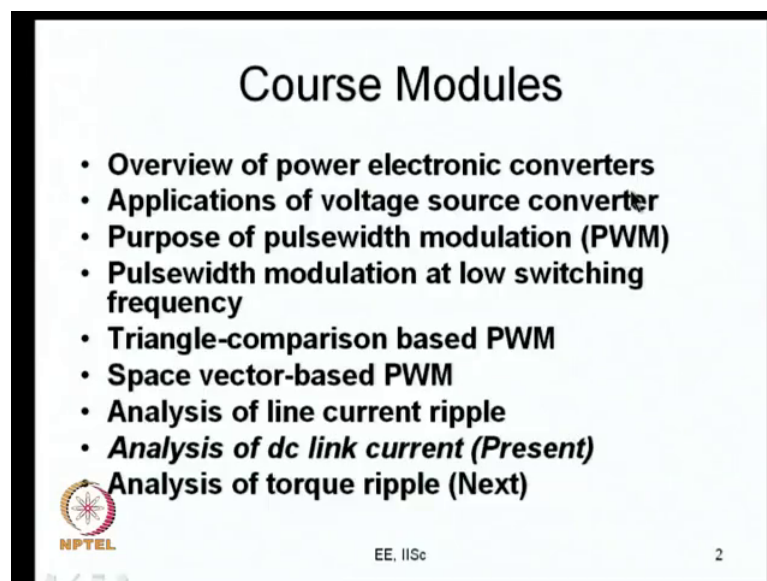


**Pulsewidth Modulation for Power Electronic Converters**  
**Prof. G. Narayanan**  
**Department of Electrical Engineering**  
**Indian Institute of Science, Bangalore**

**Lecture - 27**  
**Instantaneous and average dc link current in a voltage source inverter**

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters.

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So we have been going through several modules in this course as we have seen before like we have had an overview of power electronic converters, and we looked at to the voltage source converter different applications of voltage source converters such as modern drives and power quality at different converters and so on. And we looked at modulation at low frequencies, then we have been looking at modulation at higher switching frequencies that is if switching frequencies being much higher than the modulation frequency. And we looked at how do we produce PWM mega force based on triangle comparison and how do you do it based on the so called space vector approach, and we also saw that there are some advance bus clamping PWM methods which are possible especially with the space vector approach.

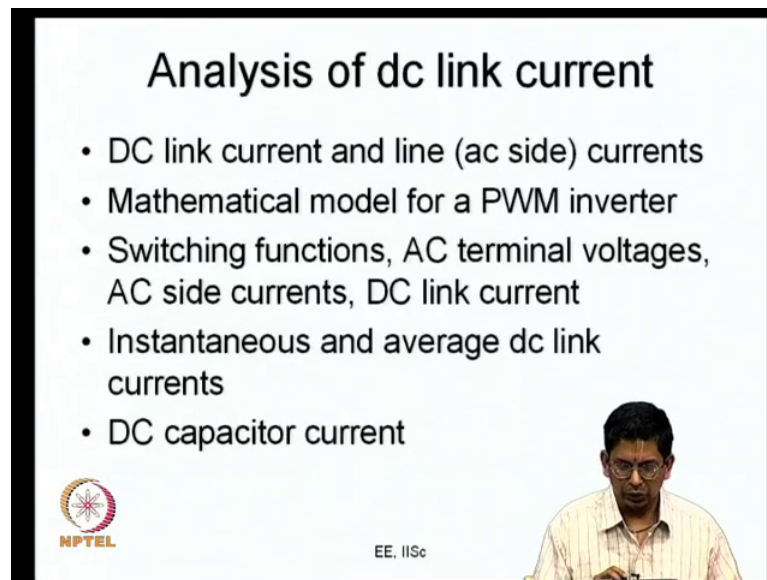
And after that we were focusing on line current ripple in the last module, that is you know there is PWM inverter you have a DC voltage, and you have certain modulation

index and that produces certain amount of AC voltage, but what happens is it does not produce only the AC voltage, it produces certain harmonics also and we were looking at those harmonics we looked at them in one lecture in the frequency domain point of view we looked at the harmonic spectra, and then in the subsequent lectures we were trying to evaluate the arms line current ripple. Let us say in motor drives platform voltage was inverter you may have a sinusoidal current and only that on top of the sinusoid you may have some ripple current, we were trying to estimate that R m s is line current ripple.

So, we were kind of into this space vector here for generation PWM and generation and then we were into the space vector domain for analysis here, now and we were also focusing in the previous module more on the harmonic distortion and such aspects and we would continue that in a later module on analysis on torque ripple. Where in the last module we analyzed line current ripple, we will have a subsequent module on torque ripple which is in some sense related and we will look at more of that now. Right now what we do is we take a break, and we will look at you know things which are more fundamental in the sense you would look at the relationship between the AC side current and the DC side current and the AC side fundamental currents to the I mean to be specific and the DC link current.

So, this module is going to be on analysis of DC link current. So, in all the previous modules we were looking at how do you produce PWM wave forms and what is the PWM wave form, and what are the harmonic components and things like that, our focus was entirely on the AC side, but now it is moving to the DC side. So, we just going to see why it is that now right.

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**Analysis of dc link current**

- DC link current and line (ac side) currents
- Mathematical model for a PWM inverter
- Switching functions, AC terminal voltages, AC side currents, DC link current
- Instantaneous and average dc link currents
- DC capacitor current

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The slide features a small inset image of a man with glasses and a striped shirt, looking down at a device. The NPTEL logo is in the bottom left, and 'EE, IISc' is in the bottom center.

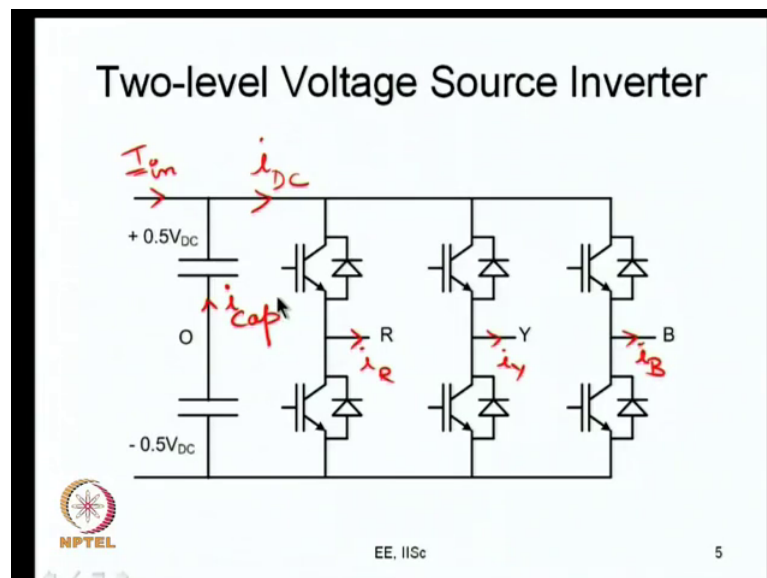
So, you know we will see some kind of how the DC link currents and the line currents are AC side current are related among the various things in this one. And we will we will work out a mathematical model for a PWM inverter, let us say you want to solve an inverter using like an equation solver package you want to do some simulation. So, you need you need a simple model, we will try to come up with a simple model which you can use in a in equation solver to solve that now.

And we will define switching functions for the three-phases and we will relate the DC voltage switching functions as in the AC terminal voltages. And then we will look at the relationship between the AC terminal voltage and AC side current essentially these are related in terms of the load, and then with the AC side currents and the switching functions how to derive the DC link current is what we going to see. And we will look at both the instantaneous and the average values of DC link current.

So, at any particular instant there is certain amount of current flowing through the DC link that is the instantaneous DC link current and that can be averaged over half carrier cycle or a sub cycle and that is what we call as average DC link current. It is same as what we use before an average pole voltage, the average voltage vector, average current and all those they are seen similar to that now we are going to be looking at the average DC link current also. In this module we will also look at the DC capacitor current. So, what is the amount of current that will flow through the DC capacitor?

