**Pulsewidth Modulation for Power Electronic Converters**
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**Lecture - 25**
**Evaluation of RMS line current ripple using the notion of stator flux ripple**

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters.

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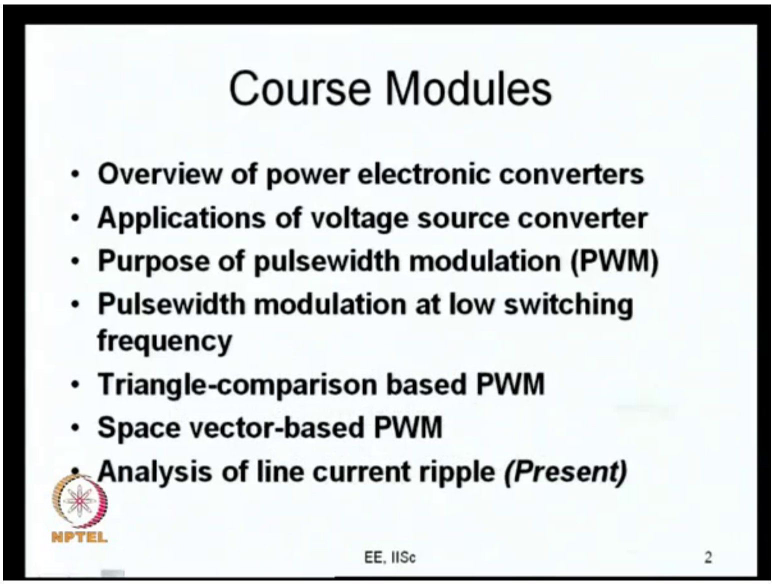


We have been going through different course modules. We looked at an overview first had an overview of various power electronic converters. Then we looked at the applications of voltage source convertors. Then we looked at purpose of Pulsewidth modulation. Then we slowly looked at Pulsewidth modulation at low switching frequency; with a very low like such like a couple of one or 2 switching angles per quarter and a very few switching angles in a quarter. And in the subsequent 2 modules we were looking at PWM generation at a fairly high switching frequency, where the switching frequency is much higher than the fundamental or the modulation frequency. And there are 2 popular approaches we looked at both the popular approaches of producing PWM. One is triangle comparison, other one is the space vector based PWM. And in the space vector based PWM, we saw the space vector based PWM is more general than the triangle comparison based PWM.

So, you have continuous and discontinuous modulation methods. Which can be implemented either using the triangle comparison approach or with the space vector approach. You also had some advanced base camping PWM methods, which can only be implemented using the space vector approach. And now after the PWM generation, we have begin to look at the waveform quality, well we are producing PWM waveforms all right. We are able to control the fundamental voltage is the purpose of the PWM is two fold, you know one is to control the fundamental voltage.

And the next is to control to also have a reasonably good waveform quality; that is make sure that the harmonics and their harmful effects are reasonably low. And so, we are getting into this of analysis of line current ripple which as what we have been focusing in the present module knob.

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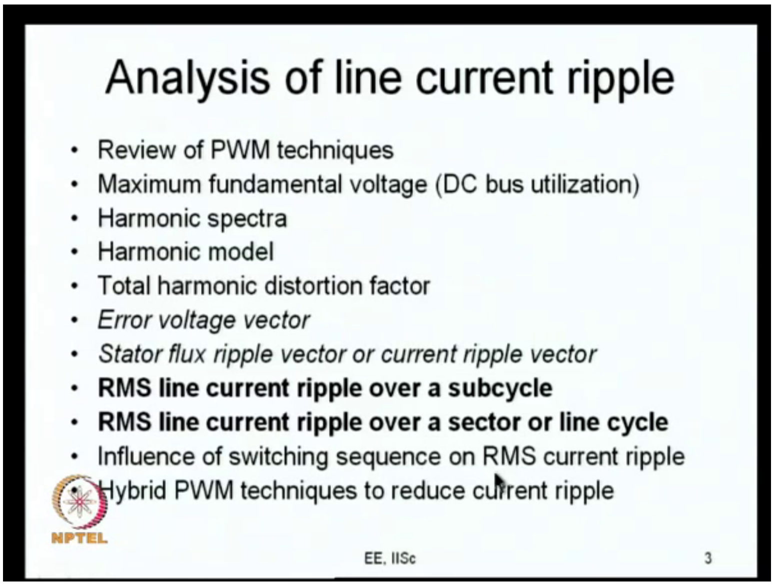


So, actually the present module the analysis of line current ripple includes a set of a things, these items or topics.

So, one of them was we had a review of the PWM techniques which we had seen in the last 2 modules, and we looked at the dc bus utilization. That is how much fundamental or ac side voltage can be produced with a given dc voltage. And we looked at the harmonic spectra, typical harmonic spectra of the different PWM waveforms, when you are implying different PWM method sinusoidal PWM or conventional space vector PWM etcetera. We get we developed some idea about that, and as I told you that our focus is not mean not so much on the harmonic spectra, and I also gave you some references where this the harmonic spectra are discussed in much greater detail. And we looked at the harmonic model and the total harmonic distortion factor etcetera. When the harmonic model as you can quickly recollected is actually for I mean for an induction motor, I mean drive the harmonic voltage you see the induction motor as a as it is leakage inductance. So, that is the harmonic model. And the then this is a measure of the waveform quality. We looked at the total harmonic distortion factor. Which is basically what is the RMS current divided by the fundamental current.

So, one way to come up with this total harmonic distortion factor is to you know calculate the individual harmonic components, and then I mean of the voltage. And from the individual voltage harmonic components, you can compute the individual current harmonic components, and from there the RMS current ripple, and hence the total harmonic distortion factor. What we have been doing is the other way that is we consider the error voltage vector. The error voltage vector is a representative of the harmonics in the voltages now.

You integrate that error voltage vector and that is; what is the stator flux ripple vector, which is a measure of the current ripple vector. So, since the harmonic model is that of an inductance. So, if you apply voltage across an inductance there is certain current through the inductance. The current through the inductance is proportional to the integral of the voltage applied; that is why if you have error voltage vector and you integrate that indicative stator flux ripple vector are equivalently your current ripple vector.

And so, we would kind of review these 2 things which we discussed in the last class. Error voltage vector and the stator flux ripple vector, and we would focus on how the RMS line current ripple over a sub cycle, and also the RMS line current ripple over a sector or the line cycle. So, sector is one 6th of the line cycle. And we are also going to look at the influence of this switching sequence on the RMS current ripple, and how you can construct hybrid PWM techniques which use a combination of switching sequences, that we will do in the next lecture. So now, we will be focusing on the calculation of RMS line current ripple.

So, today we are going to look at evaluation of RMS line current ripple using the notion of stator flux ripple. As I mentioned just a while back, this RMS line current ripple is can be evaluated from the harmonic spectrum also. You can calculate the voltage harmonic components, and from the from the voltage harmonic components you can calculate the current harmonic components, and from the current harmonics you can evaluate the RMS current ripple. And the other end what we are going to do is we are going to use the notion of stator flux ripple and what is stator flux ripple, is the time integral of the error voltage or the voltage ripple.

So, we are going to use this to evaluate RMS current ripple. You primarily in this lecture.

