Pulsewidth Modulation for Power Electronic Converters
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Lecture - 28
DC link current and DC capacitor current in a voltage source inverter

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters. So, we have been looking at power electronic converters particularly DC to AC converters and you know how exactly, you do the pulsewidth modulation for them that is what the whole lecture series is given about.

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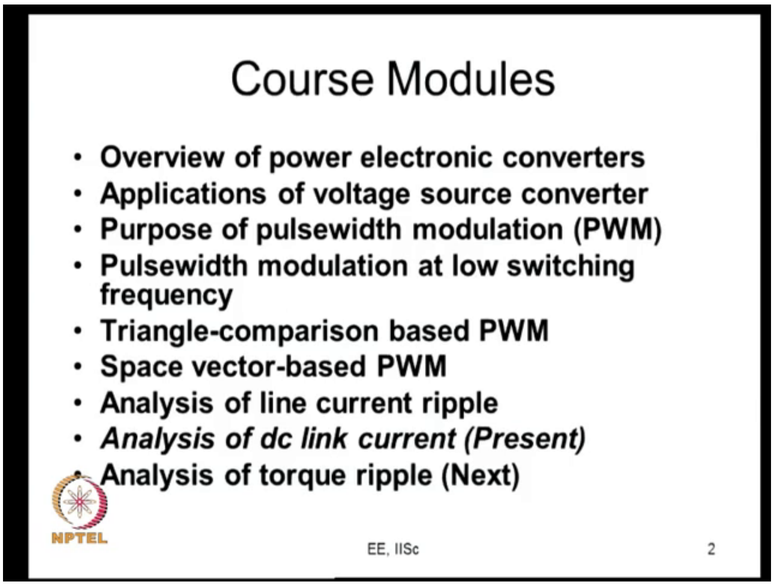

And the overall course module is just for a quick recap, first what did was we has an overview of various power electronic converters, where we looked at a DC DC converters then we looked at DC to AC, and in the DC to AC we had two level I mean voltage source current source and two level multilevel and so on so forth. Then we looked at say this application of voltage source converter in the second module. So, such as motor drive or the static reactive power compensation, and active front and converter etcetera then we started looking at the purpose of pulsewidth modulation basically you want to have the desired fundamental voltage and you want to keep the harmonics and their effects low. So, that is the purpose of pulsewidth modulation we revived the basic things modulation such as the Fourier series and so on and so forth.

Then we try to working on pulsewidth modulation at low switching frequency that is we have only switching frequency like 5 time of fundamental frequency or 7 time of fundamental frequency and so on how do you go about doing things like that. So, we looked at the pulsewidth modulation low switching frequencies and the other module.

Now then subsequently we have looked at the triangle comparison based PWM; when you are generating pulsewidth modulation, one of the common ways of doing it as triangle comparison. That is what you have is you have 3 phase modulating signals and you compare them with a triangular carrier, and you compare and the and based on the comparison you switch your inverter like if you say that you know your modulating signal is greater than the carrier and the top devices on, other ways the bottom devices on and so on and so forth as we have already seen.

So, today we may have to have a quick look at some triangle comparison methods also and also may be space vector based PWM today for the purpose of our talk here. So, we know the space vector based PWM is another way of generating PWM signals. So, these two are the widely used on approaches for generating, and as we have discussed you know like most methods can be done in both ways like sign triangle or third harmonic injection etcetera, they we normally use triangle comparison; conventional space vector PWM etcetera bus clamping PWM etcetera. So, these are all this is basically these two are two different options like how you go about generating the PWM wave forms.

Whereas we have a class of bus clamping PWM techniques, which we call as advanced bus clamping PWM methods which will require space vector based approach for PWM generation. So, that is one burden of our the entire course is that, we have been trying to say that space vector PWM is more general than triangle comparison PWM. So, whatever you can do a triangle comparison PWM, you can do it with space vector, but certain things that you do with space vector cannot be done with triangle comparison PWM. So, these 3 modules we focused on how you generate the PWM signals this is at the low frequency and these two are at the high frequencies 1 by triangle comparison and the other 1 by so called space vector approach now.

After that we have got into the analytical part of that. First is we wanted to see how to analyze the line current ripple, that is when you are having a fairly higher switching frequency you know you really cannot do your Fourier series kind of analysis there are too many switching angles, there and what we are also interested normally is the RMS value of this current ripple. So, we tried doing some analysis and how exactly you can arrive at that. So, that you can evaluate and compare different PWM methods based on the line current ripple, and you know this line current ripple as you know it comes because of the non-sinusoidal components in the applied voltage.

Then after analysis of that, we have gone into this analysis of DC link current which is what we have being doing presently, we did it in the last lecture and we are going to consolidate on that we had a few discussions on that we are going to consolidate those discussions and take it to some kind of logical conclusion in this particular lecture. And after that we will be going into this analysis of torque ripple that is when you apply these kinds of outputs of an inverter at to an induction motor drive, the fundamental component produces of the output voltage produces this fundamental current and fundamental flux and that produces steady torque.

Whereas all the harmonic voltages they end in generation of certain ripple. So, this is what we will do in the next module, which would be in the next couple of lectures now. So, as I said here now we are focusing on the analysis of DC link current.

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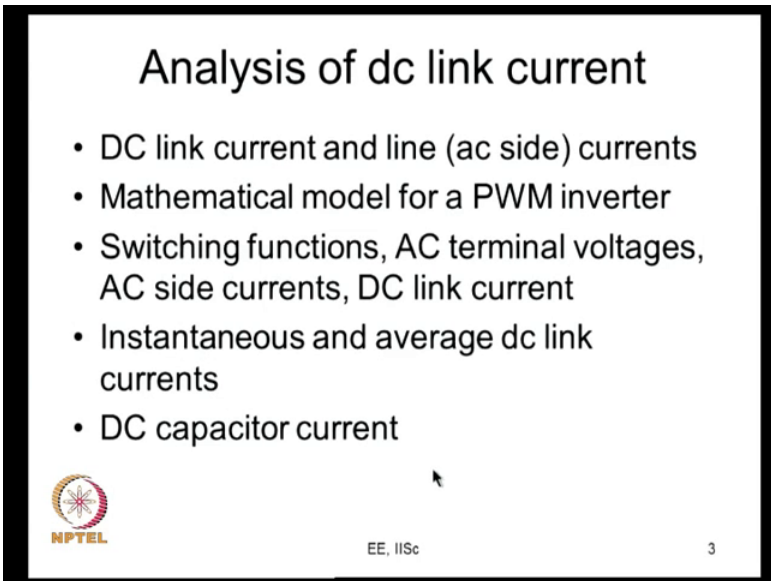

And in this there is various things that we are trying to cover is, we just trying to cover the relation between the DC link current and the line currents the AC side currents now. What I have observed while teaching this course at the Indian institute of science and otherwise that, students generally are very familiar with this line current they get very quickly familiar with this. Because the DC side voltage is fixed and we multiply with some modulating signal and we generate an AC output, and once there is an AC output voltage you have a load that load may be in r l load or an induction motor load or whatever you have good harmonic module and the fundamental module available for that. So, it is easy for you to understand what the AC side currents are.

So, you know the moment the PWM output voltage is given most students are quick at coming up with a how the line currents look like whereas, students normally find it difficult to come up with them into conceive, how this DC link current would look like. It is a very important thing because this is the kind of current that is actually drawn by the inverter. So, that is 1 reason why it is important and the other reason it is important is this DC link current has a DC component, which is what you know it is a measure of power that is being consumed by the I mean power being converted by the converter. This DC link current has a very major ripple component. So, it is a high ripple component and these ripple component would flow through the DC side capacitor.

So, as you know for a DC capacitor RMS current rating is very very important current rating. When you choose a DC capacitor, you need to specify not only the capacitance value not only the voltage rating you also have to choose the RMS current rating and that really comes from the ripple components of this DC link currents, and overall for an understanding simple voltage source inverter, it is very essential that we need to link these two that is to be able to relate the line side currents with the DC link currents. So, that is something we would try to consolidate our understanding in this regard into today’s lecture.

So, to be able to relate the two side on DC link current and the AC side current, what comes in between our this switches of the inverter. So, we are actually going to do mathematical model for PWM which we did in the last class, like basically we are looking at this DC voltage and certain switching functions. So, there are bunch of 6 switches pairs of complementary switch, for every pair of complementary switch we define 1 switching function. Based on the switching function we try to derive the voltages and the inverter voltages are fed to the load model, and then the lead currents the load currents are fed back when using the load currents on the switching functions again we come back to the DC link current.

So, we had a brief discussion on the we had a discussion on the mathematical module, last time we also may have a quick review of it now. So, for this we will use switching function as I just mentioned while back. So, what are the switching functions? You basically have 6 switches as I said while back and each switch in every leg; they have two switches with switch in a complementary fashion. So, you define what is called as a switching function if. Let us say if switching function is one particular switch on or it is 0 otherwise. So, that is the idea behind a switching function and so on. The AC terminal voltages AC side currents and from AC side currents and the switching functions, you can actually go to the DC link current.

So, we would have a quick recap on that and we would deal with this instantaneous and average DC link currents at greater length, and we would also try to come up with DC capacitor current. And we will try and see how an analytical expression can actually be written for example, DC capacitor current. Let us say you have voltage source inverter and it is being operated at a particular DC voltage at a particular modulation index and switching and you know power factor and so on so forth.

So, under circumstances, can we come up with an expression for the RMS current is what we would look at? We may not be able to do the details of these derivations, but at least we can say how to go about deriving an expression for this RMS DC capacitor current. So, an expression for RMS DC capacitor current would mean, you will have to have an expression for see RMS DC link current. So, let us just get started with today's lecture. So, which is 28th of the series and we call it DC link current and DC capacitor current in a voltage source inverter.

So, we know voltage source inverter and we just today we want to improve our understanding on these two, which are actually related closely related to each other in today's lecture.

