Pulsewidth Modulation for Power Electronic Converters
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Lecture - 29
Analysis of torque ripple in induction motor drives – I

Welcome to this lecture series on Pulsewidth Modulation for Power Electronic Converters, where we have been dealing with the pulsewidth modulation for different kinds of DC-AC converters essential.

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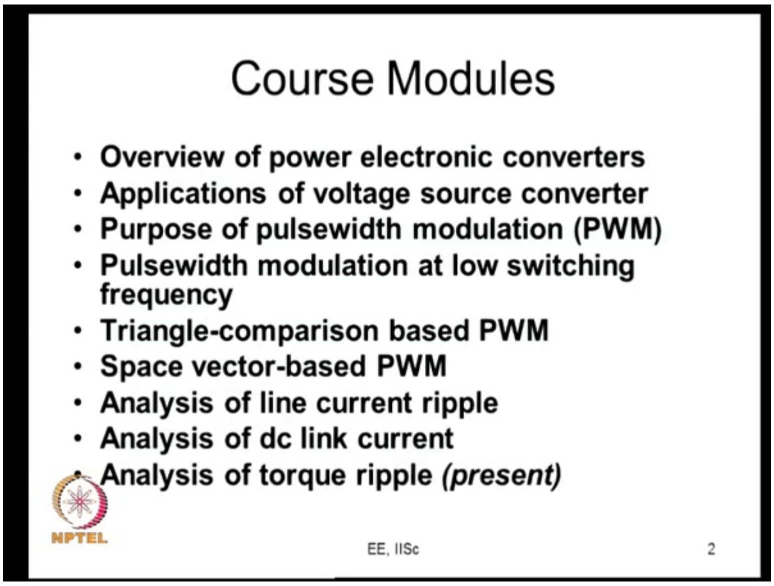

So, we have heard this various course modules, which are like overview of power electronic converters and applications of voltage source converters and so on and so forth. So, we after that you know we look at the purpose of pulsewidth modulation the law switching frequency PWM how to produce it at low frequency. This is something we look at again today in the context of pulsating torque, and in the next two things we look at the PWM generation at the higher frequency, I mean when the switching frequency of the inviter has significantly higher than the fundamental frequency, you generated by comparing three-phase triangular careers.

I mean three-phase modulated signals against a triangle career or you go by the space better approach where the voltage vector is provided to you as a revolving voltage vector and sample that and you tried to synthesize an average vector equal to the reference vector in every sub cycle (Refer Time: 01:23) doing it now. So, these two modules are essentially on the generation at the highest switching frequency side and a from now onwards we have started looking at the waveform quality aspects. For example, this line current ripple one aspect of waveform quality, you want to produce sinusoidal current. So, you want a apply a sinusoidal output voltage. However, your inverter has harmonic also the harmonic leads to harmonic current. And therefore, you have certain current ripple which is actually the sum of all the harmonic currents.

So, we try to analyze this line current ripple, we mostly did it in the space between domain and they we would you know we try to get some measure for the r m s ripple current, that is flowing to the line then in the subsequent module to look at DC side, we look at the DC link current this is a last couple of lectures we did this now. How is the DC link current and then you know to understand the nature of the current that flows the DC capacitors also. So, once again we come back to analysis of torque ripple which again on the way from quality aspect now, this is kind of closely related to the line current ripple.

So, we hear you are torque worried about the harmonic currents produced by harmonic voltages; here we are worried about the harmonic torques produced harmonics now. So, you have harmonic voltages, the harmonic voltages produce harmonic current as well as hormones fluxes. So, because of that essentially start getting pulsating torque. If only fundamental current were flowing through that and their only fundamental flux in the machine, if there will be a fixed torque there will be no ripple. So, because of harmonics in the fluxes and currents there actually torque ripple.

So, today we will start with that now. So, analyze torque ripple, we will look at two situations primarily that is one is we look at low switching frequency that is when really the pulsating torque pretty much height and we will also try to extend it at the high frequency side and will extend the analysis of the line current ripple to the analysis of torque ripple. So, this lecture that in today’s lecture we will predominately focus on the low frequency aspect and in the coming lecture will focus on the higher switching frequency case, and we will do some analysis similar to what we did for the line current ripple and go for torque ripple.

So, for now we are now going to look at low switching frequency PWM and analyzing pulsating torque now.

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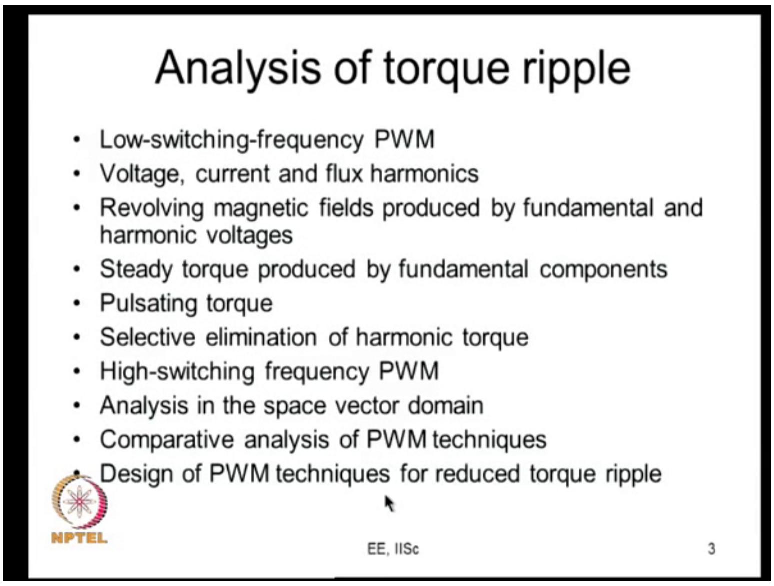

So, that is why I said that there is going to be low switching frequency PWM, you might have a quick review on the this necessary and then we look at the voltage current and flux harmonics when your feeding an inverter, you know not just a sinusoidal voltage, but also the harmonic voltages. The harmonic voltages are going to generate harmonic current in any load when in a motor, harmonic voltage are also going to lead to harmonic fluxes. So, you first take a look at those harmonic current and harmonic fluxes will develop some idea on what kind of revolving magnetic fields they produced that is what the fundamental produces and the harmonic produces.

See you should remember that the fundamental component you know there are three-phase fundamental component there are three-phase sinusoidal, they produce a revolving magnetic field. You take any other harmonic voltage also like the 5th harmonic voltage 7th harmonic voltage, that is also a three-phase balance sinusoidal voltage, but it is just at a different frequency and it might have a low amplitude. So, it will also produce revolving magnetic field. So, that is something will try to get an idea about.

So, an idea about the harmonic means the revolving magnetic field produced by Harmonic voltages. So, then we would try to understand the steady state torque produce by fundamental components like the fundamental fluxe and the fundamental current alone and we look at the pulsating torque, which normally comes from by the interaction of the fundamental fluxe on the harmonic current and the fundamental current are the harmonic fluxes.

And then we would also probably try to say you know try to do how can do a selective elimination of harmonics torque, you may recall when we did this low switching frequency PWM earlier, they looked at what called as selective harmonic elimination. What we try to do is we try to produce a PWM voltage waveform, which gave us the necessary fundamental voltage, but eliminate a specific harmonic voltage we call that as specific har elimination of harmonic voltage or specific I mean selective harmonic elimination.

Now what we do is, we will try see how we can selectively eliminate an harmonic torque will get some idea on how to do about it now. Then in the subsequent lecture most probably we will move on to this high switching frequency PWM and we will try to do analysis of a pulsating torque in the space vector domain, and may be have a comparative analysis of different PWM techniques and possibly design a space vector based PWM techniques for reducing torque ripple. So, might recollect that in space vector method you know we had apart from regular accesses conventional space vector PWM and bus clamping PWM, we also had some advanced bus clamping PWM methods some of them can actually contribute to reducing the torque ripple that something that we could see that next class.

So, now, I would call this is analysis of torque ripple in induction motor zone with particular focus on the low switching frequency.

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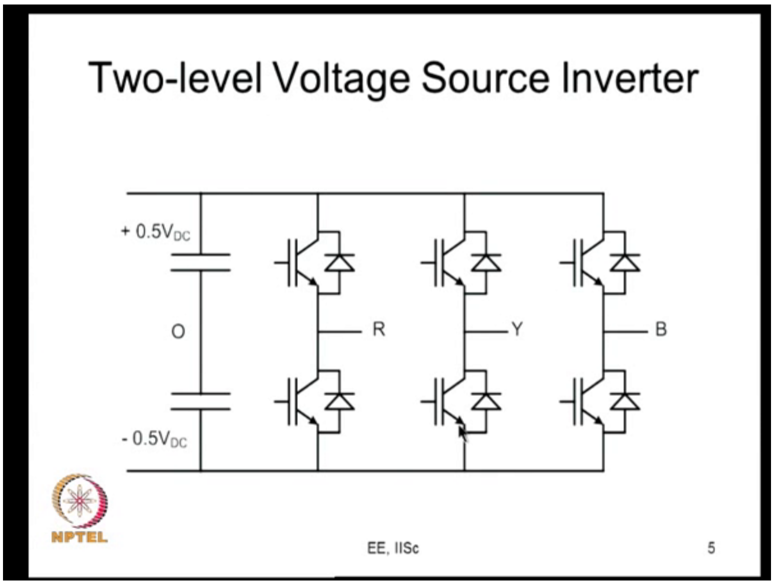

So, this is the voltage source inverter which we have been seen over the last several lectures and that the motor. Now in this case we are now clearly looking at a motor been connected these are the R Y B terminals of the motor. So, the motor is not shown here and this is the DC side, there is a DC voltage of V DC applied here and DC midpoint o.

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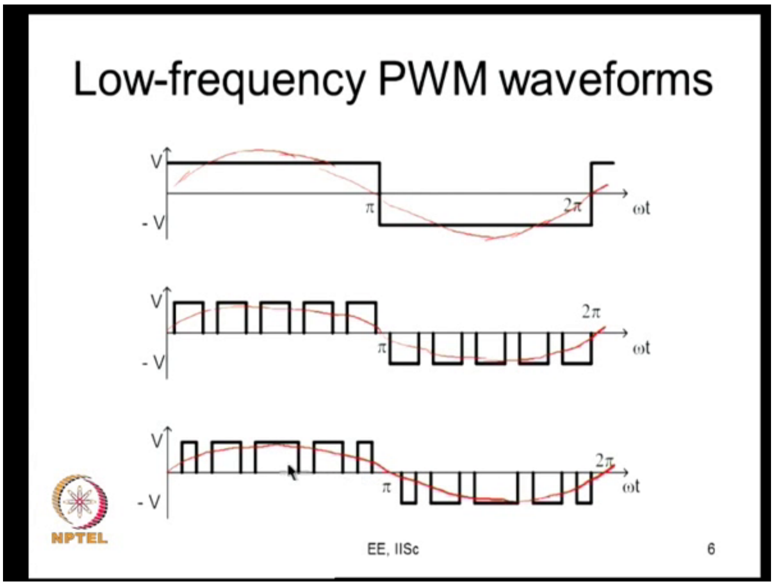

So, you can look at some examples of low frequency PWM waveforms remember these are actually been given the context of a single phase inverter it does not matter.