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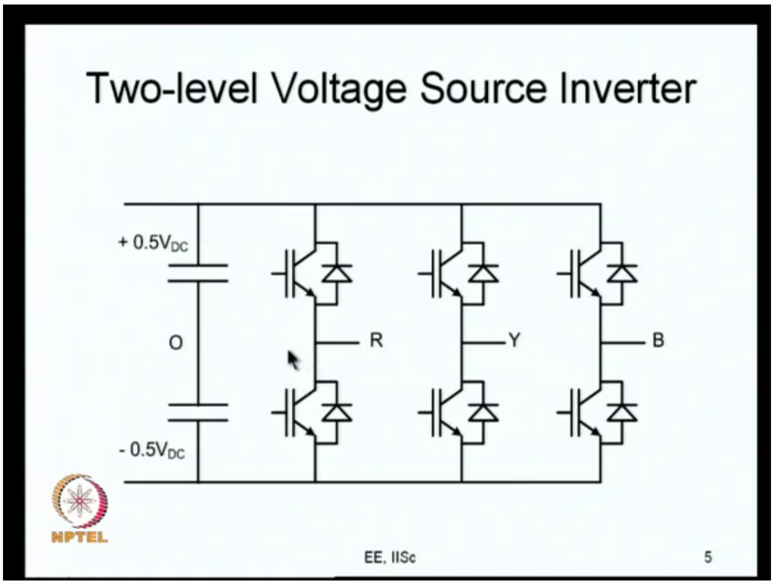


So, here is a quick review of a voltage source inverter. So, there are 3 legs corresponding to the 3 phases in each leg we have 2 devices, which switch in a complementary fashion. And it is a essentially everything is a single pole double throw switch, and the midpoint is what we call as pole. And we when we generally say pole voltage. We are talking about the voltage at this midpoint, with respect to the dc bus neutral.

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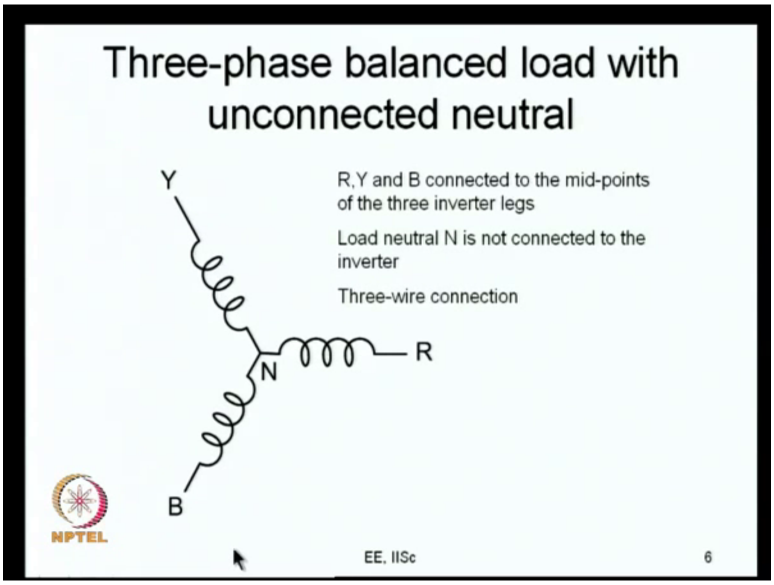


So, we are looking at 3 phase loads this RYB are the midpoints of the are the load terminals they are connected to the midpoints of R phase leg Y phase leg and B phase leg. And n is not connected anywhere particularly. So, the RYB are the ones that are connected. So, this is something what is called as an isolated star connected load with isolated neutral. So, sometimes you may have 3 phase 4 wire connections. Where the neutral is also connected, somewhere here what we are talking of is a 3 phase 3 wire system load.

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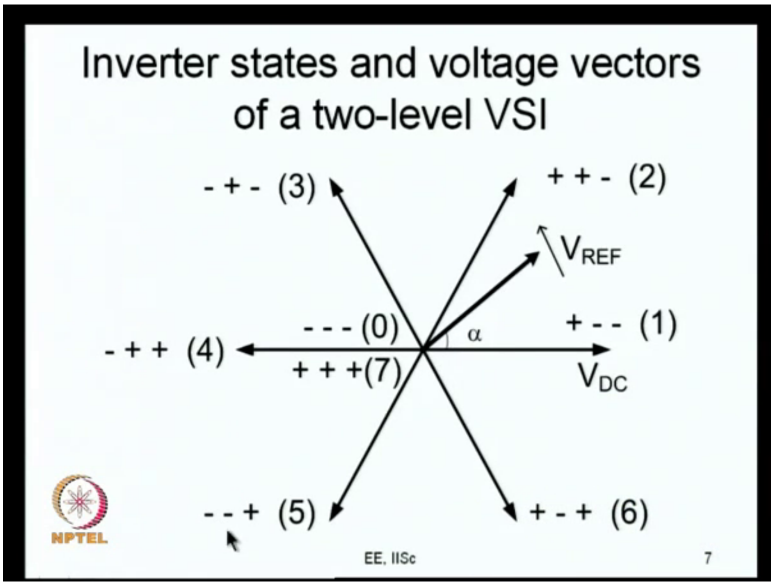

So, this is the kind of load, and these are the different voltage vectors. And since you had you know there are every phases positive or negative; that is every phase is like a single pole double throw switch. It can be either connected to the positive throw or to the negative thrown. If you look at R phase, it is all positive here. And it is all negative here. Similarly, here it is positive, here it is negative. So, Ry and b can be either connected to positive or negative and therefore, there are 8 inverter states as we discussed before. And of these 2 inverter states produce a null vector as we discussed earlier. Then the remaining 6 states produce this 6 active voltage vectors now.

Then you can in the space vector based party modulation, what you do this; you have a revolving reference vector this is revolving reference vector. It revolves at the fundamental frequency and it is magnitude is indicative of the magnitude of fundamental voltage that you want. And you sample it in every sub cycle. Once you sample it in a sub cycle, you produce it using a set of the nearest vectors.

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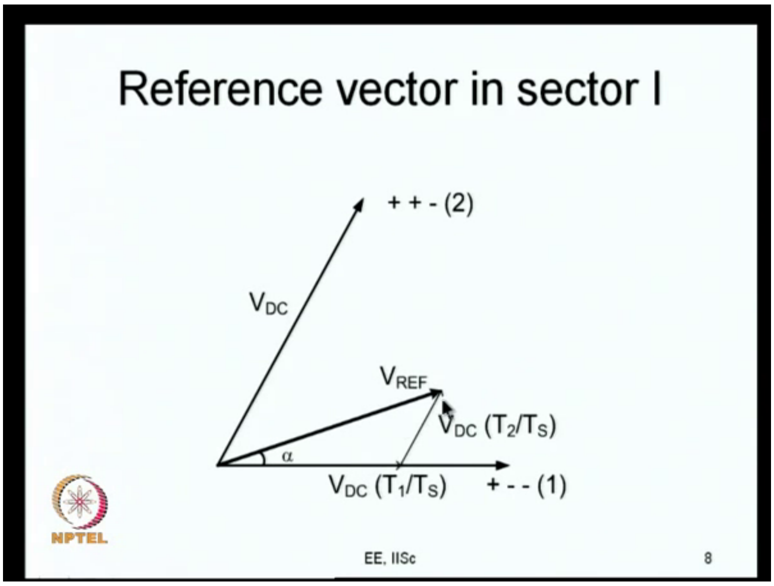


So, let us say you have sampled, and that sample lies between these 2 vectors. Active vector 1 and active vector 2, as you can recall we will apply active vector 1 for some T 1 seconds in active vector 2 for T 2 seconds, and the null vector for the remaining T 2 seconds to produce an average voltage vector equal to this reference vector. So, that is what is illustrated in this diagram.