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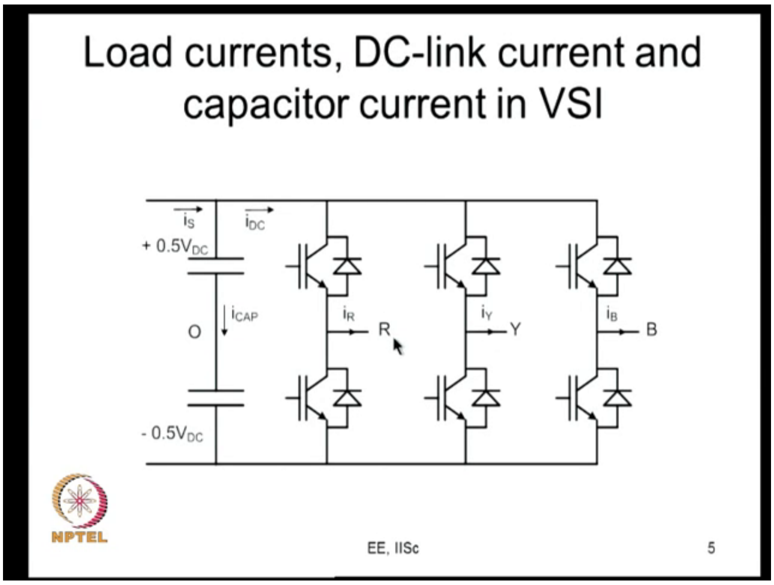

So, let us just get into this now. So, let us look at voltage source inverter. So, this is a 3 source voltage inverter, as I have said n number of times before every leg is a single pole double through switch and in the pole is the midpoint of this leg and this is connected to leg which is usually inductive and the through of the switch are connected to across the DC terminals and so this single pole double through switches are really as using two complementary switches and the switches are capable of conducting in both the directions.

So, you have R phase leg you have Y phase leg and you have B phase leg and the load is connected this is 3 phase load. So, it is R Y and B terminals are actually connectivity and the load is not shown here, and the load currents are going to flow here this is the indicative positive direction. So, if i R for example, is positive this is the direction it is current flows if i R is negative then the current is actually flowing in the opposite direction. So, and as we also discussed while we are discussing voltage switch inverters we can see how you know which device conducts. For example, we have transistor and anti parallel diode in the top, you also have a transistor and antiparallel diode in the bottom.

So, the question is who will conduct when. So, let us say this is the direction of current flowing here that is from my left to right as shown by the arrow mark here. So, in that case let us say if the top devices been switched on, the bottom device will be switched off and the top transistor will actually conduct this current. So, let us say then when the top switched off and the bottom switched on, in that kind of a scenario you have difference in this now. So, who will be conducting? The top transistor can no longer conduct. So, the bottom diode diode will come into conduction. So, for this direction of current it would be the top transistor or the bottom diode.

For the opposite direction of current it would be the top diode or the bottom transistor. So, whether top or bottom conducts difference on whether the switching whether the gating signal to the top is higher, gating signal to the bottom is high. Otherwise you know whether in the IGBT or the diode conducts depends on the direction of the current. So, we have said that you know this is been you know required bidirectional current conduction and so on and so forth.

We have also mentioned that some point of time that voltage source inverter, any voltage source convertor basically means at this pole you should be able to apply the this voltage are this that voltage regardless of the direction of current excuse me.

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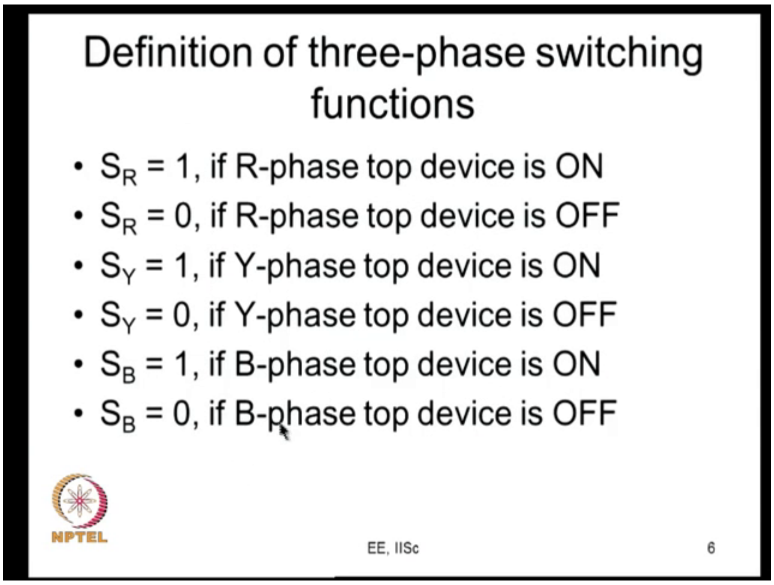

So, this is what we have been now. So, what I am trying to say today is let us say we are we considering fixed DC voltage. In actual cases there will be certain amount of ripple on the voltage, but we are ignoring all that and we are considering it to be fixed voltage now. Then what are doing you are modulating the inverter? Basically we are going to be switching all these that is you are this is switched in a complimentary fashion, and the way you switch the R phase you would switch the Y phase in the same way. Except that the gating signals to be shifted exactly by 120 degrees at the fundamental frequency and the way you switch the Y phase the same way you will be switching the B phase except that the B phase gating signals would be switched, when I will be shifted by 120 degrees from that of the Y phase. So, this is what we are going to be doing now.

As a result of this switching action so, you will get voltages here which will be you know plus v DC by 2 minus v DC by 2 and so on so forth. You will get line to line voltages which may be plus v DC or 0 during the positive of cycle and minus v DC and 0 usually during the negative off cycle and through the load will have currents flowing through the AC load is will have current flowing there now. So, you will have an AC voltage available and that AC voltage, but not necessarily sinusoidal, it will have a fundamental component plus it will have switching frequency components and that will drive these currents i R i Y i B through the load and which also have sinusoidal fundamental I mean fundamental and harmonic components now.

So, now we are now trying to say these are the 3 phase load currents, for most of our analysis regarding the DC linker and we would ignore the harmonics part in that. As we will see that part is actually pretty small. So, we would consider let us say mainly the fundamental component in doing this now. So, the next question is if this current i R is what is flowing here. So, what is the current through the top device or what is the current to the bottom device let us say; what is the current to the top device. So, this i R flows through the top device if the top devices on. So, otherwise it flows to the bottom device that is as simple as that now ok.

So, now what is this i DC? It will be this top device current plus that top device current that sum of all this thing is called the DC linker current and please remember in our discussions here we call this as i dc and this is what is really we call it as the DC link current here. So, this DC link current then you have this i S which is we use a term s because this comes from the source like it comes from the DC source now. So, the difference between these two flows through the capacitor current. This current as you can very well make out it is going to have a DC component and the ripple component.

So, most of the ripple component is going to flow through this or we are going to assume the entire amount of ripple component flows to this and the DC component cannot certainly flow through this that comes from there now, and a you know this is how the overall things are. So, this is the DC link current and the DC link current can actually obtained as we will show mathematically by product of switching function of this current for this leg this leg and that leg. So, let us just look at that mathematically now.

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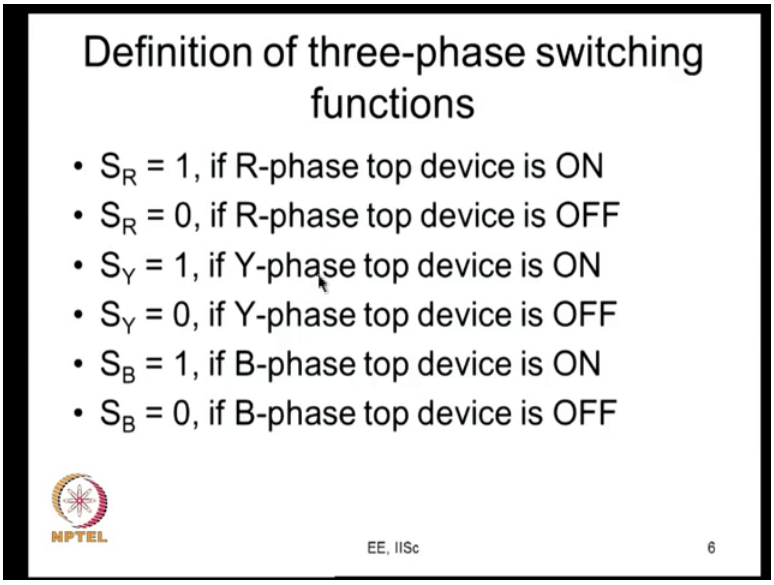

So, we will start with what is switching function as we did in the last class we just recap here. So, we say that switching function is indicated by S and whether the switching function corresponds to R Y or B phase likes this is indicated by subscript R Y B and when S R is equal to 1. We mean that the top device is on. R phase top device is on when S R is equal to 0 we mean the R phase top device is off that is the R phase bottom device is on. The same thing is valid for S Y and you know the Y phase and the B phase switching function S Y and S B respectively.

So, these are the switching functions which we defined for our convenience. So, the switching function is going to actually indicate; let us say if you say S R 1 is equal to 1 S Y is equal to 0 and S B is equal to 0 what we mean is, S R 1 equals 1 means R phase top device is on, S Y is equal to 0 means Y phase bottom is on S B is equal to 0 means B phase bottom is on. So, this is the way to define the different switches which are actually conducting.