So, in the DC link current we have a DC component which flows from the mains, and it has a ripple component which primarily flows from the DC capacitor. So, that is something we will probably look at in the next lecture, but as part of this module and we would get an understanding of DC link current in this now. So, this lecture is particularly titled instantaneous and average DC link current in a voltage source inverter, voltage source inverter the voltage source inverter has a DC side and the AC side and this is the DC side current that is flowing into the 6 bridges here and what is it is instantaneous value on how do you evaluate it is average value that primarily the question we looking at in this particular lecture now.

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So, this is the voltage source inverter just for you to take a re look at these are the DC terminals and these are the 2 switches and in every leg, and in every leg the top is on bottom will be off if the bottom is on the top will be off. So, you know that everything is like a single pole double throw switch. So, what we are trying to do here is these are currents we can define them as let us say  $i_R$ ,  $i_Y$ ,  $i_B$  these are the line currents or the AC side currents and this is what we are calling as the DC link current  $i_{DC}$  this is this is what want to do.

So, generally you know many situations I found that you know students are also more comfortable coming up with this the a the AC side current, because in an inverter the DC volt is fixed and then if the modulation signal are given or the PWM signals are given it

is easy to define the three-phase terminal voltages and then if you have a proper load model you can predict the three-phase currents.

So, students are you know generally very comfortable predicting this three-phase currents, but I found them often having some difficulty in doing this DC link current and this is very important from certain other points of view also. So, why is DC link current important. So, let us say there is this DC link current there is some power supply which will primarily provide a DC component. This DC link current also has a ripple component and that will be supplied by the capacitor which I call as  $I_{cap}$  now. So, this  $I_{cap}$  is also very important, that is one of the reasons why you should do this is you should know what is the rms value of this. When sizing the capacitors there are two things, one is you have to make sure you have big enough capacitors such that the voltage ripple is very very small.

So, that depends on the value depends on what is there on the DC side whether you have rectified and all those issues also so, but in general the philosophy is the value of  $C$  should be chosen such that the voltage ripple here should be very small excuse me and the next rating now important rating of a capacitor is rms current. You have to make sure that the capacitors are have enough rms current that I have chosen here now. So, that is in that sense also it is important now. More importantly we are now looking at it is just only understanding of the voltage source converter; we are doing this under this exercise. Firstly, to get a better hold over the voltage source converter and get a better understanding of voltage source converter and we will also use this idea to come up with a how to evaluate the capacitor current alright.

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**Three-phase balanced load with unconnected neutral**

R, Y and B connected to the mid-points of the three inverter legs

Load neutral N is not connected to the inverter

Three-wire connection

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So, the kind of load we have been assuming is a three-phase load R Y and B, and it is a balance three-phase load this neutral is not connected to any other point electrically other than through the line terminals. So, this R is connected Y is connected B is connected. So, it is a three-phase three-wire load as I have mentioned before, this is the kind of load we have always been considering in this course now.

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**Definition of three-phase switching functions**

- $S_R = 1$ , if R-phase top device is ON
- $S_R = 0$ , if R-phase top device is OFF
- $S_Y = 1$ , if Y-phase top device is ON
- $S_Y = 0$ , if Y-phase top device is OFF
- $S_B = 1$ , if B-phase top device is ON
- $S_B = 0$ , if B-phase top device is OFF

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So, let us define the switching function now, see there are 6 devices they are on they are off they are on they are off you know they somewhere they has to be represented now.

So, they can be there is one convenient way of representing them is switching functions now. We can define switching function three-phase switching functions  $S_R$   $S_Y$  and  $S_B$  these are actually digital signals they can take the values of just 1 or 0 and  $S_R$  is equal to 1 if R phase top device is on and  $S_R$  is 0 if R phase top device is off or the bottom device is on that is all. They are it is it s a complementary logic between the top and the bottom. So, top is on bottom is going to be off and vice versa.

So,  $S_R$  is equal to 1 if the top is on  $S_R$  is equals to 0 if the bottom device is on and similarly  $S_Y$  is equal to 1 if y phase top is on and  $S_Y$  is equal to 0 if y phase bottom is on. Similarly you have  $S_B$  is equal to 1 or 0 depending on whether the top device is on or the bottom device is on. So, you have the three-phase switching functions which are now being defined now.

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The slide displays a three-phase inverter circuit with three legs (R, Y, B). Each leg consists of a top IGBT and a bottom IGBT. The DC bus is split into  $+0.5V_{DC}$  and  $-0.5V_{DC}$  relative to a neutral point 'O'. The pole voltages are defined as  $v_{RO}$ ,  $v_{YO}$ , and  $v_{BO}$ . The line voltages are defined as  $v_{RY}$ ,  $v_{YB}$ , and  $v_{BR}$ .

Equations shown on the slide:

$$v_{RO} = (S_R - 0.5)V_{DC}$$

$$v_{YO} = (S_Y - 0.5)V_{DC}$$

$$v_{BO} = (S_B - 0.5)V_{DC}$$

$$v_{RY} = v_{RO} - v_{YO}$$

$$v_{YB} = v_{YO} - v_{BO}$$

$$v_{BR} = v_{BO} - v_{RO}$$

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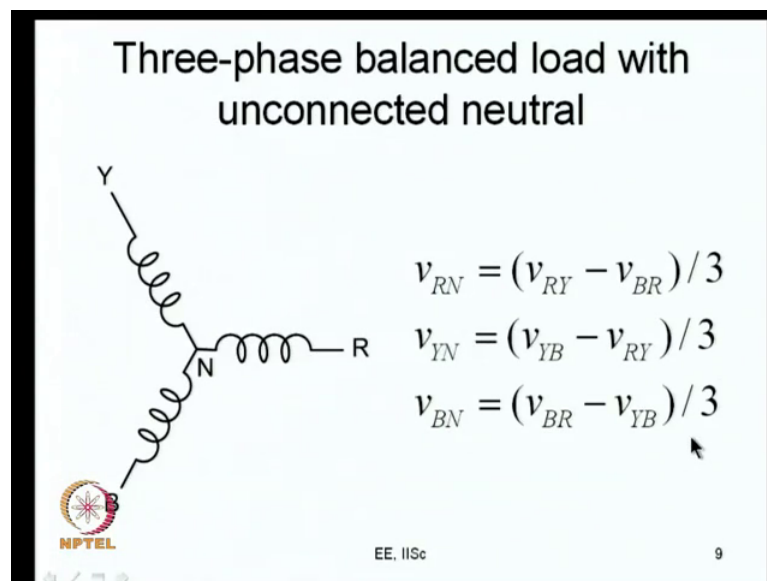
So, with these switching functions you can define on the DC bus voltage, you can define what are your voltages now. So, you have the switching defining function as we defined before and then let us say you have the pole voltages, the first you thing you have to come up with pole voltage that is the voltage at this pole R with respect to o. So, what is that going to? When the top device is one that is when  $S_R$  is equal to 1, the pole voltage is going to be  $0.5 V_{DC}$ .

So, this is going to be  $S_R$  minus  $0.5 V_{DC}$ . So, when  $S_R$  is equal to 1 it becomes  $V_{DC}$  by 2 and when the top device is off or the bottom is 1  $S_R$  is equal to 0 and that is equal

to minus 0.5 V DC therefore, the pole voltage  $v_{RO}$  can be expressed in terms of  $s_R$  and  $V_{DC}$  leg this  $s_R$  minus 0.5 V DC and  $v_{YO}$  can be written as  $s_Y$  minus 0.5 times  $V_{DC}$  similarly  $v_{BO}$  can be as  $s_B$  minus 0.5 times  $V_{DC}$ .

So, given the DC voltage and the switching functions  $s_R$ ,  $s_Y$  and  $s_B$  it is possible for you to write down  $v_{RO}$ ,  $v_{YO}$  and  $v_{BO}$ . Next you have the line voltages here see you have  $v_{RY}$  what is  $v_{RY}$ ? it is  $v_{RO}$  minus  $v_{YO}$ . And similarly  $v_{YB}$  is  $v_{YO}$  minus  $v_{BO}$  and  $v_{BR}$  is  $v_{BO}$  minus  $v_{RO}$ . So, you have the pole voltages and line voltages I will define legs now, you will slowly evolving a mathematical model if you were to stimulate voltage source inverter what you can do is you can assume all the devices to be ideal and you have their DC bus voltage and then you can start with the switching functions and the switching functions you know you can generate them easily for example, by comparing a sin and a triangle or whatever for whichever way suitable for you can do this now. So, start from the DC voltage and the switching functions feed for these devices, you can start saying; what are the AC side voltages.