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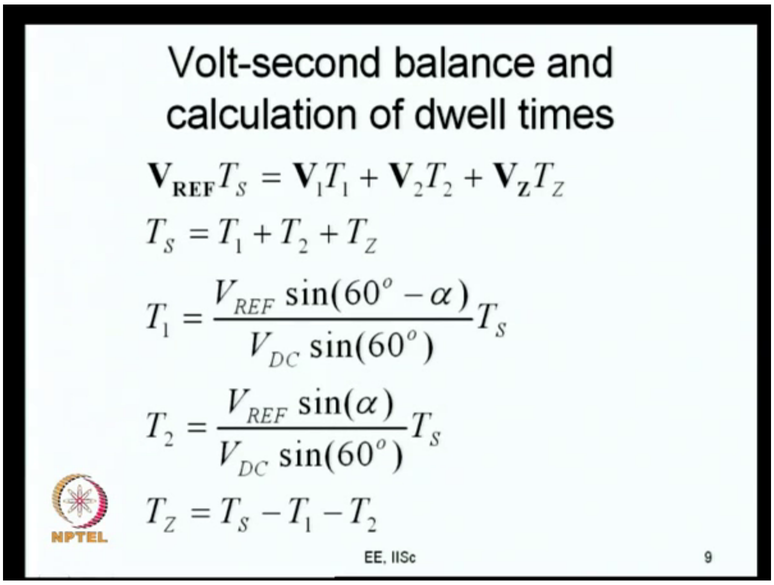


And when you are doing that, how you are going to calculate your T 1 T 2 T 2 is what is discussed here. So, this is the reference volt seconds, and this is the applied volt seconds. The reference volt seconds should equal to the applied volt seconds, and the sub cycle duration within the sub cycle you apply 3 different vectors; active vector 1 active vector 2 and the null vector. So, their times add up to T s.

So, you have 2 equations here, being a vectorial equation and this is a third equation. There are 3 equations and 3 unknowns T 1 T 2 and T 2 you can solve for them, as shown below here. And you can calculate a T 1 T 2 and T 2; this is what you do in all space vector base PWM.

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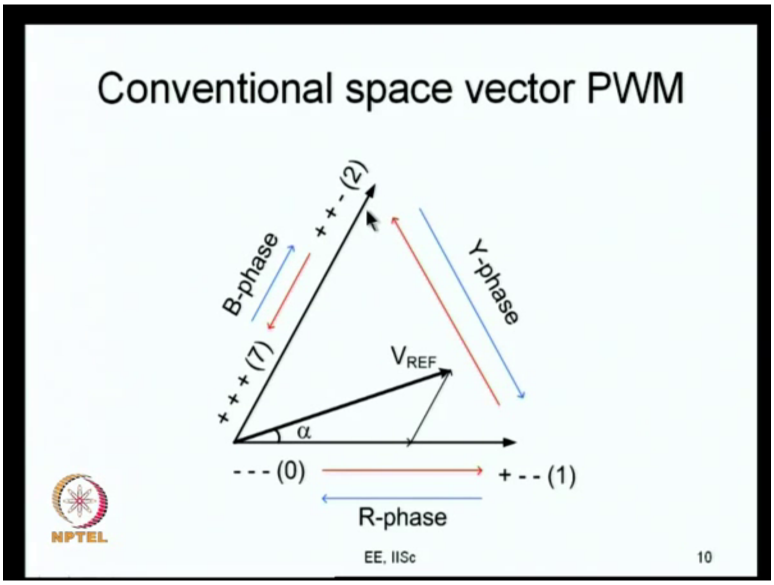


Particularly when you are talking of the conventional space vector PWM, you have this null vector time T 2, and there are 2 different 0 states 0 and 7. So, what you do is you apply this 0 state 0 minus minus minus for half the time, and plus plus plus for half the time that is T 2 by 2 and here it is T 2 by 2. You start here with minus minus minus. Let us say stay here for T 2 by 2, and switch the R phase to go to active state 1. Stay here for T 1 seconds, and from here you switch the Y phase and go to active vector 2. Plus, plus minus, you stay here for T 2 seconds, then switch the B phase and you can go to the 0 state, plus plus plus for T 2 by 2.

You can do the same thing in the reverse fashion in the next sub cycle. So, you can do this in the alternate fashion, and this is what you call as conventional space vector PWM. Similarly, when your reference vector it moves you know moves and moves and goes to an next vector, instead of vector a sector now this is called sector 1, it would move to sector 2 when it moves to sector 2, instead of active vectors one and 2 you will start active using active vectors 2 and some other vector 3 which will be like this.

You will start using that and this is conventional space vector PWM just for your recollection now. We are going to be dealing with the calculation of RMS current ripple exclusively for conventional space vector PWM today, though the process is quite general you can use the same process for the other PWM methods also, which is something we will discuss in the next class.

So, today the focus will be on RMS current ripple particularly for this convention space vector based PWM. So, you can probably take this as an illustration of how to calculate p RMS current ripple with this space conventional space vector PWM as an example.

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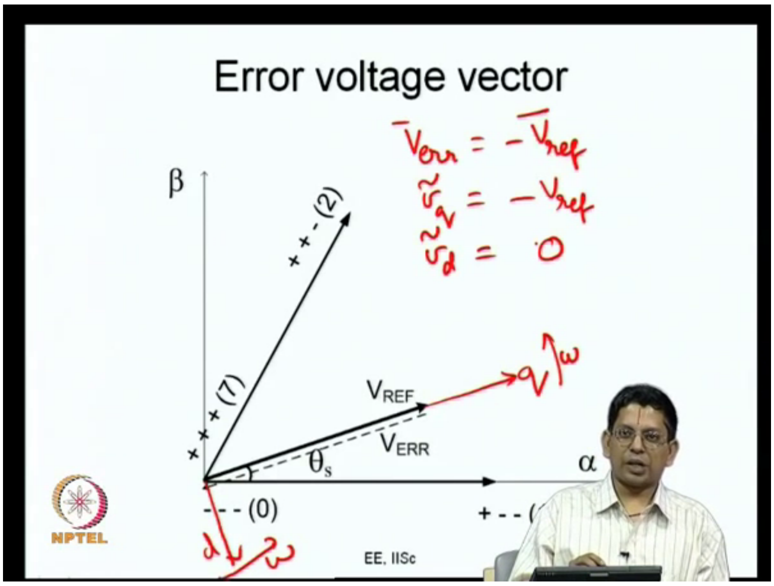


So, let us move on. So, this is now this is to review error voltage vector that I say said before. So, there is something you know there is always a difference between what we want and what we can apply in the context of this inverter. So, what we want is a sinusoidal voltage, but you do not get this. So, what you do is you actually synthesize sinusoidal voltages by applying dc pulses, that is what you do in an inverter. So, instead of a sinusoidal voltage you keep on applying dc pulses positive and dc pulses.

