switching it at some duty ratio d.

So, it is V in into d that would be the pole voltage average pole voltage in this case. Now you want dc output. So, you want the pole voltage to be equal to your desired dc voltage. So, this is how you want it to be, in the case of a dc d c chopper now.

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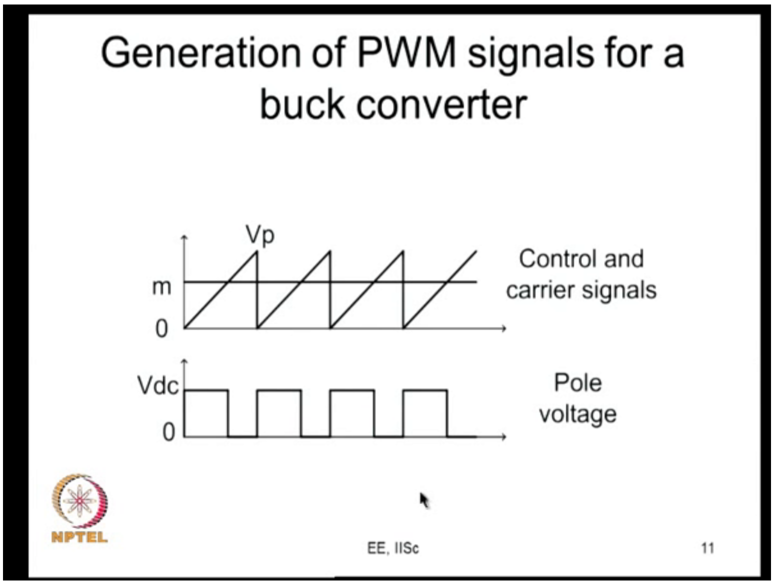

And if you want this, how do you achieve that? You achieve that by having or carrier signal and a control signal like that, and you compare the 2 and this would be a getting pulses that are fed to the active device. So, if you want more amount of fundamental voltage higher duty ratio you take this up, if you want lower you just bring this down. So, you achieve this by a comparing a dc modulating signal with such carrier to produce this now.

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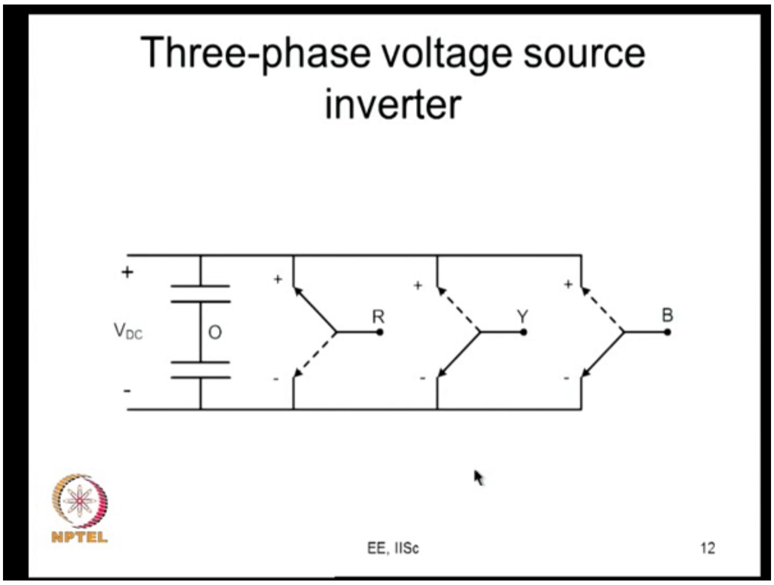

What is the situation in a sine in a sinusoidal inverter? Here 3 phase voltage you desire a sinusoidal voltage between V R Y. Again, you desire V Y B. So, again between V B R you desire sinusoidal voltages. Like, you can look at average voltage V R Y average what do you mean by V R Y average; averaged over a sub cycle. And similarly, V Y B is V Y B averaged over a sub cycle; we want these wave forms to be sinusoidal. You can ensure these to be sinusoidal if V R O average sinusoidal V Y O average sinusoidal and V B O average sinusoidal ok.

So, you further want these 3 to be 3 phase symmetric, V R Y average V Y B average and V B R average. So, you can achieve this by having V R O average to be sinusoidal V Y O average to be sinusoidal and V B O average to be sinusoidal and also symmetric. So, what we actually want is we would want the V R O to vary like that.

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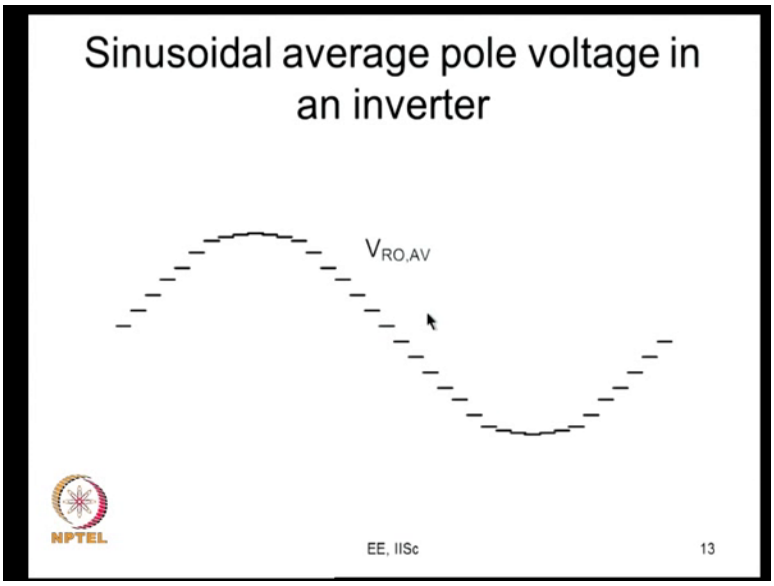

So, in one sub cycle if this is your V R O, the next sub cycle this is your V R O, and the third sub cycle this is your V R O 4th sub cycle. So, this is V R O average goes on increasing sub cycle by sub cycle. Here when it goes on increasing what it effectively means is the top device is on for longer time, and the short is on for shorter times. So, here the duty ratio is maximum. Again, it reduces and comes back to close to 0, when it is close to 0, the top and the bottom devices are both on for equal durations of time, and then you go down. And when you have your most negative V R O average your duty ratio is your minimum, and then it goes back now.

So, you switch that in such a fashion that you know in in different sub cycles the average values are different. And these average voltages vary in a sinusoidal fashion, they vary in a sinusoidal fashion. And how can you ensure this? If one mean that you can ensure by comparing triangle with a sine as we will see now.

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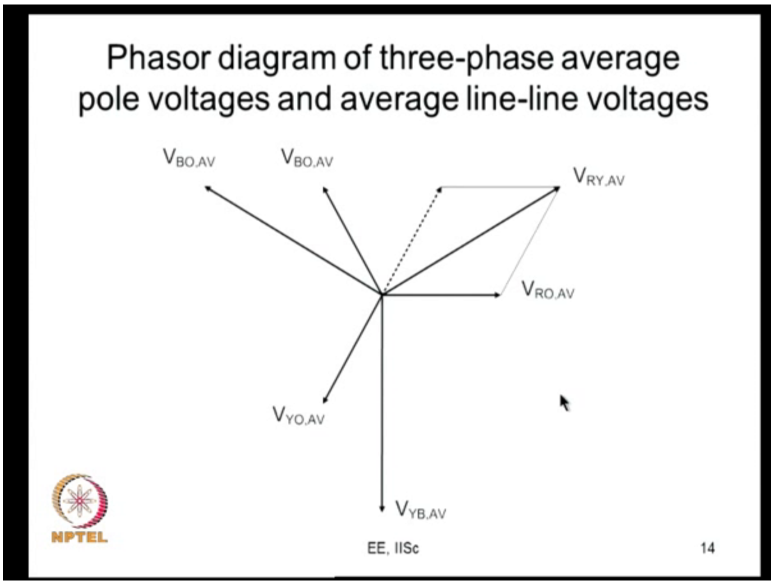

If let us say you are able to you produce such a V R O average, and V Y O average has given by this phasor and V B O average given by this phasor, then you can get your V R O average as desired, by the difference between that. Again, V Y O average minus V B O average will give you this V Y B average, again this V B O average minus V R O average will give you your V B R average. This should be there is an error here. It should be V B R average thank you. So, you ensure them you want these to be sinusoidal.

So, you want these line to line voltages to be sinusoidal, one way of ensuring, but not necessarily the only way is to make sure that V R O average and V Y O and V B O average are sinusoidal as shown by these phasors.

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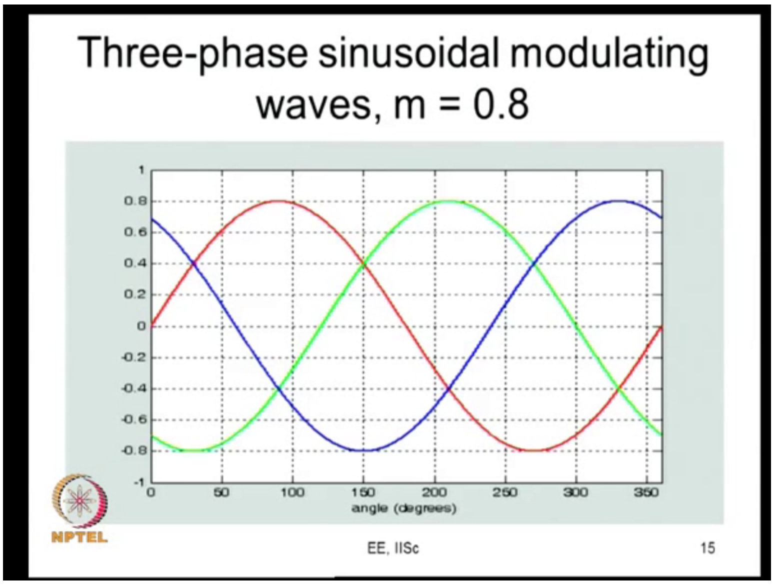

So, how do you get them? You different modulating signals m R m, Y and m B and you compare them with a high frequency triangular carrier; which goes up and down like this and the triangular carrier is not been shown in this figure. So, whenever the modulating signal is greater than the triangular carrier the top device is on, otherwise the bottom device is on. So, it goes on like that.

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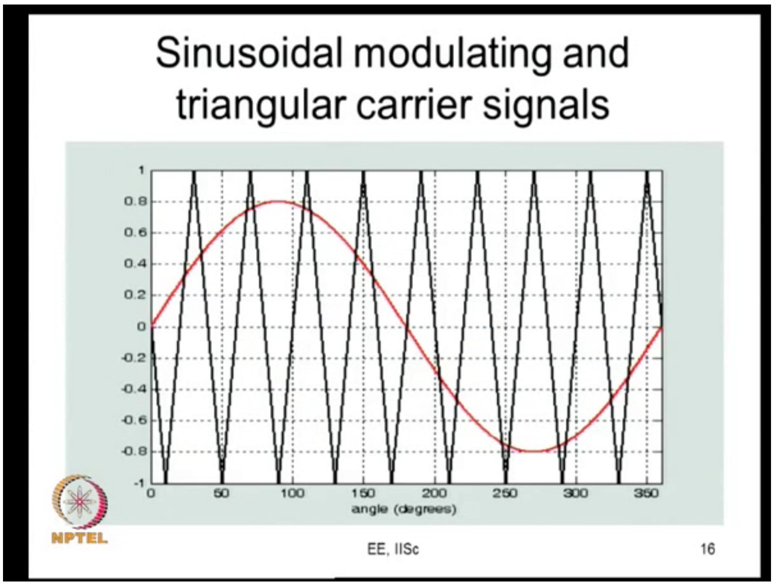

So, you go about doing this. So, this is an particular I am I am showing only one R phase signal, and I am showing the triangular carrier. Of course, the carrier is shown to be of lower frequency here, this is only for illustrative purposes. And you typically the carrier frequency will be much higher than what it is been shown now.

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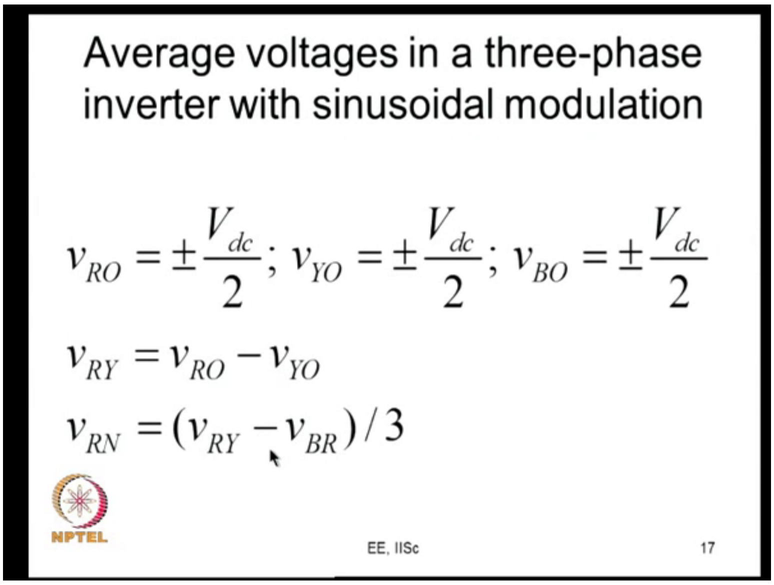

So now if you look at R phase, the top device can be on or the bottom device can be on. So, with accordingly V R O can be either plus V dc by 2 or minus V dc by 2. Similarly, V Y O can be plus V dc by 2 or minus V dc by 2, V B O can also be plus V dc by 2 or minus V dc by 2. So, V R Y equals V R O minus V Y O V R N s equal to V R Y minus V B R by 3 assuming 3 phase balanced load. So, this is how these are pole voltages, these are line to line voltages, you similarly you have V Y B and V B R. These are line to neutral voltages operate on the load. First neutral voltages operate on the 3-phase balanced load now.

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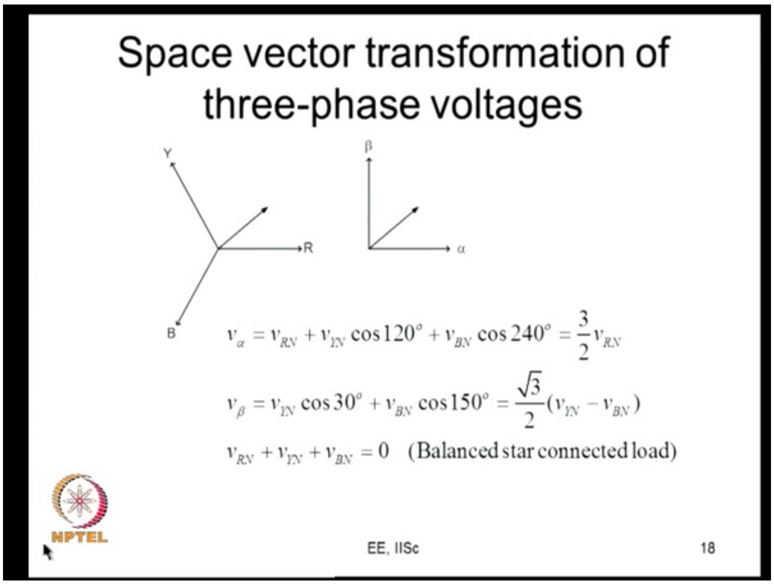

So, you can do a trace space vector transformation. Once you have V R and V Y and V B and which add up to 0 like this, you can have some V alpha which is equal to 3 by 2 tends V R N, and V beta which is equal to root 3 by 2 times V Y N minus V B N. This is what we have seen before.