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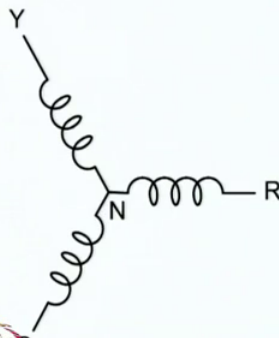


So, now we have come up to the line voltages now let us go to the line to the neutral voltage, we have said it is a three-phase balanced load star connected load. In this kind of situation it is possible to say that  $v_{RN}$  is  $v_{RY}$  minus  $v_{BR}$  by 3. Similarly  $v_{YN}$  is  $v_{YB}$  minus  $v_{RY}$  by 3 similarly  $v_{BN}$  is  $v_{BR}$  minus  $v_{YB}$  by 3.



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### Equations for an RL load

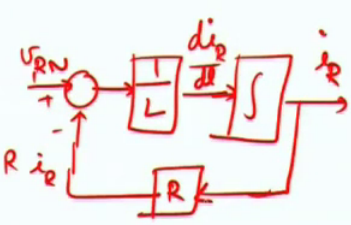

$$v_{RN} = Ri_R + L \frac{di_R}{dt}$$
$$v_{YN} = Ri_Y + L \frac{di_Y}{dt}$$
$$v_{BN} = Ri_B + L \frac{di_B}{dt}$$

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So, we have discussed this even earlier now, first you can reach up to  $v_{RN}$ ,  $v_{YN}$  and  $v_{BN}$  now. So, the three-phase voltage is separate on the load are available, let us assume an RL load just for the time being I mean just as an exercise, then you have  $v_{RN}$  is equal to the resistive drop plus the  $L \frac{di_R}{dt}$ ,  $v_{YN}$  is equal to similarly  $Ri_Y$  plus  $L \frac{di_Y}{dt}$  and  $v_{BN}$  is equal to  $Ri_B$  plus  $L \frac{di_B}{dt}$ . So, this would be your load equation right.

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### Solution of load equations

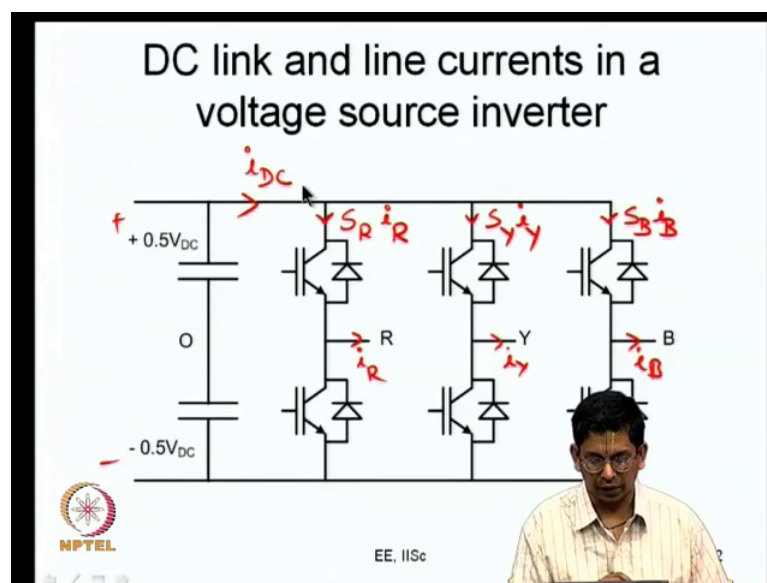
$$\frac{di_R}{dt} = \frac{v_{RN} - Ri_R}{L}$$
$$\frac{di_Y}{dt} = \frac{v_{YN} - Ri_Y}{L}$$
$$\frac{di_B}{dt} = \frac{v_{BN} - Ri_B}{L}$$


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So, you have to solve the load equation how can you solve the load equation? You can rewrite this as  $\frac{di_R}{dt} = \frac{v_{RN} - Ri_R}{L}$ . So, any you this you can integrate now what you do is let us say you have  $v_{RN}$ , and this  $v_{RN}$  you can subtract  $i_R R$  from here you subtract that will give you  $L \frac{di_R}{dt}$  you divide it by  $L$  you get  $\frac{di_R}{dt}$  and you integrate that, this will give you  $i_R$  you can multiple this by  $R$  this now. So, this is how you do that now, when you want to solve the equation something like this you don't do a differentiation, but you reconvert the problem into one of integration and you basically use an integrator as shown here into this knob.

So, this is a first order equation and you require a single integrator to that you can solve this similarly this is how you go about solving now. If you solve them what are you going to get you are going to get  $i_R$ ,  $i_Y$  and  $i_B$ .

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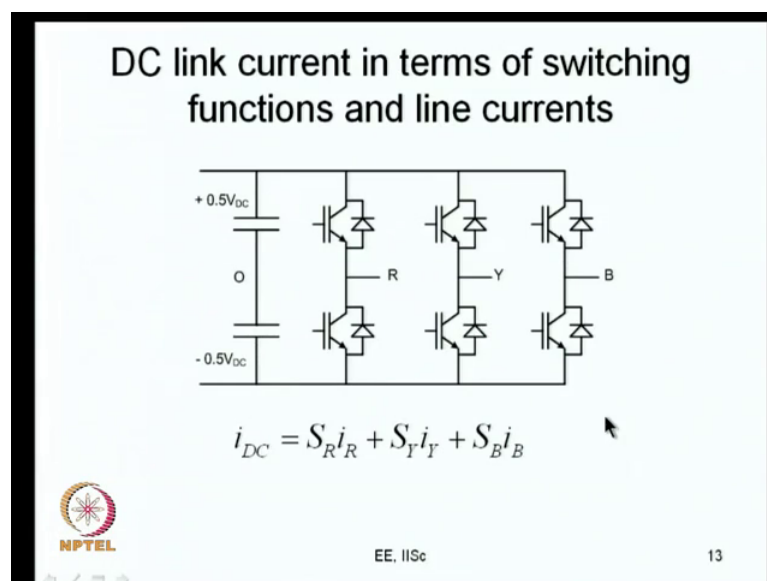


So, now with  $i_R$ ,  $i_Y$  and  $i_B$  you have to find out  $i_{DC}$  that is a next step. So, let me say this is  $i_R$ , this is  $i_Y$ , this is  $i_B$ , that is starting from the DC voltage and the switching functions you have determined the three-phase voltages  $v_{RO}$ ,  $v_{YO}$ ,  $v_{BO}$  or  $v_{RY}$ ,  $v_{YB}$ ,  $v_{BR}$  or  $v_{RN}$ ,  $v_{YN}$ ,  $v_{BN}$  and from the three-phase AC voltage we have come up with the three-phase AC currents are also available  $i_R$ ,  $i_Y$ ,  $i_B$  and now what is that? All that now the next step for you is to find out what is  $i_{DC}$ . As you can very clearly see this is  $i_{DC}$ , it is the sum of these three currents here now the question what is that current.

The current as I have indicated here can take 2 different values 1 is it can be equal to  $i_R$  if the top device is conducting or the current can be equal to 0.

So, you can say that the current here is  $S_R$  times  $i_R$ . So, when  $S_R$  is equal to 1 it is  $i_R$  when  $S_R$  is equal to 0, it is 0. And similarly you can say that this is  $S_Y$  times  $i_Y$  and what is this current  $S_B$  times  $i_B$ . And therefore your  $i_{DC}$  is simply the sum of these three quantities  $S_R i_R + S_Y i_Y + S_B i_B$  and it is this k c Kirchoff's current law you get  $i_{DC}$ .