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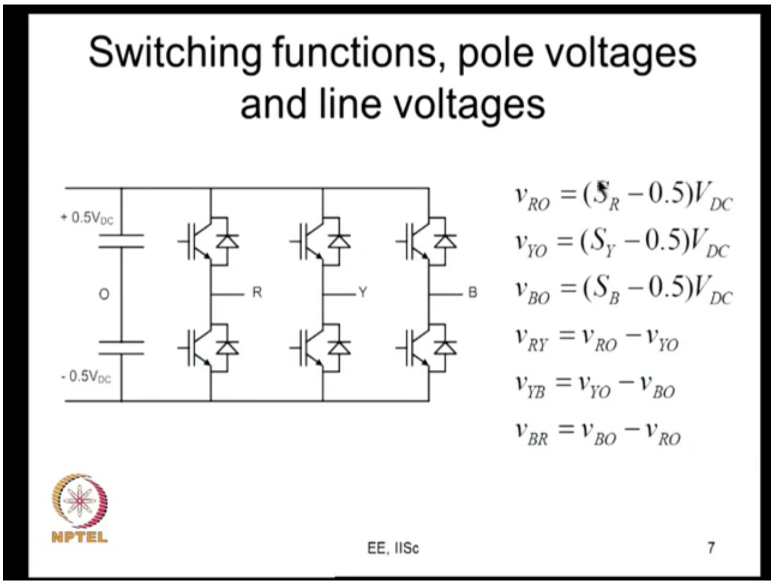

So, now you have that here, you can write down your pole voltages in terms of these switching functions now. If it is a S R if S R is 1 1 minus 0.5 is 0.5 0.5 times V DC. So, your top devices on you get your pole voltage to be 0.5 V DC; if the bottom devices on then S R is equal to 0. So, you will get minus 0.5 V DC. So, you can see that it is fairly clear from that the same way you will get v YO to be plus 0.5 V DC if S Y is 1 that is the top is on you will get it as minus 0.5 V DC if S Y is 0 or the bottom is on same way about the B phase.

So, you have your 3 phase pole voltages. So, you can see that now we are related in these pole voltages on one hand with the switching function and the DC bus voltage now. So, there are some expressions on pole voltages. So, the line to line voltages as we know the differences of two pole voltages, if I looking at v RY this is v RO minus v YO similarly v YB is v YO minus v BO same way about v BR. So, you get the 3 phase line to line voltages now. So, these line to line voltages are actually applied on the 3 phase load which is not shown in this picture, right.

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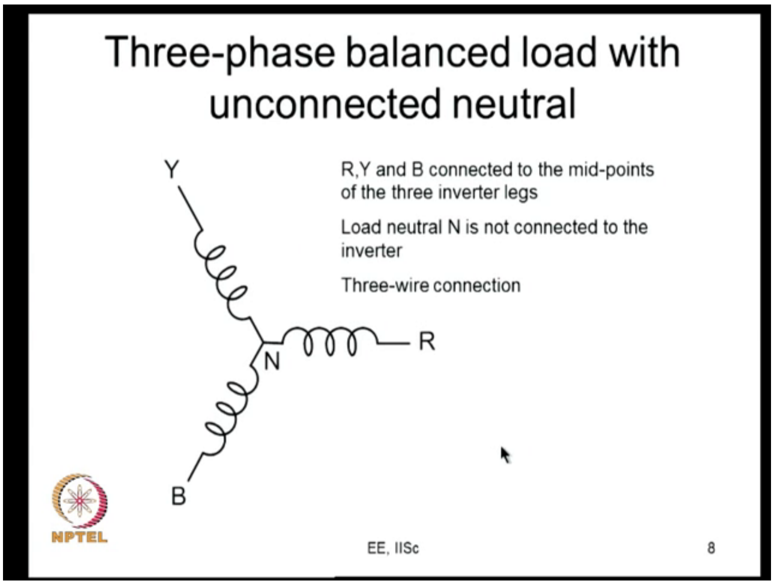

So, let us say you are considering a 3 phase balance load. So, these are R Y B terminals, we have now assumed a star connected load with the neutral point N.

But this neutral point N is not connected anywhere as we have always be assuming in this course. So, this is actually a 3 phase load, it is a 3 phase 3 wire load it is only R Y B are connected. It is not a 3 phase 4 wire and neutral is not connected. It is a 3 phase 3 wire load further it is a 3 phase 3 wire balanced load. So, like if you apply balance voltages they will draw balanced currents.

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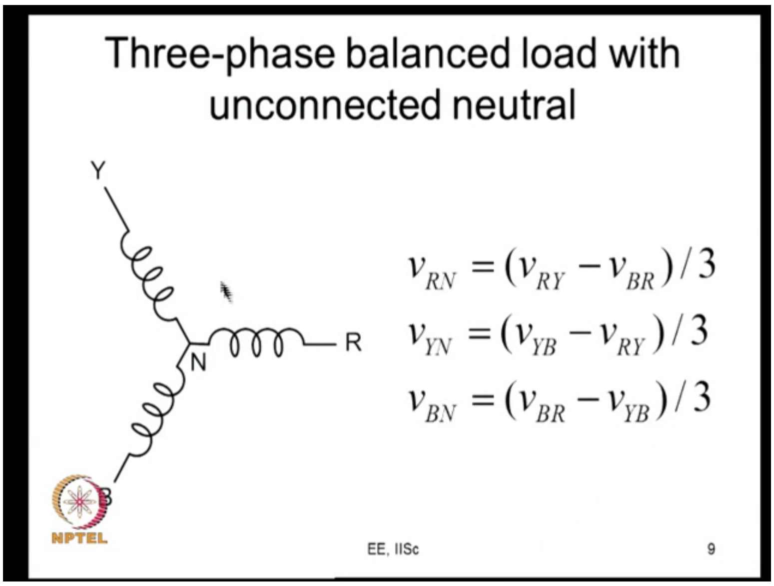

So, for just simplicity let us just consider like you know r l load as we go further, but before that if the load is balanced what you can all say always observe is you can express your V R N, this is the phased neutral voltage applied on the load in terms of v RY and V B R that is given by this equation here. So, v RY minus v BR the whole divided by 3 would give you v RN whenever the load is balanced. So, similarly you have v y n and you have v BN. So, you have expressions for this. So, you can now also have the line to neutral voltages for the 3 phase load.

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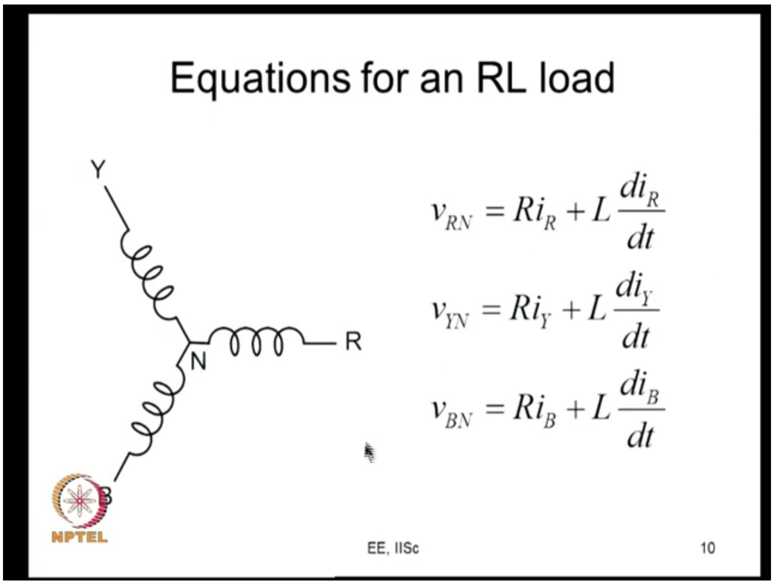

if you are considering RN load for example, this v R N v Y N v B N are fit to the load if the load happens to the RN load as I mentioned previously this is the circuit equation.

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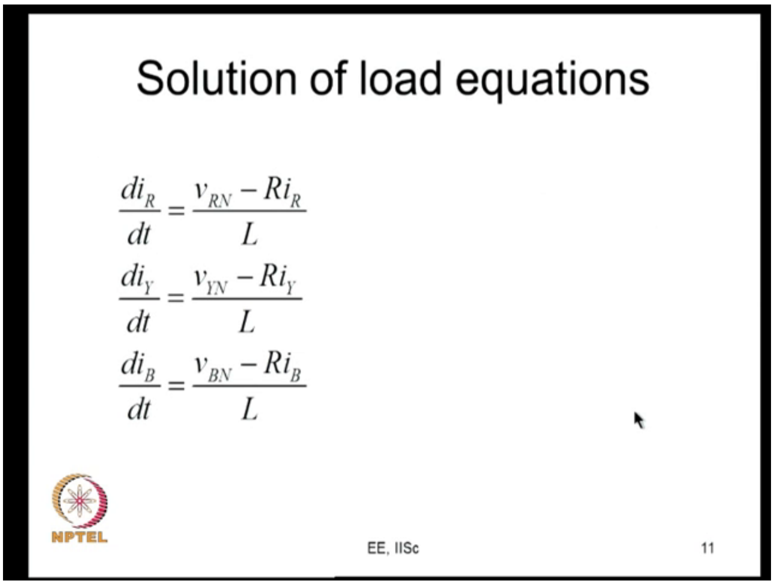

So, you can solve this circuit equation that is d i R by d T etcetera. So, you can integrate d i R by d T that give you i R and. So, once you have a i R you can multiply it by r and subtract from v RN the whole thing can be divided by L. So, you can you will have a loop you will have you know the first order dynamic system.

So, this is be another first order system this would be another first order system, you can just solve them they will be give you i R, i Y and i B.

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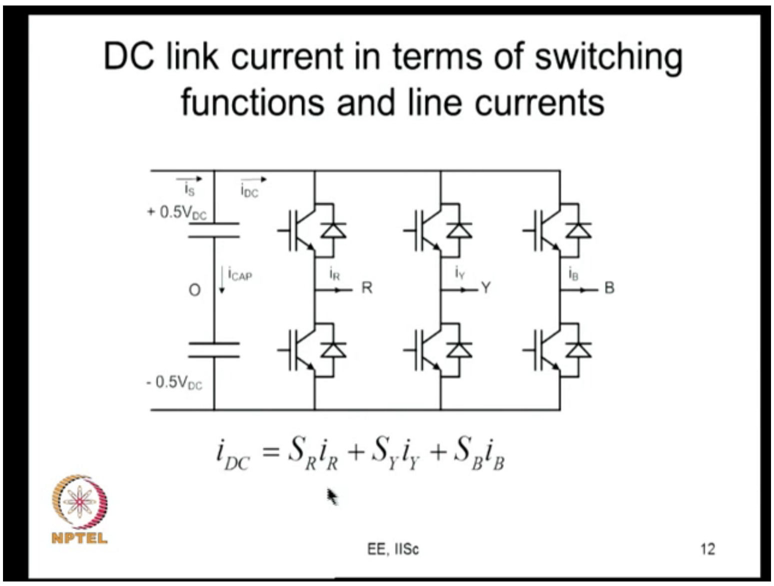

So, these i R i Y and i B can now be used. So, this i R i Y and i B they come from the load. So, the load it is not necessarily it is an r l load, you can it can also be a induction motor load. So, in this case this equations etcetera can be more complex, but you know there are courses and there are motor drives etcetera we would have studied about modeling of a motors induction motors and so on and you will know how exactly to simulate from those courses.