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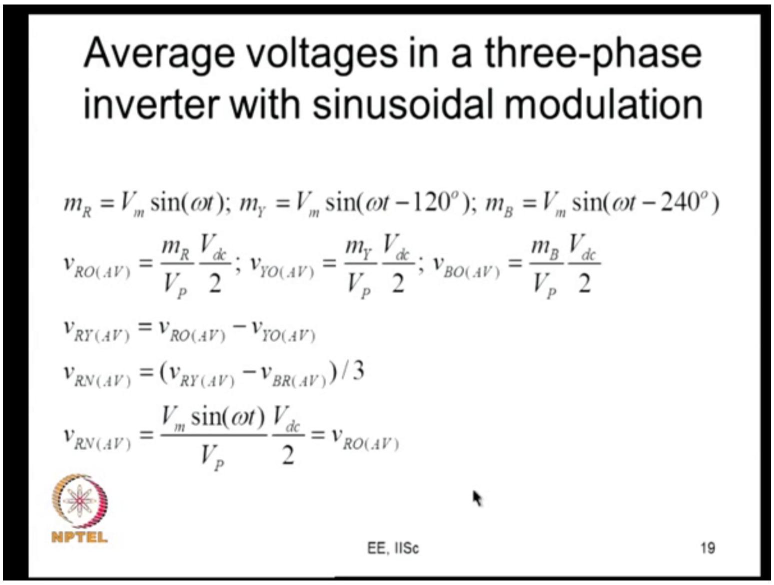

So, next what we can do is; we will start looking at the average quantities, average do over the sub cycles. So now, you have m R. M R is V m sin omega T is sinusoidal modulating signal for R phase, m Y sinusoidal modulating signal for Y phase. Same amplitude, same frequency, phase shifted by 120 degree. M B is the sinusoidal modulating signal for B phase, same amplitude same frequency phase shifted by 240 degrees from m R. So, this is what you have.

Now you are going to compare them with carrier and then produce the signals and do that. So, your V R O average what will be V R O average be it will be m R upon V p times V dc by 2. So, if m R is equal to V p it will be V dc by 2. If m R is equal to minus V p this will be minus V dc by 2. So, this V R O average will be anywhere between minus V dc by 2 and plus V dc by 2, it cannot exceed plus V dc by 2. That is what is going to cause it is change.

Here now even let us say you have m R is equal to V p you will restrict m R to V p when you are doing your linear modulation. In your over modulation is m R will go higher than that, but even if it goes higher than that it will not cross that is what is not exactly shown here, but this is it is been written more in the context of linear modulation. V R O average cannot exceed plus V dc by 2, and it cannot go below minus V dc by 2.

Similarly, V Y O average during linear modulation is m Y by V p times V dc by 2. And V B O average is also m B by V p times V dc by 2. When B phase modulating signal becomes higher than V p, V B O average is clipped to plus V dc by 2. When m B becomes lower than minus V p, then V B O average is clipped to minus V dc by 2. Then what is V R Y average? Whatever is V R O average minus V Y O average; the same way you can define V Y B average and V B R average. And V R N average is one third of V R Y average minus V B R average.

So now, you look at this V R N average. This V R N average will be actually V m sin omega T by V p into V dc by 2, which is equal to V R O average if you are looking at sinusoidal modulating signals. So, if you are using sin triangle PWM and you are in linear modulation then V R N average and V R O average are equal to 1 another. What happens if you have common mode added? This will contain the common mode components and this will not contain the common mode components as we will just see now.

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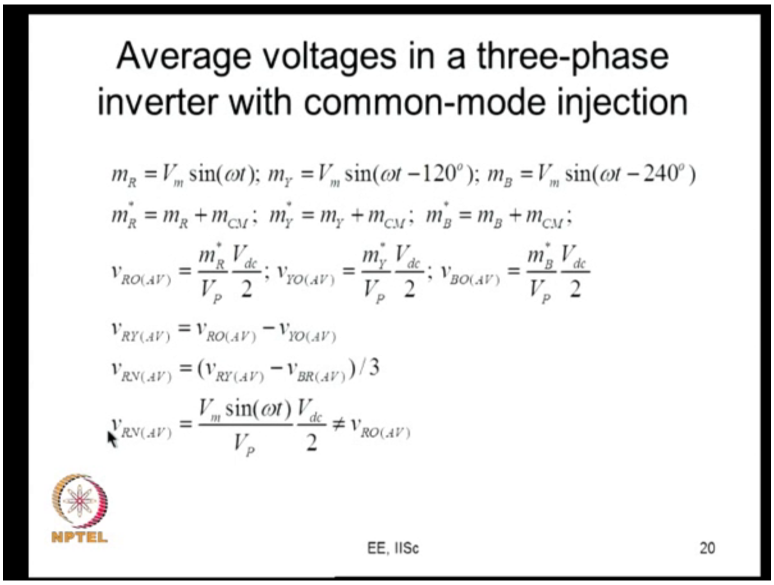

So, this is a case where common mode has been added, 3 phase sinusoidal modulating signals as before, some common mode signal m CM has been added to all these 3. To get m R star m Y star and m B star. Now m R star is compared with a carrier to produce PWM waveforms. Similarly, m Y star and m B star. So, your V R O average will be m R star by V p into V dc by 2, but this is valid only as long as m R star is within plus V p. And minus V p if m R star for some reasons goes beyond plus V p as it happens in over modulation, then this will be equal to V dc by 2.

Similarly, it may m R star might go lower than minus V p, in that case that will get clipped to minus V dc by 2. So, this expression is exactly valid for linear modulation. If it is over modulation then if m R star greater than 1, then or greater than V p V R average is plus V dc by 2, if m R star is less than minus V p it is equal to minus V dc by 2. That is that is a difference it will essentially make. And so, this is your V Y O average which depends on m Y star, and it is basically you need to scale the modulating signal with respect to the carrier peak, and you multiply by V dc by 2. That is what it is. Similarly, it is m B star by V p into V dc by 2. What is your V R Y average it is V R O average minus V Y O average.

Now you see when you are doing this you are actually subtracting m R star minus m Y star, the whole by V p into V dc by 2. When you subtract m R star minus m Y star there is m CM here and there is also m CM here that gets cancelled. So, that gets V R Y average will not contain the common mode components at all, and then when you do your V R N average you will come out to V R Y average minus V B R average whole divided by 3. So, your V R N average would contain only the fundamental component. And the common mode components which were present in V R O average would now be absent. So, they are not exactly equal.

So, if you are looking at linear modulation, V R O average will be an exactly replica of your modulating signal, it is a scaled version of your modulating signal. It would contain the fundamental and the triple m frequency components, whereas V R N average would contain only the fundamental components. So, it will not have the triple m frequency; however, if you are going into over modulation, all these will be non-sinusoidal V R O average will be non-sinusoidal. And therefore, V Y R Y average will also be non-sinusoidal V R N average also will be non-sinusoidal as we will see now shortly.

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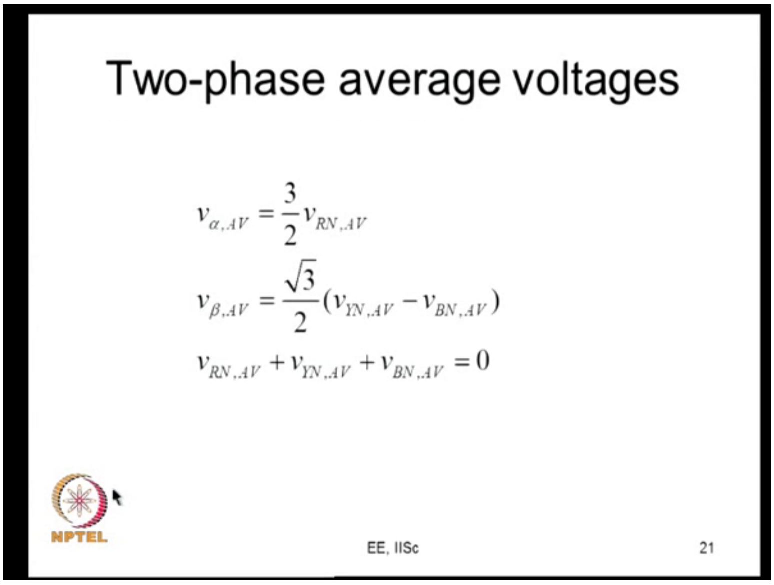

So, once you have these 3 phase average voltages, again this is what is V R N average. It is instantaneous V R N is been averaged over one sub cycle. Similarly, V Y N has been averaged over one sub cycle, V B N has been averaged over one sub cycle. Now you add you know you do will you do your space vector transformation on V R N average V Y N average and V B N average. So, this is your space vector transformation and this gives you what is carolled as V alpha average and V beta average, these are the components of the voltage space vector or the average vector.

So, this is V alpha average and V beta average now. So, you know these are so we will start referring to them as 2 phase voltages. So, when I when we say 2 phase voltages, we have used this term before also, but we have not used this this term in you know number of lectures in between. So, whenever we use a term what we mean is basically the 2 components. The 2 orthogonal components of the average voltage vector along alpha axis and the beta axis. So, these are 2 phase average voltages.

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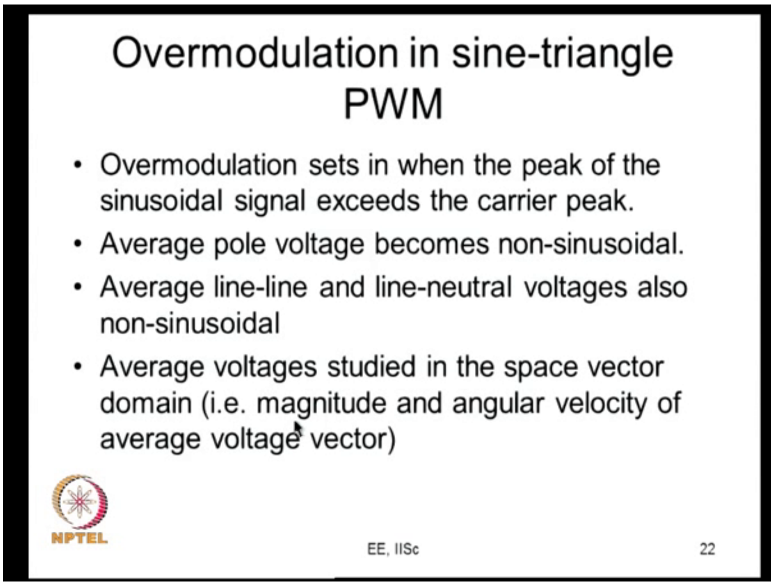

Now, when you go into over modulation sine triangle PWM, what do you mean by this? Let us put our basic understanding in. Over modulation sets in when the peak of the sinusoidal signal exceeds the carrier peak which is obvious which is a starting point. So, what we would expect we would expect the average pole voltage to become non sinusoidal, why, because the modulating signal goes above V peak. Whenever it goes above V peak average pole voltage will be clipped plus V dc by 2. Similarly, when the modulating signal goes below minus V peak this will be clipped minus V dc by 2. So, a clipped sinusoidal wave form, right? A peak clipped sinusoidal wave form is certainly not sinusoidal. It has low frequency harmonics. It will have third fifth 7th and what not. So, you would expect your average pole voltage to be non-sinusoidal.

When average pole voltage is non-sinusoidal, and particularly when the average pole voltage could contain components other than the triple m components. It is not going to contain only third ninth it is going to contain fifth 7th 11th 13th also. In that case the average line to line voltages will also have harmonic components. And when they have harmonic components the line to neutral voltages applied on the lower would also have those harmonic voltages. So, this would be low harmonics. So, you the load will get something like apart from the fundamental, it will have some fifth 7th 11th 13th kind of harmonics applied on to this that is because of over modulation. This has certain things that we can expect it just common sense. Average voltages now what we are going to look at we are going to look at these, look at certain cases particular examples and see how they are varying and get a clearer understanding. And we are going to look at this in this space vector domain.

We are going to see if these kind of a non-sinusoidal average voltages are transformed into this space vector domain, how are they going to look at that is one of the things that we are going to look at today.

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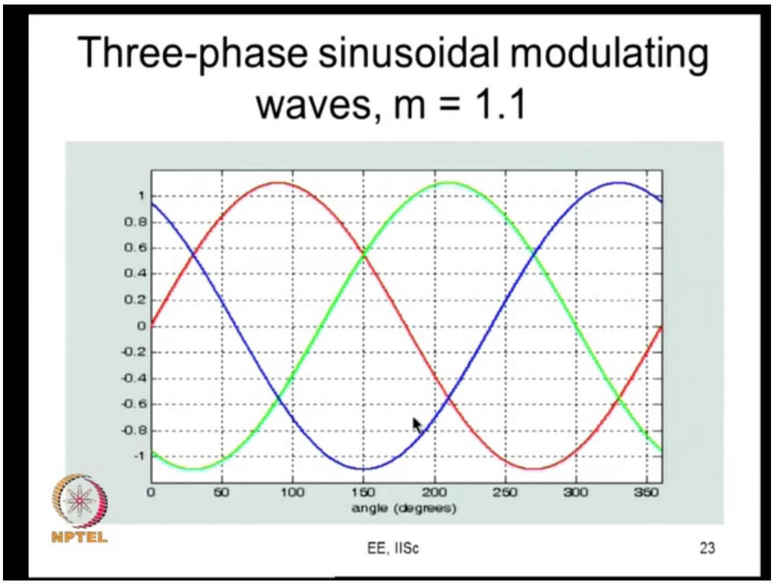

So, let us get inside now. So, this is over modulation, you see I have get m is equal to 1.1 what is m here? Here m stands for the ratio of the peak of the sinusoidal signal to the peak of the carrier. I am taking the peak of the carrier to be a one here and the peak of the sinusoid is 1.1 all right. So, you see that it goes higher than the carrier peak here. Again, it goes below that here. And the same story is to the same thing is true for all these 3 modulating signals. So, this is one example of over modulation. I would say you are not deep into over modulation. You are slightly into over modulation. What you have done is you have just gone above little above that. One is permissible, you have gone little above that.