So, let say you have an a c load, but you have a DC voltage available; what you can do is you can apply this DC voltage initially and then with a sign reversal like this and go about doing it. So, you can apply it as a square wave thrown inverter. So, what could you called as a square wave single phase inverter and you can produce such an output R now. As we already seen this as a fundamental component that this is v, that fundamental component will be 4 v upon pi that would be the peak value of the fundamental component which is go or ridden come like this part now. So, in case you want me to draw that I can do that. So, it will have a fundamental component we will go slightly higher than that and come back and go and take it over like this.

So, you cannot control this fundamental voltage and that is the reason why you would want such kind of pulses. So, by such pulsation what you can do? You can reduce the fundamental amplitude like what I have shown here. These are all certain things we have discussed in good details when we did the purpose of PWM and the low switching frequency PWM now. So, the second waveform gives you some way of controlling that, but it still has high harmonic content. So, the third way from for example, can give you a very good way of controlling the fundamental voltage and also it controls the harmonics much better.

You can see that you look at the sinusoids, it will be very clear that you know I mean pulsewidth are equal, here the pulsewidth modulated in a particular fashion insensately they are modulated in the sinusoidal fashion here the width very high and it is low you know it the varies in sinusoidal fashion and you will see that you 5th harmonic could be very high here for example, whereas in this case that will be pretty low. So, the harmonic content improves. So, a low frequency harmonics they are able to reduce significantly, it will result in a very good quality waveform current waveform through the load. So, these are certain example of low frequency PWM which you know in like a you can look at it this as possible outputs of single phase inverter.

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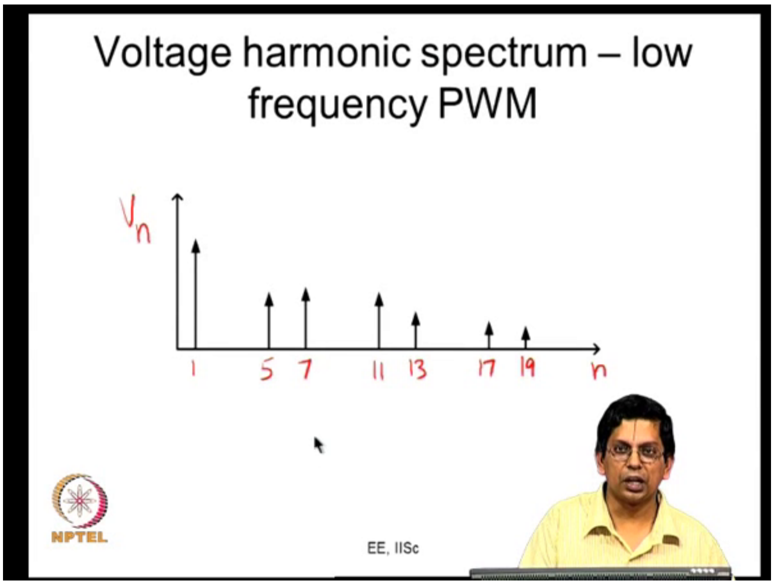

Now, let us take an three-phase inverter output, some typical voltage low switching frequency PWM waveform.

So, I have not indicated the harmonic orders here, we are trying to look at some low voltage harmonic spectrum low frequency PWM. So, it is switching at some frequency I mean there are some PWM being produced. So, the PWM form voltage form applied now we are looking at the inverter output. So, the line clean voltage will not have the third harmonic etcetera, first what we find here is this the fundamental component this is fundamental component. Then you have 5th and 7th harmonics, then you will have 11th and 13th harmonic, then you will have seventeenth and nineteenth if necessary I can write them for you write it down this is one fundamental, this can be 5 and 7 and this would be 11 13 this would be 17 19.

So, this is the harmonic order in and this is the amplitude of any harmonic voltage we will call it V n. So, this just a typical waveform that you might have, where you know this the PWM waveform has been representatives in terms of this. So, what you want only some fundamental voltage which was amplitude may be called as v 1, but what you are getting is you know all this V 5 V 7 v 11 v 13 v 17 v 19 etcetera also getting operate on the load. So, this V 5 certainly going to cast I 5 when applied to a linear load, this V 7 is going to cost certain I 7 when applied to a linear load.

So, this is all going to call 5th harmony current, 7th harmony current 11th harmony current 13th harmony current and so on for example, when you feed them to r l load now we are going to feed it to an induction motor load. So, these will also produce in an induction motor also this 5th harmony voltage will produce 5th harmony current, 7th harmonic voltage will also produce 7th harmony current. Not just that the 5th harmonic voltage will also produce 5th harmonic flux and the 7th harmonic voltage will also produce 7th harmonic flux. So, we are going to take a look at these current harmonics and flux harmonics, review the current harmonics another and look at the current flux harmonic and some detail today.

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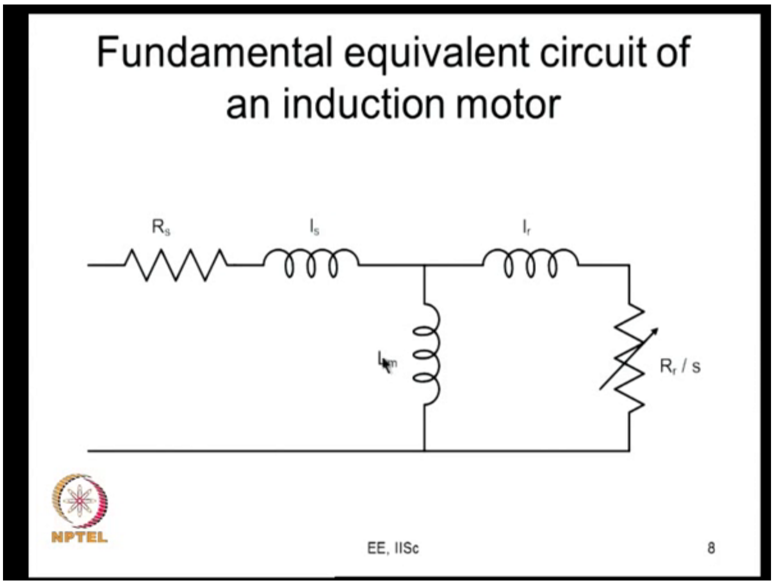

So, if you want to look at what is the effect of the applied voltage, the applied voltage as a fundamental component as harmonic components. So, as far as fundamental component is concerned it is a sinusoidal voltage, it is you know look at the preface equivalent circuit it a you know fundamental is balanced three-phase sinusoid, you can look at the preface equivalent circuit of the induction motor. This is the rotor side you know winding resistance and the leakage inductance, there is a magnetize inductance I mean this is a for the state and this is the rotors leakage and this is the rotor resistance R r divided by resistor slip as u know slip is what is slip is a synchronous speed minus the router speed divided by the synchronous speed.

So, the rotor does not rotate at the synchronous speed it rotate slightly lower than that, the measure of that is the slip now. So, this is the fundamental equivalent circuit. So, can apply your fundamental voltage to this and whatever current drawn by this circuit is the fundamental current that you should expect your motor to draw, this is something that we have seen even before.

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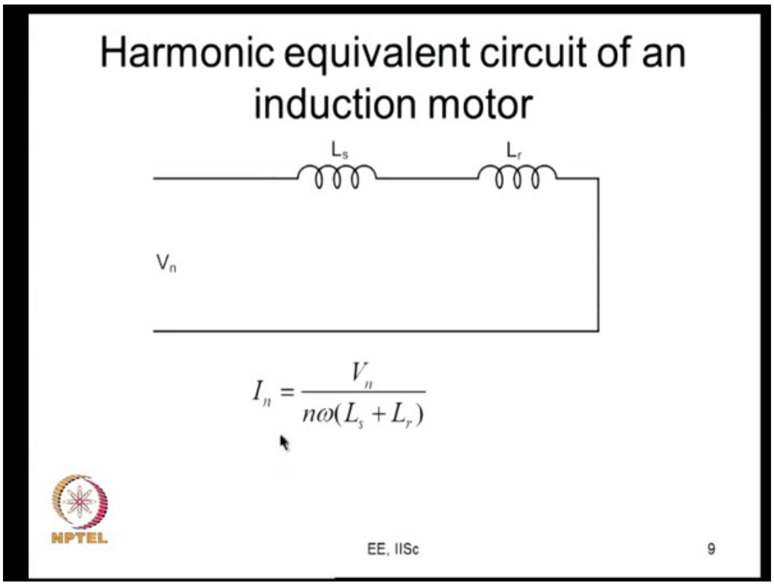

Now, if you apply the harmonic voltage you consider some unit harmonic voltage this V n and the machine model now reduces to Ls and Lr as we have seen also in earlier cases, what happens when you are applying harmonic voltage the slip very high close to 1 we will also probably have a chance to relook at it now today in the short R from now. So, this s normally for the fundamental will be very low like 0.02 0.03. Therefore, R r by s will be very big where as for harmonic this s will be almost close to 1 and therefore, R r by s is almost R r and which is actually small quantity.

So, if this is effectively for the harmonic it is Lr and Lr coming parallel L m Lr is very small and therefore, it will dominate over L m and therefore, you only Lr. And once again at high frequencies this reactance is going to dominate over this resistance and therefore, the motor will be effectively seen as only the sum of it is two leakage inductance and that is what is shown here. So, it is just the two leakage inductance and it is a model it is a harmonic model of the induction motor or harmonic equivalent circuit and this equivalent circuit if V n is your amplitude of the nth harmonic fundamental, Ls and Lr are your leakage inductance V n upon n omega times Ls plus Lr is going to be your nth harmonic current amplitude. So, this is your very convenient way of calculating what is your nth harmonic current now.

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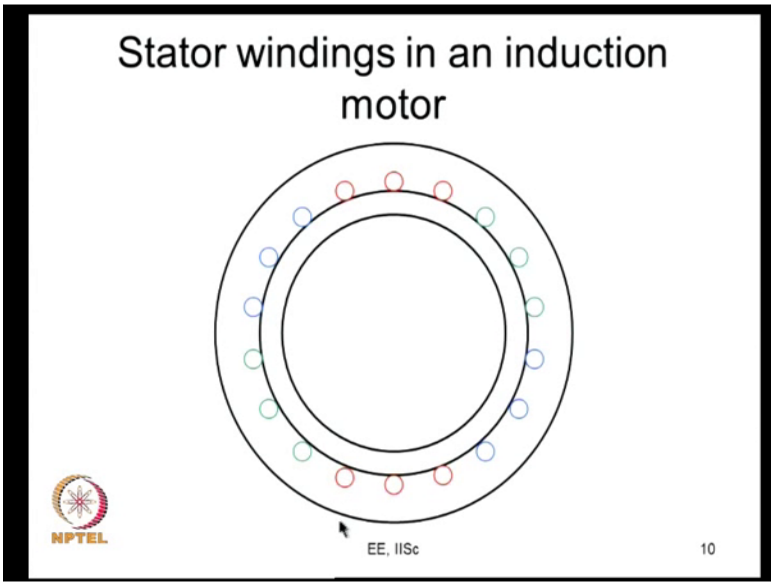

So, let us slightly get into the details of the motor now. So, this is the state winding the outside the state and we have the windings the conductor sitting in the inner periphery of that and this is the rotor and what you have in between a air gap. So, you look at this, this is your r phase winding that is the coil is going coming out may be going coming like this and then going there and then coming like this now. So, it goes on like this. So, effectively what happens is, this is a coil whose axis is like a here and this is your y phase is coil I have shown it in green color instead of yellow color right.