So, those i R i B i Y and i B from the load has to come back here. So, this is what you are using now. So, now, that i R multiplied by this switching function S R is going to give you the top device current. So, if S R is equal to 1 then the top device current is i R. If S R is equal to 0 the top device current is 0. So, the top device current here is S R i R again what is the top device current here? It is i Y multiply it by S Y. So, S Y times i Y the same way here the top device current is S B times i B. So, the sum of these 3 top device currents it is your DC link current and i dc can be written like this as a sum of this product of switching function and line current now.

So, this is the i dc let us say if S R is 1 S Y is 1 and S B is 0, it is basically i R plus i Y or let us say S R alone is 1 others are 0. So, i dc is equal to i R. So, now, in i R you have fundamental component you also have the harmonic components and you are considering fairly high switching frequencies. Now the harmonic components are going to be very low ripple point of this going to be fairly low. So, this i dc expression you will get it as reasonably valid you know it will be correct if even if you approximate this i R by it is fundamental component i Y also by it is fundamental component and i B also by it is fundamental component.

So, what we will do here itself is that, you will actually consider only the fundamental components to make the calculations easier because the contributions of the harmonics flowing in R Y and B the harmonic themselves will be low when you are talk of high frequencies. Low meaning low compare to the fundamental component and their contribution to the i dc is going to be negligible. So, this is the equation for the instantaneous DC link current, in terms of switching functions and the 3 phase line currents. So, we go further now.

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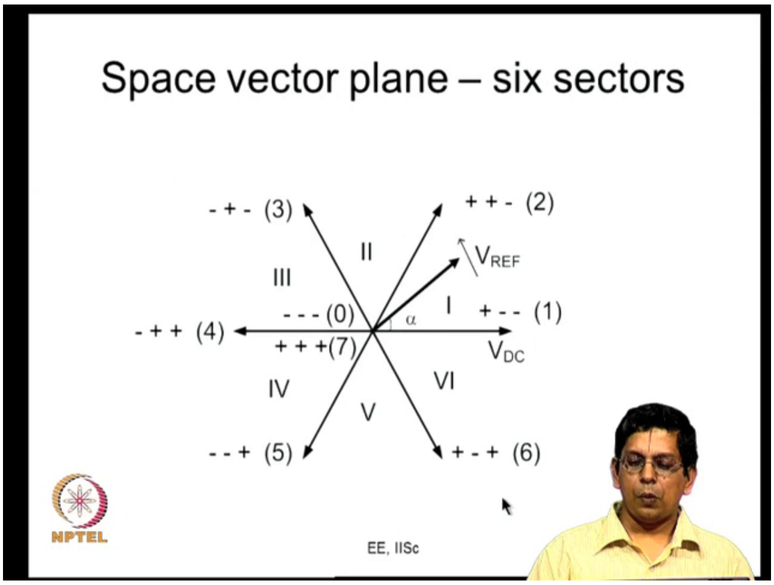

So, before we go into studying how the DC current is going to vary, it is better that we take a quick look at this space vector plane. The about the 6 voltage vector active vectors and the 0 vector, there are you know 8 different inverter states; this is when all the bottom are on this is when all the top are on. So, in this states you have a null vector getting applied.

Now, what you have is there are 6 other states and in these states there some these are actually called active state as we have seen earlier. So, these active state produce active vectors which are like this and based on the conventions that we assume, the active vectors are actually of magnitude v DC. As I have also mentioned before if let us say you use the slightly different kind of space vector transformation, you may have a constant multiplied by v DC. Some constant like you know it could be root 2 by 3 under square root and so on so forth. So, things often, for our kind of space vector transformation is vectors give you a voltage v DC and a for the active vector all produce voltage vectors of magnitude V DC, but the angles are different as we have seen before now.

For today's context actually this v DC does not matter so much, we are actually looking at you know why we want to look at is a we are shortly going to relate the DC link current with the inverter states. The DC link current we have related to the switching functions. If you put the 3 switching functions together that is the inverter state. So, it is better that we take a look at this.

So, whenever the voltage vector is here, that is the voltage vector if the voltages provided as a space vector. What if the voltage reference provided as 3 phase sinusoids or 3 phase sinusoids common mode added; what you do is, you transform those 3 phase sinusoids into this space vector domain and you will get a reference vector. So, if that reference vector happens to be in sector 1, you would normally be applying these to active vectors plus minus minus and plus plus minus and you will be using this null vector. So, this we call as sector 1 as the region between active vector 1 and 2. We call this as sector 2, sector is also known as hextant.

Because there are you know these divide pair into 6 portions that is the region between active vector 2 and 3. So, similarly this is sector 3, this is sector 4, this is sector 5 and this is sector 6. The operation of the inverter symmetric; as you can shortly see the DC link current will also actually be symmetric and this actually will have periodicity of sixty degrees as we will see. So, first let us start looking at sector 1 and see the how the DC link current is.

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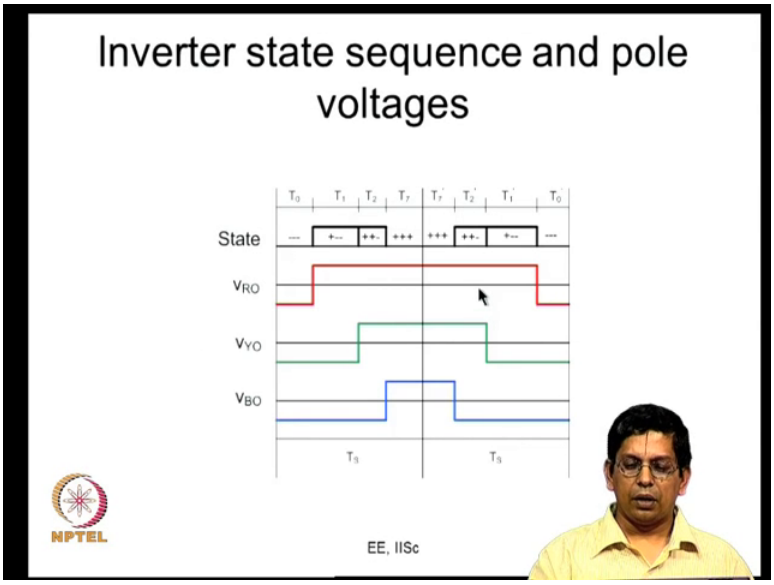

So, sector 1 as I mentioned you are going to apply active vector 1 and active vector 2 which are indicated by this plus minus minus and plus plus minus. So, I mean this may be a little cramped, but you can certainly see when it is plus minus minus you can see R phasevoltage is high, Y phase is low and B phase is low. So, the inverter state here it starts from 1 0 state minus minus minus, that is all v RO, v YO and v BO are negative and it goes to plus minus minus where v RO switches pass to v YO and v BO are 0 again you have this T 2. So, plus plus minus right then you have plus plus plus were all of them are on. So, this is start from 1 0 state goes through 1 active state then the second active state and finally, goes back to the other 0 state.

The next sub cycle are half carrier cycle it will have the reverse sequence plus plus plus then B phase switches and plus plus minus, then Y switches then becomes plus minus minus then R switches and becomes minus minus minus. And if you look at 3 phase pole voltages this is how they are R Y B, B Y R that is the sequence in which they are actually going to be switching. So, this is just to you know look at the inverter state sequence and this is the sequence in which the inverter states are applied by all the continuous PWM methods wherever your modulation signal is a continuous function of angle.

So, for example, sign triangle PWM, third harmonic PWM conventional space vector PWM etcetera will have these kind of sequence 0 1 2 7, 7 2 1 0 and if T 0 is equal to T 7 that is conventional space vector PWM So, conventional space vector PWM make sure that these 2 0 sates are applied for equal durations of time. So, what is actually listed? So, it is actually for the conventional space vector PWM.

So, now the next step is we are going to see when the inverter states are varying like this, how is the DC link current is going to vary that is the reason why we are now having recap on this ok.

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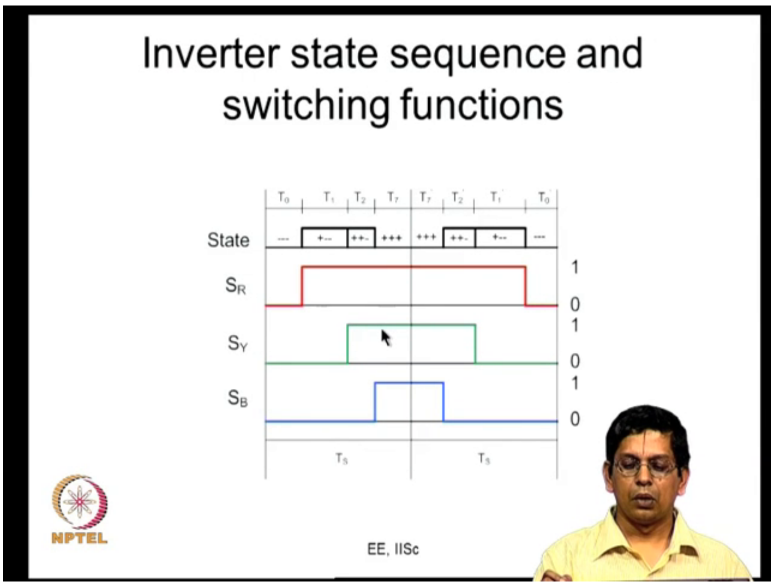

So, before we go in to that, we looked at as pole voltages now let us just look the look at them as a S Y S R and S B switching functions. So, when you go through inverter states 0 1 2 7, 7 2 1 0, now 0 to 1 transition S R goes from 0 to 1 again this is the switching function for Y phase, S Y goes from 0 to 1 here again S B goes from 0 to 1 and then S B switches from 1 to 0, here S Y switches from 1 to 0, S R switches from 1 to 0. So, in this interval R phase top devices is on in this intervals R phase bottom device is on the same way about Y phase here the Y phase top is on here the Y phase bottom is on.