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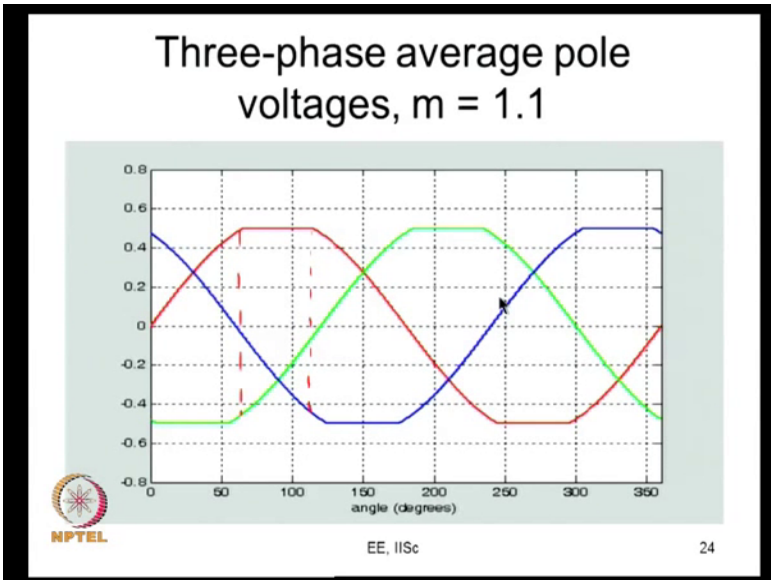

Now, let us see what happens. This is clipped, this is should have been sinusoidal ideally, but this is clipped. Why? That is because it is plus V dc by 2, it cannot exceed plus V dc by 2. Similarly, it is clipped here, why, because minus V dc by 2 is the most negative average pole voltage that can be applied. The bottom device is continuously on, and therefore, your pole voltage is minibus V dc by 2 throughout a sub cycle. In that case the average pole voltage is minus V dc by 2 it cannot be more than that. The same way you can see that R phase it is true about Y phase it is true about B phase.

So, we are going to look at these things. Here our depth of over modulation is not very high I would say. So, let us see this is the region during which R phase is not switching. So, you will see pulses on the R phase PWM wave from here, but you will see no pulses here. The top gate device will be continuously on. Similarly, you look at the R phase PWM pulses, you will see that it is continuously off here. We the top device gating signal will be continuously low the bottom device gating signal will be continuously high. And so, there are pulses here and there are no pulses here, and that is why it is called pulse dropping.

So, this pulse dropping begins and this kind of modulation index 1.1 there will be only one phase which will undergo pulse dropping at a given point of time. If this increases 2 phases for example, here it is pulse dropping for R phase, pulse dropping for B phase starts a little later, then goes on till. Then the pulse dropping for you know the Y phase starts a little later now. So, this is the nature of your 3 phase pole voltages.

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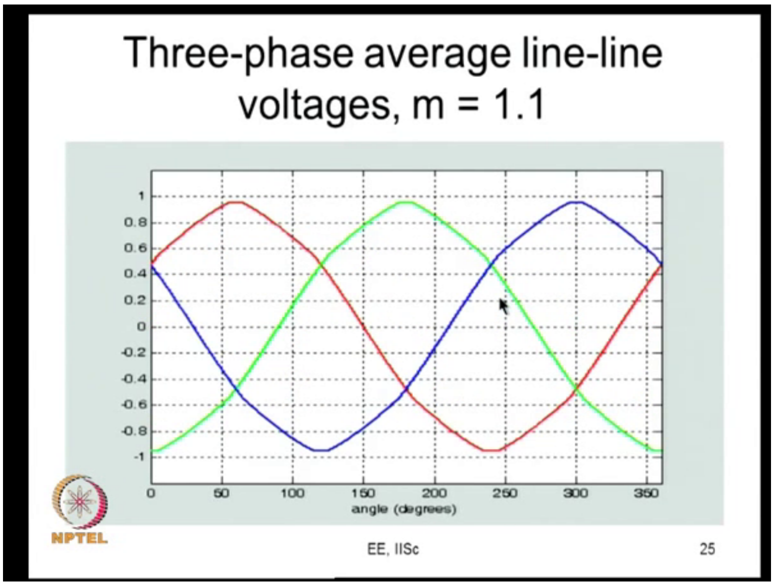

Now, you go about the procedure is clear I have already dealing here at the procedures. So, when I subtract that what happens now? Now you look at these are sinusoidal voltages now. So, what has happened between earlier thing and now, whatever were the harmonic components the triple m once have got knocked off, they have been removed. And all the other components like first to fifth 7th 11th 13th etcetera, they get multiplied by a factor root 3 I am there here.

If you do harmonic analysis of this, you will find fundamental fifth 7th 11th 13th etcetera, and these harmonic amplitudes will be root 3 times the corresponding amplitudes for V R O average and etcetera. So, you have your signal like this. Now you can certainly see that it is non-sinusoidal. It is kind of going and saturating here somewhere here. It is lower than one, one stands for the dc bus voltage V dc. So, it is a little lower than one. So, you know it is it is goes somewhere there and it comes around like this now the same way you have this V R Y average this is V Y B average, and this is V B R average.

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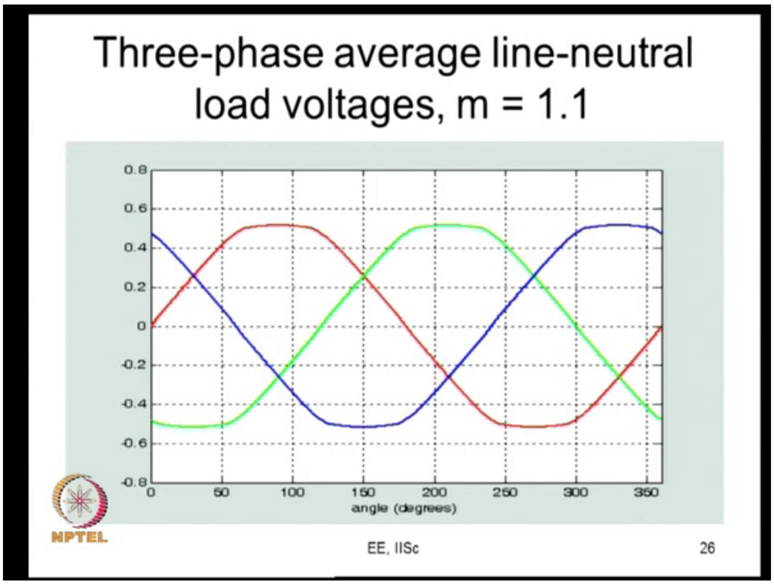

Now, let us go back. So, what is this? This is V R N average. This is the average line to neutral load voltage now. So, this is not exactly clipped. You see that it is slightly going above 0.5. So, this is V R N average. And this is this V Y N average, and this is V B N average. You should have ideally applied sinusoidal voltages here, but instead of in the linear modulation this should have been sinusoidal. Instead of that there is certain amount of harmonic distortion in these wave forms. These are low frequency distortion like fifth 7th 11th 13th kind of components are here, and these are going to cause corresponding harmonic currents, and could cause pulse rating torque and other issues here now.

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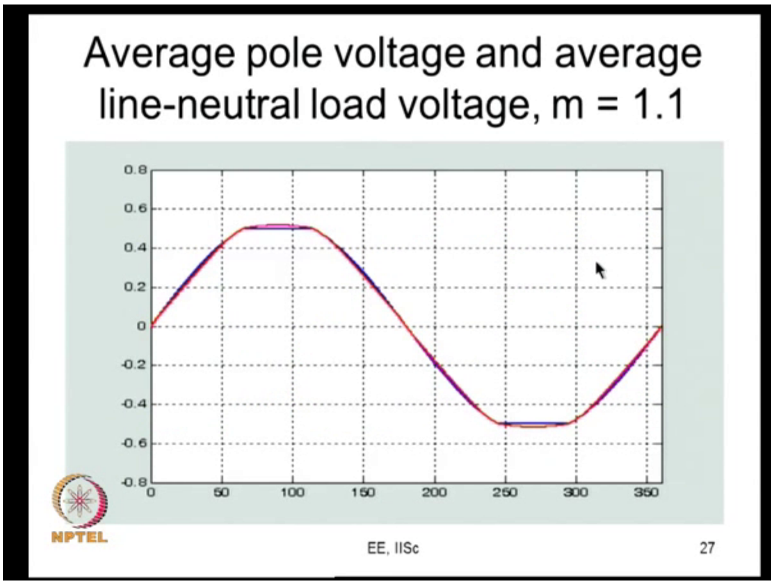

So, if we transform now for example, that V R O average and V R N average are not exactly the same that is what I was trying to tell you. So, this blue wave form is the V R O average, you can see that it is flat here and that is at 0.5 V dc. Whereas, the V R N average slightly goes above that here it goes below this now. So, the difference between the 2 lies in the triple m frequency components some small amount of triple m frequency component which is there in V R O average is not there in V R N average. So, you get that there now.

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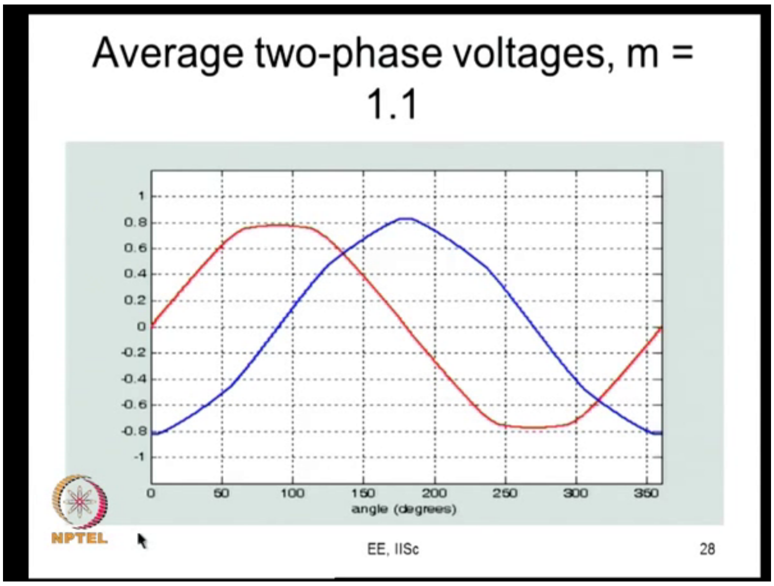

So, we transform those 3 phase voltages, V R N average V Y N average and V B N average to get V alpha average what is this V alpha average. And this is V beta average. Very clearly these are non sinusoidal. And therefore, what happens the magnitude on the angle. So, ideally this should be sinusoidal, and if these are sinusoidal the magnitude of the voltage vector will be constant. And the voltage vector will move at uniform speed.

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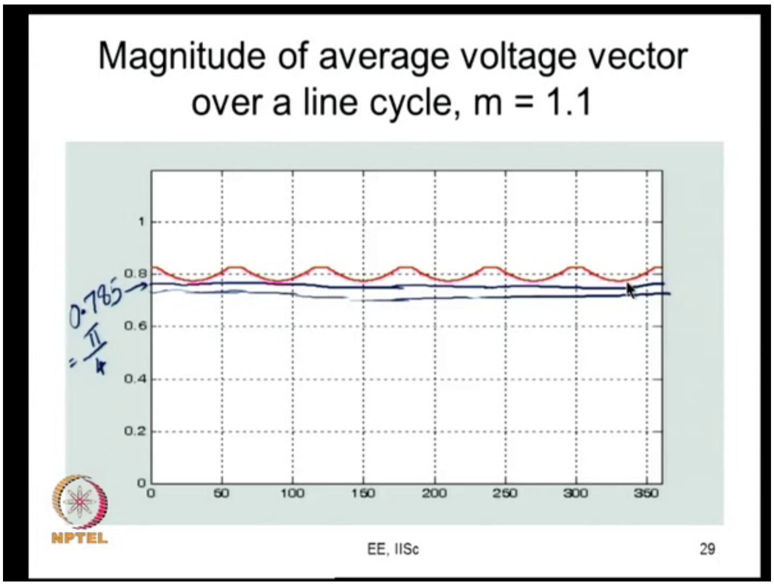

Now, what happens? The magnitude varies like that. Then m is equal to 1. When m is equal to 1, this would have been I will I will take a different colours. So, that it does not mix up with that at m is equal to 1 it is not am not drawing it very clearly it should be really above this. This is how it would have varied. And this would be constant, and it will go to like. So, this is for a m is equal to 1.

So, this one will be actually equal to 0.785, which is pi by 4. So, that would have been your voltage. That would have been the average value of the vector now, but once you cross that what happens the magnitude goes on changing like this. You can see a ripple on that magnitude it is not constant now. And you can see that it is actually 6th harmonic. Every 60 degree it repeats.

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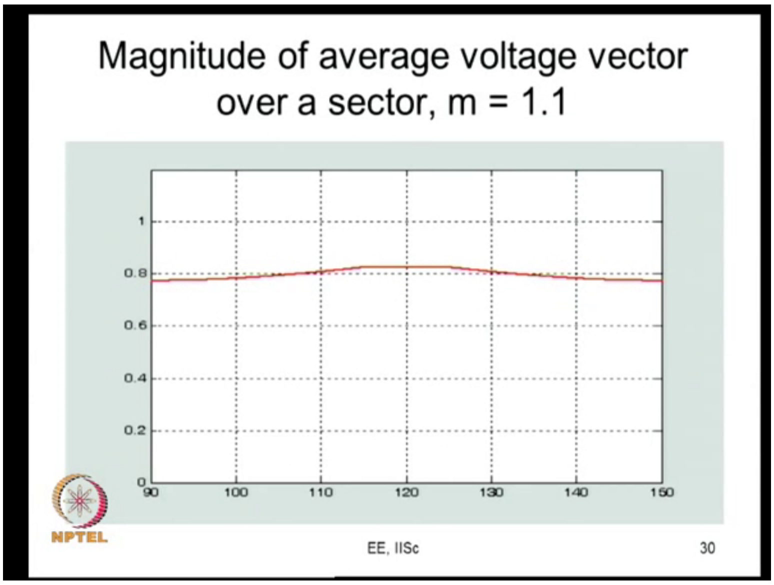

So, it is repeating in every sector. Therefore, you look at only one sector, you can see that this is how it varies this is sector one. Where omega T equals 90 degree stands for the positive peak of R phase voltage, and omega T is equal to 150 degree stands for the negative peak of B phase voltage. So, this sector I am just showing here you can see that the magnitude is no longer constant it is varying with the fundamental angle.

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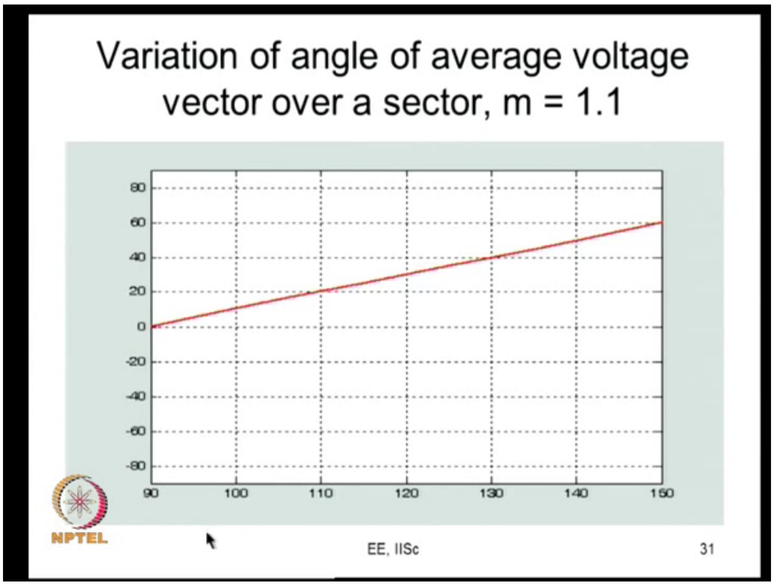

Now, how about the angle? The angle of the vector is more or less linearly with time. So, the angular velocity is more or less constant.