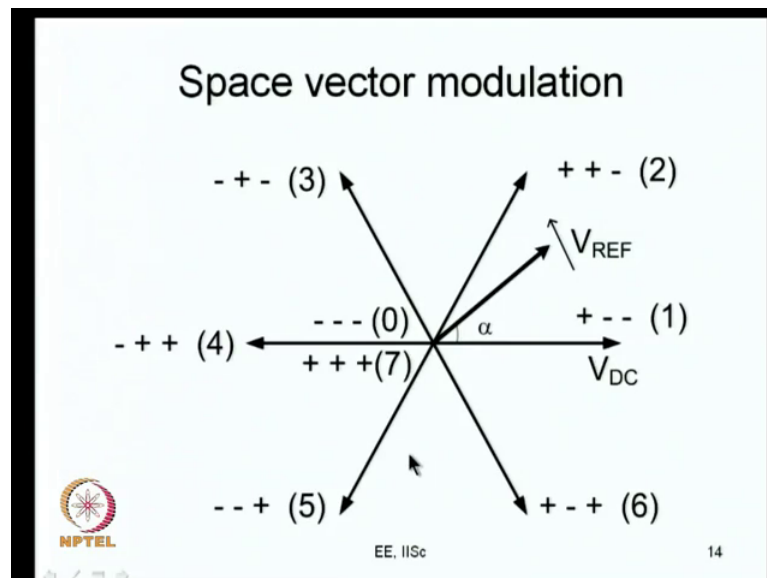
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So, this is what is indicated here,  $i_{DC}$  is equal to  $S_R i_R + S_Y i_Y + S_B i_B$ . So, you have the DC link current available in terms of switching functions and line currents. Now let us say we move on, this is just an instantaneous relationship we have come up with the mathematical model for that if you go back from one together that is initially what we did? We know we are taking a voltage source inverter like this and this is a three-phase three-wire balanced star connected load we are assuming like a in this three-phase three-wire load, and we have defined this switching functions  $S_R S_Y S_B$  like this and with this with the switching functions on the DC voltage, we write the three-phase pole voltages like this and then we write the three-phase line to line voltages as I have shown here and from here what we do is, you can write down the three-phase line to neutral voltages as I have shown there.

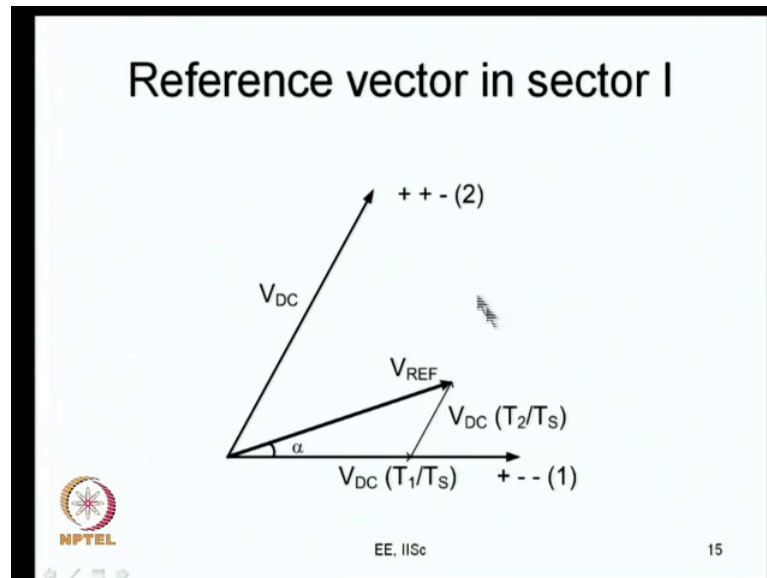
So, these are the different you know up to this you come to the AC voltages, once you got the AC voltages you require the load model. Now considering an R L load you can write down the three-phase equations for the load and you can solve them and you by solution gives you  $i_R$   $i_Y$  and  $i_B$  and once you get your  $i_R$   $i_Y$   $i_B$  you can multiply them respectively by the switching functions  $S_R$   $S_Y$  and  $S_B$  and you can add them up to get your DC link current. So, this kind of gives you a whole picture of what is required and you know you know most part of to stimulate an inverter for example, alright the only other thing we have to do is actually divide the current between the capacitor and the input, which we will do it later and also this basically relates the DC link current and the three-phase currents view and so.

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Now, with that model what we try to do is, let us say the PWM inverter has been switched is being switched using space vector modulation. As we have been seeing in the last our lectures, space vector modulation basically means the voltage vector is you know a voltage vector is provided instead of three-phase voltage references and the voltage vector is as certain magnitude  $V_{REF}$  and that is proportionate to the decide fundamental voltage right and it is revolving at a constant frequency, and it is frequency is  $v$  modulation frequency it is a desired modulation frequency, and this is sampled and every time you sampled once in a sub cycle  $T_s$ .

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And within that you do a time averaging, you apply v 1 active vector for T 1 seconds and active vector 2 for T 2 seconds. And you apply the null vector for the remaining time and you produce a this quantity now.

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### Volt-second balance and calculation of dwell times

$$\mathbf{V}_{REF} T_s = \mathbf{V}_1 T_1 + \mathbf{V}_2 T_2 + \mathbf{V}_Z T_Z$$

$$T_s = T_1 + T_2 + T_Z$$

$$T_1 = \frac{V_{REF} \sin(60^\circ - \alpha)}{V_{DC} \sin(60^\circ)} T_s$$

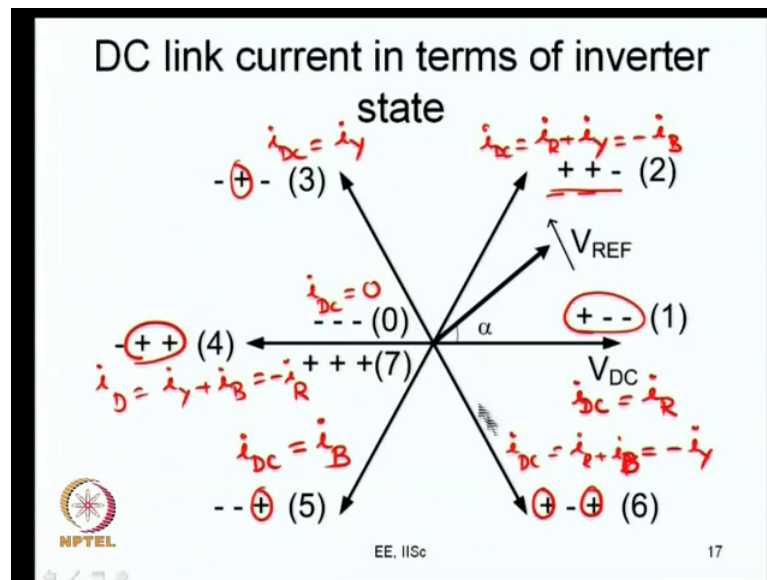
$$T_2 = \frac{V_{REF} \sin(\alpha)}{V_{DC} \sin(60^\circ)} T_s$$

$$T_Z = T_s - T_1 - T_2$$

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And these are how to calculate T 1 T 2 and T z that depends on the values of V REF alpha and then this is how we do it as we have been looking at in the last several lectures right.

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So, now what do we do? Let us say try and relate the DC link current and the inverter state. So, at any instant of time the R face has some switching function as is the top device is on or bottom device is on, S R is 0 or 1 similarly y phase S Y is 1 or 0, the top device is on or the bottom is on same case about the SB also.

Now, what we are going to do, we are going to look at the inverter state and then try to relate what is the DC link current how do we do that alright maybe it may be worthwhile to look back here. So, now, if you see if you take a case S R is also 1 S Y is also 1, S B is also 1 what happens; all the three top device are on and the bottom devices are off; that means, an inverter state plus plus. So, in this case what happens  $i_R$  plus  $i_Y$  plus  $i_B$  and that will be equal to 0 because that is what is flowing to a three-phase three-wire load therefore,  $i_R$  plus  $i_Y$  plus  $i_B$  is equal to 0.

So, in the inverter state when the zero when the top devices are all on, it is zero what happens if all the bottom device are on? S R is 0 S Y is 0 and S B is 0. So, whenever the inverter state is 0,  $i_{DC}$  is 0 and this is what we mentioned at the time we were defining zero state what is zero state is? It a state of the it is one of the states of the inverter where there is no connection between the DC side, there is no power flow between the DC side and the AC side that is  $i_{DC}$  is equal to 0 that is what we meant alright. So, we can we look at the inverter states one by one and write that down let us see this here. So, in this

case let us say if you consider the zero state minus minus minus R plus plus, you have  $i_{DC}$  is equal to 0 as we established just a while back.