So, now this is the inverter state sequence over two sub cycles and this is how the switching functions vary S R, S Y and S B are vary.

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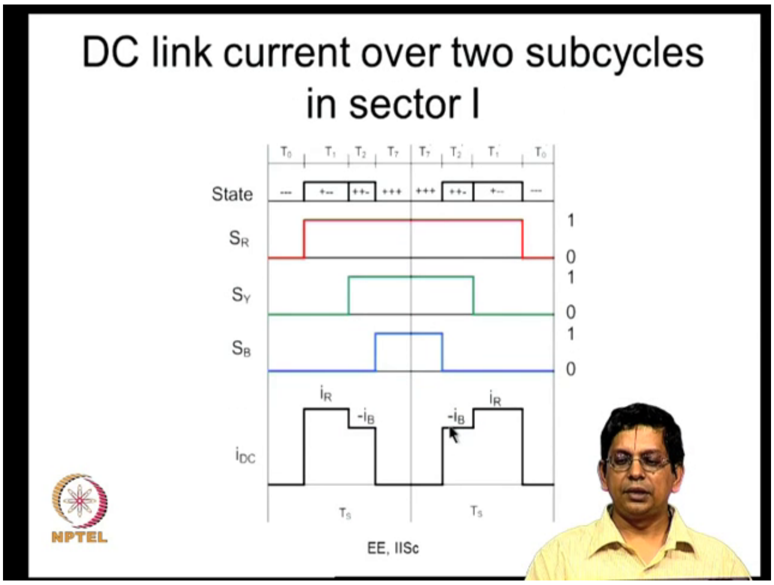

Now, how do the DC link current vary. So, we have just reproduced this state and the switching functions from the earlier graph. Now what happens you have minus minus minus what does minus minus minus means? All this switching functions S R S Y and S B are 0 that is all the 3 bottom devices are on. So, the top devices are of. So, you go by your expression S R i R that is 0, S Y i Y is 0, S B i B is 0. And therefore, the DC link current is 0 that is there is no current flowing between the DC side and the AC side and That is the reason why you call it as 0 state. During a 0 state there is no active transfer of power between the DC side and the AC side. So, the load keeps is free willing. So, the 3 phases of the load are shorted and water is a stored energy is just kind of free willing, there is no power flowing from the DC side to the AC side. So, that is a reason we called 0 state and this is now what you get here the DC link current is 0.

Then we move on to the active state what happens in the active state? You have plus minus minus now S R is now 1, S Y is 0 and S B also is 0. So, you have your S R i R is equal to i R this is the R phase current the R phase current is actually a sinusoidal quantity it will vary over a long cycle, but what are we looking at? We are looking at over a 1 sub cycle this could be let us say 100 microseconds kind of a interval and your this is the another sub cycle this could be another microseconds also.

So, 50 microsecond and 50 microsecond kind of a interval, while i R can be periodic over 50 microseconds or 15 or 25 milliseconds 10 milliseconds whatever. So, you can see that we are looking over lower a short interval and therefore, in as I mentioned already in i R we have ignored the harmonic component, we are considering only the fundamental component which sometimes you would call as i R 1. So, the fundamental component and the fundamental component is sinusoids we are viewing over a small interval of time. So, that looks like a flat line. So, this is your i R.

Now, what happens when S Y switches up S R is already high S Y is also become high, S B continuously below. In this interval it is S R i R plus S Y i Y and the S B is 0. So, S R i R plus S Y i Y is simply i R plus i Y and it is a 3 wire load, it is a 3 wire load therefore, i R plus i Y plus i B is equal to 0 or i R plus i Y is equal to minus i B. So, the current is minus i B, the DC link current that actually. If you look at the B phase current now R phase current is passed. So, B phase is now let us say it is closer to the positive peak then B phase kind of you know 120 degree ahead of that. So, this is B phase actually negative. So, minus i B would be a positive quantity here. So, you have this current flowing the second active, state is applied the amount of DC link current is different.

But you know there is some DC link current. Once again after B also switches what happens now all the top devices are on. So, S R i R is equal to i R S Y is 1. So, therefore, S Y i Y is equal to 1, S B i B is equal to i B. So, i R plus i Y plus i B is 0 because it is again a 3 wire system the sum of 3 is 0. So, once again you see there is an inverter state being applied. So, actually if you look at DC link current it is very difficult to measure a DC link current you know why because you really cannot put a current sensor. There the

DC link current we are actually why are we are measuring we just go back to the previous slide and take a look at that this is between this capacitor and here.

And we want the actual parasitic inductance to be extremely low. So, we would all the connections of the terminals of the capacitors and those of the devices have to be connected in such a fashion, that the stray inductances are very very low. This is because we want to protect the devices from over voltage spikes. If there are stray inductance during d i by d T you will have over voltage spikes and because of over voltage spikes the devices will get killed, and therefore we should do not happen. So, you will have a normally you know very tight design, there will be a sandwich bus bar kind of a design which will make sure the stray inductances are very very low. And therefore, it is almost impossible to measure current area, you need to make some special arrangements you use some co axial current transformer or some such arrangement to really make sure that current there.

If you measure the current, what you will actually see is how this is. If you actually measure this you will see that the current goes through this 0 at this start and end of every sub cycle. From these 0 themselves you can make that this is 0 state is being applied and whenever the DC link current is nonzero, you can know that an active state is actually been applied now. So, this is something which we actually covered in the last class, but this this time I just wanted to sync in. So, we are just doing it in some greater amount of detail. So, this is i R and this is minus i B.

So, this is again now what happens in the next sequence you are going to 7 2 1 0. So, here again it is 0 here again it is 0 it is minus i B and i R what is a difference between this sub cycle and that sub cycle. Obviously, the state sequence is the reverse it is 0 1 2 7 and this is 7 2 1 0. Other than that you can see that the time intervals have slightly change, because the 3 phase references are the reference voltage vector are slightly changed.

Under steady state condition the magnitude of the reference vector would not change, but the angle will change by an angle equal to omega T s where omega is your fundamental frequency and T s is the sub cycle time. So, because that angle alpha has changed and the 3 phase references have changed slightly, you have your active vector time T 1 slightly change to T 1 dash T 2 would have slightly changed to T 2 dash and similarly T z would have changed to T z dash and now we are calling this T o dash and T 7 dash.

So, there are some slight changes that is because the reference vector angle has changed now. Other than that what is a difference i R itself would have changed slightly, which is not very visible from this curve. So, you are talking of i R is equal to some ii m sign omega T minus 45 degrees that is let us say is your fundamental current now. So, you are now looking at as some angle for example, something like omega T c equal to 100 degrees.

So, here this angle may be something like omega T c equal to 105 degrees are so. So there is a small difference that you might actually get here which I have not shown clearly on this curve. So, this i R and this i R will be slightly different again this minus i B and this minus i B would be slightly different at remember this minus i B is i R plus i Y. So, if you look at it over a 60 degree interval are much longer than interval, you will see this current. If let us say we are talking of unity power factor load than your current and voltage are in phase the voltages in sector 1 it has already crossed the 0 curve, it is positive peak. So, the current would have also cross the positive peak.

So, as you go in this direction if you are talking of unity power factor load, you will find this i R is actually dropping slightly. So, this level will go on dropping, similarly if you go little earlier you will see that this level is actually going on increasing. So, you can actually see those variations out of this 3 phase currents really here, which I am not showing I am showing you only two sub cycles which is actually very very small duration within this sector 1 ok.

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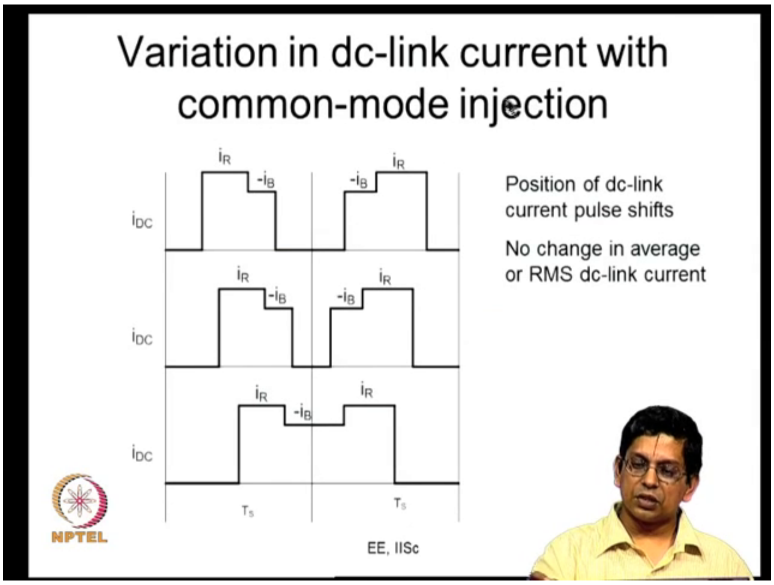

Now, how does with vary from common mode injection, that is you are generating a reference voltage, you are applying your active vector 1 2 etcetera active vector 1 2 time you have calculated null vector time, you have calculated you have applied them equally, which means actually some common mode injection you can divide them on equally also many ways. So, T 0 can be 0.1 T z T 7 can be 0.9 T z for example. So, which that is you can divide this zero vector time in different ways which is equivalent to adding common mode voltage to all the 3 phases.