The same things if you look at it in the space vector domain you have this reference vector. So, applying 3 phase sinusoids means applying a voltage vector of constant magnitude undervaluing at a uniform speed in the space vector domain. So, we are showing this reference vector here at a particular snapshot of the reference vector, or a particular sample of the reference vector now. So, this is the reference vector. So, in a particular sub cycle this is the commanded reference, vector you need to produce this reference letter how are you supposed to produce this reference vector? This V ref vector was not available to you, the closest option is available to you are V 1 vector V 2 vector and V z vector though V 1 vector and V 2 vector are close to this; both in terms of magnitude and angle, they are not equal to them.

They are different from this. So, this null vector is also substantially different in magnitude from this now right. So, what do we see? So, so what are we doing? We are producing this V ref vector in an average sense. We are doing it in an average sense by applying this for certain time T 1 and this for certain time T 2 in this for certain time T z. So, there is a difference between what you apply and what you want now, what I want is V ref, but what I apply is null vector that is the particular example that I am considering. What I am applying is null vector? And therefore, there is an error vector. What is that error vector? The error vector is applied vector minus the desired vector, which is minus V ref vector.

So, let me just write it down quickly. So, you have this V error vector here is equal to minus p ref vector. This is what we have now. So, we can look at it in the dq domain also. This is I have just generally written it in the you know in the polar coordinates, I can write this in alpha beta I can also write it in the dq domain right. So, this is what the dq domain is what we you know discussed in the last class. What we can do is we can do the analysis in instead of alpha beta axis in d and q axis, which this is d and this is q; both are revolving at fundamental frequency omega where omega is the fundamental frequency now.

So, there is an error voltage vector those error voltage vector can be resolved along q on T axis; obviously, you can see that the error voltage vector is entirely aligned along that q axes. So, you can say V q tilde is equal to minus V ref, and V d tilde is equal to 0. So, these are the components of the error voltage vector, when 0 vector is the applied vector now. So, sometimes you apply the 0 vector, and sometimes you apply active vector 1 or active vector 2.

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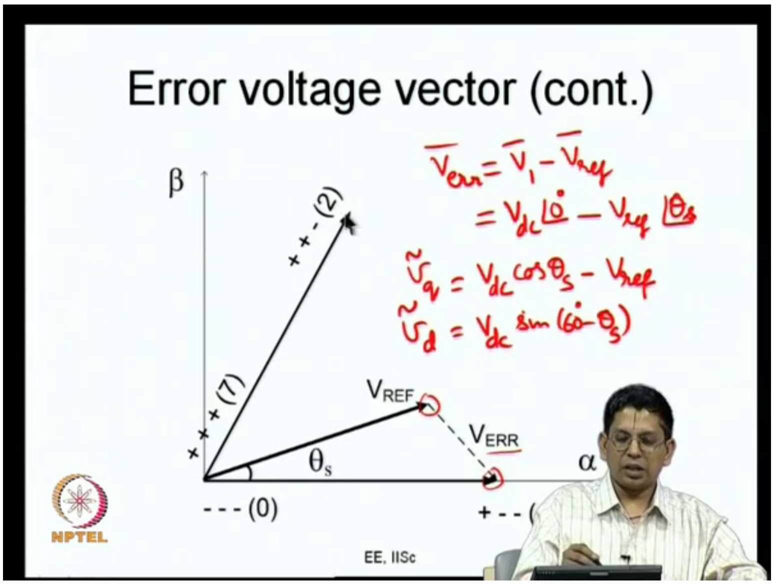


Now, let us say you have applied the active vector 1 therefore, what is what you have applied is this what you want is the reference vector. And therefore, there is an error between the 2. And that is what is given by this V error vector now. So, what is this V error vector in this case? V error vector is equal to V 1 vector which is minus V ref vector. The other way I can write it is V 1 vector is actually, this is V dc angle 0 degree in the alpha beta plane I can polar coordinates, and minus V reference angle theta s, as I written here now.

So, I can write this in the V q and in the in the in the dq reference frame also. If I take V q tilde, now what is V q tilde it is going to be V dc times cos theta s minus V reference. This is your V q tilde. And what is V d tilde? There is V dc, instead of cos it is going to be sin of 60 degree minus theta s; that is going to be your V d tilde right.

So, these are the error voltage vectors, when your reference vector is what is what is what it is as shown here, and the applied voltage vector is 1. This is the error voltage vector, the components of the error voltage vectors along the q axes and d axis are as indicated below.

Now we have covered the case of null vector we have covered the case of active vector 1, and the only thing remaining is the active vector 2. So now, let us say look at the active vector 2. So, let us go to this active vector 2.

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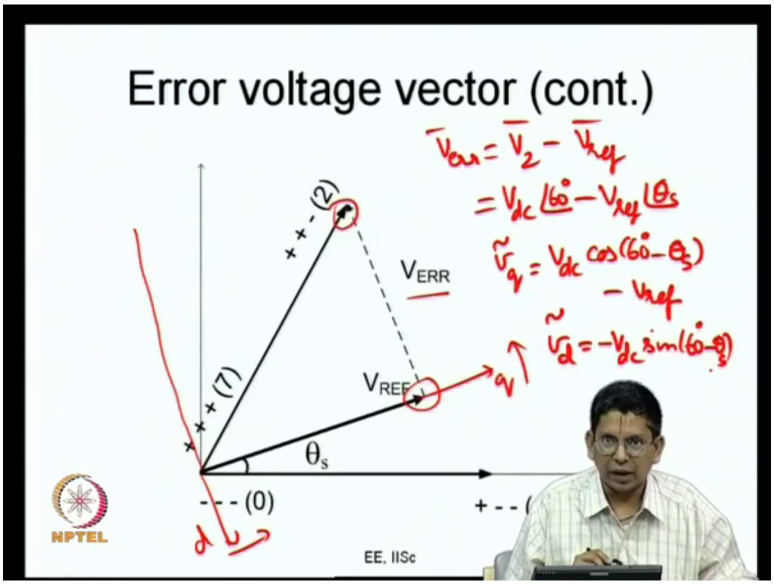


So, here what we have is, again let us see that we can consider this are your q axes, and this is the d axis, and both these are revolving at omega, but now you are considering a snapshot; that is, they revolve at the fundamental frequency your sampling time is much shorter is smaller than the fundamental frequency. So, you can say your V reference vector is aligned with the q axes, and then this is your d axis now. So, your error voltage vector is the applied vector is this. This is the applied vector. The this side vector is this and the error vector is the difference between the 2.

So, as I have said before V error vector equals V 2 vector minus V reference vector which I can write it as V dc angle 60 degree minus V reference angle theta s. I am taking this alpha axis as the reference when I when I am writing those equations here, all right. So, let me go further, I can write my V q tilde and V d tilde as before. And what is V q tilde? V q tilde is V dc times cos of 60 degree minus theta s. This is the component of the applied voltage vector; applied voltage vector is V dc cos 60. The along the q axes this is the vector that is applied; so minus V reference.