Now let if you take the inverter state plus minus minus what happens, you have a case where  $S_R$  is equal to 1, but  $S_Y$  is equal to 0 and  $S_B$  is also equal to 0. And therefore, your  $i_{DC}$  is simply equal to  $i_R$ ,  $i_{DC}$  is equal to  $i_R$  what happens in the when you are talking of the inverter state plus plus minus you see this plus plus minus. So,  $S_R$  is 1,  $S_Y$  is 1,  $S_B$  is 0. Therefore, in this you have  $i_{DC}$  equals  $i_R$  plus  $i_Y$  which is also equal to minus  $i_B$ . So, when you inverter state applied is plus plus minus inverter state 2 then  $i_{DC}$  is equal to minus  $i_B$ . Let us do a similar exercise for three. So, only  $S_Y$  is equal to 1 in this case.

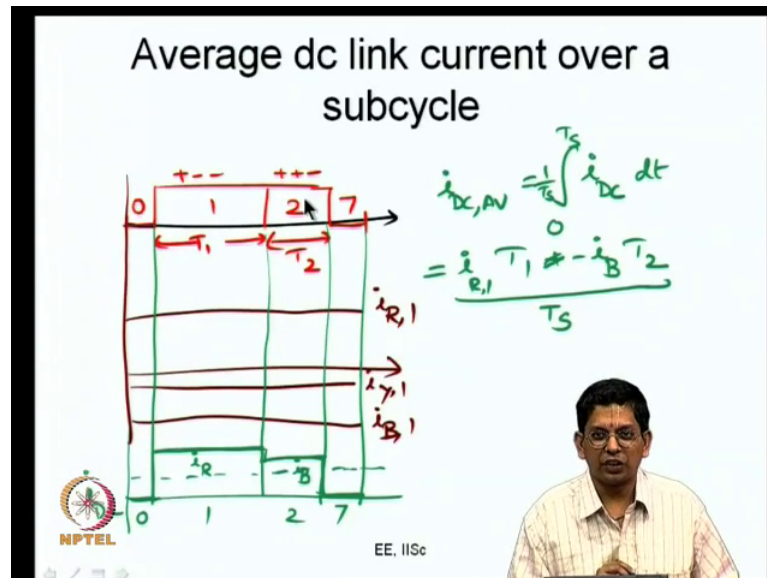
Therefore you have  $i_{DC}$  equals  $i_Y$  then about inverter state 4, this is  $S_Y$  is equal to 1 and  $S_B$  is equal to 1 and  $S_R$  is equal to 0 therefore, you have  $i_{DC}$  equals  $i_Y$  plus  $i_B$  and what is that equal to? Minus  $i_R$  alright then there only 2 other inverters state left this is 5. So, only  $S_Y$  is 1. Therefore, you have  $i_{DC}$  is equal to  $i_B$  and similarly here these 2 states are higher then when the top devices are on in this you have  $i_{DC}$  equals  $i_R$  plus  $i_Y$  is the right I am sorry  $i_R$  plus  $i_B$  which is equal to minus  $i_Y$ .

So, the picture is complete. So, when the zero states are applied  $i_{DC}$  is equal to 0 when the active states are applied it is equal to one phase of the current or the negative of that. Here if you see  $i_{DC}$  is equal to  $i_R$ ; when inverter state 4 is applied  $i_{DC}$  is equal to minus  $i_R$ . Similarly when inverter state three is applied  $i_{DC}$  is equal to  $i_Y$  when inverter state 6 is applied  $i_{DC}$  is equal to minus  $i_Y$ .

Similarly when inverter state 5 is applied  $i_{DC}$  is equal to  $i_B$  and here it is equal to  $i_{DC}$  is equal to minus  $i_B$ . So, you can also you can also relate to this now, this is really the positive peak of R phase this is the negative peak of R phase this position stands for the positive peak of y phase this is the negative peak of y phase. Similarly this is the positive peak of b this is the negative peak of b. So, you see a there is also correlation there now.

So, have now try to relate the DC link current in terms of the inverter states. So, let us go forward. So, this is the instantaneous current and what do you mean by the average current how can we calculate the average current. So, what happens within a inverter state. So, within a sub cycle; so let us say we have a sub cycle.

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And in that sub cycle you have inverter states, let me say you have states this is the sub cycle duration, inverter state zero is applied here let us say the inverter state 1 is applied 2 is applied and 7 is applied and this is the whole duration  $T_s$ . So, this zero means all of them are negative 1 means plus minus minus, 2 means plus plus minus let us say this is what is applied in 1 inverter cycle. So, this is for duration  $T_1$  this is for duration  $T_2$ .

So, now happens you have three-phase currents, in this sub cycle you will have three-phase currents. So, what are their values it depends on the power factor also. Now let me say this is my  $i_R$ , this is my  $i_Y$  let us say this is the my  $i_Y$  and this is  $i_B$ . There you know you can consider them to be constants over the sub cycle and you can ignore the ripple because for all this purpose the ripple if you consider it is much smaller than the fundamental you can take this as the fundamental component  $i_{R1}$ ,  $i_{Y1}$  and  $i_{B1}$  these are the values of currents. So, what do you have? You are looking at what is the DC link current at every particular this thing.

So, the DC link current is let me use a different color, during this interval the DC link is 0, during the last interval also the DC link current is 0 this is  $i_{DC}$  why zero states are being applied. Now you have this do you know what is it that what is that equal to that is equal to  $i_R$ . So, this current is equal to  $i_R$  and how about next to interval  $T_2$ ? That is going to be  $i_R$  plus  $i_Y$  or it is minus  $i_B$ . So, this minus  $i_B$  is something like this, this is minus  $i_B$  the rest of the time is 0. So, this is the nature of variation of the DC link



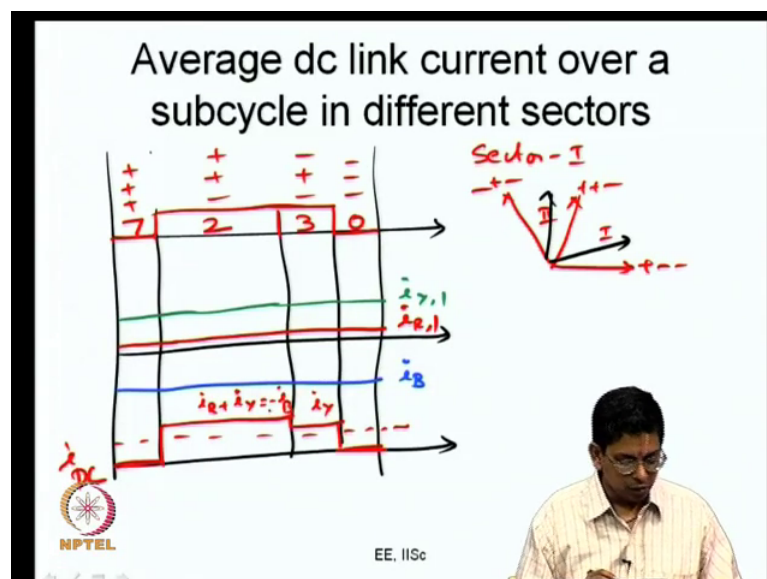
current over a sub cycle. So, this is how it varies over a sub cycle. So, when zero is applied this is when 1 is applied when 2 is applied and when seven is applied and.

This has certain average value what is that? Some average value like this, some average value and how much is that average value? Let us call this as  $i_{DC}$  average what is that  $i_{DC}$  average? That is equal to whatever is your instantaneous  $i_{DC}$  and let us say  $d$   $t$  integrated over 0 to  $T$  s and divided by  $T$  s it is the average value over a carrier cycle and what is that equal to? That is equal to  $i_{R1}$  multiplied by  $T_1$  plus or rather minus  $i_B$  multiplied by  $T_2$  the whole divided by  $T$  s.

Now, this is the average current, now this is this average DC link over that now. So, what will happen we move from one cycle to another cycle instead of one you go into the next cycle what is going to happen now? Your  $i_{R1}$  value will change in minus  $i_B$  will also change and the  $T_1$  and  $T_2$  values will also change. So, the DC link average current will go on changing from one sub cycle to the other sub cycle, and this is the case we have considered in sector 1 and then if you go to another sector you will see a similar picture and which we will just do in a short while now. So, what you will see is you will see that is actually repeats over every sector as we will see a little later now.