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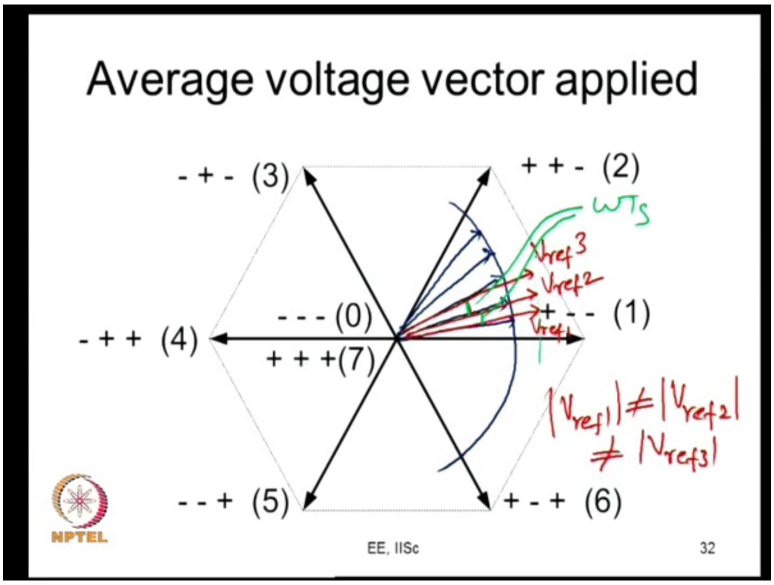

Let us go further. So, what does it really mean? Let us say, if we are looking at linear modulation, this would have been a circle like this. This would have been circle and in the different sub cycles you would have applied vectors like this. Now what is happening? You may have applied some vector like this. And some other vectors like this. Again, some other vectors like this. Let us call this as V ref 1, let us call this as V ref 2. Let us call this as V ref 3.

Now the magnitudes V ref 1 is not equal to the magnitude V ref 2, and that is not equal to magnitude V ref 3. This is because of over modulation. This is what our analysis shows. And the next thing, these angles that you may have between that let me use a different colour to indicate the angle. This angle what is it black threat in colour. This angle and this angle they are they are more or less equal to omega T s. The angles are almost approximately equal to omega T s, also this angle.

So, the vector this magnitude is changing. It is not really sinusoidal it is, it is changing somewhat, but it is still moving at a uniform velocity; that is what it means now.

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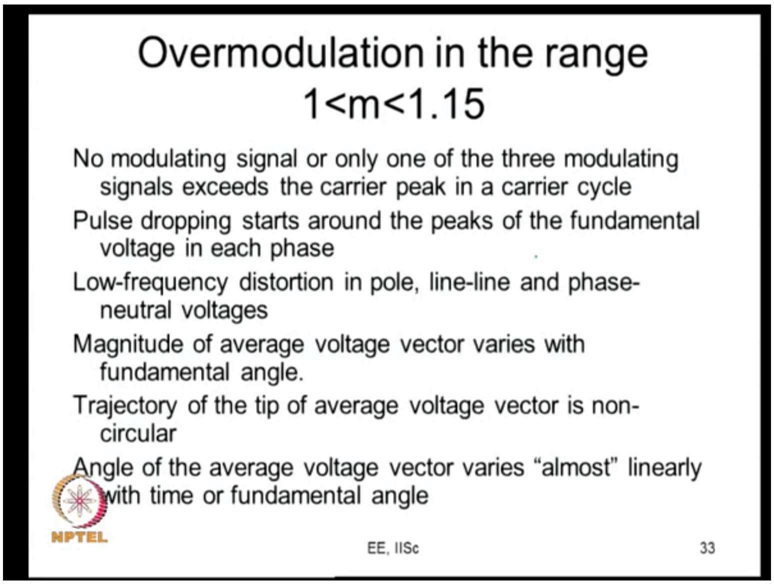

So, let us summarise some of our findings for this region. Whatever we found is true for this region 1 less than m less than 0.15. So, 0.15 is actually 0.2 root 3. That is what I have written as 0.15 now if you are taking a modulation index like this, no modulating signal or only one of the 3 modulating signals would exceed the carrier peak in a given carrier cycle. So, that add this one. So, that one will be clamped the other 2 will continue to switch. Sometimes all the 3 may not, might not exceed and therefore, all the 3 might switch. So, you will see that 3 phases are switching or may be 2 phases are switching.

And if you look at a phase you will see that the pulse dropping has started. And where does it starts it starts around the peaks of the fundamental voltage. That is where the sinusoidal signal goes beyond the carrier. So, the pulse dropping of each phase would start around that, and you will see that pulse dropping now. But the overall pulse dropping duration will be like you know less than 60 degree in length. So, in reverse case could be like from 60 degree to 120 degree for R phase, that will happen when you are look at m is equal to 2 by root 3.

Now, because of this what happens, the V R O average is a peak clipped sinusoid. And therefore, there is some average pole voltage, there is some distortion here. And therefore, the same distortions, but for the triple m frequency components are carried over into the line to line components and into the phase neutral components. Because of this what happens the magnitude of the average voltage vector varies with fundamental angle. It is no longer constant, it varies. Then what happens further? The trajectory of the tip of the average voltage vector is non-circular. I mean, if the if the magnitude has been constant then it is trajectory would be circular.

Now if you look at the tip of the average voltage vector one sub cycle average voltage vector in the next sub cycle and the subsequents of consequent sub cycles, you plot that that trajectory will not be circular, because this magnitude is no longer constant. Then you look at the angle of the average voltage vector it varies almost linearly. That is a small approximation there. So, it is almost linearly with time. You can say it is more or less linear there is there is nothing there is not so much of deviation here, but as you go to higher and higher modulation indices you will see that there is more deviation. And at closed to the very high modulation indices closed to 6th step you will see that, this is no longer really linear, but it becomes closer to piece wise linear.

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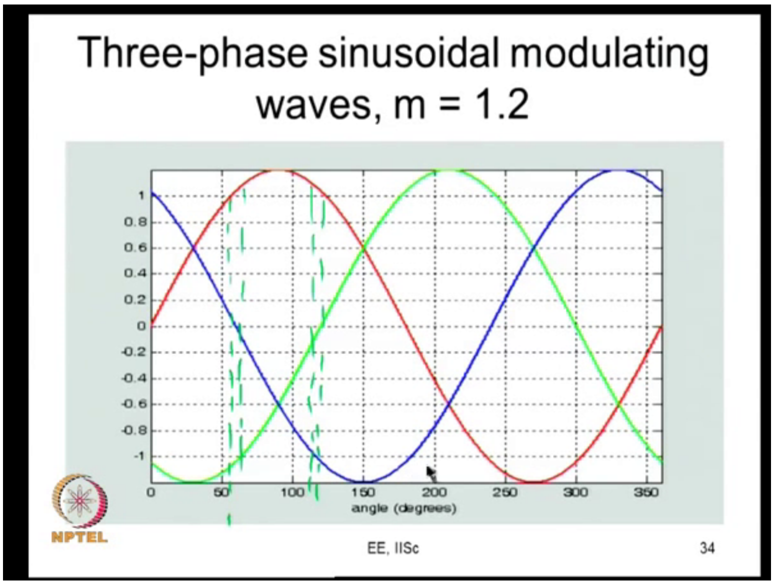

So now I am to going to consider another example of 1.2. So, when I have taken 1.2 what happens? Here R phase goes above the signal peak value let me just draw some lines there so that, so where does R phase go above the peak? So, in this region R phase is above. So, in this region you can see partly this Y phase is also lower than minus 1. So, there is going to be a small region where both of them are going to be clamped, where both of them are going to be clamped.

For example, here, in this region you will see that both of them are going to be clamped the same way here. I have not drawn this very accurately, you will also find a band here. So, during these intervals both the phases are clamped, whereas in this interval only R might be clamped, in this interval both R and Y. Here may be R and B and here only B phase is clamped. So, it goes beyond this now.

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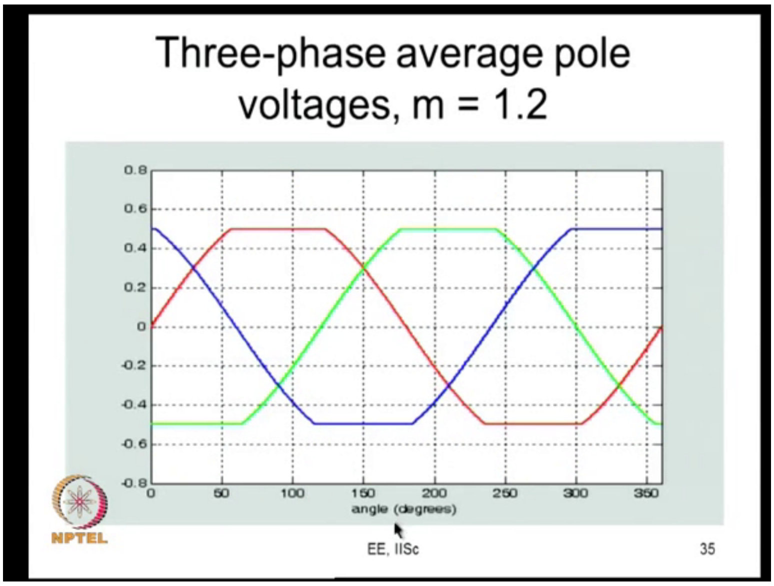

So, what happens to the average pole voltage? Clipped to 0.5 V dc here it is clipped to minus 0.5 V dc. This looks roughly trapezoidal, because you know this is a linear this is a sine waveform, closed to 0 crossing looks more like a straight line. So, it is roughly trapezoidal this is R Y and B, V R O average V Y O average and V B O average now.

So, this has triple m frequency fifth 7th and other harmonics.

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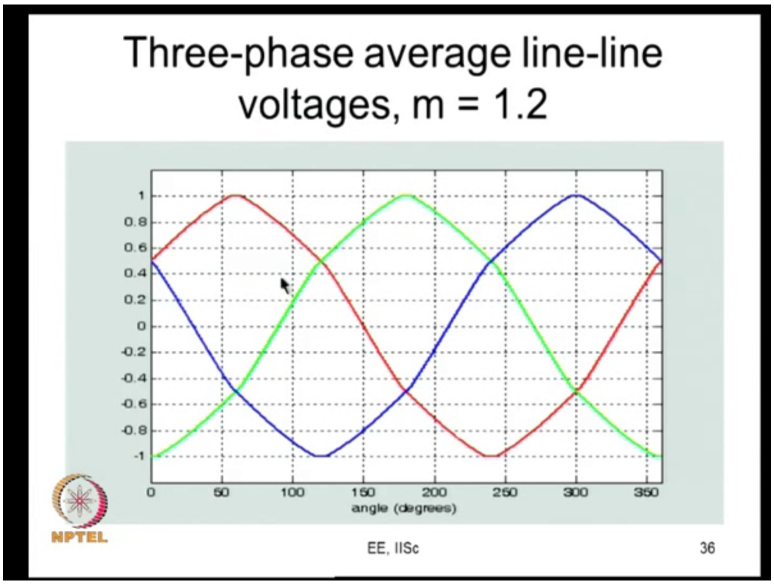

When you do this you know, when you go to the line to line voltages the triple m frequency components are absent. And you can see that these waveforms look more sinusoidal than for example, this waveform. That is because the triple m frequency components have gone, and therefore, it looks more sinusoidal, but you can very clearly see that it is no longer sinusoidal. It is much worse than when you saw that at m is equal to 1.1. If you do a harmonic analysis you will see that fifth 7th 11th 13th etcetera, would have increased in some sense. So, this is V R Y average V Y B average and V B R average now.

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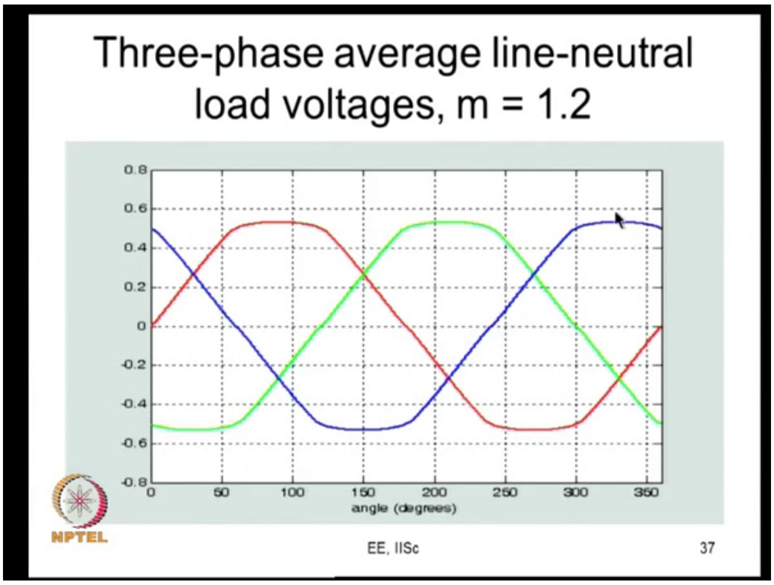

So, from here let me go to V R N average V Y N average and V B N average, you can see that there is a significant amount of low frequency distortion has set and now.

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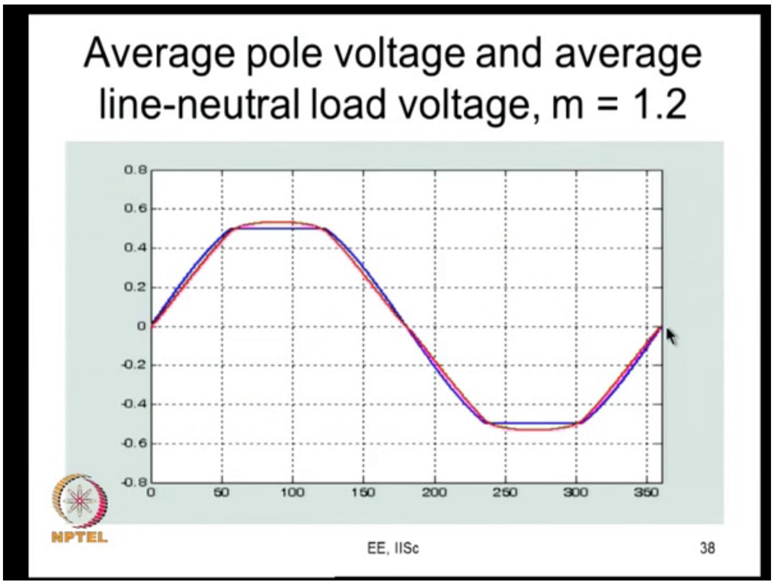

So, here to distinguish between V R O average, this is V R O average and this is V R N average. So, V R N average is a little more rounded and goes slightly above 0.5 V dc, and does not contain triple m frequency components, whereas V R O average is clipped at 0.5 V dc and minus 0.5 V dc and looks more trapezoidal in nature.