So, if you are m R m Y m B or your 3 phase modulating signals, you can add some m c m some small quantity to all the 3 of them. So, what will happen now? As we have already see, if that if a small component is added for example, then all this switching will actually shift to the left by equal distance. If some small positive value subtracted then all negative value is added, then you know you may see that all of them will shift to the right, but by the same extent.

Therefore what will happen is here you will see that the sum of this. So, it is a same inverter same DC bus voltage same modulation index and a same load everything same. So, and we are also looking at the same sub cycle, the only difference we are looking at is we are considering a case, where the null vector division is equal and we are considering. Another case where the null vector division is kind of unequal that is the common mode added here is different; the common mode added here is something else.

So, when you do this you are now done within such way fashion that T 0 has been increased and T 7 has been decreased. So, you what happens is you get same kind of wave forms for the pulse, except that this is just shifted a little this is when T 0 increases at the expense of T 7. Now on the other hand if T 7 increases the expense of T 0, you will find this whole thing shifting to the left. So, I have shown that this is shifting to the right.

What happens as an extreme case if T 7 becomes 0? If T 7 becomes 0 then this pulse and that pulse they come and combine with one another and you get a pulse of this nature. And now within every two sub cycles you will see one such large pulse here and the here you will see 1 pulse in every sub cycle.

So, in continuous PWM scheme such as conventional space vector PWM if you can look at the DC link current, you can actually say what the switching frequency is from that. So, this region from 0 because it is going through 0, this is 1 half of a switching cycle 1 sub cycle. So, this is actually 1 complete switching cycle all right. Now if this happens like. So, this goes around. So, what is the difference? The pulse position it actually shifts, but does the average value of i dc over the sub cycle changes no. So, whatever is the average value here, the same thing is the average values here also the same thing in average value is something also.

So, the average value of the DC link current depends on the active vector times, which have not changed as we discussed in the last class. So, this is the active trip time for T 1 this is T 2 that is not changed; what is changed even T z is not changed T z is divided between T 0 and T 7 that has changed, but T z is equal to T 0 plus T 7 that is still same. So, you have the same average values and also if you try to look at other things you will you will still get the same thing. So, this average value actually really it does not change for you within this.

So, what happens you can look at the RMS value, this RMS value is not going to change why I am talking of RMS over a sub cycle you are going to square this is going to become i R square this is going to become i B square and this is i R square multiplied by time T 1 this is i B square multiplied by time T 2 and this is what you are going to divide by T s to get your mean square value integrated now. So, here again this is i R square i B square. So, you try to evaluate this RMS value you will get i R square into T 1 plus i B square into T 2 the whole divided by T s. So, the whole thing under square root would be your RMS value. So, this RMS over thing also does not change. So, this is true about this sub cycle this is true about every sub cycle, which is at to come and every sub cycle which has been crossed.

Therefore what we can actually say is one thing that we can see is the PWM method like you know the you add this common mode and all that. So, that changes the harmonics in the line currents, but it does not significantly change the DC link RMS current. So, whether you talk of sign triangle PWM or third harmonic injection of course, you have the DC bus utilization is different that is another point so, but if you look at you know within that linear range of operation. So, they will not actually cause give you a considerable difference as for as the DC link current is concerned and it will follow from there that the capacitor current will also the RMS value of the capacitor current will also not change very significantly.

So, what will change, you can see that there is a small difference in the harmonic components. So, the pulses are actually same everywhere, but you can see the angles are slightly different. So, if you actually take harmonic spectrum. In these cases what they do is they do not use Fourier series, but they use double Fourier series to calculate such things. So, now, if you actually look at the harmonic spectrum, you would see that this harmonic spectrum and this harmonic spectrum will be slightly different that is all that (Refer Time: 39:51), but the overall RMS current does not change very significantly up with the PWM method to PWM method that is one important thing.

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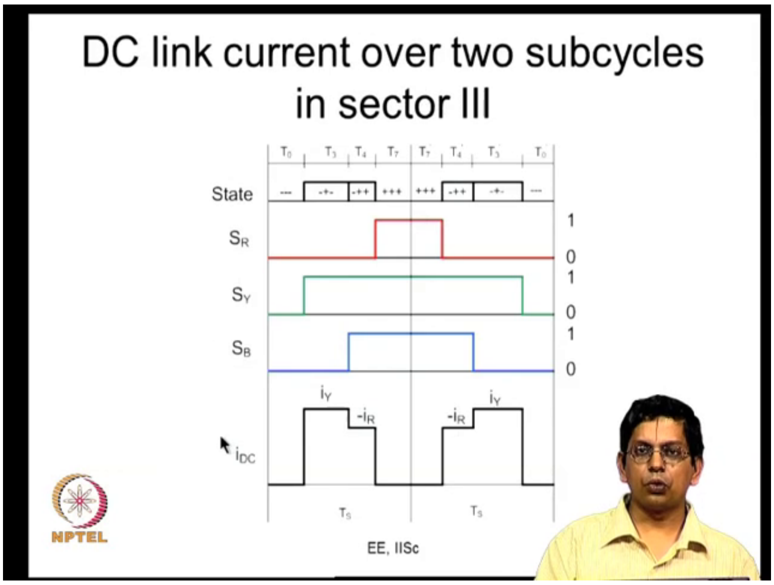

So, let us go further; now we have actually looked at sector 1 this is sector 1. Now we are going to look at sector 3 why? We are actually looking at the same thing 120 degrees later that is all that same reference vector same magnitude, but the angle is been increased by 120 degrees. So, we are now in sector 3 or the 3 phase modulating signals have changed. So, m Y is become what m R was before. So, you actually have 3 phase symmetries. Now, what happens is instead of 0 1 2 3 you 7 you have 0 3 4 7 like this and you have 7 4 3 0 applied in the other way now. That is in terms of inverter state that is minus minus minus it goes to minus plus minus, then minus plus minus and plus plus plus.

So, you look at the switching functions or the R phase is it switches last whereas, Y phase switches first like R phase used to switch in sector 1 now Y phase is switching now. Then B phase switches then it is R phase. So, the functions S Y S B and S R are like this now. So, what is your DC link current? It is still the same thing it is S R i R plus S Y i Y plus S B i B. So, if you take this 0 state as I mentioned before the DC link current is 0, again what happens at the end? It is plus plus plus everywhere. So, it is S R is equal to 1 S Y is equal to 1 and S B is equal to 1 therefore, DC link current is i R plus i Y plus i B which is again equal to 0 and how about in between? It is minus plus minus. So, the current is equal to i Y and how about this interval? Here it is S Y is 1 S B is also equal to 1. Therefore, the current is equal to i Y plus i B which is equal to minus sorry. So, this is i Y plus i B which is equal to minus i R, ok.

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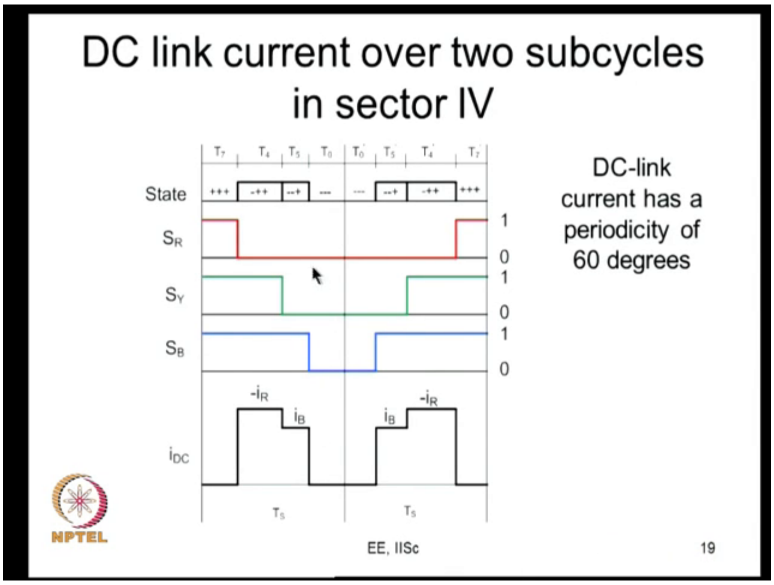

So, this is i Y this is minus i R this is minus i R i Y, but you can see this same shape is getting maintained. So, we have come 120 degrees later. So, now, i R has that value what i Y has that value what i R had 120 degrees before. So, there is 3 phase symmetry here. So, the roles of the phases have changed, but you can see that the DC link current is not changed that is if whatever DC link current you have in one sub cycle now, you will have very similar DC link current 120 degrees later this is because of 3 phase symmetry.

So, if you want to analyze the DC link current, you do not have to analyze it for the entire 360 degrees it is enough if you analyze for 120 degrees. In fact, it is not even 120 degrees it is only 60 degrees as I am going to show now. Now what I am going to look at is I am looking at sector 4 that is diametrically opposite from sector one. So, whatever was the reference vector I considered in sector 1, now I am considering the same magnitude, but the angle is 180 degrees added to whatever it was. So, if I consider something like angle ten degrees, I mean now I am going to look at 190 degrees right. So, I am now in the fourth sector now.

So, the sequence is it is like 7 4 5 0 0 5 4 7 it is like plus plus plus and then it is minus plus plus minus minus plus minus minus minus. So, it goes on like this now. And you can see that it switches in the same sequence R Y B, B Y R this is just the reverse of what it was. So, once again you look at when you have the 0 state, the DC link current is 0 again you have the 0 state the DC link current is 0. So, what you have here now? It is minus i R i B why it is minus i R? You have S R is equal to 0, but S Y is 1 and S B is equal to 1. Therefore, DC link current is equal to i Y plus plus i B and i Y plus I be is equal to minus i R and similarly if you look at here S R and S Y are 0, S B is equal to 1 the DC link current is equal to S B into i B which is equal to simply equal to i B. So, this is minus i R i B and 0.

But you can see that the current pulse this is same i dc as not really changed. So, here also you see that i dc is not changed. So, whatever was your DC link current in sector 1 some 120 degrees later it is same and again at 180 degrees also it is same. So, whatever the current was at some angle theta and you know theta plus 120 and theta plus 180 they are same. So, that actually means in every sector it is going to repeat. So, it has actually got a periodicity of 60 degrees. So, whatever it was at some angle 60 degrees later also it is a same thing.