So, let us say we are looking at the average DC link current over a sub cycle in different sectors. So, that is now the earlier one we looked at sector one where you used the active state 1 and active state 2.

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Now, let us say you do differently let us say in sector 2 now. So, let us say if you want to do in sector 2. So, what am I doing I am considering some sub cycle and let us say the inverter states are 7 2 3 0 in what is called as sector 2 that is you have the other region endure plus minus minus, plus plus minus this is minus plus minus this region is sector 1 this region is sector 2. So, we are going into sector 2 now.

So, here the inverter state is plus plus plus, the inverter state is plus plus minus the inverter state is minus plus minus this is minus minus minus if you are going to do like it that. So, let us say what are the three-phase currents now. So, we are in sector 2 and you can see that in sector 2 your R phase is got a as crossing. So, you may have a R phase current which is like this, this may be your R phase current, this is  $i_R$  I can call it  $i_{R1}$  in the fundamental component we can ignore the harmonic component here now, and let us say we go to  $i_Y$ . So,  $i_Y$  we will be somewhat positive now, this is the y phase for a melt current now and then let us say we look at the b phase current this is  $i_B$ . So, now, these are the three-phase currents these are the three-phase currents now. So, when I want to draw my DC link current what I can say is during this time it is 0 this is also it is 0 here I am drawing the DC link current.

Then what happens during 2? It is  $i_R$  plus  $i_Y$  this is  $i_R$  plus  $i_Y$  here it is  $i_R$  plus  $i_Y$  or minus  $i_B$  and here what it is? It is equal to  $i_Y$  this is  $i_Y$ . So, this is the nature of current during that right. So, if you take 2 instance which are 60 degrees away, let me just show that here. If you take 2 instance which are 60 degrees away like here, this is  $V_{REF}$  that is  $V_{REF}$  the same value of fundamental angle the angle is 60 degrees then what will happen is you take these 2 cases this duration you will have a suiting sequence like this, this duration  $T_1$  and  $T_2$  will be equal to these durations  $T_2$  and  $T_3$ . And similarly these currents what are  $i_R$  and minus  $i_B$  they will be equal to what are  $i_R$  plus  $i_Y$  that is minus  $i_B$  and what is this  $i_Y$ .

So, let me just write this down also this is  $i_Y$  this is minus  $i_B$ . So, you have to see that in a 60 degree time things change, is  $i_R$   $i_R$  is so much. So, 60 degrees later minus  $i_B$  will have the same value similarly, here minus  $i_B$  is so much 60 degrees later  $i_Y$  will have the same value mean. Please remember these are all we are considering only the fundamental components and the harmonics are very small we have seen the peak to peak ripple current is small they are all negligible now. So, what do you get? You see that the same pattern of the DC link current ripple repeats here and there, this and this what I

have drawn in green and what I have drawn here in red, this is sector 2 this is sector 1 both of them are the same. If you are considering one sub cycle in sector 1 and another sub cycle exactly 60 degrees away in sector 2.

So, this DC link current has a periodicity of one sixth of the fundamental cycle or just one sector. So, that is the periodicity of that and when you want to evaluate the this DC link current directly average value. You basically can just look at evaluating over one sector that is one you know good way of doing it now. So, this is what we have now come to. So, we can just start going into the expression for an average DC link current now let us start writing down an expression.

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### Expression for average dc link current

Sector: I ( $90^\circ \leq \omega t \leq 150^\circ$ )

$$i_{dc, AV} = \frac{i_{R,1} T_1 - i_{B,1} T_2}{T_s}$$




$\omega t = 90^\circ + \alpha$   
 $V_{ref} = \frac{3}{2} V_m$   
 peak phase fund volt.

$$i_{R,1} = I_m \sin(\omega t + \phi)$$

$$i_{B,1} = I_m \sin(\omega t + 120^\circ + \phi)$$

$$T_1 = \frac{V_{ref}}{V_{dc}} \frac{\sin(60^\circ - \alpha)}{\sin 60^\circ} \cdot T_s$$

$$T_2 = \frac{V_{ref}}{V_{dc}} \frac{\sin \alpha}{\sin 60^\circ} \cdot T_s$$

So, what did we find let us say taking the case of sector 1 any sector you can consider let us consider in case of sector 1. So, we found that this  $i_{dc}$  average that is equal to  $i_{R,1}$  that is the R phase fundamental component multiplied by  $T_1$  that is the time for which the active vector 1 applied then minus  $i_{B,1}$  which is the fundamental component of b phase current multiplied by  $T_2$  and the whole thing divided by  $T_s$ .

Why are we doing like this? When the zero states are applied the DC link current is zero when the active states are applied when active state 1 is being applied, that is applied for  $T_1$  duration and the  $i_{dc}$  link current is  $i_{R,1}$ . When active state 2 is applied it is applied for duration  $T_2$  seconds and the DC link current is minus  $i_{B,1}$  therefore,  $i_{R,1}$  multiplied by  $T_1$  minus  $i_{B,1}$  multiplied by  $T_2$  the whole divided by  $T_s$  and you will

see that in this case  $i_{B1}$  is negative and therefore, minus of  $i_{B1}$  is positive. So, you this is what you have.

So, you know you can go ahead like this you can evaluate this term. So, can we evaluate right expressions for this how do we write expressions for this that is the next question now? So, we can reproduce some of the older expressions that we have had and we can write correspondingly new expressions what can we do? This is  $i_{R1}$  what is that that is the fundamental component how can you write what is the fundamental component? Let us say you have a fundamental has some peak value  $I_m$  and that is going to be  $\sin(\omega t + \phi)$ , which is  $\phi$  is the profile triangle. Similarly you can say  $i_{B1}$  is equal to  $I_m \sin(\omega t - 240^\circ)$  or  $120^\circ + \phi$ .

So, you have them. So, what are this  $I_m$  is the peak values of the fundamental currents  $\omega t$  gives present instants,  $\omega$  is the angular frequency and  $t$  is the present instant of time. So, we are considering  $\omega t$  in the range  $90^\circ$  less than or equal to  $\omega t$ , less than or equal to  $150^\circ$  this is the range we are considering this is sector 1. So, sector 1 starts from the positive peak, this is the positive peak of R phase fundamental voltage. And this is the negative peak of b phase fundamental voltage sector 1 extends from the positive peak of R phase to negative peak of b phase this is what you have now this can be substituted what else? Then you have  $T_1$  and  $T_2$  what you can do about  $T_1$  and  $T_2$  is you can write some other expressions,  $T_1$  is what is  $V_{ref}$  divided by  $V_{dc}$  times  $\sin(60^\circ - \alpha)$  divided by  $\sin(60^\circ)$  multiplied by  $T_s$  and what is  $T_2$ ?  $T_2$  equals  $V_{ref}$  divided by  $V_{dc}$   $\sin(\alpha)$  divided by  $\sin(60^\circ)$  multiplied by  $T_s$ .

So, you have  $T_1$   $T_2$ . So, now, next what you can do? You can try multiplying all of them and do this now. So, there a few issues now what are they what is  $I_m$  is a peak value of current what is  $V_{ref}$ ?  $V_{ref}$  is the magnitude of the fundamental I mean it is the magnitude of the reference vector and that is related to the fundamental voltage how is it related to the fundamental voltage. So, you recall our earlier discussions based on our space vector definitions and so on,  $V_{ref}$  is equal to  $\frac{3}{2}$  times the peak value of the fundamental phase voltage is peak value of fundamental phase voltage now.

So, this is  $V_{REF}$  and here you have  $\omega t$  and here you have  $\alpha$  and this relationship also I can quickly write down. Now, you are in sector 1, in sector 1 whatever

is your alpha omega t is equal to 90 degree plus alpha, is the relationship between omega t and alpha I sector 1. Further v reference is equal to 3 by 2 times V m where V m is the peak phase fundamental voltage this is the peak phase fundamental voltage.

So, when you substitute all these quantities you will get, you can kind of deduce an expression for the average current let us just do this one more step in the following one right. So, let us say we want to do i R 1 T 1 minus i B 1 T 2.