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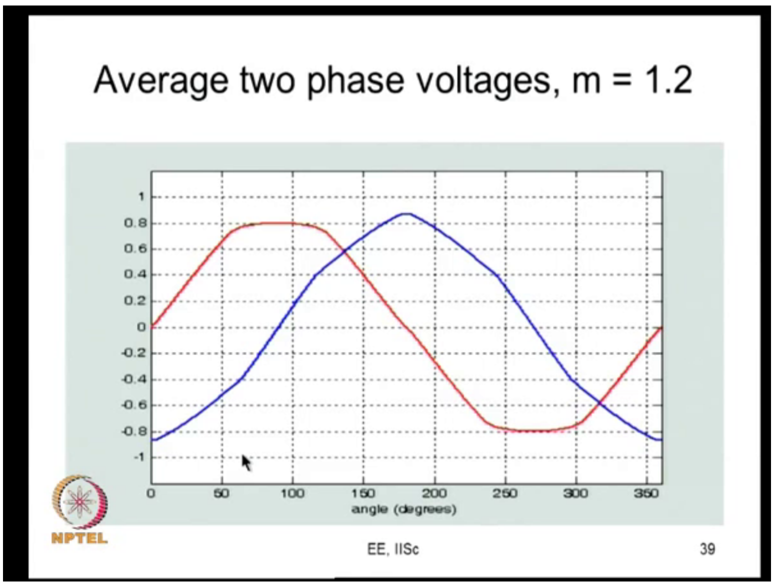

So, when the 3 phase average voltages are transformed into this space vector domain your V alpha averages like this, you can see that V alpha is kind of flattening here. And here also it is flattening here, and you can see is V beta average you can start seeing some kind of straight lines here every 60 degree you can start seeing the straight lines.

So, a little latter this will actually become exactly straight lines, and this will also probably become straight line like this. You will see this becoming a trapezoidal one and you will see this. This is all piece wise linear lines, at a slightly higher modulation index than now.

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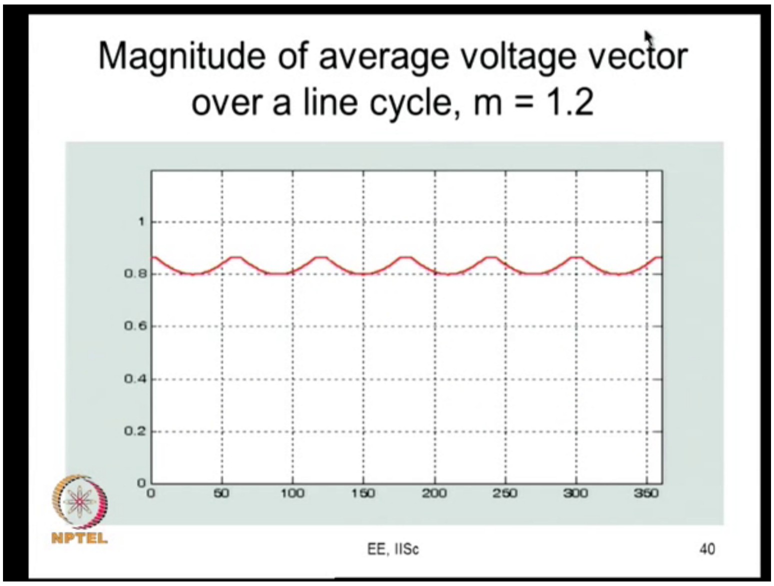

So, if I have considered this V alpha average and V beta average, you will look at the magnitude. What is the magnitude? V alpha average square, plus V beta average square, under root if I do that this is how the variation is. Whereas, the average voltage is somewhere in between that it is some between somehow 0.8, and this is now going like this. So, this is 0.866 to 0.8 it that is the variation here.

If the highest value is right now it should be 0.866, we cannot say that you have to look into that and see this now. So, you get this kind of variation, and you can see that the voltage vector is actually constant here and it kind of varies here.

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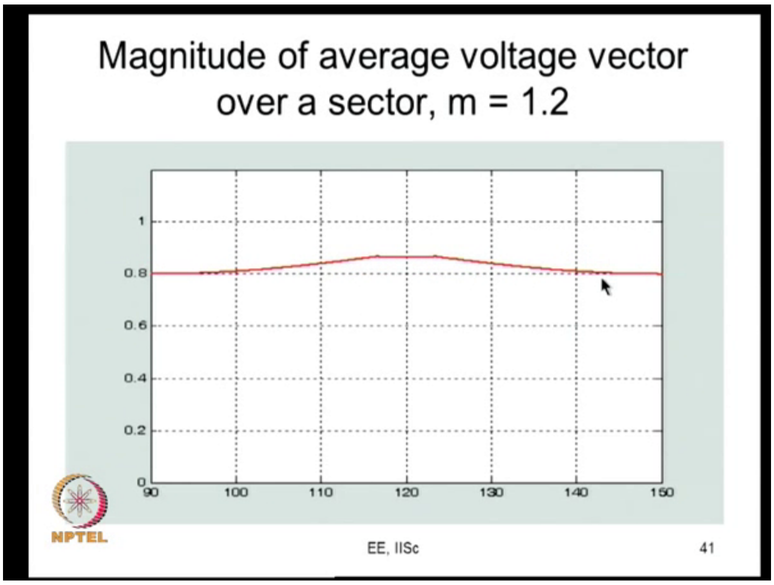

So now this is has is periodic over every sector. So, it is enough we are going to look at one sector. So, we are going to look at this 90 to 150 degrees, which is what we call as sector one. So, this corresponds to the positive peak of R phase, this corresponds to the negative peak of B phase. And here we find that this is how it goes all right. So, this is the magnitude of average voltage vector over a sector now.

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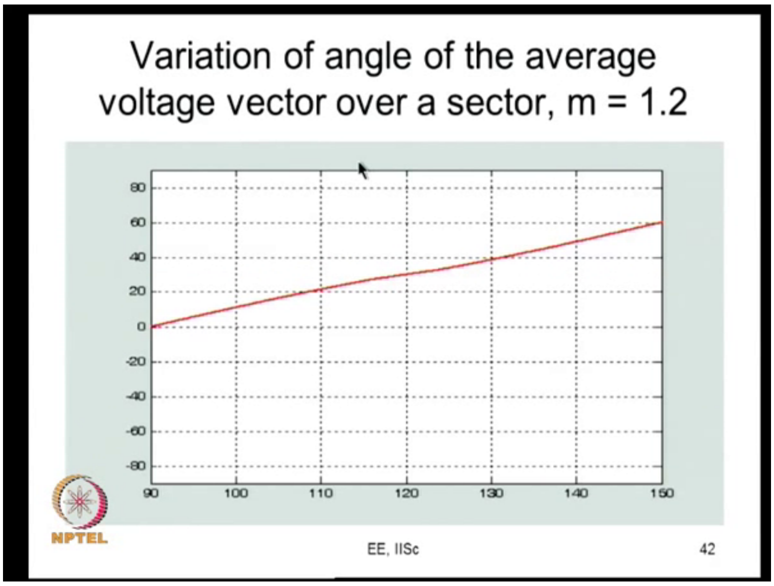

Now, it is no longer constant. How about the angle? Angles still rises reasonably linearly with this thing if you if you plot the constant straight line joining these 0.2s you will see some slight deviation between the 2, but that is not very high it is still more or lesser straight line, and you can say that the average voltage vector is moving at a uniform angular velocity.

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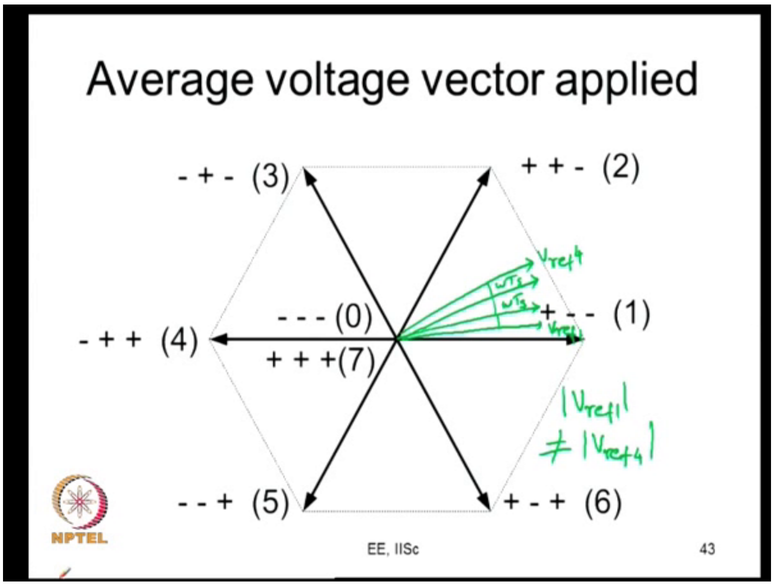

So, if I look at the voltage vectors, what happens now? I might come to slightly higher modulation indices, like this. You can very evidently see that if I call this V ref 1 then I call this V ref 4, their magnitudes are very, very different magnitude of V ref 1 is not equal to magnitude of V ref 4 or V ref 2, whatever the other things that you can really look at now. So, the magnitudes go about changing, but how about the angles? The angles are still these angles are still equal they are equal. How much they are? There that is equal to omega T s. This angle is also roughly equal to omega T s may not exactly be, but it is roughly equal to omega T s. This is how the nature of variation is.

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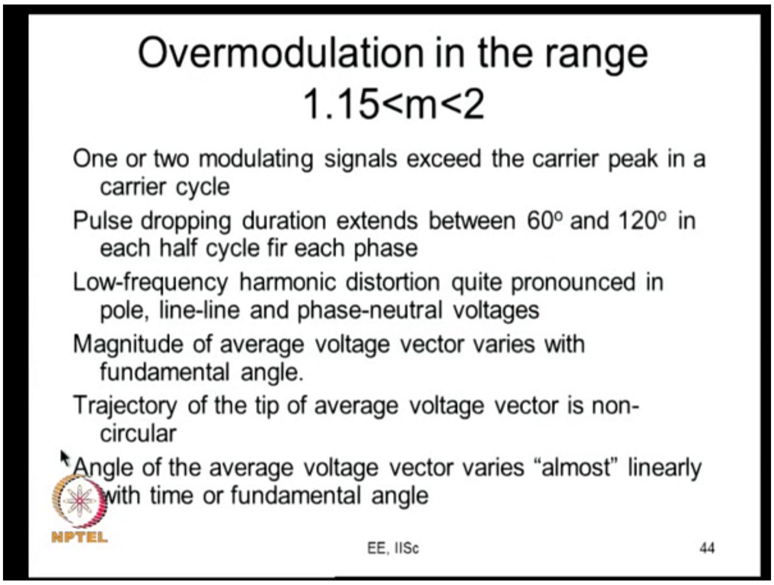

So now I am going to look at this 0.15 m. This m m into 2. So, this is the range now. So, for this range now 1.2 was really representative. What happens is 1 or 2 of the 3 modulating signals they exceed the carrier peak. In a carrier cycle you consider any arbitrary carrier cycle, you will sometimes find one modulating signal exceeds the carrier peak, sometimes you will find 2 modulating signals exceeding the carrier peak now.

So, this pulse dropping is there. So, pulse dropping is for longer than 60-degree duration, but shorter than 120-degree duration. In fact, if it is 0.15 or 2 by root 3 your pulse dropping duration will be 60 degrees for each phase, in each half cycle. If m is equal to 2 you will see that the pulse dropping duration is 120 degree for each phase in each half cycle. This f I R must be f O R you could correct it now. F O R all right. So, low frequency harmonic distortion where is low frequency harmonic distortion, but it is quite pronounced when compared to what you founded m is equal to 1 or in the range one to 0.15, now it is much more pronounced in goes on increasing as m goes from 0.15 to 2.

Now, if you look at the magnitude of the average voltage vector it varies the fundamental varies with fundamental angle, and this variation is also more pronounced than it to ask in the range one to 0.15. Again, the trajectory of the tip of the average voltage because this magnitude is varying the tip of the average voltage vector you know, it is trajectory is no longer circular I mean it is more non-circular than it was before if I can use the term. Then angle of the average voltage vector. However, continuous to be almost linear with time, so it is more still it is more or less equal now.

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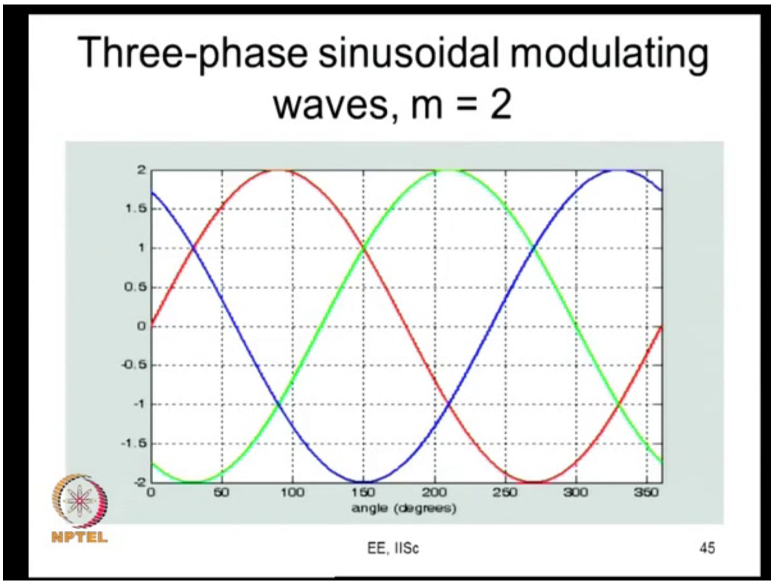

So, let us look at the particular in case of m is equal to 2, where you can see that this is the peak of the carrier you are going twice the peak of the carrier now. So, what is happening is this is 0 to 30 degree, from 30 degree to 150-degree R phase modulating signal is higher than the peak. I mean the triangular peak. And similarly, you can look at here also, it is from 2 10 to 3 30 degrees the same way you have for Y phase and B phase now. So, this 120 degree you will see that for example, you take this 30 to 90-degree R phase exceeds the positive peak of the carrier, the same 30 to 90 degree the Y phase exceeds the negative peak of the carrier. Similarly, here R phase exceeds the positive peak of the carrier here B phase exceeds the negative peak of the carrier. So, 2 phases are clamped concurrently in any carrier cycle which to depends on what is the angle of your fundamental now, so that you can just be evident from this figure.