So, this is like a coil you know this goes around like this; so not around. So, this is like a coil whose axis is something of this nature and then you look at the blue phase, they are indicated in blue colors now. So, you have the coil running around like this, this something on the here and there a no at the end it goes over. So, this is like a coil you know whose axis is in this direction now.

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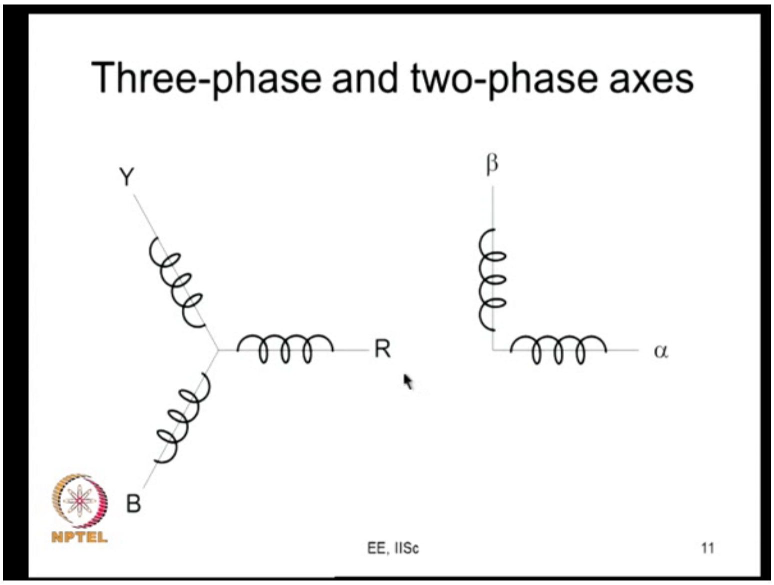

So, the three-phases axis are actually like this. So, you have you have the R phase axis, you have the Y phase axis and we have the B phase axis. And for purpose of analysis now we always of course, we have a machine, which has three windings and you know along three axis like this. It is possible to think of the machine as one having just two windings call them alpha and beta long this two axis.

So, you know what we have seen before is we can exit this with some three-phase voltages, here we should can excite them so call two-phase voltages or two-phase current both sinusoidal as the same frequency have a phase shift of 90 degree now. So, a three-phase machine can be seen as an equivalent two-phase machine.

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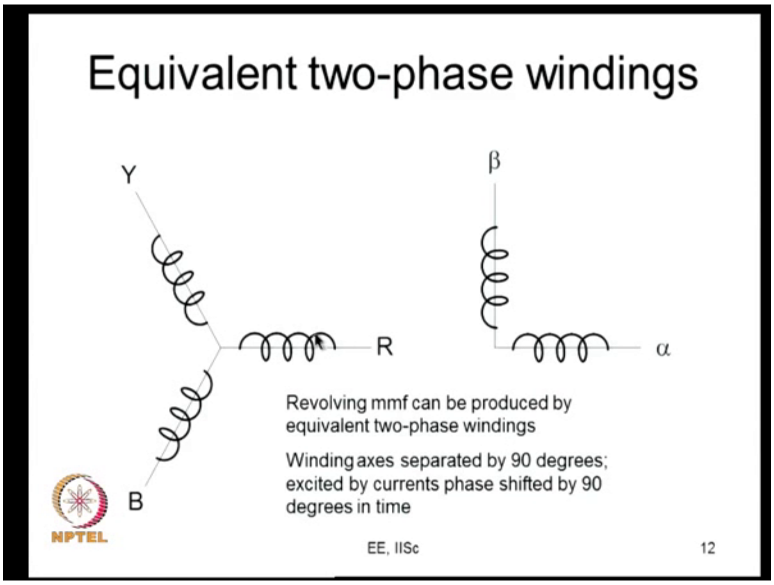

So, this is further here. So, how there is a revolving m m f, which is produced by the three-phase winding. Something the same job can be done by the two-phase winding also. So, whatever i R i Y and i B when applied we will do the same job can be done by i alpha and I beta. So, winding access, but here you see the winding access are separated by 90 degrees, they should be excited by current which are also phase shifted by 90 degrees in time. So, we have seen this we just having a review of this.

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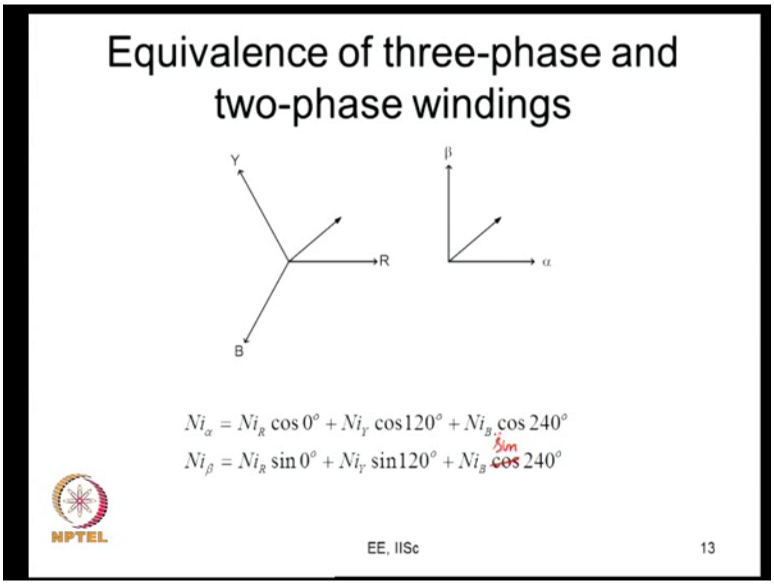

So, that you know we can look at this space vector transmission which we will need in a short while now. So, the three-phase winding are the axis are represented by the arrow marks here and this can be MMF here and this alpha beta, there are the two winding axis this is the MMF now.

So, you can look at this MMF as a result of the current flowing through R Y and B windings, the same MMF being produced by current i alpha and i beta flowing through alpha and beta windings now. So, you can equivalent this N i alpha that is along the alpha winding which is number of turns, certain current i alpha flows and that produces at amount flux which is actually equal to N i R cos 0 plus N i Y cos 120 plus N i B 240 and again this beta axis winding and there some beta i beta current flows to that, that produces n a MMF n i beta in this direction, the same thing is produced by the three-phase winding as given by here N i R sin 0 plus N i Y sin 120 plus N i B cos 240, I mean this there should be sin sorry there is an error here please change it out that now ok.

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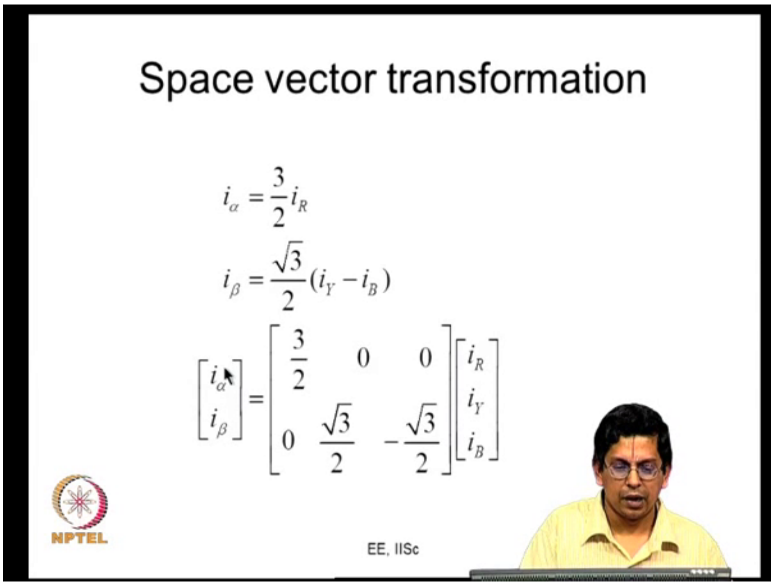

So, thank you. So, you can write down the space between transformation like this, i alpha can be equated to i R the condition that i R plus i Y plus i B is equal to 0 it a three wire load. So, this is your i alpha it related to i R, i beta is related to i Y and i B like this thus the three-phase current i R i Y i B can be transformed into two-phase alpha beta as shown here, again you can do the same thing on the three-phase voltages.

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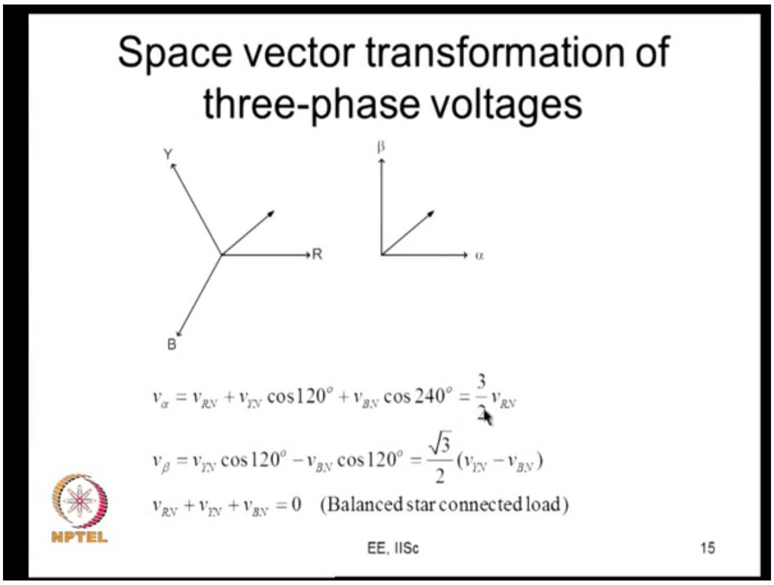

So, you have v RN, v YN v BN and when we have balanced load we have v RN plus v YN plus v BN is equal to 0 balanced star connected load. So, you can write your v r alpha like this as 3 by 2 times v RN and v beta as root 3 by 2 times v YN minus v BN this is the space vector transformation of three-phase voltages which we had seen in some earlier lechers now.