This is the V q tilde vector. The applied component minus the desired component, and then what is V d tilde? And V d tilde is basically you have along the d axis. So, what it is applying is you have something applied along the q axes. So, that is nothing but minus V dc sin of 60 degree minus theta s right. So, this is what it is now. So, you have V q tilde and V d tilde, we did this in the last class also. I am just having a quick recap on that. So, because what you are going to is you are basically going to integrate these vectors now. And that is going to be the stator flux ripple vector which is the measure of current ripple vector.

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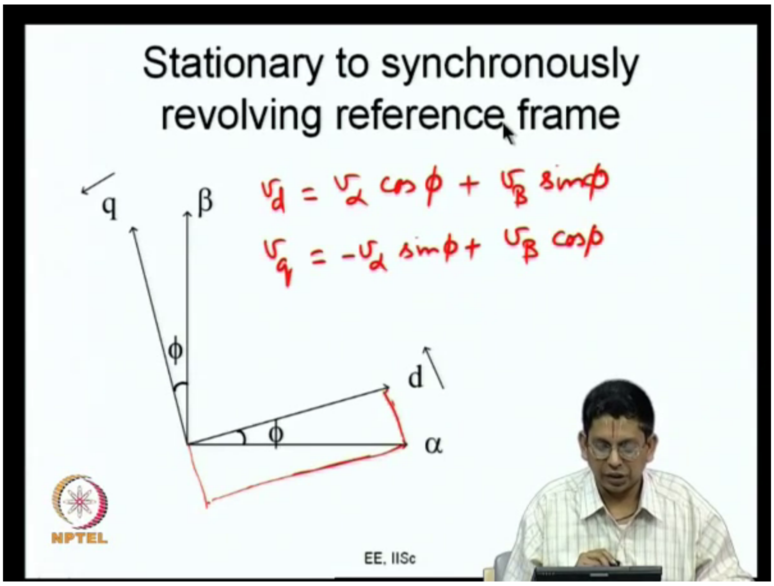


So, let us get going with that. So, this is stator flux ripple vector, and I have done this in the stationary to synchronously I mean reference frame there are 2 frames. And we are just choosing the dq reference frame to do the analysis here. Because you know, the end I mean the benefits will be little apparent when we want to calculate the RMS current ripple little later. So, if you just to have a quick recall of the you know transformation between the alpha beta and dq, what you can look at this you can say that this d axis voltage if you have any that will be V alpha times cos phi plus V beta times sin phi. So, given V alpha V beta and phi; this is what you have now.

Similarly, you have V q, V q is going to be V beta times cos c and if you take V alpha. So, V alpha would have minus V alpha sin phi as the component along that; so V alpha if you resolve it along that, you can do this and like this. So, this would be the 2 components of V alpha. So, this is V alpha cos phi, and this will be minus V alpha sin phi. So, that is what I have written now.

So, given V alpha V beta and sin phi, V alpha V beta and this angle phi; you can compute V d and V q. In the same way you can very easily write the inverse of this transformation. Given V d V q and phi you can compute V alpha and V beta. So, you can move from the stationary reference frame to the synchronously revolving reference frame, or back from the synchronously revolving reference frame to the stationary reference frame now.

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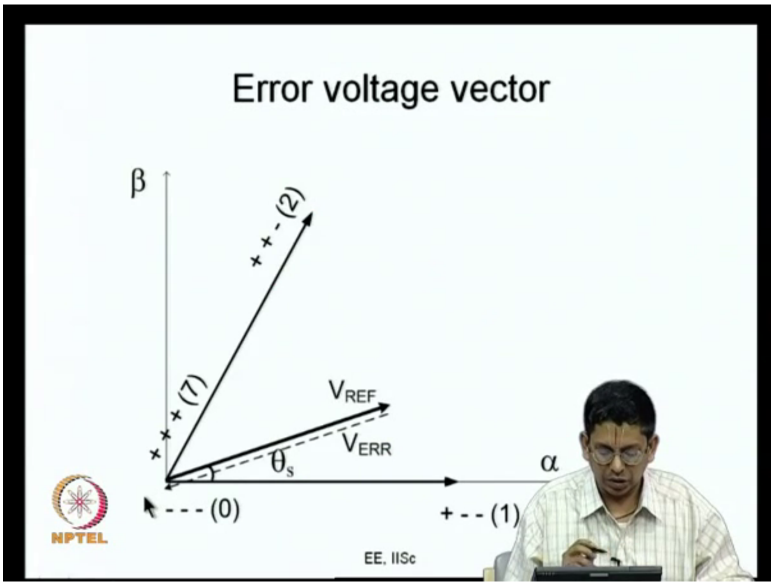


So, this is let us just keep this because we have primarily been able to cover this in the previous cases themselves. So, these are the error voltage vectors which you can look at both the frames now.

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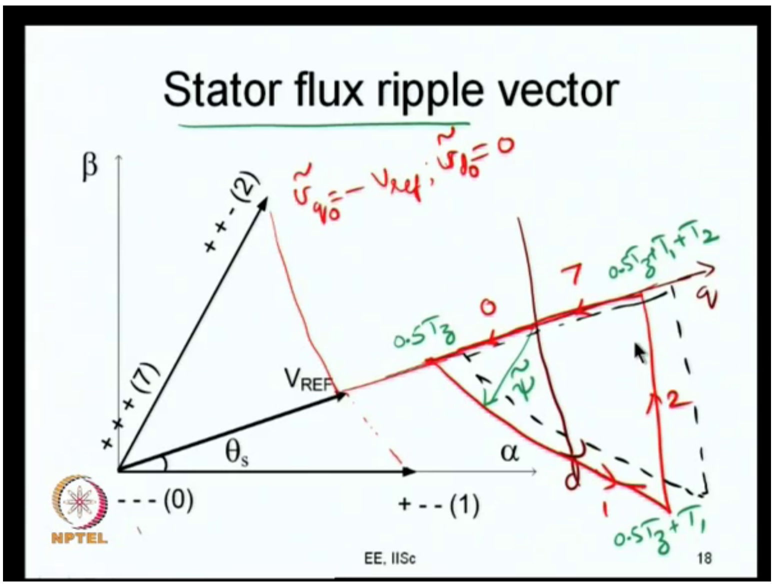

So now, let us go on to the stator flux ripple vector. So, the stator flux ripple vector is something like this now, excuse me. So, what we need to do is we have to integrate the error voltage vector. So, as we saw, the error voltage vectors if you really want to just look at it the error voltage vector is sometimes, the negative here the negative of V reference and when null vector is applied.