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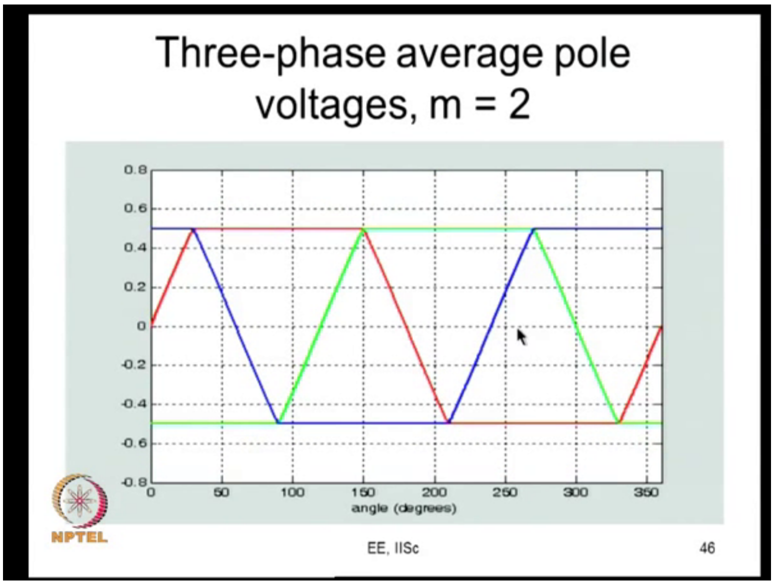

This is how you use here V R O average pole voltage. And now this is your V B O average, and this is your V R O average. They all look almost trapezoidal waveforms. Because this is sinusoidal actually, but sin around this 0 crossing is more or less is a straight line. The sin theta is approximately equal to theta. Therefore, sin x approximately equal to x therefore, you get this kind of thing here now.

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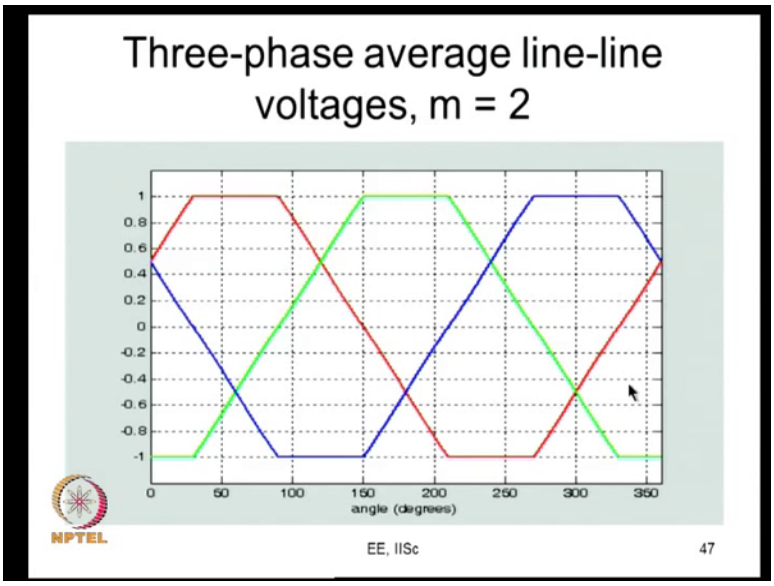

So, when you do this, you get some things this is are the V R Y average V Y B average and V B R average. This look a little more sinusoidal than the previous once, but nevertheless you can see that they are flat. What do you mean by they are flat really? It is V dc. So, during this region this is R Y is equal to V dc V R Y is equal to R phase is always connected to the top device Y phase is always connected to the I mean the bottom switch is always on. So, V R Y is always positive here. And if you take this region for example, R Y take this region what does this is V B R? V B is the lower device is always on the R phase the top device is always on.

And similarly, you look at here V Y B is V dc. So, V Y V phase top device always on, and B phase bottom device is always on during this now. So, you can see that they are started switching lower and lower. So, actually if R phase you know any particular phase for 120 degree it does not switch. It switches only during the 60 degrees around the 0 crossings; the first 30 degree and the last 30 degree in every half cycle.

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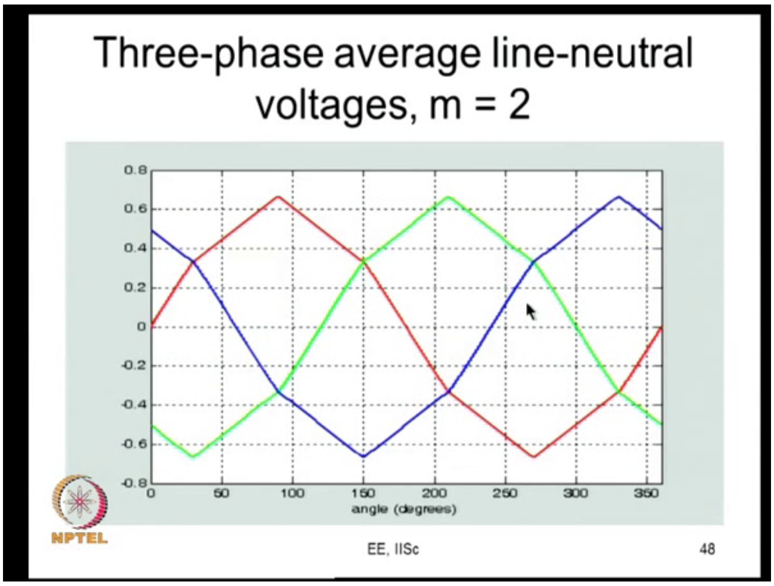

So, you see that there is more distortion, and now these are the V R N waveforms V Y N average and V B N average you can see that they are almost like straight lines; piece wise linear lines.

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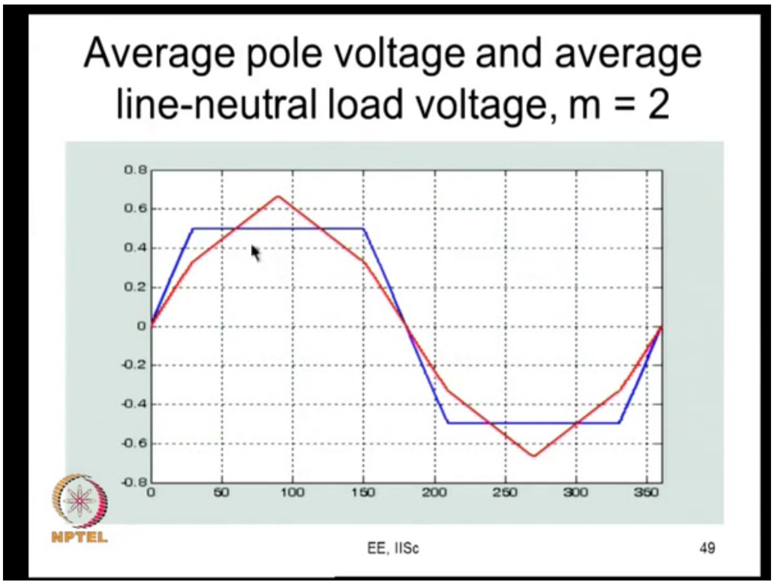

And V R N average and V alpha average is nothing this is the comparison of V R O average and V Y N average. Whereas, V R O average is trapezoidal V R N will average looks like this, it is a piece wise linear line. And the triple m frequency components are absent here; this is what m equals 2.

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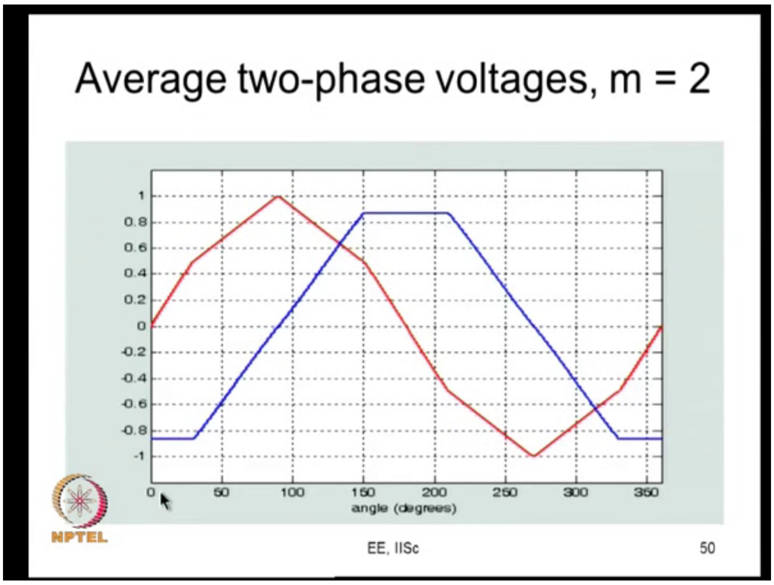

So, this is V alpha average. V alpha average will look very similar to V R N average, why? Because V alpha average is 3 by 2 times V R N average is just one 0.5 times that. And so, you get the wave form here. And this is your V beta average now. So, you can start seeing that you know these are flat and this is a straight line for 100 degree it rises flat for 60 falls for 120 degree linearly again flat for 60. Here what happens it dries a double the slope for 60, at some single slope for 30 degrees I mean for 60 degree again single slope for 60, it falls. Double slope for 60 and again single slope for 60 degree duration. This is how the nature is now.

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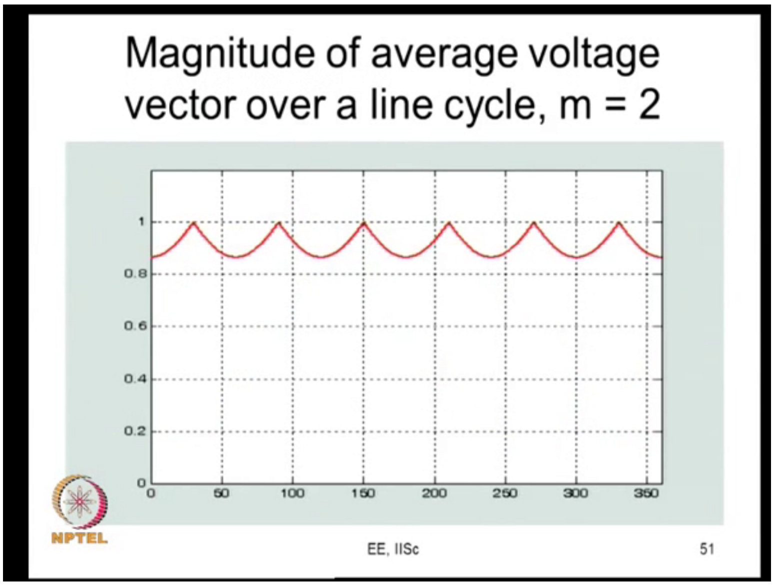

So, if you look at the magnitude, the magnitude is no longer constant, but the magnitude is goes on varying like this. The variation the magnitude is very much pronounced. So, it varies like this.

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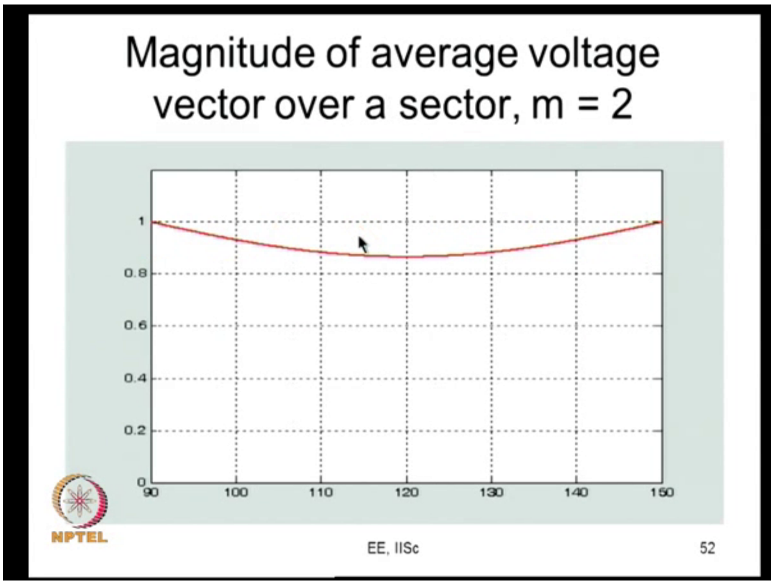

And if you look at just one sector, the magnitude starts from one, and goes and ends here now.

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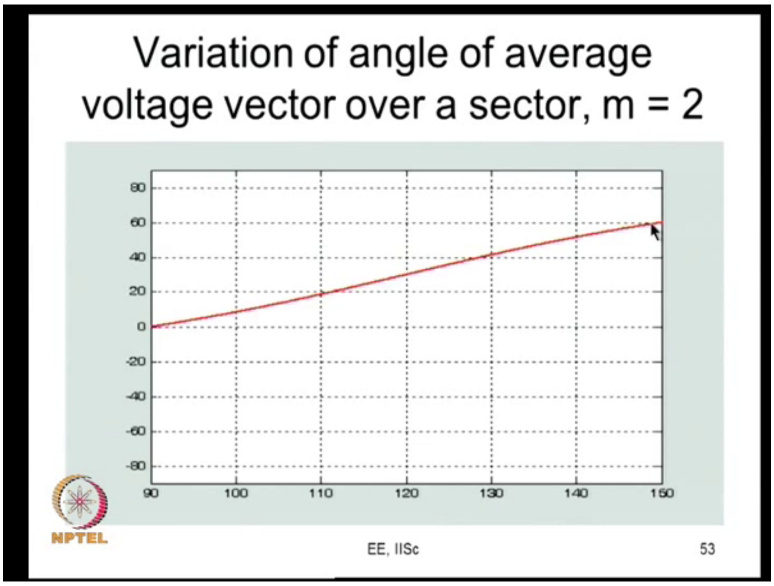

So, what happens now? But if you look at angle alpha, this alpha is varying like this. It is still more or less linear; so I as said before.

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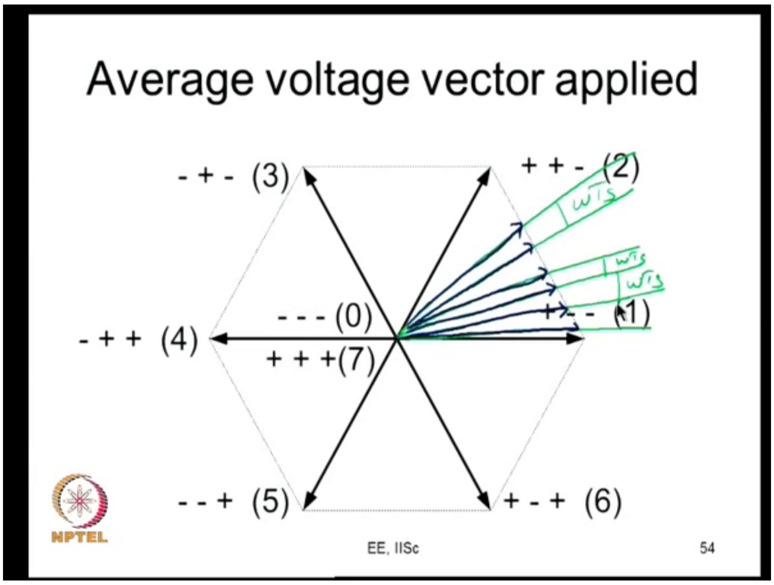

So, what happens? Now how are the vector looking like? If you go to the earlier slide, the angle goes on changing linearly. So, what it does mean it is you know very angular step. Now for example, if I have 1, if I have this is the reference vector in the first one, this is the reference vector and the other one. These are all uniformly phases what it says? The reference vectors are uniformly spaced though I am not able to draw it as neatly.