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**Expression for average dc link current (cont)**

$$i_{dc,av} = \frac{i_{R1} T_1 - i_{B1} T_2}{T_s}$$

$$= \frac{3V_m T_m}{2V_{dc} T_s} \frac{1}{\sin 60} \left[ \begin{array}{l} \sin(\omega t + \phi) \sin(60 - \alpha) \\ -\sin(\omega t + 120 + \phi) \sin \alpha \end{array} \right]$$

$f(\alpha, \phi)$

So, what do we have here? We have i dc average is equal to i R 1 T 1 minus i B 1 T 2 divided by T s here you have v reference for T 1 and T 2 is v reference is 3 V m by 2 then you have I m here. So, this is I m which is common here and a again T s is common. So, what I can see is divided by T s and you also have divided by V DC that is three v m you have V REF by V dc which 3 V m by 2 times V dc and I m, I m is what is common between i R 1 and i R 2.

So, you have taken I m out there you have divided by T s and what else there, there is also a factor of sine 60 degrees 1 by sine 60 degree. Now what I have is from i R 1 I have sin of omega t plus phi that is multiplied by for T 1 you have sine of 60 minus alpha, then for minus i B you have minus of sin of omega t plus 120 degree plus phi multiplied by sin of 1 alpha.

So, this is what you have now, and you can that is actually an expression and these are all constants under the given operating condition, you have a particular modulation index a particular DC voltage and then you have particular modulation index then you will have particular value of  $V_m$  and you will have particular load and therefore, the peak variant of  $I$  is  $I_m$  and  $T_s$  is your sub cycle duration. So, all these are constants now then this is what the time we required and you can see that there are parameters coming here this is what is that this is the power factor angle  $\phi$  and this is  $60^\circ$  there is  $\alpha$ ,  $\alpha$  can be replaced in terms of  $\omega t$  you can say  $\omega t$  is  $90^\circ$  plus  $\alpha$  I suggest wrote a while back and similarly here also you can replace that.

So, this whole thing can be replaced in terms of  $\omega t$  can be replaced in terms of  $\alpha$ . So, you are going to get an expression in terms of  $\alpha$  and  $\phi$ , this part is going to be a function of  $\alpha$  and  $\phi$  and this part is going to be there. So, you can see that your  $i_{DC}$  depends on various parameters, one is the depends on the DC current and excuse me and there is the DC voltage and then there is the modulation index  $v_m$  and then the peak value of current and power factor angle and so on and so forth.

So, you can deduce this expression, you can do a similar exercise considering sector 2 instead of sector 1 and check whether you come up with an expression you complete this expression which is just some more Arithmathematics and you try doing a similar exercise for sector 2 and see if you come up this thing this is something that you can just take it as a home assignment for this. This exercise will kind of help you to correlate the DC side current and the AC side current better now then you should understand that this 2 level voltage source inverter is one of the simplest configuration of voltage source converters. Here you should be able to relate the DC side and AC side better only then you know there are other applications I mean there are multilevel converters then so on and in those cases you will be able to relate them in a much simpler fashion. So, this is an useful exercise in that regard and you can just go about doing this now.

Now, what we will do is we will try certain alternate ways of doing it, now I started off saying that after establishing this whole thing that is writing down all the equations pertaining to the inverter model is said we will take space vector modulation, and in the space vector modulation what happens the entire fundamental cycle is divided into individual sub cycles and for every sub cycle some reference vector is provided like this

and an applied voltage vector whose average is equal to the reference vector is applied now.

So, you apply vector 1 2 0 and all for this time and you can come up with that and these are the times for which (Refer Time: 42:33) based and this we went ahead and calculated. We found out for every particular state what is the corresponding value of DC link current and then the inverter state changes in a particular fashion and so, you know every time it changes we change the current and  $i_{DC}$  is now equal to  $i_R$ , it is equal to minus  $i_B$  and so on and so you get these kind of wave forms here and. So, we try to do this from that particular point of time point of view. So, this is more the space vector modulation, it is from the space vector point of view that is we have assumed it to be a space vector modulated or rather we are looking at this you know and then we are trying to derive here.

So, we are looking at how the inverter state changes from one to the other. An alternate way of doing this would be to look at the modulating signals, that is let us say you go here let us say you look at this what is this?  $S_R i_R$   $S_Y i_Y$  and  $S_B i_B$  now, this is the instantaneous value. Now if you are looking at average that can also be determined by the average of these correct. So, that is the other way of doing it. So, let us go to the end here right. So, what we do is let us restart this exercise ok.

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$$\frac{1}{T_s} \int_0^{T_s} i_{DC} dt = \frac{1}{T_s} \int_0^{T_s} (S_R i_R + S_Y i_Y + S_B i_B) dt$$

$$i_{DC,AV} = \frac{i_R}{T_s} \int_0^{T_s} S_R dt + i_Y \cdot \frac{1}{T_s} \int_0^{T_s} S_Y dt + i_B \cdot \frac{1}{T_s} \int_0^{T_s} S_B dt$$

$$= d_R i_R + d_Y i_Y + d_B i_B$$

$$d_R = \frac{T_{ON,R}}{T_s} \quad d_Y = \frac{T_{ON,Y}}{T_s} \quad d_B = \frac{T_{ON,B}}{T_s}$$

$m_R = V_m \sin \omega t \Rightarrow d_R$   
 $m_Y = V_m \sin(\omega t - 120^\circ) \Rightarrow d_Y$   
 $m_B = V_m \sin \omega t + m_{CM} \Rightarrow d_B$

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So, what we have is let us start saying we have  $i_{DC}$  equals  $S_R i_R$  plus  $S_Y i_Y$  plus  $S_B i_B$ . Now both can be integrated 0 to  $T_s$  with respect to  $T$  and you can divide by  $1$  by  $T_s$  on both the sides. So, when you do that what do you get this is nothing, but  $i_{DC}$  average and what is that equal to? It is basically equal to  $i_R$  multiplied by integral of  $S_R dt$  0 to  $T_s$  divided by  $T_s$ . Similarly it is  $i_Y$  into  $1$  upon  $T_s$  0 to  $T_s$   $S_Y dt$  then  $i_B$  times  $1$  upon  $T_s$  integral  $S_B dt$  0 to  $t_s$ .

So, what is this equal to? The question is what is equal to? This is the switching function it takes values of 1 and 0. So, when it is 0 the integral is not affected whenever it is 1 that is only when affected and so, it is going to be 1 for certain amount of time and what is that? That is really the duty ratio. So, that is really the duty ratio for which. So, it is on for certain duty ratio time  $t_r$ . So, this becomes rewritten as  $d_R i_R$  plus  $d_Y i_Y$  plus  $d_B i_B$  is it right. So, you can write that in terms of the duty ratios, the time for which the top device is on what do you mean by duty ratio of R? This is the on time of the R phase top device divided by  $T_s$ .

So, this is what you have now. So, you get this, this for time for which it is on and when you that is and similarly  $d_Y$  is the time for which the on time for the y phase top device divided by the total sub cycle time and  $d_B$  is defined similarly as  $T_{on}$  times B plus divided  $T_s$ , and this approach is useful and is very very valid. So, what you can now do is you see that  $d_R d_Y d_B$  are all there, all this  $d_R d_Y$  and  $d_B$  can be expressed as some functions. You already have  $i_R i_Y$  and  $i_B$  and  $i_R i_Y$  and  $i_B$  can be approximated by  $i_{R1}$ ,  $i_{Y1}$  and  $i_{B1}$  as I said before that is their fundamental components I am sorry this is  $i_{B1}$ .