So, this actually repeats that is the periodicity of 60 degree. You can actually verify this mathematical now we are actually trying to work out illustrative examples, and we are trying to understand the problem, because understanding the problem is very essential part and then you can write down the equations correctly it is a question of deriving things. So, we are now trying to do it. So, we are taking to time do this. So, you can see that.

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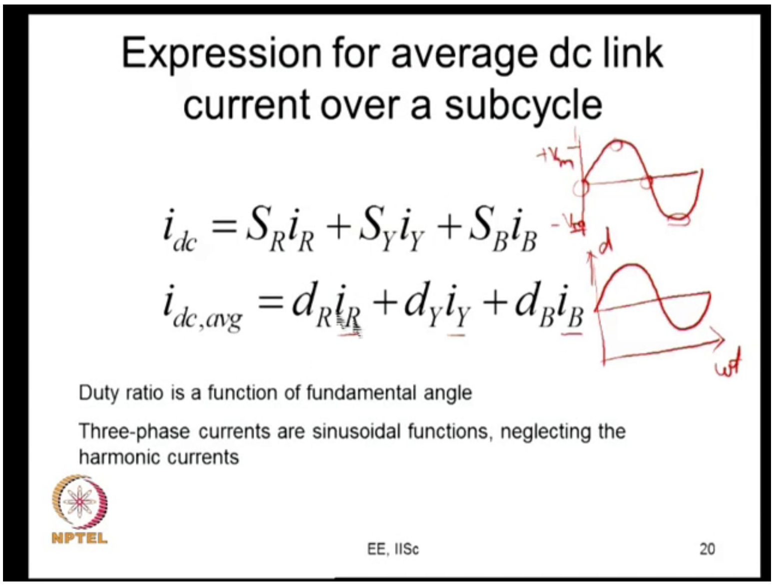

You know the kind of periodicity you can appreciate, which you can actually do mathematically which is what we can get started off with.

This is an expression which I have been I have said at least 100 times now. This is the instantaneous DC link current, all these are instantaneous quantities as I mentioned before in i R i Y i B we have neglected all the ripple of the harmonic components and we are considering only the fundamental components and S R S Y S B are instantaneous switching functions. So, they are discontinuous the moment it switches 1 becomes 0 or 0 becomes 1. So, this is how it is going to be and you get i dc. i dc will also be a discontinuous function of time because S R S Y and S B are discontinuous functions of time, on the other hand i R i Y and i B are continuous functions of time.

So, the first averaging that you can do is over a sub cycle, what you do is actually over a sub cycle. One way to look at it is like what we did before, you can look at whatever is this current for example, if this is your minus i R, minus i R can be multiplied by T 4 plus I B multiplied by T 5 the whole divided by T s is an average value of this DC link current. This is looking at the vector time the active vector times on the null vector time. The other way to do that is look at them individually look do it in a per phase basis, you are looking at the inverter state the totally, but you are going by phase by phase.

So, i dc it has these 3 contributions from the R phase top device I mean Y phase top and the B phase top device. Now if you average the i dc over a sub cycle we call this i dc average. So, what has been done? This is been averaged over a sub cycle duration that is i dc average. So, if you average all these quantities over a sub cycle duration, i R does not change at all because sub cycle duration is very small it is I m sign omega T. And let us say it is got like the time period is something like 20 milliseconds or 25 millisecond or 30 millisecond, now whereas the sub cycle duration we are talking of is something like is 100 microsecond or 50 microsecond you can see it is much smaller than that.

So, now this i R is practically constant. So, what you do is you are actually averaging S R. So, S R is sometimes 1 and it is sometime 0 and the average of this S R is the duty ratio d R. Similarly the average value of S Y over the sub cycle is the duty ratio d Y. So, it is the duty ratio of Y phase top device in that particular sub cycle. Similarly this S B when you average it over a sub cycle it is gives you d B. And therefore, your i dc average over a average over a sub cycle is like this.

So, what this is the product of duty ratio and current or the fundamental current. So, this is for one particular phase the same thing is added for all these 3 phases now. So, as you can very clearly see these 3 phase currents are sinusoidal currents and what about this d R d Y and d B they are also they are related to the modulation function. If the modulating functions are sinusoids then they are also sinusoids otherwise there is some common mode component there is some common mode components added now.

So, for example, if I am looking at sinusoidal PWM and I want write my d R how it is going to be the modulating signal is going to vary like this the modulating signals varies like this. So, this may be from plus V m to minus V m. So, what happens to the duty ratio? The duty ratio is 0.5 here and the duty ratio is maximum here it is 0.5 here and the duty ratio is minimum here. So, the duty ratio actually varies about 0.5. So, it goes through a it is 0 it goes through a maximum, it goes through a minimum this is how the duty ratio is as the function of the fundamental angle.

So, you can see that the duty ratio can actually be expressed as a function of omega T of the fundamental angle these are all expressed as functions of fundamental angle. Therefore, you can have i dc average also that. So, mathematically we know that the i dc and i dc average if this has been average over a sub cycle, what are the actual differences? This i dc will have all of switching frequency components. So, some of those switching frequency components will actually be absent when you do this i dc average so, but it retakes all the other components. The major component here would actually be a 300 hertz kind of thing you know it will look at I mean this is actually product of sinusoids. So, it is actually pretty interesting if you are you have two different sinusoids. So, this is 50 hertz and that is 50 hertz.

So, when you multiplied two different 50 hertz sinusoids what you will get you will actually get DC and you will get a 100 hertz sinusoids or you multiply sign omega 1 T and sign omega 2 T, you are going to get a product you are going to get sign of omega 1 plus omega 2 similarly another sign function which is omega 1 and minus omega 2. If you are taking both omega if you multiply omega and omega it will be 2 omega and 0. So, the product of this will give you a DC component, and a as I can double frequency. So, the same thing this will also give a DC. In fact, they are symmetric. So, it will be the same DC. So, you will also have a DC now. So, the DC component will all get added up now.

So, the question is about the AC component and in the case of a single case rectifier, you will see that there is a 100 hertz ripple. So, here what you would actually expect is a actually a 300 hertz ripple, but that 300 hertz ripple would theoretically come out to be very very I mean theoretically 0 or turn out to be actually very very small anyway. So, this actually equation is it takes you one step closer to that, because your d R d Y and d B. So, from now we can see i R i Y and i B can be easily written, we can see how to write d R d Y d B though I have already indicated here in this figure, now I am just going to write it mathematically.

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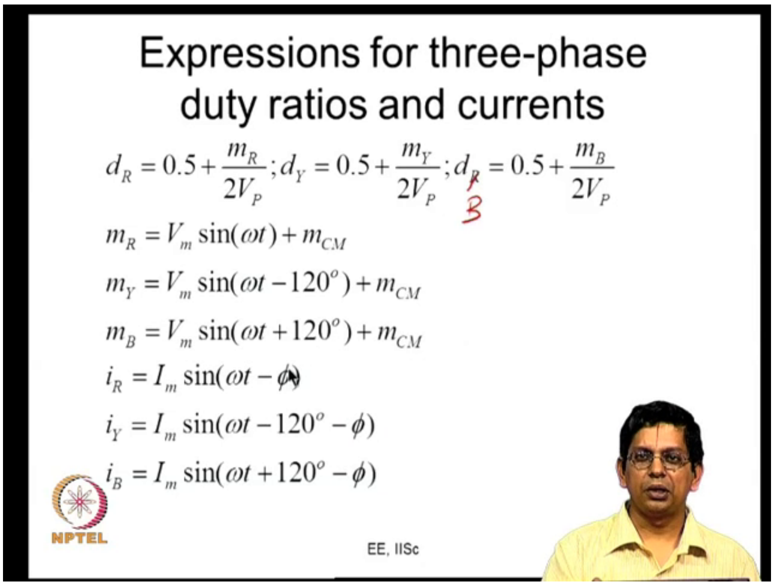

So, if you have d R you have the modulating signal m R. So, that m R divided by 2 V P that where this is the peak of the carrier. The carrier is assumed to vary from plus V P to minus V P. So, m R by 2 V P plus 0.5 that is duty ratio; the same way of duty ratio of Y phase can be given as 0.5 plus m Y by 2 V P I am sorry this should be d b please correct that that is called to be d B. So, this is d B; thank you. So, this is. So, you can say m R m Y m B are 3 phase sinusoids they are balance sinusoids. So, d R b Y d B are also going to be like that except that their level shifted by this 0.5.

Then what is m R? If we are talking of sign triangle PWM m R is some V m sign omega T and where omega is your fundamental modulation frequency, if f is your modulation frequency it is 2 phi f and V m is the peak value of this sinusoids and V m by V P is what you would normally call as the modulation index. So, m R is your V m sign omega T and m Y would be your V m sign omega T minus 120 and m B is equal to V m sign omega T plus 120, but it does not have to be sinusoidal PWM, you may have some common mode also added to that. So, in that case you can add the common mode m CM to all the three.

So, your modulating signal is 3 phase sinusoids with the common mode added to that. So, you can now mathematically express your d R in terms of m R. So, m R again has been written in terms of this sign omega T and so on therefore, d R you can actually express in terms of omega T and the modulation index. So, this is V m by V P is your modulation index then. So, d R gets expressed in terms of the modulation index and omega T how about i R? I r get expressed in terms I m sign omega T minus phi this is the peak value of current flowing through the load and this phi is the power factor angle we have considered a lagging power factor angle somethings lags by phi.

So, i R i Y i B are as I have said before the fundamental component alone is shown the harmonics have been neglected. So, your now able to express both your d R d Y d B and i R i Y i B in terms of omega T and the modulation index and the peak value of current I m right.