So, they are they are still the same spacing, with the it is moving at the same thing, but what is the length of this reference vectors; obviously, you cannot be producing reference vectors longer than that. You can very clearly see, what is the magnitude? This is 1, this is 1, and it goes on like this. So, the magnitude of the average voltage vector is really changing like that. What exactly happens? Alpha changes in magnitude is falling like this, what it exactly means is; it stops here.

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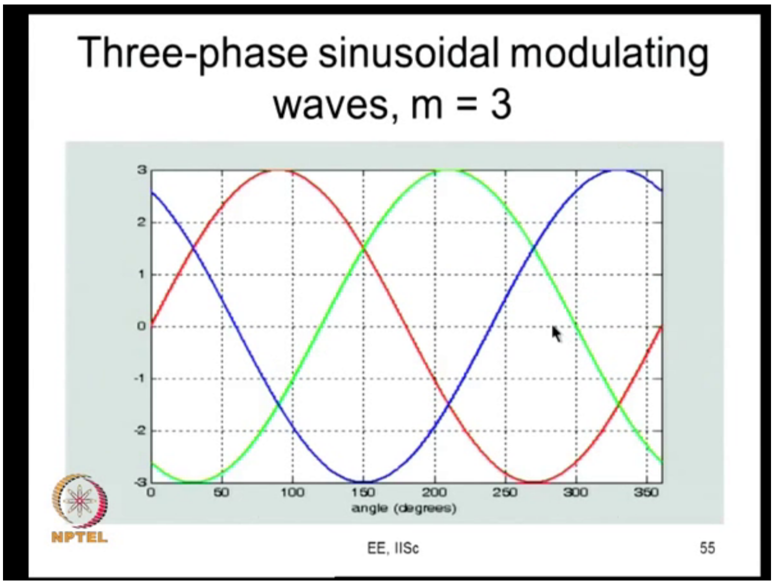

Let me choose a different colour. This is how the vectors are. There is no null vector at all applied, is that clear? How can you say that? You can see that, only one phase is switching. Which phase switches?

Now, this is 90 to 150 degrees right. So, the only phase that switches will be Y. R is not switching and B is not switching. If you want you can go back here, and look at this 90 to 150 degree, let us look at the average pole voltages. This is 90 to 150 R phase is not switching. Again 90 to 150 B phase is not switching. Who is switching? Only Y phase is switching. And therefore, the voltage has been shift switching between this active vector plus minus minus and plus plus minus. So, the resultant active vector it is tip will always lie on this. If T 1 is applied for longer durations it will be closer here. If T 2 is applied for longer duration it will be closer here. This is what happens now.

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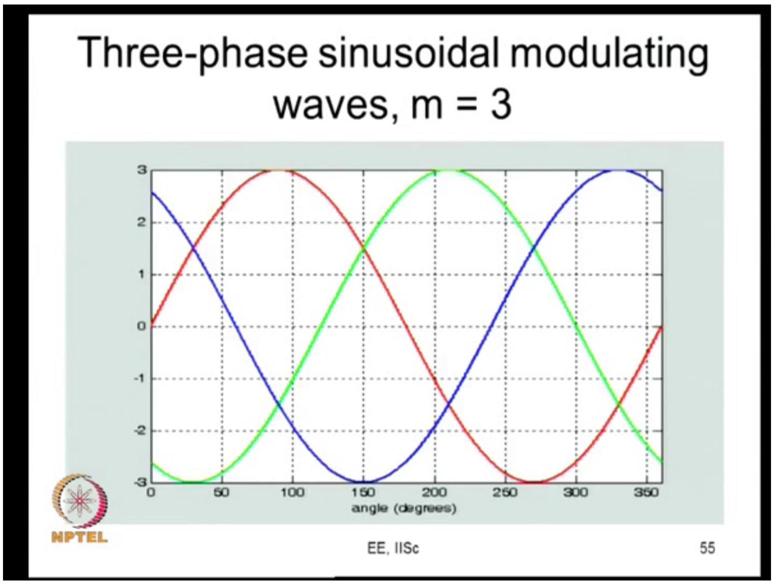

The next interesting variation is if you go to m is equal to 3 beyond 2 what happens? There is it is clipped for longer time, not 120 degree. It is longer than 120 degree.

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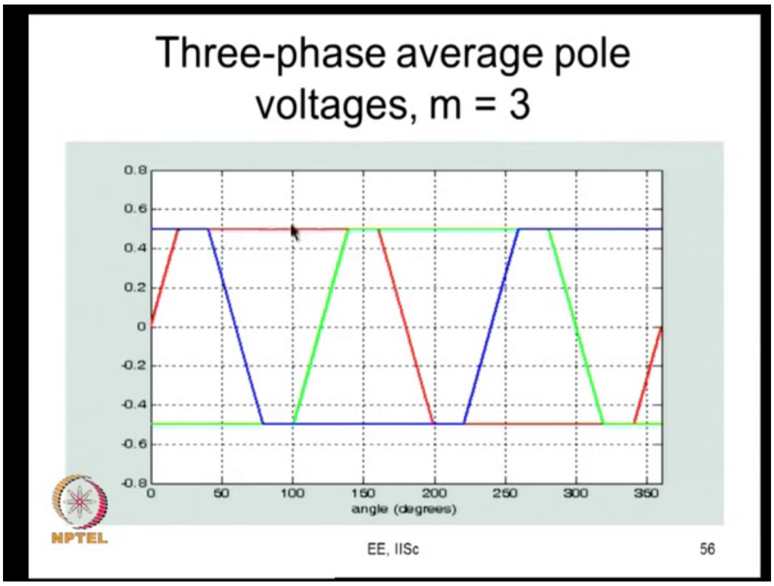

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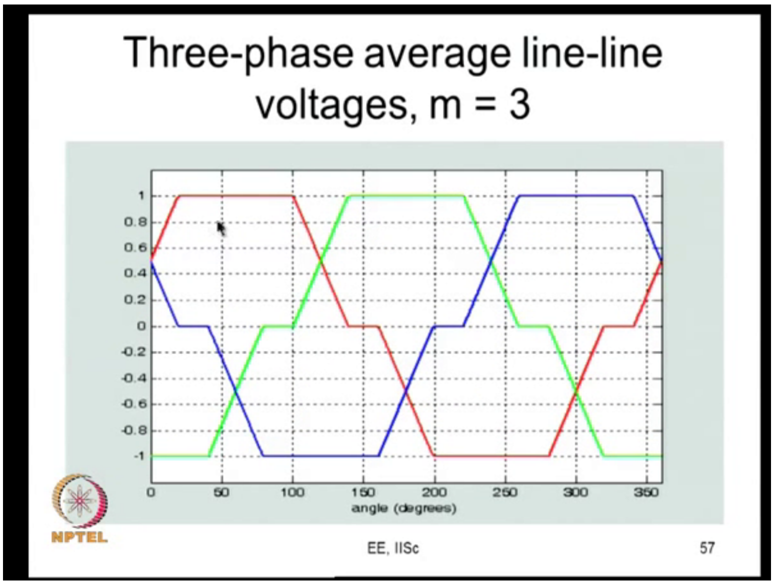

So, you get your average line to line voltages varying like this.

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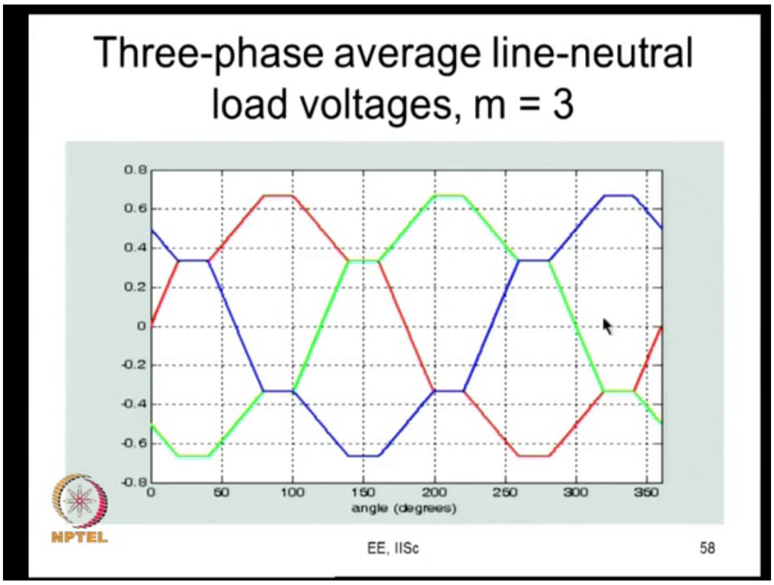

And if you look at your V alpha average and V beta average, they go about doing like this.

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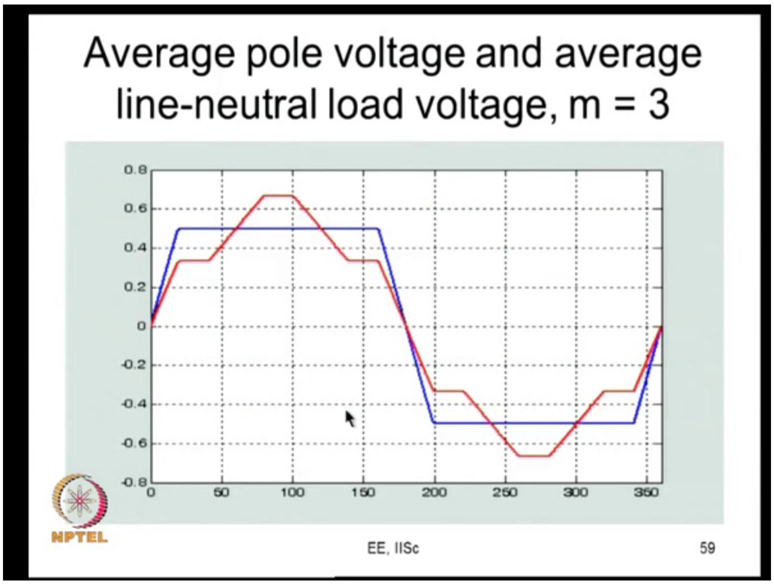

So, this is your V R O average, and V R N V R O average and V R N average. So, you can see that it is it is here clamped to some peak values and here it is clamped to peak values. And this is actually 0.5 V dc. This is 2 by 3 V dc.

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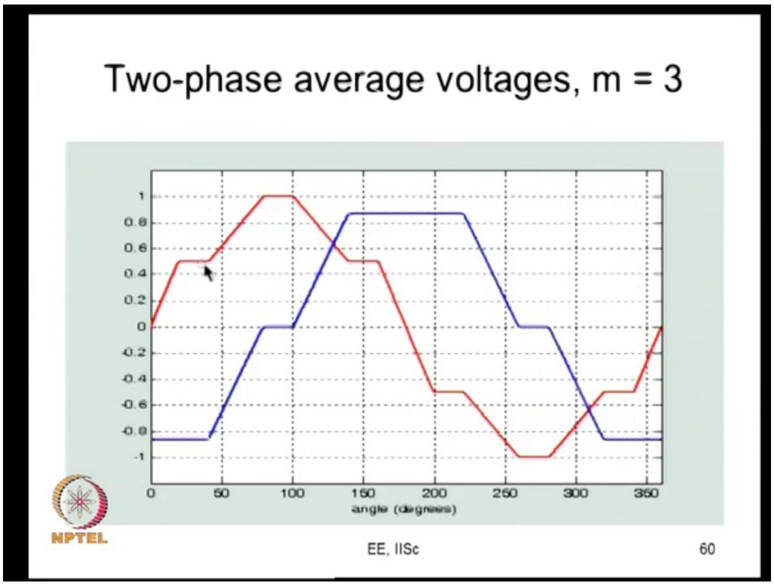

This is V dc by 3 this is also V dc by 3. So, if you look at the magnitude, the magnitudes are 1.

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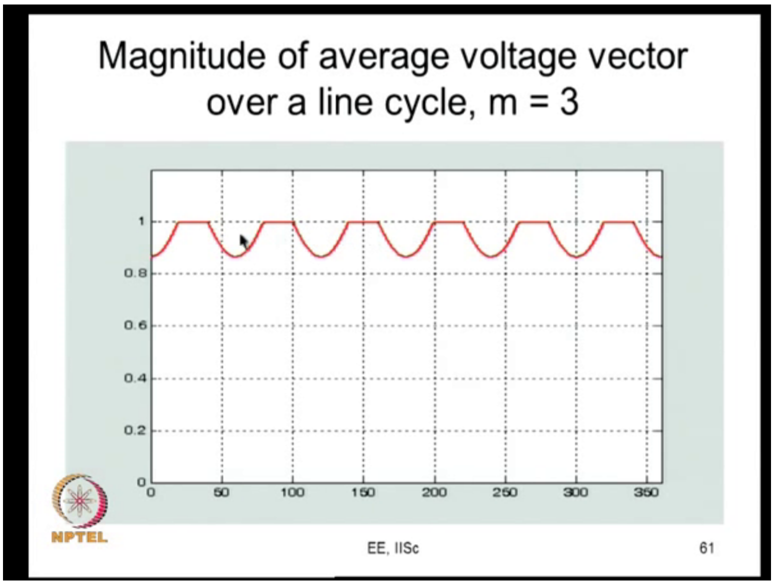

If what do you mean by magnitude being 1, an active vector is actually being applied. Here it is another active vector is being applied.

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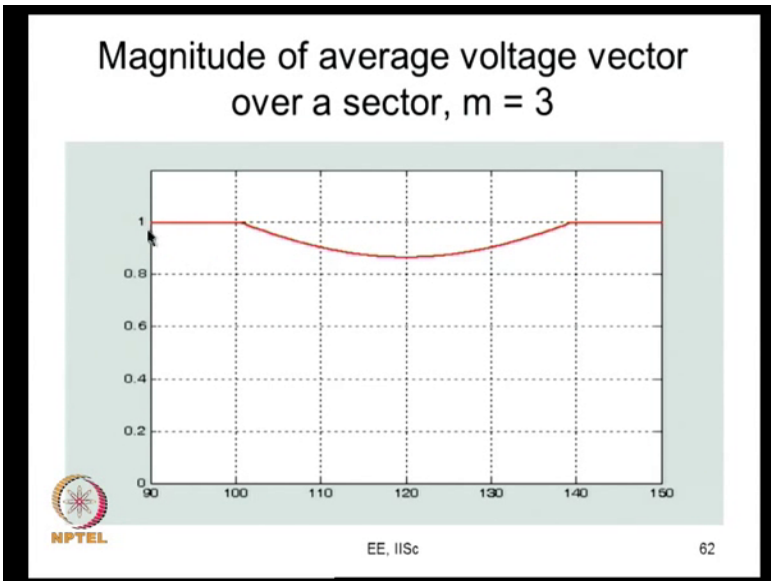

So, if you look at just one sector, up to this it is active vector one is been applied. And then here active vector 2 is applied. And what is happening in between? There is switching between active vector 1 and active vector 2. So, in all these sub cycles we switch for active vector 1 and 2 only. Null vector is never applied.