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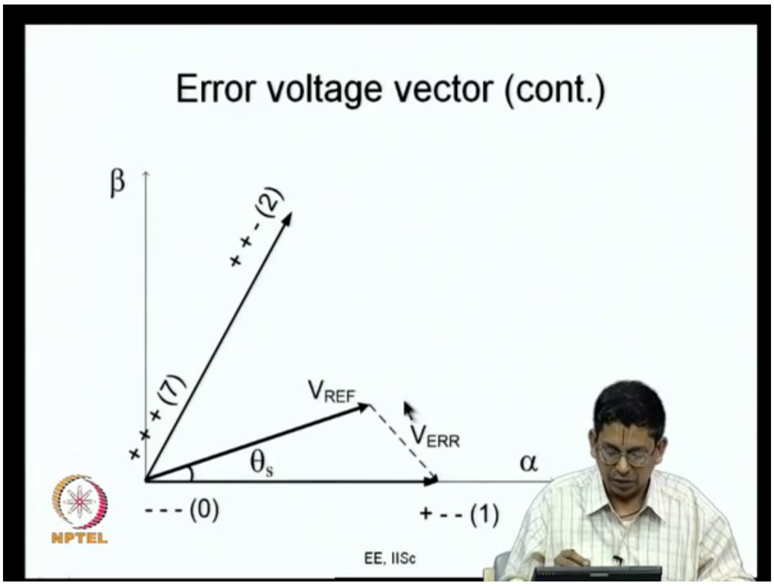


And it is V 1 vector minus V 2 vector, when V 1 vector is applied.

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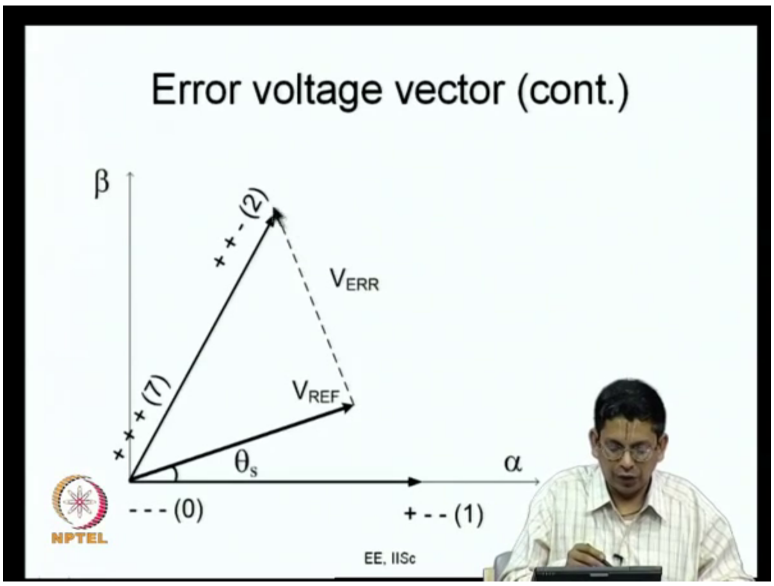

And it is V 2 vector minus V reference vector when active vector 2 is applied. So, what is going to happen is; in a sub cycle you are going to change, you are going to apply the null vector first and then active vector 1 active vector 2; and again, the null vector or the reverse fashion. So, this is what you are going to do and therefore, your stator flux ripple vector is going to change in a particular fashion. So, let us get started with that. So, let us say let us define this axis, this alright. So, let me say that this is a reasonable representation of the q axes. And let me say this is the d axis at an angle of 90 degrees now. Instead of drawing d axis here I am just taking the (Refer Time: 20:24) here just for convenience to draw.

First what is your error vector, the error vector has been null vector is being applied. Therefore, what happens is let me just change over to the other color right. When 0 vector is applied, your error vector is like this. And this is when 0 is supplied. This is what your error vector is minus V reference vector. Or if you look at if you recollect it. V q tilde is equal to minus V ref, and V d tilde is equal to 0. So, for this you get your error going around like this, something of this sort now.

Now, after that let us say you apply your one. So, let me also make this clear as V q 0 tilde and V d 0 tilde. Why I am a saying this? This is V q tilde corresponding to the null vector or the 0 state 0 being applied. Now the next time when I do that the error voltage vector is along this. It is V 1 vector minus V ref vector. So, whatever I will have? Let us say this ends here is going to go something like this. This is going to move like this, as I discussed in the last class. This is going to be parallel to that; so, ignore my drawing inaccuracies.

And finally, this is your other error voltage vector, and this man will move parallel to that and you will come back to the origin. So, this is 1, this is 2, this is 7. Let me just tell you again. So, this is the origin of the dq. So, the time is 0; this is the in starting instant of the sub cycle duration. The first T 2 by 2 seconds in the sub cycle you are applying the 0-state minus minus minus. When you are applying the 0-state minus minus minus, you are applying the null vector. And the desired vector is V ref vector. So, what is the difference; null vector minus V ref vector which is actually minus V ref vector. So, the error voltage vector is minus V ref vector.

And you integrate that you get an a vector which really grows like this. You integrate that, you really get a vector that grows like this now. This is at the instant T 2 by 2, let me write that down. This is at the instant 0.5 times T 2. Then beyond that what happens? The reference vector and let me call this as psi tilde vector, when I integrate that what I get is the stator flux ripple vector. So, that is what I am calling as psi tilde vector now.

So, this vector the tip of the vector once I go on doing the integration, the tip of this vector moves parallel to this, error voltage vector 1. So, as I said just ignore my drawing inaccuracy. So, just you can follow this error voltage vector, it goes on like this now. And subsequently what happens? You are applying this is at this instant, this is at the instant 0.5 T 2 plus T 1. Then between 0.5 T 2 plus T 1 2, 0.5 T 2 plus t 1 plus T 2 between these 2 duration you apply the active state 2.

Therefore, your applied voltage vector is here, this decide vector is here, and this is your error voltage vector and this error. So now, your tip of the vector you are integrating the error voltage vector. So, the tip of psi tilde vector moves like this, and reaches this point at 0.5 T 2 plus t 1 plus T 2. And subsequently when you apply the error voltage vector it moves back in this direction now.

So, if you see that this you know this base is equidistant in this side you know this vertex of the triangle or this vertex of the triangle are equidistant from this origin, which you may call is or so now, this is the case with the conventional space vector. PWM if you are using let us say something like sin triangle PWM or some other third harmonic injection PWM and so on and so forth. What will happen is; you know you will generally that is within the limit. So, that is sin triangle PWM is applicable only up to certain amount of fundamental voltage. Let us say consider within that limit. So, what would actually happen is; you would it will it will just move a little parallel, that is let us let me take like this.

So, T 0 may get shortened and T 7 make get longer you know lengthened in, be because of common mode addition, if it happens like that what will happen is just this will move like this, and this will move like this. And it will anyway come back to the same point. So, this is what I was trying to show in the dashed line is the case when these 0-state division is unequal. This is when the 0-state time is equal. So, you can you know get a feel for this right. Now those if the 0es time is equal, then this peak value and that peak value are equal. On the other hand, if the 0 trusted times are unequal, then this peak value and that peak value are actually greater than that.

That is, I mean r this peak value is lower, but this peak value is greater. That actually means that the RMS value of this ripple along the q axes is higher. This is just a point that I am trying to make still qualitatively, I would say that you know why conventional space vector PWM is better than sin triangle PWM in terms of harmonic distortion. The reason is this, that is you have this unequal divisions and I will I will stress on this point probably in the next lecture more, when I would like to discuss certain alternatives to conventional space vector PWM and I would say how they are different. But for now, I am just making this simple point you can see that there is an error in the q axes ripple, and the peak value of the q axes ripple is more.