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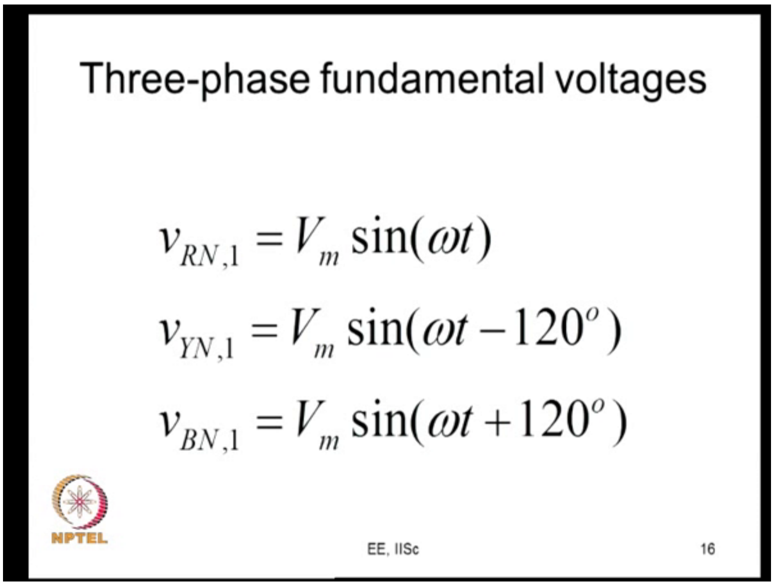

So, what we are going to do now is going to look at the components one by one, we are applying some voltage v RN v YN and v BN on the machine. So, let us consider the fundamental component alone of v RL. So, we would call that as v RN comma 1, 1 representing fundamental component. The same you know there amount of fundamental voltage is applied along y axis also with the phase shift of 120 degree.

So, we would call that as v YN comma 1, this v BN is the b phase line to neutral voltage applied on the load v BN comma 1 is the fundamental component. So, let us call this is a v m sin omega t. So, this would be v m sin omega t minus 120 this would be v m sin omega t plus 120.

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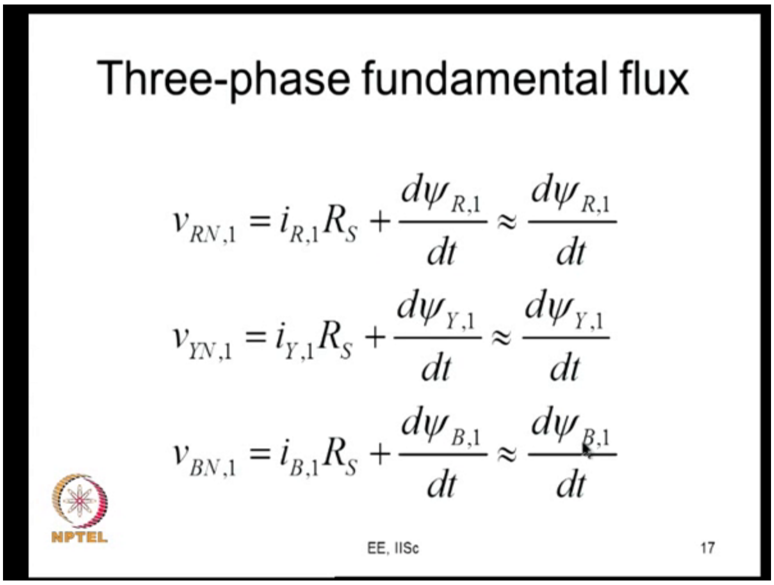

So, what would be the fluxes that is what we are going to look at now. Now if you have your applied voltage v RN 1 that is the sinusoidal in the fundamental voltage applied that is equal to i R drop, this R s is a straighter resistance and i R 1 is the fundamental current through r phase. So, this is i R 1 R s plus this i R 1 by d t, this is the rate of change of flux linkage now.

What we can do in most cases is, this is term is negligible compared to the second term we will simply call this is d psi it is approximately d psi R 1 by d t. So, that is the r phase voltage fundamental voltage is approximately equal to the rate of change of r phase flux leakage. So, this neglecting i R drop is a stranded assumption which is actually valid at higher speeds, this basic you know that v wave drive idea is based on that constant volts per hertz drive is actually based on that. If we ignore i R drop, you have your flux linkage and this would turn out to be the product of flux and frequency. So, your fluxes approximately proportional to voltage by frequency, that is why you do your constant v b f control.

Those who may be familiar with the constant v b f or volts per hertz will know that constant vbf would have works well at fairly high speed, but when it comes to lower speed there is some kind of flux weakening. So, certain voltage boost is given. So, what happens there is this i R drop become significant at low speeds of the motor drive or low modulation indices this quite a few of you have done something on motor drives would be fairly familiar with that. So, except so, if you know unless you considered low modulation indices and low modulation frequency drive, i R you know is negligible is a fairly valid assumption that all that I am trying to say now.

So, these voltages d psi by dt the same thing can be about the y phase fundamental voltage and the corresponding y phase flux linkage. The b phase fundamental voltage the corresponding b phase flux linkage now, alright.

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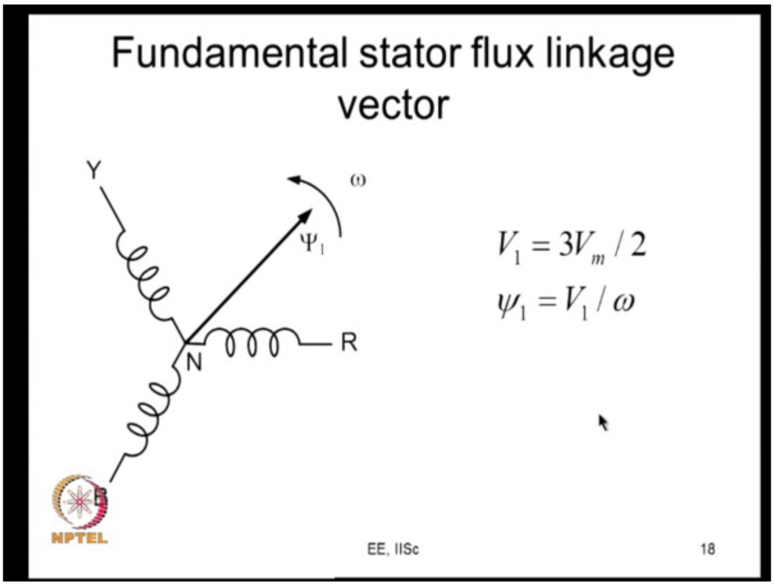

So, these are about the fundamental fluxes now.

So, these three-phases when you do a space vector transformation. So, you get a voltage vector V 1 of magnitude 3 V m by 2. So, if you go back to that you can see this here. So, you are v alpha is equal to 3 by 2 times v RN, if your v RN is v m sin omega t v alpha is going to be 3 by 2 times v m sin omega t and you can see that the magnitude of a vector is 3 by 2 times the peak value v RN that is what you are getting here. So, the peak value of v RN fundamental is V m. So, the voltage vectors of magnitude 3V m by 2.

So, the flux vector that is the three-phase flux linkages are there fundamental fluxes are there, you can actually transform them into the space vector domain to get your psi one. So, this psi one will be nothing, but whatever is magnitude as a voltage vector V 1 divided by omega that is what it will be. So, this produces a magnetic field revolving magnetic field, we are taking of only the fundamental components at this point of time the other component we are going to take a one by one. So, the fundamental component alone what happens it produces flux linkage whose magnitude is given by that, and it rotates at the fundamental frequency omega.

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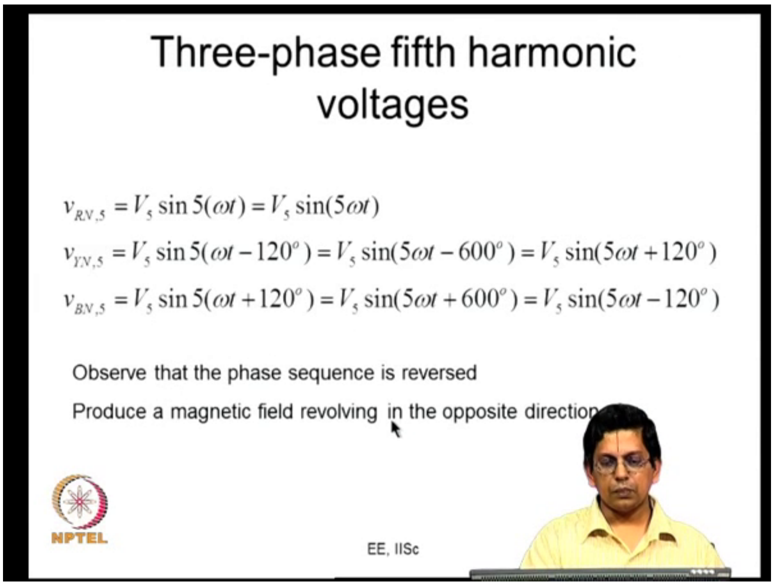

So, next if you look at the 5th harmonic voltage; other than the fundamental there are many harmonic components. So, let us just look at the 5th harmonic for now. So, the 5th harmonic let us say it has some peak value V 5. So, v RN 5, v YN 5, v BN 5 is the 5th harmonic components of v RN, v YN, v BN respectively and these are of course, balanced three-phase sin the since all of them have same amplitudes they are at the same frequency there actually phase shifted.

So, what you are actually doing is from v RN and v YN, v YN is time shifted from v RN by 120 degree. So, if you replace omega t in v RN by omega t minus 120 you are going to get the corresponding y phase there. So, we are we say that it is V 5 times sin of 5 omega t minus 120 5 times omega t minus 120. So, again your 5th harmonic in v BN would be V 5 times sin of 5 multiplied by omega t plus 120. So, this is what you have now.

So, if you do this, this is V n sin 5 omega t this is 5 times omega t minus 120 is sin of 5 omega t minus 600 and we now the periodicity is 360 degree now. So, r is 720 So, this is same as 5 omega t plus 120 degree, because I am just adding 720 degrees to this 5 omega t plus 120. The same way if I do this here, this is 5 of omega t plus 120. So, what I have is V 5 times sin 5 mega t plus 600. So, I can subtract 720 degrees here. So, it becomes V 5 times sin of 5 omega t minus 120.

So, what happens here if you look at the earlier case it is omega t y 5 is omega t minus 120 b phases omega t plus 120. So, now, if you look at here y phase is omega t 5 omega t plus 120, b phase 5 omega t minus 120. So, you can see that the phase sequence has been changed. So, what happens when the phase sequence changes? Many of you who know basic induction motor control certainly know this, if you just interchange two of the terminals the motor will start running in the opposite direction. So, the same way what they mean essentially means that this fellows the 5th harmonic voltages are going to produce magnetic field which actually runs in the opposite direction, because you know they are they give you different phase now.

So, they produce a magnetic field in the opposite direction now, which will look at it in little more detail as we you know go by.