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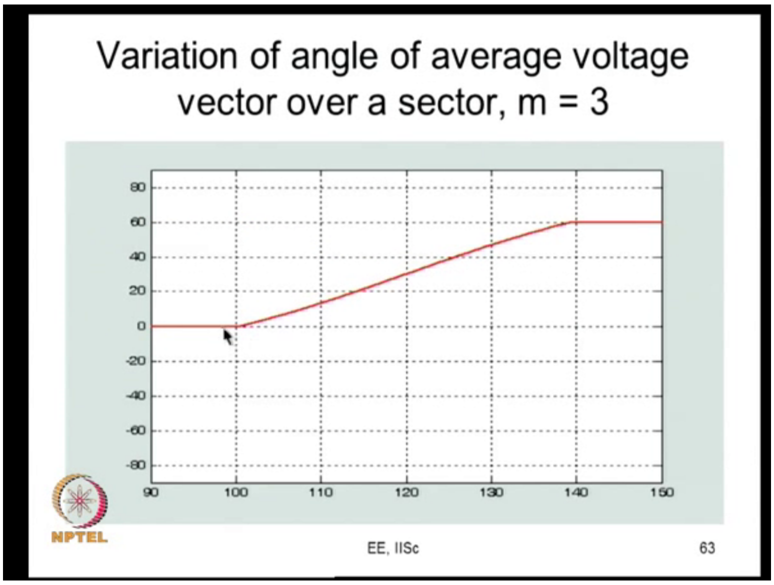

And so, but if you look at the angle, because active vector one alone is applied here, it is not moving at all it is 0. Here active vector 2 alone is applied it is not moving, in between it moves more or less like a straight line not exactly. But this can be approximated as a piece wise straight line.

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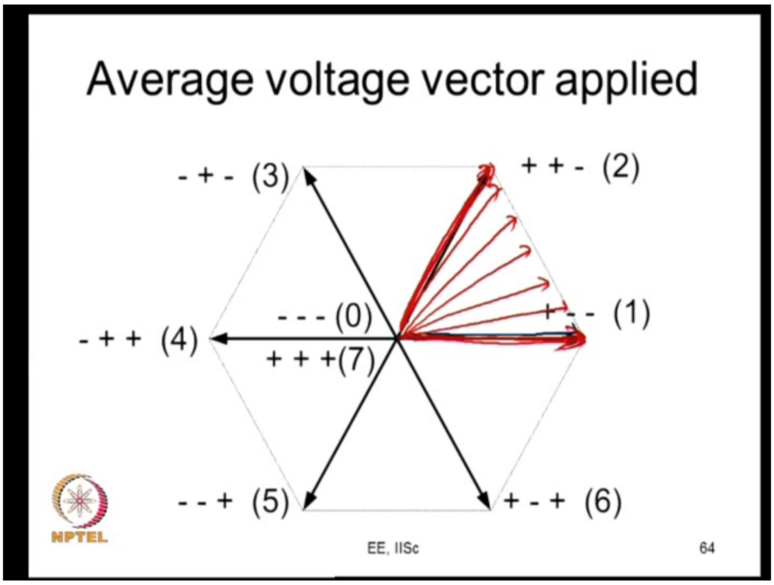

Therefore, when you go close to that the m range greater than 2, this PWM what it does is; you it moves little like, first it is here for longer time. Let me choose a red colour. So, it stays here for a number of then it moves like this. Then it moves like this. Then finally, it goes and stays here it stays here. That is how the average voltage vectors are moving.

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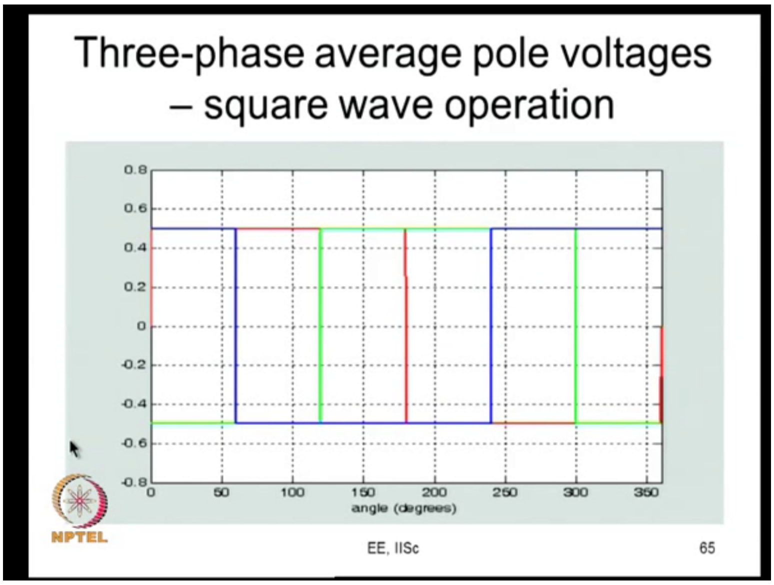

So, this is very important for us to understand, how it is going to operate in the you know square, I mean in the voltage space vector plane.

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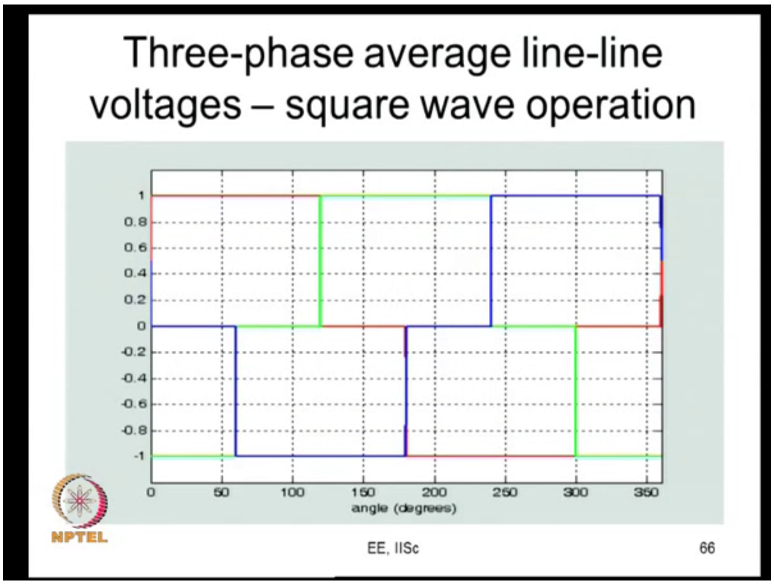

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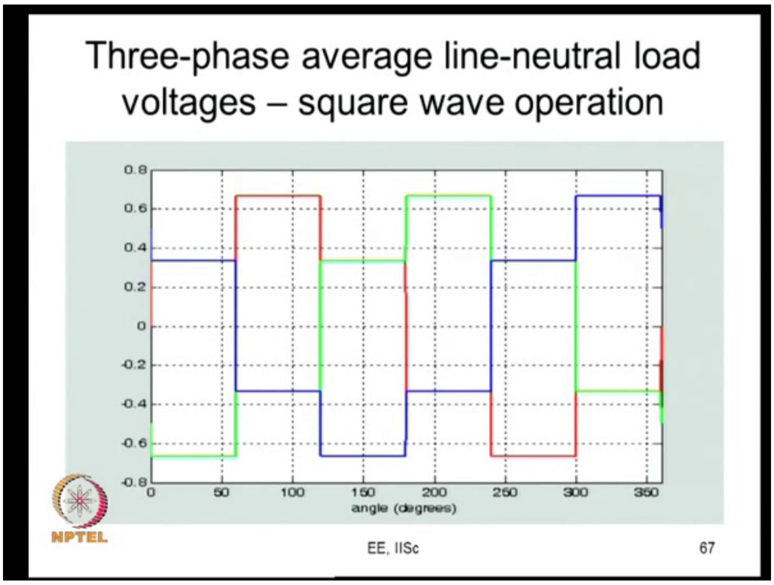

So, this is I have illustrated the average line to line voltages square wave mode. So, you can see that it is average pole voltage for square wave. And these are the line to line voltages. And these are the 2 phase average voltages.

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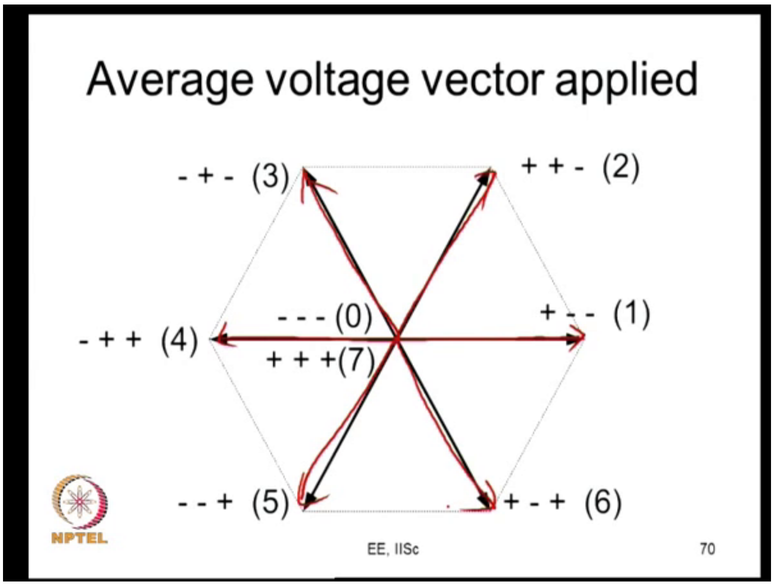

In the case of square wave what exactly happens is; only one vector is applied, this vector is applied for entire 60-degree duration. This vector for another 60, this vector for another 60, this is for another 60, this is for another 60, this is for another 60 that is square wave operation now.

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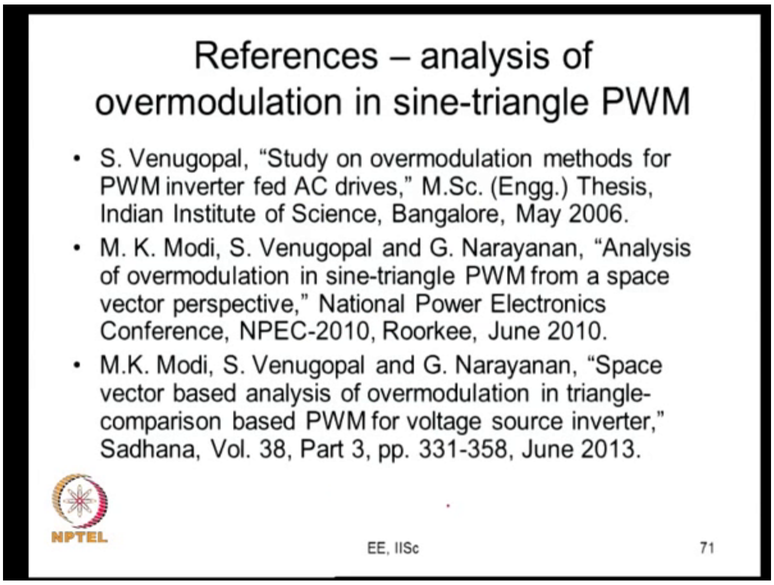

And these are good references for you. Firstly, especially these 3 references, all these 3 references deal with the analysis that we presented today. So, these are chapters in the thesis and there is a conference paper and this is later journal version of this paper. So, I would suggest this as your references.

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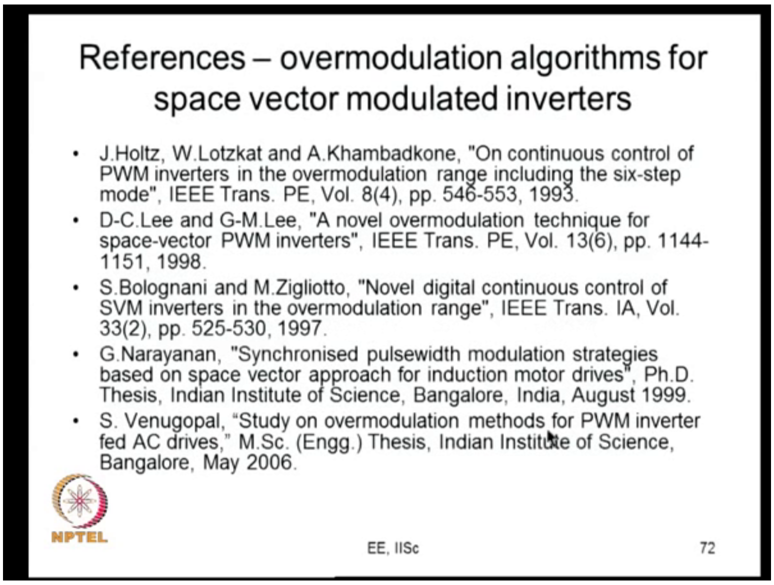

There are other references for the next part of our lecture, which I would discuss about in the next lecture. So, thank you for your interest here.

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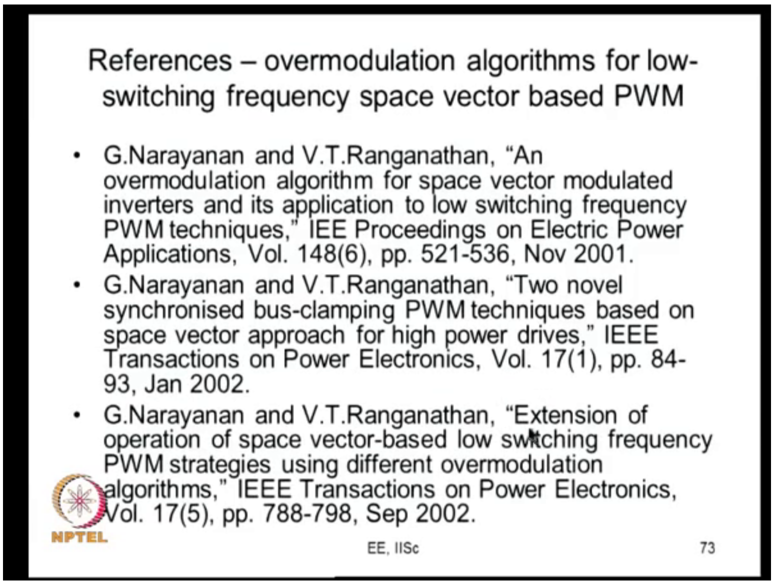

And I hope this was useful to you. And hope to have you again in my lectures.

Thank you very much, bye.