And therefore, the RMS value of q axes ripple will also be more. When you add when 2 when 2 sin sinusoidal PWM you have one 4th third harmonic injection, what happens is it becomes very close to conventional space vector PWM. And the division of null vector time is almost equal. And therefore, you get the same thing as the red one. And that is the reason why when you have a third harmonic injection whose amplitude is one 4th of the fundamental, you get your THD is minimized. But that is not the case when let us say you have one 6th or some other third harmonic the when the ratio when the percentage of third harmonic added is something other than 25 percent all right.

So, let us just you know stick back to our conventional space vector PWM for the time being now. So, this is how the error voltage vector is. So, I have just shown that trajectory of this error voltage vector, along the q axes and the d axis. And now it is time for us to break that into the d and q axes components. So, let us do this again now. So, what I will do is, I will first draw this.

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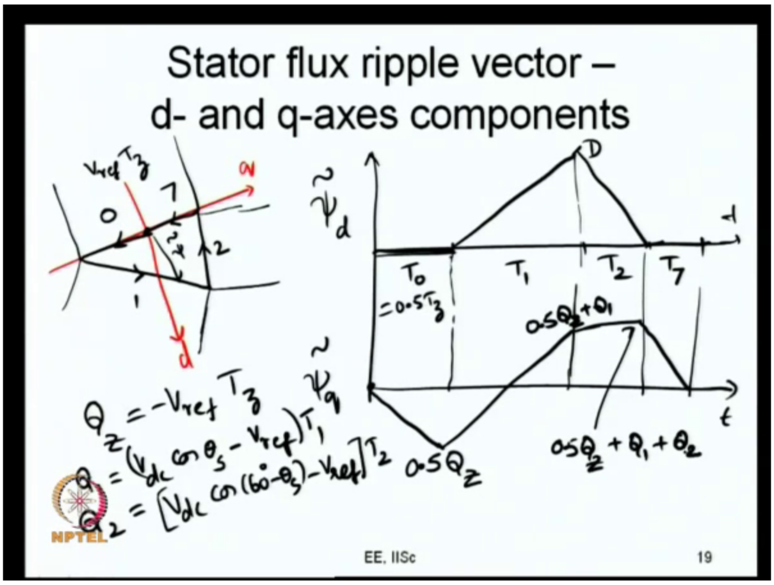


Now, let us say this is the q axes, and then this is the d axis. And I am trying to reproduce that stator flux ripple vector trajectory that I had drawn previously. So, let me just change the ink color to something else. Now let us say what we had was something like that. What we had was something like this. So, this is when the 0 state 0 is applied, this is when the 0 state 1 is applied, this is when the 0 state 2 is applied, and this is when the 0 state 7 is applied. Now I what I am going to do is I am going to resolve them into the d axis and the q axes components against time now.

The time information is kind of hidden in this diagram. So, I am going to plot the d axis ripple and the q axes ripple against time now. So, let me just get started with that now. The d axis ripple is a little easier to draw. Therefore, let me first take that d axes ripple first. I am calling the d axes ripple as psi d tilde this is the d axes component of the psi tilde vector. Now this vector that we have plotted here we will call this the psi tilde vector that is the d axes component is this now. So, the initial time, when the 0 state 0 is bidding applied, there is no change in the d axis ripple there is no change in the d axis ripple. Then subsequently for certain duration of time, when active state 1 is applied, the d axis ripple changes and changes linearly. And which is some value like this now. And subsequently when active state 2 is applied, it falls like this, and then it is 0 here. So, this I would call it as T 0, this is T 1, this is T 2, and this is T 7.

Of course, this is this is the time for which 0 state 0 is applied T 0, T 1, T 2 and T 7. And I have drawn for the 4 different durations, and this also should be a straight line let me try making this closer to a straight line. So, it is almost a straight line it falls like this. Why are these straight lines? Why are these straight lines? This is a straight line that is straight line why is that? Because this is the integral of V d tilde, what is that? It is some value we had got in the previous 1. What is that some value? It would be something like V reference sin of 60 minus theta. So, that is what it is now. So, that is a constant value it actually changes, but we are now considering over sharp small interval when the q and d axes are wherever they are.

And so, there is no change in the reference vector. So, the applied voltage what you are looking at is the difference between the applied voltage vector and the reference vector. So now, that you are resolving along the d axes and the q axes component now. So, within this interval the error voltage vector is unchanged. It has a d axis component and some q axes component; the d axis component is 0. And even this interval T 1, again your error voltage vector is constant though it is different it is V 1 vector minus V reference vector and it has certain d axis component. And that d axis component is the slope that you see here. And again, in this interval there the error voltage vector is constant, it is equal to V 2 vector minus V reference vector, it has a d axis component that d axis component is a constant now. And therefore, you get it as a straight line, while it is increasing here it is decreasing here. So, V d tilde is positive here, and V d tilde is negative here and it come back to this now. And it reaches some peak value right, some peak value it reaches. And let us probably call that peak value as some capital D. Let me call this as some capital D.

Alright, now let me try and go on plot this is versus time, this is versus time, let me plot psi q tilde. How am I going to get psi q you tilde? So, let me just project all this down. So now, what happens is when I start from here when I apply null vector, psi q is 0 and it actually goes in the other direction. It actually goes in the other direction. Then when I apply active vector 1, then V q tilde is actually positive. And it goes about increasing that something like this. Then when I apply active vector 2, V q tilde is a small positive value for the specific case, we have considered and it changes like this now.

And subsequently when the other 0 vector is applied, that is T 7, I mean the 0 state 7 is applied, this falls like this, comes to (Refer Time: 31:47) this is what we have. So, psi q tilde has a variation as shown here. So now, we can give some more values to this now. What are those values? So, let us say we call this let us project this, that is what that magnitude of this is actually V reference multiplied by T 2, V reference multiplied by T 2. Or it is actually minus V reference into T 2 now.

Let us define some quantity let us call this Q z is equal to minus V reference multiplied by T 2. What is the significance? We are talking of the interval when the 0 vector is getting applied, and the error vector is minus V ref, when 0 vector applied and 0 vector is applied for a total duration T z. So, this is error volt second difference now. And what you have here is; this T 0 is equal to 0.5 T 2.

And therefore, this is going to be 0.5 times Q z. This is going to be 0.5 times Q z, all right. Then you have other one; that is, you have this quantity, you have this quantity which is the length of the other side of the triangle. What is this length of this triangle? This is again the error voltage vector. So, what is the error voltage vector corresponding to this side it is V 1 vector minus V reference vector that multiplied by T 1 is the length of this particular triangle. So, if you look at it is q axes component, it is q axes component is something like, Q 1 is V dc cos of theta s minus V reference times T 1. So, what would be a q axes value at the end of it will be 0.5 Q z 0.5 Q z plus Q 1 that would be the value there now. Again, you take this one, another side of the triangle what is that side equal to? That corresponds to the state 2. So, there is the an error voltage vector is V reference minus V 2 vector, I am sorry V 2 vector minus V reference vector multiplied by T 2, that is what is this time. And it is q axes component you can call this as q 2 would be V dc cos of 60 minus theta s minus V reference. That is the error voltage component along the q axes that multiplied by t 2.