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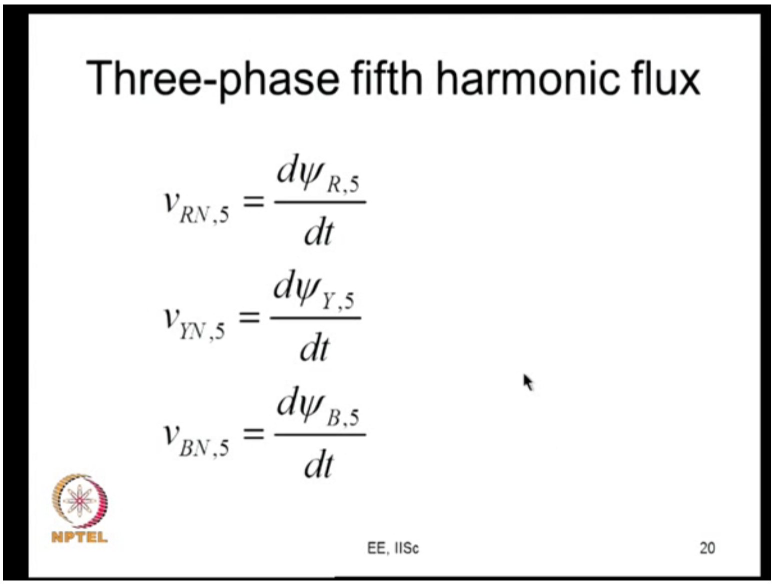

So, this is the three-phase 5th harmonic flux how is it related? The i R drop is negligible is even more valid here, you know because there is not so much of current and resistance itself is negligible compared to be reactance. So, the 5th harmonic voltage is equal to the rate of change of the 5th harmonic flux. So, the same thing is for r phase y phase b phase. So, we get some d psi r y etcetera now and psi R 5 is sinusoidal. So, d by dt of that is again sinusoidal of course, this is sinusoidal. So, what you will only have is you will have omega term coming up here.

So, you will have the psi R 5 amplitude will be v RN 5 by omega. So, this can roughly be written as omega multiplied by certain amplitude here. So, you have this equation.

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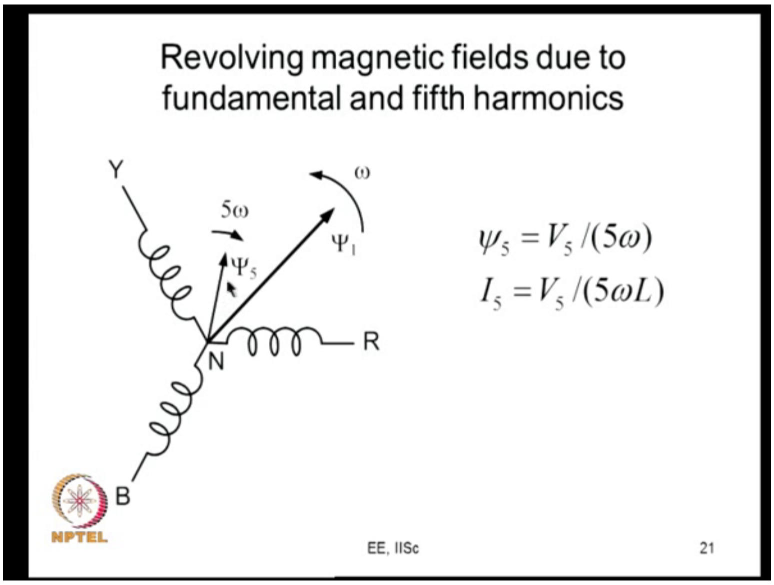

So, as I mentioned before it produces certain amount of flux and that revolves in the opposite direction. So, this psi 5 is going to be V 5 times V 5 divided by 5 omega that is what you get from here. So, you get this psi 5 is equal to V 5 by 5 omega and this 5th harmonic as we had seen earlier is also causing currents, you know 5th harmonic current the 5th harmonic current can be given by V 5 by 5 omega L, where L is the stator leakage inductance plus the rotor leakage inductance of the induction machine.

So, this is what in some sense this two equations some of the effect of the 5th harmonic voltage applied on the machine this is the 5th harmonic flux and this is the 5th harmonic current. So, the flux it interesting to see you know there is a magnitude, what is the frequency and what is the direction. As we already saw the direction of the flux is opposite to that this is in the anticlockwise direction where this is clockwise direction.

So, this revolves in this direction and at what frequency it is 5 omega, because that is the up let frequency. So, this revolves at 5 omega; so the relative speed if you look at is 6 times omega that is the relative speed of the 5th harmonic fluxes with res are the 5th harmonic magnetic felid with respect to the fundamental magnetic field the 6 times omega. So, this the fund causes what is called as 6 harmonic torque as we would see a little later.

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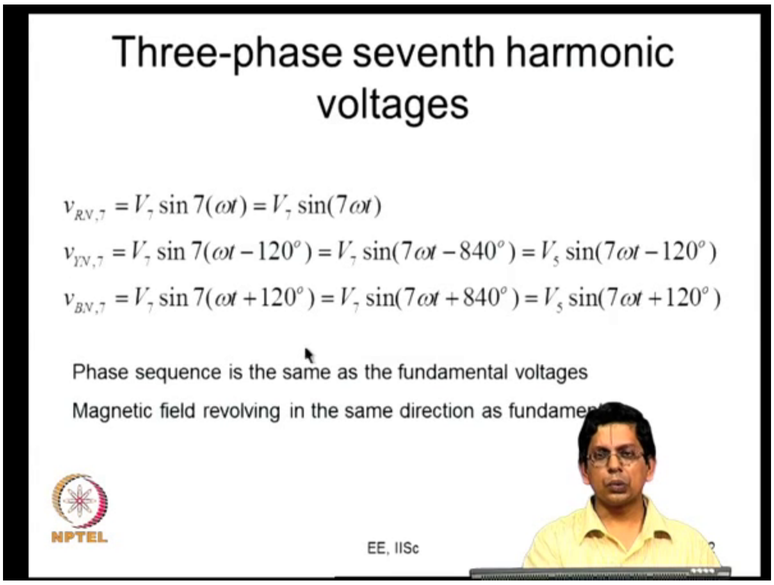

Now let us as also at the 7th harmonic voltage. So, 7th harmonic voltage let us see the r phase volt we will called as v RN comma 7 as we have seen now v RN stands for the r phase line to neutral voltage the instantaneous voltage, this 7 stands for the harmonic order.

So, this is the 7th harmonic component the same way v YN 7 and v BN 7 now. Let us say the amplitude of the 7 harmonic component is V 7. So, this is some V 7 times sin 7 seven omega t is let us say is the v RN now and what is v YN of 7? It is V 7 sin 7 multiplied by omega t minus 120 and that is V 7 sin 7 omega t minus 840. So, you can add 720 degrees to the 360 or 720 whatever because it is periodic over 360 the whole thing is periodic over that. So, if you add 720 degrees to that, you get V 5 sin sin omega t minus 120 degree. Now if you look at the 7th harmonic of b phase that is going to be V 7 sin of 7 times omega t plus 120 which is actually V 7 sin 7 omega t plus 840. So, this is plus 120 degree. So, you can see that the phase sequence is same as fundamental, this is r phases 7 omega t, y phases 7 omega t minus 120 and the b phase it is phase angle is a 7 omega t plus 120 degree.

So, this is same as a fundamental the phase sequence is same as fundament, but the magnetic field. So, it also revolve in the same direction as fundamental. So, you can do this analysis for 11th and thirteenth. For 11th you will find that it is like the 5th harmonic it produces the magnetic field in the opposite direction where is for 13th like 7th it will be producing magnetic field in the same direction as the fundamental. So, same way you can look at 17 and 19, 23 and 25. So, you can actually generalize all this I know this is what you will get if you go looking by harmonic after harmonic. So, all the harmonics which are like 6 k minus 1 where k is an integer like 5, 11 etcetera they would produce you know revolving magnetic fields in opposite directions in the other one should produce in the same direction is this now.

So, let us look at only 5th and seventh. So, that they are representative of the rest of the harmonics and also 5th and 7th are the most prominent once low frequency case, because higher the harmonic voltage the harmonic currents are anyway better filtered therefore, the problems of higher harmonics are little lower than the problems caused by the lower frequency voltages.

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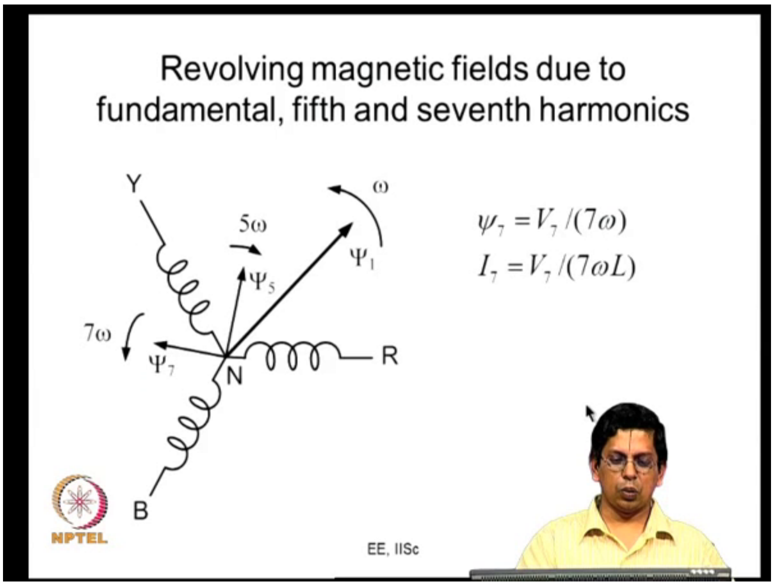

So, let us focus on 5th and 7th here and go little further. So, the 7th harmonic certainly drive certainly amount of I mean prove the magnetic field and that is like V 7 by 7 omega. So, the 7th harmonic voltages the three-phase voltages you transform them into space vector, you will get some voltage vector V 7 magnitude. So, this V 7 by 7 omega would be would be the magnitude of the 7th harmonic flux vector and the 7th harmonic voltage also causes some I 7 is equal to V 7 by 7 omega L. So, this causes the you know there is certain amount of for 7th harmonic flux, there is certain amount of 7th harmonic current both are being caused by the 7th harmonic voltage being operated.

Now if you look at here this is 7 omega, and this what I am giving psi 7 it is it is revolving in the same direction as the fundamental, but at a speed 7 omega. So, interestingly the relative speed between psi 7 and psi 1 is same as relative speed between psi 5 and psi 1. Here the relative speed is 6 times omega here also the relative speed is 6 times omega excuse me therefore, this contribute this also contributes 6 harmonic torque alright.

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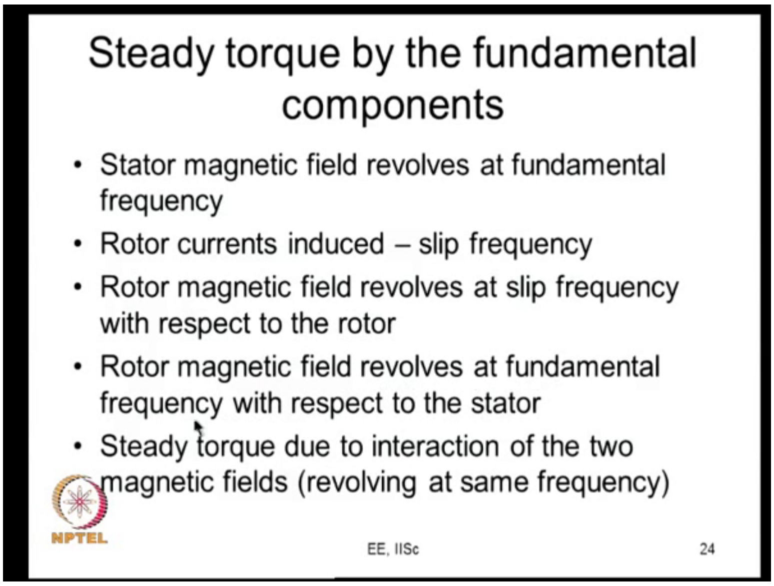

So, let us look at a little closure, what is this torque generation. So, we have looked at voltages we have looked at currents and today we have started looking at fluxes and now we are talk of torque. So, what is this torque? Let us first look at the steady torque produced by the fundamental components that is you have sinusoidal machine induction motors with fed sinusoidal voltages they produce steady torque right.