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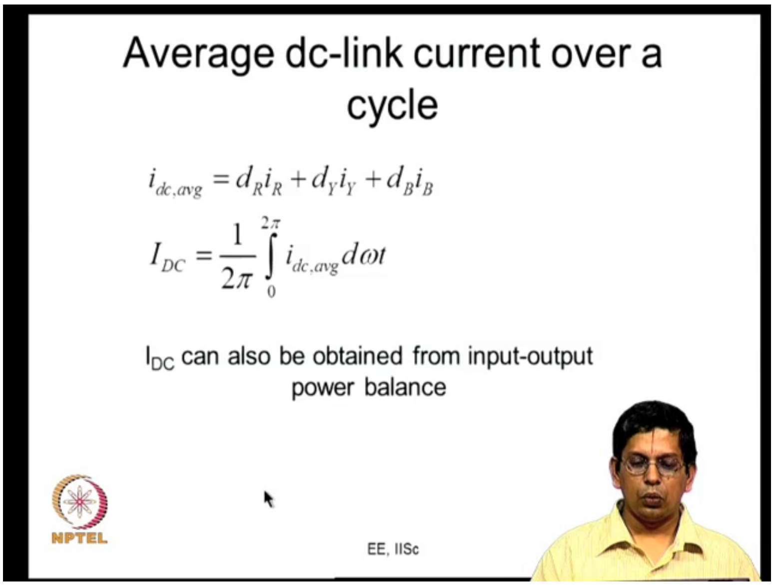

So, you can bring all of them together which I have not really shown. So, from this i dc average d R i R you can do this. So, the 3 phase currents they are whatever if you actually change your modulation signals and so on you will see that this d R d Y d B has to be changed accordingly from that now. So, you can get your, i dc average over a sub cycle. Then what you can actually do is you can average this i dc average over a sub cycle over entire line cycle I have taken 0 to 2 phi, but you will see that there are several symmetries. So, you really do not have to consider the entire cycle, it you can consider 120 degree and we may consider only 60 degree as we discussed before.

If you do this, anyway you have this available as omega T you take the average value you get your i dc what is this i dc? This is the DC component in their so called DC link current and this is the DC link current averaged over the entire line cycle and this is really measure of the amount of power that is flowing into the voltage source inverter or the power that is being converted by the voltage source inverter from DC to AC. So, this V DC into i dc is the power flowing from the DC side, and the same would be flowing out of the AC side if you assume that the inverter is I d l and therefore, you can arrive at this i dc also using the so called input output power balance; what do you mean by input output power balance. Now this v DC into i dc is the power flowing in the DC side and the whatever flows in as to flow because you assume your inverter to be efficient you know 100 percent efficient now.

Therefore what do you have on the AC side? The AC side you have depending on your DC voltage and modulation, you have some AC side voltage and a there is a 3 phase load it draw certain amount of current at certain power factor. So, you can always right that; if V m is your peak phase voltage I m is your peak phase current and phi is your power factor angle; so V m I m into cos phi. So, this whole thing multiplied by 3 by 2 would be the AC side power. So, you can look at it from that.

And the AC side power and the DC side power you balance you can come to I DC. So, i dc can be expressed as the AC side power divided by whatever is your v DC. The AC side power is available from DC bus voltage modulation index and the load current amplitude the load power factor angles. So, that is also done which is what something we discussed towards the end, and I left it as an exercise for you to be done in the last exercise.

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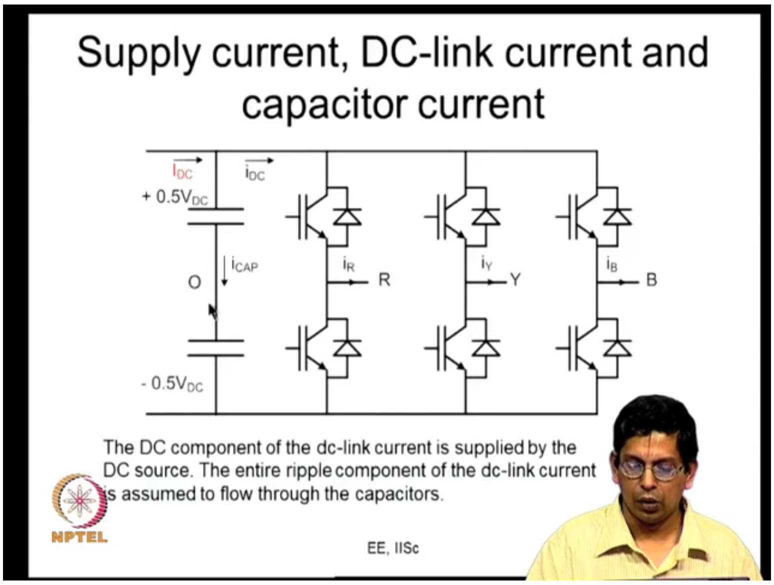

So, now what is the important assumption we are going to make, here we have a DC component the rest of that are ripple component now. So, what we are going to assume here is, what the source applies the DC has to come from there because the DC component in this cannot flow through the capacitor, the capacitor can take only the ripple. So, the DC entirely flows, but what we are assuming is only DC flows of this source, the entire amount of ripple is flowing through that. So, that way you can link up this i dc and average.

So, this i dc is the instantaneous i dc or i dc average over a sub cycle. If you if you subtract this capital i dc from that you are going to get I CAP that is from the instantaneous i dc you multiply this capital i dc you will get instantaneous I CAP. This is what I am trying to say. So, you can quickly do this and this is the way of arriving at I CAP. So, only the AC ripple flows through that now.

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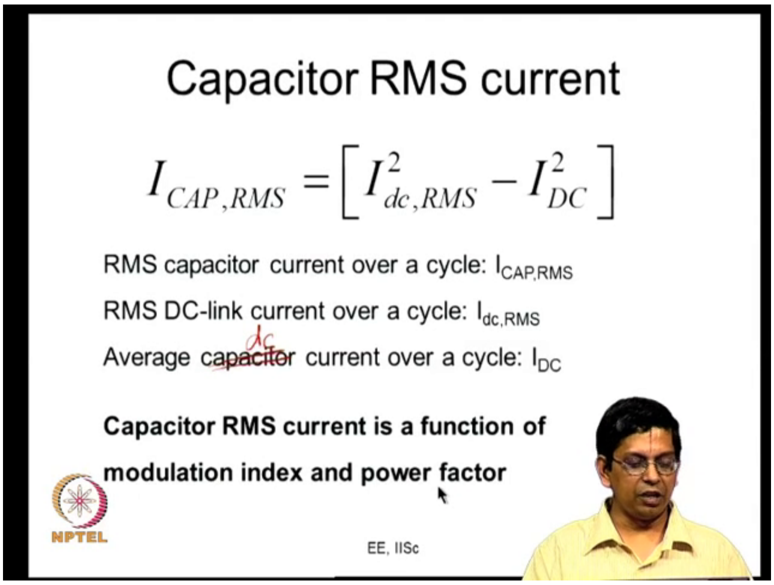

So, now this i dc the DC link current has certain RMS value you can evaluate it by integrating squaring this i dc and averaging it over a line cycle and taking as square root under that that is your i dc RMS it is the RMS DC link current over a cycle.

Now, then you have this i dc which is actually the average I mean it is not the average capacitor current I am extremely sorry. So, it is the average DC current, which actually flows through that there can be no average current through the capacitor I am sorry about this error. So, you have this now if this i dc RMS square you subtract i dc square from that, you are going to get the ripple part of it that is the I CAP RMS out of that now. So, this is going to give you RMS current expression.

So, this way you can actually calculate, you can see that this is really a function of all these both of these are actually a functions of modulation index load current amplitude and power factor. So, you will also see this capacitor RMS current is a function of modulation index power factor and also the load current amplitude. So, it is possible for you to actually derive close from expressions, it is little difficult to be done here, but it can actually sit and do it yours of the major steps we have really covered here and to help you there are these references now.

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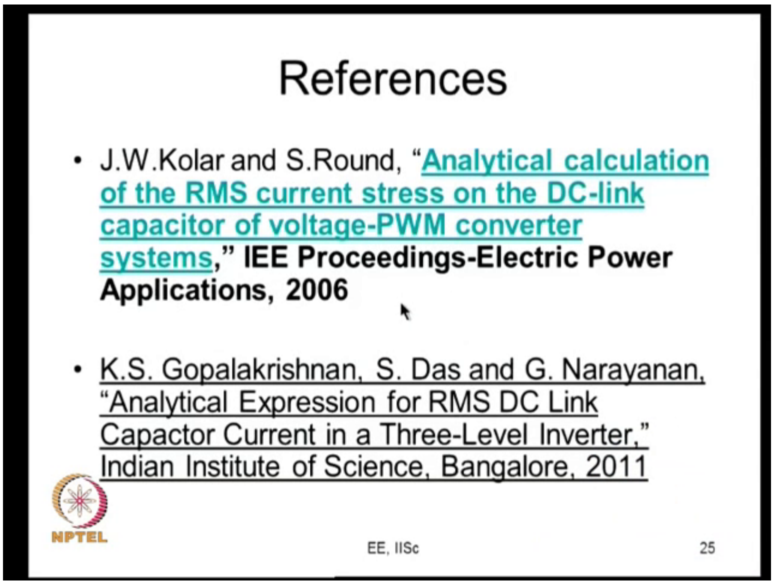

For example this is a reference where the two level thing has actually been two level inverters RMS calculations has been done this is by kolar and round, analytical calculation of the RMS current stress that is basically the RMS current flowing through the capacitor on the DC link capacitor of voltage PWM converter system, this is published in I double E proceedings in 2006 and this is a more recent work, but it is on a 3 level inverter.

So, never the less it is useful. So, it is a little more complex than a two level inverter, but once you know what are 3 level inverter is once you know how to calculate the RMS current for two level it is possible to do this now. So, this is another one which is actually presented in the centenary conference on the Indian institute of science in 2011. So, both these are available on the web. So, these references will actually be helpful to you to do is calculations now.

So this is, I bring my discussions on the capacitor current to an end and in the next week we will actually be discussing on the pulse setting target. Thank you very much for your interest in this lecture series and hoping to hope that you would follow the subsequent lectures also.

Thank you.