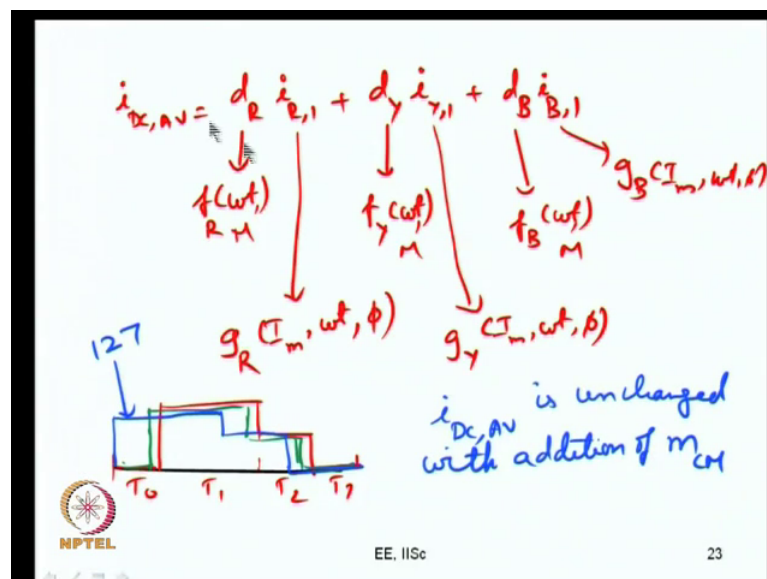
And the next thing is you can look at  $d_R d_Y$  and  $d_B$  and what are these?  $d_R d_Y d_B$  are also functions of the fundament I mean the change  $d_R d_Y$  and  $d_B$  change with the fundamental angle how do they change that depends on the PWM method. So, if you are talking of sinusoidal PWM method what happens is  $d_R d_Y d_B$  change in a sinusoidal fashion. So, you will have some way of writing this up. So, let us try doing that now. So, let us say we go to sinusoidal PWM now. In case of sinusoidal PWM what do you have you have your modulating signal  $m_R$  equals  $V_m \sin \omega t$  is that right this is what you have and what is your duty ratio of R phase? This is something we had done in an earlier class now.



So, can now see that this  $m_R$  would vary from minus 1 to plus 1 corresponding to your duty ratio variation from 0 to 1, this you can relate to the duty ratio you can relate to the duty ratio. So, when  $m_R$  is equal to 0 the duty ratio is 0.5 that is how you will get a relationship now. So, from here you can express duty ratio as a function now. Sometimes what happens similarly from  $m_y$  is  $V_m \sin$  of  $\omega t$  minus 120 and from this you can express duty ratio as a function of  $\omega t$ , and the same way you get  $m_B$ . Now you may now use sinusoidal PWM, you may sometimes use let us say a third harmonic PWM or something else. So, you will have your  $m_R$  star is some  $V_m \sin \omega t$  plus  $m_{CM}$  and this would give your duty ratio of R phase.

So, the duty ratio of R phase is now a different function on the fundamental angle than it is now. So, what you can do is you can also evaluate the duty ratios of R phase y phase and b phase and you can go about doing that. You can write this as sinusoidal function y as this and b as the other one and from there you can come to this whole point, that is you can write this as some function let me just write it down.

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So,  $i_{DC,AV}$  equals  $d_R i_{R1} + d_Y i_{Y1} + d_B i_{B1}$ , this is a function of  $\omega t$ . I will call this as  $f_R$  of  $\omega t$  and this is another function of  $\omega t$ , I will call this as  $f_Y$  of  $\omega t$  this is another function of  $\omega t$  I will call it as  $f_B$  of  $\omega t$ . Now how about the other ones  $i_{R1}$ ? So, this also depends on the modulation index. So, this also depends on the modulation index.

So, one way I can say that this is  $\omega t$  comma  $m$  also  $\omega t$  comma  $m$ , this is  $\omega a t$  comma  $m$  now. And this is a function of whatever the peak value  $I_m$ , and this also varies with  $\omega t$  and also the power factor angle  $r$ . So, let us call this not  $f$  for example, let us call this as some  $g$ , let us call this as some  $g_R$  and this is some other function  $g$  I am sorry  $g_Y$  of this  $I_m \omega t n \phi$  and this is some  $g_B$  of  $I_m \omega t$  and this power factor angle  $\phi$ .

So, this can be expressed now. So, this part is not dependent on the PWM method this is three-phase quantities we have ignored the fundamental component I mean we are ignoring the harmonic components the ripple, we are assuming that the ripple is much smaller than. So, that is ruined most cases in PWM inverters. And therefore, you can ignore them. Once you have ignored it is only three-phase currents it is  $I_m \sin \omega t$  plus  $\phi$ ,  $I_m \sin \omega t$  minus  $120$  plus  $\phi$ ,  $I_m \sin \omega t$  minus  $240$  plus  $\phi$ . So, these are three simple functions now. So, what depends on the this PWM signal? This depends on the PWM signal  $d_R$   $d_Y$  and  $d_B$  and this varies from 1 PWM method to another PWM method also.

So, there will be some variations you must expect when you do these kind of studies here, but there are always some ways to say that there may not be a very significant difference, it may appear to you that the DC link currents are all very different when you change from one PWM method to another PWM method, I can very simply prove that it is not really so, I mean there are some variations, but the overall you know for the particularly if you look at the in terms of the capacitor DC link current, you will find that they do not vary significantly. What is more important is we must be able to get a value for that  $R_m s$  current ripple.

So, that we can estimate that it is not very very you know it is not very much different that may not be very apparent from here because this suggests that these change with that and we are not very sure how much it is going to change now. Let me tell u you if instead of this three-phase modulating signals, if you go back and look at it as you know inverter states rather than three-phases separately, you look at it as a state that is three-phases combined together you have that advantage now let me draw that for you now right.

So, let us say this is what you have in a sub cycle and let me say this is this is what you have your  $T_0$   $T_1$   $T_2$  and  $T_7$  now what is going to change? The same invert same

operating condition. So, the same operating condition basically means  $V_{REF}$  and  $\alpha$  are same and the  $I_m$  values are same the fundamental currents these values are all same and  $V_{REF}$   $\alpha$  same meaning  $T_1$  and  $T_2$  are same.

So, this is what it is now. Here  $T_0$  and  $T_7$  can be equal, instead of let us say conventional space vector PWM I have some other harmonic injection PWM or something like that and therefore, the values are slightly different. So, what is going to happen? Let us say  $T_0$  decreases slightly  $T_0$  decreases slightly, then this is what is going to be the whole thing is going to be shifted to the left a little. So, it is not going to change the average only, the instantaneous value of the DC link current is changing, but the average value of the DC link current over  $T_s$  is not really changing. So, that the point that you have to note now.

Let us say, we go to bus clamping PWM method, not apply one of the zero state at all zero is not applied we use only 1 2 and 7. So, let me look at it in a different color right it now. So, if you are doing like that. So, this is going to become zero is going to be as indicated in the blue ink, this is sequence 1 2 7. So, this is a bus clamping method, but it is a same sub cycle duration we are maintaining. So, here what do you see? That  $i_{DC}$  average is unchanged with addition of common mode component this is something you can really see.

So, it does not matter. So, this is what you find. So, this is one reason why we look at every many things in the space vector point of view. We need to do in both variations we need to be able to handle it like this and also able to handle it like that, but you know when you are able to do it in both different ways well they have their own relative advantages. If we look at it in space vector the one advantage you are getting is you are able to see all the phases together and that gives you a significant advantage now. So, this similar approach of  $dR_iR$  etcetera we will be using when we go to evaluation of switching losses or conduction losses.

So, in conduction losses particularly you will have three-phase currents and corresponding to every phase current, there will be also some amount of voltage drop and the average voltage drop in a device would depend on the duty ratio, this you will see some similarity there you that we will do that in one more module later, that is the next module we will be doing the torque ripple analysis and the module after that

we will do this now. So, this kind of analysis will also be helpful now. So, today what we have try to do is we have just tried to get a feel for this problem and how we can relate the DC link current and the AC links current that is how do you relate the DC link currents and the AC side currents, we have just primarily got now and what actually remains to be done is basically to come up with this  $I_{cap}$ .

So, for this what we have to do is essentially, we need to get a clear expression for  $i_{DC}$  and once you have this  $i_{DC}$  average you can what you can do is you can get a further average  $i_{DC}$  over the sector that is it is periodicity and that will give you whatever is the DC current and this DC current flows from the supply, and we can assume the rest of the ripple to be flowing from here now.

So, if you can plot  $i_{DC}$  average first as a function of  $\omega t$  for 60 degrees, and get the average value of  $i_{DC}$  average this is  $i_{DC}$  average is average over the sub cycle  $i_{DC}$  average can be varied I mean over a sector you can get it is average over a sector and that gives this current. If you subtract this one from  $i_{DC}$  average you would get the corresponding  $I_{cap}$  and from there you will be able to evaluate that, this is something we will discuss in the next class before we go on to the module on pulse rating torque.

So, I thank you very much in your interest in this lecture, I hope that you found it helpful and hope you will follow the next lecture, which is going to be on the calculation of capacitor (Refer Time: 57:44) and also the subsequent modules.

Thank you very much.