So, this fundamental voltages we are applying, they cause fundamental currents and fundamental fluxes this fundamental current and fundamental fluxes would also produces steady torque, this is what we are first try to get understanding of how does it happen? It is of course, it is qualitative, but let us try to understand something here the stator magnetic field revolves at fundamental frequency what are you doing? You are applying a three-phase voltage on a three-phase winding, and it produces a revolving magnetic field at what frequency? It at the fundamental frequency, the amplitude of the I mean strength of that magnetic field is given by v by omega that is the ratio of your stator voltage to the stator frequency and the magnetic field itself rotate at fundamental frequency.

So, what happens now is it rotates because of it is rotation there something you know there is rotor currents there is induced in this is slip frequency what is happening to the rotor the rotor see an MMF wave moving fast. So, there is some EMF induced in the rotor and the rotor is started and therefore, certain currents are produced. And at what frequency is this these currents are produced that is at the slip frequency. We know that the rotor is not rotating at the same speed as the stator magnetic field, the rotor speed is lower than that in the motor operation is lower than that.

So, the rotor is lowing there something called slip that is the synchronous speed minus rotor speed divided by the synchronous speed. So, that is slip. So, there is certain slip frequency, if let us say the fundamental frequency 50 hertz and the rotors speed is equivalent to 14 hertz slip frequency is 1 hertz. So, the rotor see an MMF wave, you know at 1 hertz frequency and there is an induced hertz frequency therefore, there are rotor currents induced which are at this slip frequency 1 hertz. let us say. Now these are currents flowing through rotor. So, there cause magnetic field. So, they this magnetic is again going to be revolving why essentially rotor is also equivalent to three-phase winding and three-phase currents are flowing through that so, but the only thing is current follow of slip frequency.

So, the rotor magnetic field will rotate at slip frequency with respect to the rotor, slip frequency with respect. So, the rotor magnetic field will routed at something like let us say 1 hertz with respect to the rotor, but you see that the rotor is already rotating at 49 hertz frequency you mean speed equivalent to 49 hertz. So, the rotor magnetic field revolves at the fundamental frequency with respect to the stator. So, the rotor magnetic field it actually revolves at slip frequency with respect to the rotor, the rotor itself is rotating. So, if you look at you know stator as the reference the rotor magnetic field also revolves at fundamental frequency.

Therefore you have the rotor magnetic field rotating and also the stator magnetic field at the fundamental frequency. So, you need two magnetic fields, which are actually revolving at the same frequency and that is what produces stator torque why because the torque is thing, but kind of cross product of these two magnetic field. So, cross product means it depends on the strength of the first field and it is strength of second field and sin of the angle between the two fields. And if in a steady state situation, you are there are two steady magnetic fields you know are the relative speed the equal they have a constant angle between the two let us call the angle as delta. So, some sin delta, that sin delta is a constant where as if these two magnetic field the revolving a different frequencies what happen that delta will keep on changing therefore, sin delta will keep on changing and so, over a period of time it will average to 0.

So, if this two magnetic fields are not the same frequency, they are going to produce a torque which is actually pulsating in nature this is what we are going to steady because the harmonic volt voltages are actually causing that. So, in when you when you talk of sinusoidal operation of the machinery or when you talk of the effect of the fundamental voltages on the machine, what happens? The stator magnetic field is at the synchronous frequency, the rotor magnetic field is also at the synchronous frequency. These two together work and the producers steady magnetic field I mean producer steady torque.

So, they introduces steady torque due to the interaction of these two fields now. So, this is this also explains why you are induction motor self starting, this is the question that is often asked by student, student are asked this question now. So, one way to look at it is let say the induction motor set stand still, let us say it is a stand still let us say some 50 hertz is been applied here this is fundamental frequency let say sinusoidal frequency applied it says 50 hertz is being applied now. So, 50 hertz is being applied the rotor said stand still.

So, what is rotor current frequency what happens to the rotor? The rotor see the 50 hertz way of moving faster and therefore, there is an induced name of which is 50 hertz frequency and the rotor is as got a shorted path is a shorted one. So, therefore, the rotor current are going to flow and what is the rotor current frequency? 50 hertz. So, the rotor will now produce magnetic field, which rotates at 50 hertz while at stand still therefore, the stator magnetic field is fundamental as 50 hertz, the rotor magnetic field is also at 50 hertz and these two magnetic is there at the same frequency the produce the steady torque and therefore, there is a non zero starting torque produced by an induction motor because it produces stator magnetic field and also rotor magnetic field both of them being at the same frequency. So, it starts rotating now.

So, there is a starting torque, let say motor start rotating what happens? Let say rotor has picked up some speed, let say it is running at the speed equivalent of 1 hertz. So, what happens now? The stator magnetic field continuous to be at the same 50 hertz; now the rotor is rotating at 1one hertz equivalent frequency it has slightly increased. So, now, what is happening? The slip frequency slip frequency is 49 hertz. So, 49 hertz current is actually the rotor current are now would be 49 hertz. So, these rotor current induce will produce rotor magnetic field which will rotate at 49 hertz with respect to the rotor.

So, the rotor magnetic field is rotating at 49 hertz with respect to the rotor and the rotor itself is rotating at a freq at a speed equitant to 1 hertz, then the rotor magnetic field is rotating at the freq at the fundamental frequency or mean at 50 hertz with respect to the stator. So, the rotor has picked up speed that is come to one hertz still the rotor magnetic field and stator magnetic field are both at 50 hertz and therefore, the machine continues to produce torque. So, if the machine goes on exhilarating.

So, the rotor speed may be 10 hertz, in the case the slip frequency will be 40 hertz. So, the rotor magnetic field will still be 50 hertz same as the fundamental. Rotors exhilarated further the is gone to say something like 25 hertz. So, during that time, rotor magnetic field is 25 hertz the rotor itself is rotating at 25 hertz, once again you the rotor magnetic field with respect to the stator 50 hertz. So, both the stator and rotor continue to produce the same thing and rotor goes on accelerating till when? Till it reaches to the speed which is reasonably close to your fundamental frequency that is what you called as slip it will not reach the slip, I mean it will not reach same speed as the synchronous what is called as synchronous speed.

If it reaches synchronous speed then what happens? This slip is 0. When the slip is 0 the induced EMF in the rotor actually becomes 0 and there are no currents and you know no appreciable currents and there is n appreciable torque. So, it goes to something close to 50 hertz may be 49.8 49.5 that depends on the torque demanded, how much I mean depends on the load torque. So, it goes to some frequency which is close to 50 hertz like 49 or 49.5. So, the slip frequency is 0.5 or so. So the slip frequency at which it settles down is actually dependent on the load or it is a measure of really rotor.

So, this is how you can actually explain why an induction motor is really starting or a self starting. So, where is this is not true with a synchronous machine a synchronous machine what happen? The stator frequency is stator is very similar to in induction motor. So, it is a three-phase winding, you may be applying three-phase voltages. So, it produces magnetic field at synchronous frequency, but how about the rotor the rotor is at stand steal. So, if we R DC through that it is not producing a revolving magnetic field at synchronous speed therefore, it cannot start. But if you have done something and make sure that the stator on rotor are rotating at the same speed, that is rotors also rotating at the synchronous speed the rotor carries DC therefore, rotor magnetic field will also be at the synchronous speed the stator magnetic field is will also be at the synchronous speed.

And then the synchronous machine starts producing torque. So, any machine can produces steady torque only when the stator magnetic field and the rotor magnetic field are kind of stationery with respect one and another, only then it can produce a steady torque now alright. This is a DC machine does not much simple a fashion. So, it does not rotate at all. So, there is always a stator magnetic field, there is a rotor magnetic field or the armature magnetic field they are in quadrature.

So, when the torque increases the armature field is made stronger and the torque produced there is actually an interaction of these things. So, most machines which produce alignment torque except for machines like switch reluctance machine, which actually work based on the principle reluctance torque of this is valid for all of them. That is there is there they produce magnetic fields there is stator magnetic field there is rotor magnetic field and both of them actually either a stationary or both of them are rotating at the same frequency, and that is how it produces now.

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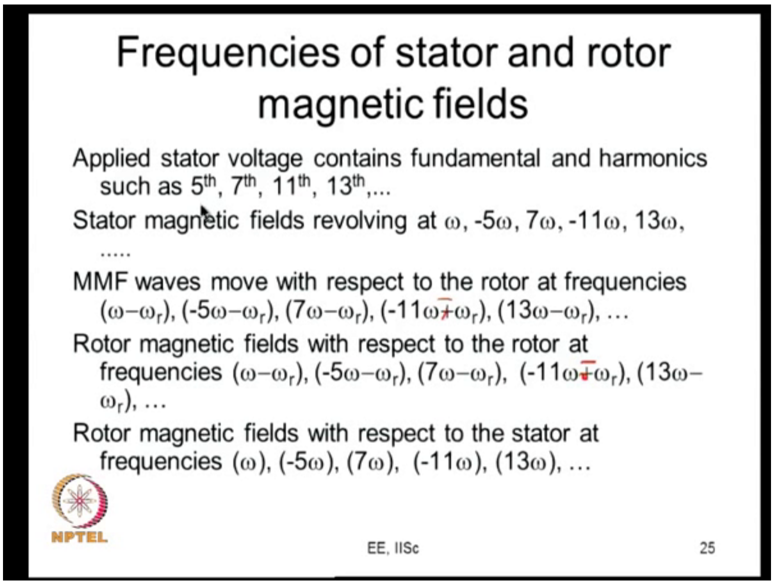

So, with this kind of basic understanding of torque production, let us look at what happens because of all the harmonics we are applying. When you are feeding an induction motor from an inverter, you are applying not only the sinusoidal the desired sinusoidal which is called the fundamental, you are also applying many of the harmonics. So, these are like 5th this is 7th 11th 13 and so on and so forth and these are considerable when your operating the inviter at a low switching frequency like 450 hertz with the fundamental itself being 50 hertz.

So, these are all funda significant. So, what happens to stator this thing now fundamental we have seen? Let us say the 5th harmonic; the 5th harmonic as we saw is going to drive some 5th harmonic current the stator going to draw 5th harmonic current no doubt about. It is also resulting in 5th harmonic flux it is also resulting in a magnetic field and that magnetic field as we saw is rotating at a frequency 5 omega and in the opposite direction hence I have indicated is as minus 5 omega. So, the 5th harmonic voltage results effectively in a magnetic field, which is rotating at 5 omega in the opposite direction.

Now, next you have the 7th harmonic; what does a 7th harmonic do? It also produces a magnetic field rotating magnetic field it is of course, it is not as strong as a fundamental, but whatever with the strings maybe it produces a magnetic field and what frequency the magnetic field rotates at? It rotates at 7 times omega and in the same direction as the fundamental and therefore, I have show it by a positive sign. Then similarly for 11 and

13 you will see that it is minus 11 omega and this is 13 omega alright. So, you have this frequencies going around like this alright now. So, if you have a harmonic pair you will find that in one of that it will be negative the other one will be positive. So, it goes on like that alright. If you have a frequency omega magnetic field omega and the rotor is rotating at omega r, the rotor see an MMF waves passed it at frequency omega minus omega r.

So, this stator magnetic field is seen as MMF wave which moves pass the rotor at speed omega minus omega r therefore, the induced EMF is at the speed omega minus omega r at this is what we were calling at slip frequency a few minutes back. So, it actually does this and therefore, slip frequency EMF and there slip frequency current and whatever flows through this now; the same time if you have minus 5 omega r. So, this also produces magnetic field, but the rotor is rotating at the same omega r.

So, you should remember one thing we say that there is a steady torque and therefore, motor r steady speed is understandable if we say there is a pulsating torque then one would expect the speed also the pulse right, but what we are assuming is though there is a pulsating torque the speed fixed, that is the motor seem to have a high inertia therefore, even if the torque produce is this you know there is a significant pulsating in the ripple content there, it does not cost the speed to change significantly is an assumption. So, we assume that the rotor is at a constant speed it is really not affected by the pulsating torque.

So, the rotor is rotating at omega r only therefore, the MMF wave you know whatever corresponding to the 5th harmonic, it is moving pass the rotor. So, it is like minus 5 omega minus omega r. So, actually if you see the induced name of the induced name of will be 5 omega plus omega r would be of that frequency in the rotor and su such rotor current of 5 omega plus omega r will be flowing through that because of the 5th harmonic. The 7th is same way the 7th harmonic with which is produce 7th harmonic in a magnetic field. So, it is 7 omega minus omega r. So, this is rotating, this 7 that is the 7th harmonic voltage applied to the stator causes some magnetic field rotating at 7 times a fundamental frequency and itself is rotating at omega r.

So, this magnetic field moves pass the rotor at a speed of 7 omega minus omega r and therefore, you know there is an induced EMF of frequency 7 omega minus omega r and induced current also of that frequency in the rotor. The same thing can be said about the other frequencies minus 11 omega there should be minus omega r there is an error here alright. So, you can see that the rotor magnetic fields are at the same frequency with respect to that this is also here R. So, now, does the rotor produce the magnetic field yes why? Because MMF is the moving at this frequencies and therefore, there are induced name of these frequencies and rotor goes at a shortest path and therefore, there are induced current said these various frequencies.

Every one of this frequency current is going to produce a magnetic field and this magnetic field is going to be rotating at a speed omega minus omega r with respect to the rotor, this is going to rotate at a speed minus 5 omega minus omega r with respect to the rotor. This is going to cost magnetic field rotating at 7 omega minus omega r with respect to the rotor this what I am trying to indicate now. So, these are all the speeds of these magnetic fields with respect to the roter what would be the speeds of these magnetic fields with respect to the stator? It is back to you add omega r to this. So, the rotor itself rotating at omega r. So, the rotor magnetic field with respect to the stator are at this frequency omega minus 5 omega 7 omega minus 11 omega 13 omega.

So, just let us take a look at it again; see this is a 5th harmonic voltage what is the is the 5th harmonic voltage do it produces the 5th harmo it produces the revolving magnetic field because it is also sinusoidal it is also balance sinusoid it is applied to three-phase winding. So, it is it has to produce a magnetic field, the only thing is the magnetic field you know the field strength is different and it rotates at 5 omega and it is at in the opposite direction. So, the 5th harmonic voltage produces the revolving magnetic field 5 omega which revolves in the opposite direction and the rotor is rotating at omega r the relative speed of this 5th harmonic magnetic field.

And the rotor is minus 5 omega and minus omega r. So, the induce EMF of this frequency 5 omega plus omega r inside in the rotor and therefore, there are induced currents of the same frequency 5 omega plus omega r and therefore, rotor produces magnetic field which rotates at this frequency minus 5 omega minus omega r with respect to the rotor and therefore, it produces minus 5 omega with respect to the stator. So, you can see that these stator magnetic fields are at some frequency, the rotor magnetic field are also there at this frequencies. The only thing is this is a strong magnetic field and this is also strong magnetic field therefore, they produce considerable torque where as this magnetic field and magnetic field there are not strong enough or any harmonic for example, take 5 5th harmonic here and 7th harmonic here, there this is low on this is I mean this is weekend is also week.

So, you could actually ignore all the interactions between a magnetic field produced by harmonic in the stator and the magnetic field produced by harmonic in the rotor, their interactions can all be ignored. We can only consider these 2; they produce the fundamental and you can consider the interaction of this field with any of these that will give you some harmonic similarly you can consider I mean that will give the pulsating torque, you can consider the interaction of this with any of these that is also going to give you the pulsating torque.

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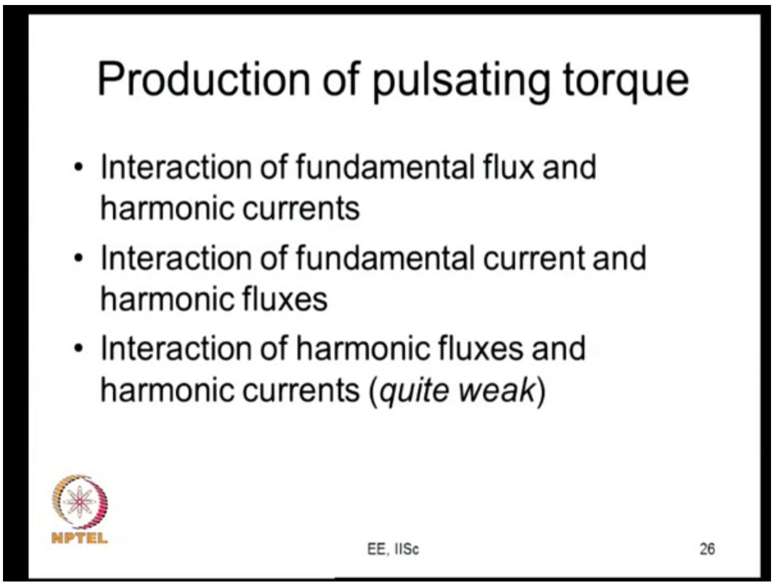

So, this is where we are going to this now. So, one way to look at it is you can look at the magnetic field you know, what do you mean by magnetic field we try to look at it. So, you can either look at it as a fluxes or certain amount of current. So, one way to do that is to look at it as the stator side flux for example and the rotor side current. So, this the stator field for example, if you looking at the fundamental magnetic field by the stator, you can actually look at the fundamental flux corresponding to the stator. On the rotor magnetic field can actually be represented by the rotor harmonic currents. So, you can consider the interaction between the fundamental flux and harmony current say the stator fundamental fluxes and the rotor harmonic currents.

Similar this will produce pulsating torque why? Because this is one magnetic field and this is another magnetic field, this fundamental flux corresponds to the steady magnetic field which is at omega. The harmonic currents they are magnetic fields which are actually rotating at different frequencies. So, the angle between these two magnetic fields keeps on changing, changing you know and sinusoidally the delta is angle between the two it varies a sin delta and that is an alternating functions. So, it averages to 0. So, there is no steady torque produced by that, but there is a pulsating torque produce by this. So, we are going to, but I mean a magnetic field I have to represent it either in terms of flux and in terms of current. So, we will normally say it is a interaction of one flux say the stator side flux and the rotor side harmonic current.

So, in the stator side flux we can say the fundamental fluxes on the stator side, and the harmonic currents on the rotor side we can look at these interactions. Similarly we can also look at interaction of the fundamental current, let us say the rotor side fundamental current and then the stator side harmonic fluxes. You can also look at the interaction of harmonic fluxes and harmonic currents. So, this is basically interaction between two harmonic fields, which is actually you know magnetic field produced by some stator harmonic and some volt rotor harmonic as I told you before this is quite weak and therefore, we can always ignore this.

This is perhaps the strongest why? Because your fundamental fluxes always near one per unit 100 percent and the harmonic current. If you look at the harmonic current what causes that harmonic current? Is basically the some harmonic voltage divided by the harmonica reactance now. In the case if you consider harmonic current and harmonic flux they are essentially same, you know because there are all both are related to the harmonic voltage. But here it is harmonic voltage divided by let us say the magnetizing inductance this is the same harmonic voltage divided by the leakage inductance. So, to put it in very simple terms, if you magnetizing inductance is much bigger and this is leakage inductance much smaller.

So, when you divide th harmonic voltage by magnetizing inductance what you get is smaller actually. So, where as this is considerable because you are dividing it by leakage inductance. If you are going to express the inductance in per unit terms, this leakage inductance is going to be very small. Therefore, that harmonic voltage divided by leakage inductance is going to be bigger, whereas harmonic voltage divided by magnetizing inductance is going to be lower. So, that a simple argument that one can give to say why this would be more dominant than that. So, what will you normally actually both could be considered, but normally this is the most more prominent term that you would actually look at now.

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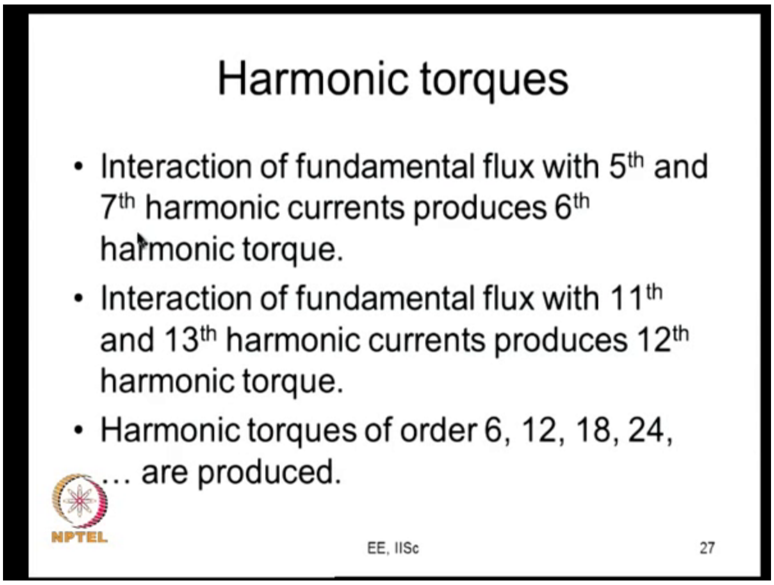

So, you can see that you have the fundamental flux and it interacts with 5th and 7th harmonic. Fundamental interacting with 5th harmonic current would for example, produce 6th harmonic torque and the fundamental interacting with the 7th harmonic would also produce the 6th harmonic torque. Again you fundamental flux it would interact with 11th and thirteenth.

So, the relative frequency is still 12 that is the interesting point. It is fundamental it has a magnetic field in one direction this is 11th harmonic is 11 omega in the opposite direction and this is the 13th harm 13th harmonic produces 13 field at 13 omega in the same direction. The relative speed between the magnetic field I know it is always 12 here. So, it produces a twelfth harmonic torque and similarly you will also 6th 12 18 24 etcetera this would be the harmonic torques produced in a motor. The motor will be applied voltages are 5th 7th 11th 13th etcetera like 5th 7th 11th 13th etcetera. So, the harmonic currents are also 5th 7th 11th 13th etcetera that interestingly the harmonic torques of order 6, 12, 18, 24 etcetera the 5th and 7th interacting with fundamental produces 6 11th and 13th harmonic interacting with fundamental produces 12th and it goes on and so forth now.

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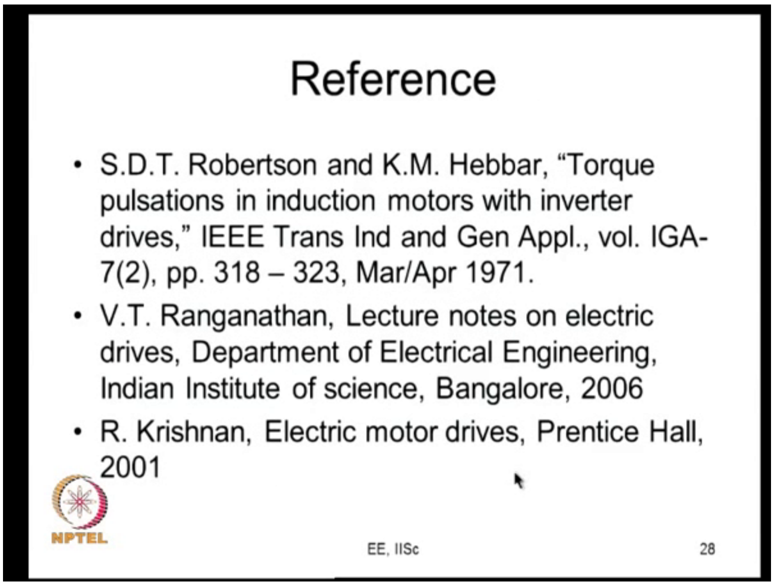

So, we have some interesting papers is actually very old it is a seminal paper original work in this area and to actually do the details with the per unit in equivalent R of an induction motor, it is possible for you to calculate how much pulsating torque is going to produce for example, has an entire percentage it is le let say when you affect with square wave or when you are affect with a particular kind of PWM waveform, how to calculate the 6th harmonic torque for example. With that it is possible to do that using equal inside of induction motor, this is one seminal work which actually explains how to do that now. I would strongly suggest this as a reference that you could go through to understand how such calculations can be done here. So, I have covered the preliminaries here there can be more detailed understanding you can actually do it quite quantitatively and the other references the lecture notes by Professor V.T Ranganathan is a lecture notes on electric drives.

So, this also deals with this topic of pulsating torque, how you go about calculating and the Professor R. Krishnan's text book on electric motor drives also discusses something about it here.

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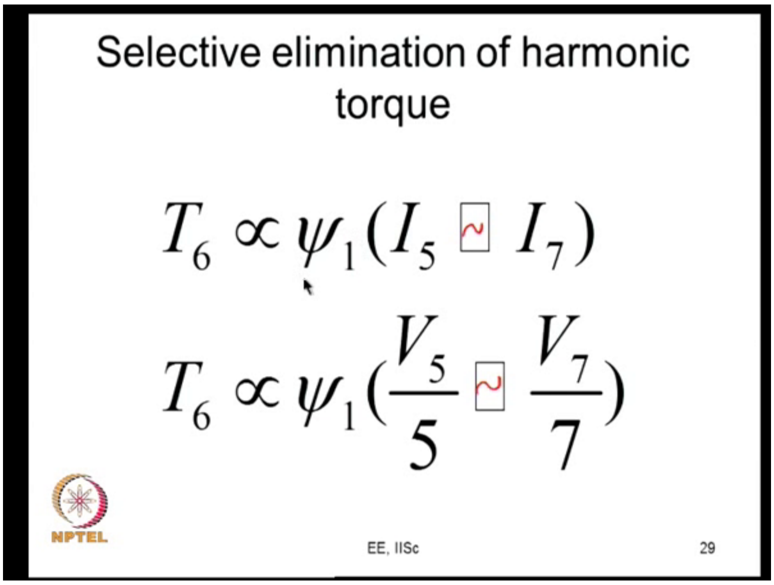

So, about how exactly evaluate you need to go into the details of the machine and which these papers can actually give you. So, this being a PWM oriented course, I would want to give some idea on how you can avoid that torque this is actually difference, this sign has come out little funnily here. So, this is actually the difference now. So, what happens is, you can look through based on the previous discussion in this reference you can see that the 6th harmonic torque is produced by the interaction between the fundamental flux and the 5th and 7th harmonic currents.

So, the fundamental flux and the 5th harmonic current produces some 6th harmonic torque, the fundamental flux and 7th also this thing. If the 5th and 7th have certain phase relationship and certain amplitude relationship then it is possible that you are T 6 can become 0. So, the amplitude relationship can be born out very very easily, if you know the 5th harmonic voltage this is going to be V 5 by 5 pro I 5 is proportional to V 5 by 5, because the 5th harmonic voltage has a reactance you know 5 times omega l this is going to see your reactance 7 times omega l.

So, I 7 is proportional to V 7 by 7, and if let us say they are actually in phase that is whenever the fundamental is crossing 0 5th also crossing 0 then 7th is also crossing 0, then you can just take the difference you can subtract this from that now. So, this part is proportional to whatever is given by the 5th harmony and this psi 1 V 7 by 7 is a measure of the 6th harmonic torque corresponding to that these could be such that if their amplitude are chosen such that you know this cancel out it is possible that you can have 0 6th harmonic torque.

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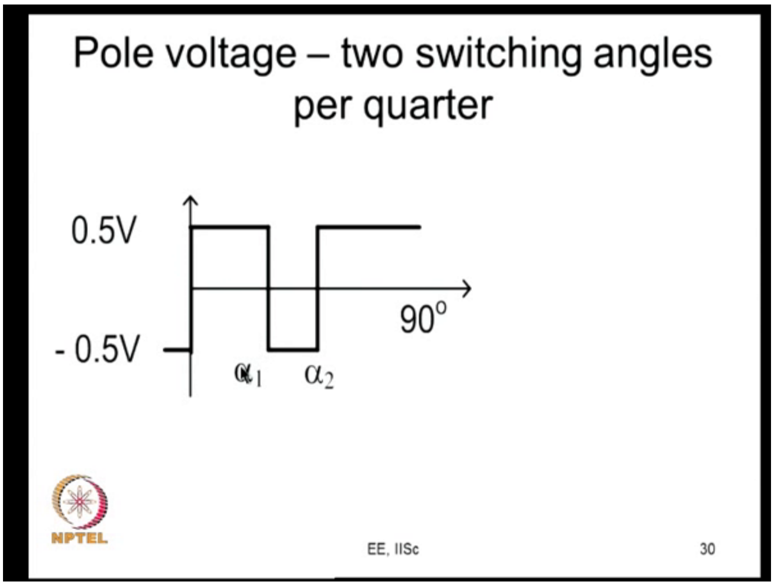

So, this is like we eliminated the you know selective harmonic voltages, if you remember the problem earlier we have two variables here alpha 1 and alpha 2 in one quarter cycle I am just showing one quarter cycle of this.

So, you have two variables you can write two equations you can write in equation for the fundamental voltage in terms of alpha 1 alpha 2, you can write down equation for 5th harmonic in terms of alpha 1 and alpha 2 and you can solve that you get your alpha 1 and alpha 2. So, if you use that alpha 1 and alpha 2 you would have manage to get the fundamental decide fundamental and also eliminate a 5th harmonic. So, it is possible for you to get your decide fundamental and eliminate one harmonic voltage by going in for this alpha 1 alpha 2.

The same way the only procedure that I am suggesting is instead of the first equation is v 1 is the decide value, the second equation instead of saying V 5 is equal to 0 we will say T 6 is equal to 0 or V 5 by 5 minus V 7 by 7 is equal to 0 that should be the second equation. If you use those two equations in solve for this alpha 1 and alpha 2 you will be ending up with this 6th harmonic elimination.

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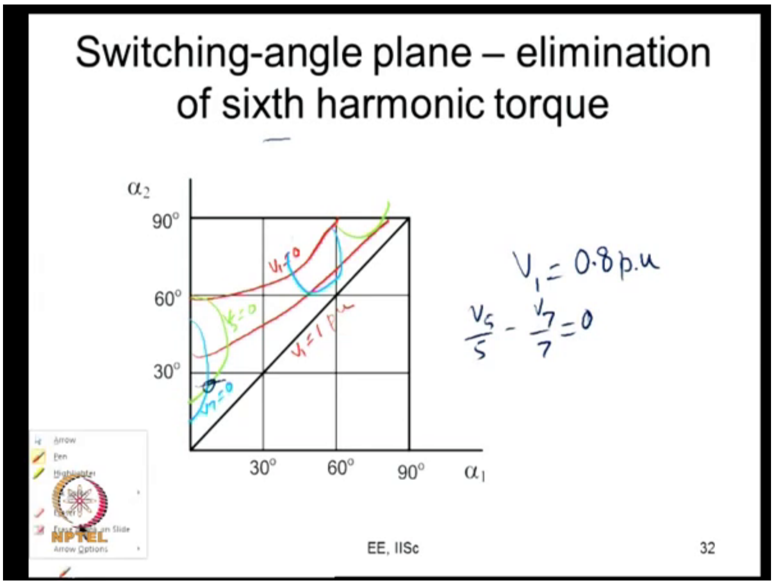

And we can actually look at it on this plane which we had looked that long back, for example, this alpha 2 and alpha 1 are the switching angles and this corresponds to square wave operation if you reconnect earlier lectures on this low switching frequency PWM, and this line would correspond to something like v 1 is equal to 0 per unit.

So, a particular fundamental voltage will go like this that is all the points along this line would actually give you particular fundamental voltage now. Then we also looked at the 6th ha 5th harmonic voltages and so on so forth right. For example, if you look at the point where V 5 is equal to 0 you will have some curve like this, this is a point where you have all V 5 is equal to 0. Again you may have some curve going like this where you are V 5 is equal to 0. So, the same way you look at V 7 is equal to 0, you will have a situation where this is V 7 is equal to 0 this is some V 7 equal to 0 these are lines were V 7 is equal to we did all these when we did selective harmonic elimination so on and so forth now.

So, on this plane it is possible for you to select points where the 6th harmonic torque is 0 an example would be this point this is an example. So, again you if you look at both V 5 and V 7 are 0, you will see some points in the infinity where V 5 by 5 minus V 7 by 7 is equal to 0. So, you can solve for this equation V 1 is equal to for example, 0.8 per unit and V 5 by 5 minus V 7 by 7 is equal to 0 to eliminate 6th harmonic torque. You can try this as a home exercise and we can anyway discuss this elimination in our next lecture before we go on to high switching frequency.

So, I thank you very much for your attention, and I thank you very much for your interested I hope you found this lecture interesting. And I looking forward to your participation in the next lecture on the topic.

Thank you very much.