So, this value you have here at this instant is 0.5 Q z plus q 1 plus q 2. I am just trying to fill in numbers for this. And finally, it becomes 0. So, what I have shown is; I have shown you the nature of variation of psi d tilde and psi q tilde for this particular V reference. And alpha if you change that V reference an alpha, I considered some value of V reference in alpha which was here; this V reference and this angle theta s if I do not consider that I will get something else. So, I will do this if I mean this triangle will be different, and the d axis and the q axes components are different. So, for a particular example we have evaluated now.

The nature of variation is shown here. You can see that this is a piecewise linear curve, and the slopes in during the null vector are equal and the other slopes during the other 2 intervals are different now. And psi d tilde is also piecewise linear, but psi d tilde is 0 during the intervals whenever the 0 vector itself is getting applied. So, this is we just to get some idea of the d and q axes components of the stator flux ripple vector, and these are also the d and q axes the correspond to the d and q axes to the current ripple director now.

So, let us say how would you get the RMS stator flux ripple over a sub cycle that is an important question now. How would I get the RMS stator flux ripple over a sub cycle? Let me go over and do that figure again now.

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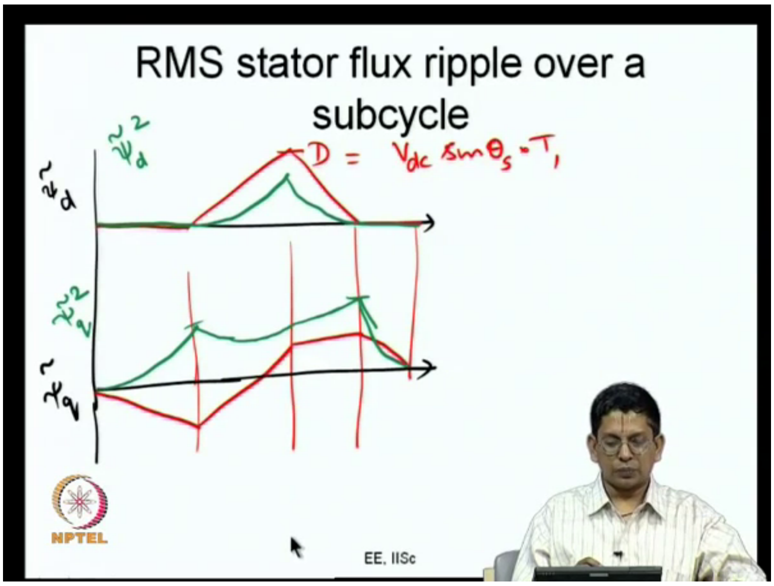


So, let us say that I have this, I have this d axis and the q axes things. So, let me call this as psi d tilde, and then call this as psi q tilde right. So, let me just choose a different color. Now what I had was; this was something like this was the nature. This was the nature of my d axes ripple and this is some capital D. And what is this capital D? This is actually the d axis error. What would be the d axis error? Something like V dc sin of theta s would be the d axis error multiplied by T 1 that is the d now.

So, if you take the q axes the q axes ripple is something like this, the q axes ripple is something like this. I am trying to reproduce that here. And we also found out the values at the various switching instance now. What I want to do now is; the root mean square value. I want to get the root mean square value. For the root mean square value what I should do? I must first square the function. Then I should take the mean value of the function, and then I should take the square root of the function now.

So, first let us square the function. How can you square the function? Let me choose a different color ink right. So now, I am going to square the first function, what happens if I square that? Psi d tilde is 0 here. So, I am square it is 0. Again, it is 0 here. So, when I square it is going to be 0. What happens in between? So, this is a straight line whereas, the psi d tilde is going to vary like a parabola, it is going to vary like a parabola. And similarly, here in the next interval also it varies linearly. So, it will be a parabola like this. This would be the nature of psi d tilde square, I am writing psi d tilde square in green color ink. So, that we know what I am doing now all right. Next what am I doing? I am going to consider psi q tilde square. So, if I square this I am going to get a parabola like this. If I square this what do I do? I will once again get a parabola let me just draw once a little better. So, I will square this I will get a parabola like this. The next time I square this I will also get a parabola, I will get a parabola which would be something like this now.

I will get a parabola which is something like this, then the subsequent one also would be a parabola and that is a section of a parabola like this and here again there will be parabola which goes down this knot. This is how the nature of variation is for psi q tilde square. So, these 2 are actually equal there is just my drawing inaccuracies. Because if these 2 are actually equal and opposite in sin. That is supposed to be. And so, this will also be equal and opposite to that now. So, why am I doing that? I am just first trying to get a feel for to calculate RMS value, how you have to square that I am just trying to square.

So, this psi tilde and psi q tilde, or I id tilde and iq tilde for that matter are piecewise linear functions, then if you square them they become piecewise parabolic functions. They are all for parts of some parabola or the other over the different intervals of time. So now, we just have to handle this. What we need to do is; we have to calculate the mean value of this. So, that is you know basically you have to calculate the area under that to do the mean we have we need to integrate that right. So, we have to calculate the area under these parabolic sections, which is what I had briefly indicated, the some basic idea behind that in the last class.

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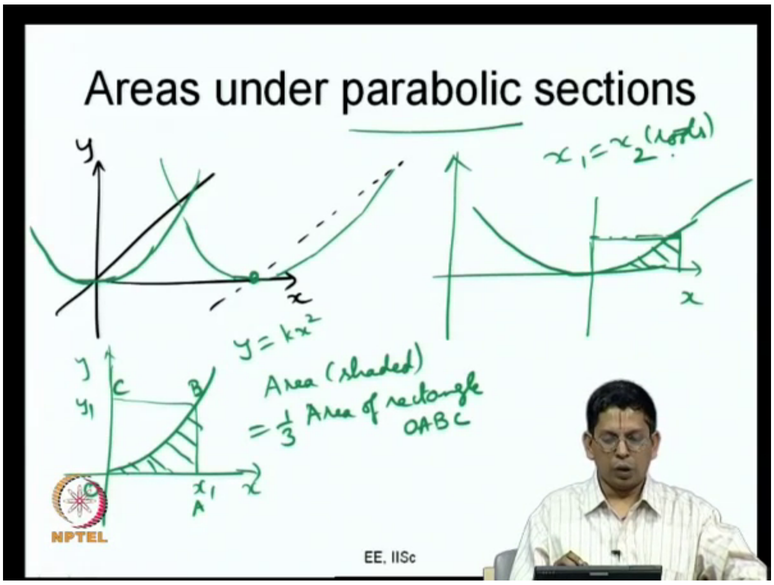


So, let us say you have a straight line like this. You have a straight line like this, which passes through the origin. This is x, this is y. So, if you square this line what happens? You will get a parabola, which is like this which would be symmetric on both sides of x axis, like this now. It does not look symmetric, let me just clear that up all right. Let me redo that. So, this will be flat 0, and the slope will also be 0 here. And this would be symmetric on both sides like this. This is how it will be now. Instead let us say if I take a different parabola. That is if I take a different straight line, that straight line let us say passes here.

This is the different straight line if I square this. So, what will happen? I will once again get a parabola, but that parabola will be like this will be something like this now. So, what we are looking at is the area under the parabola. So now, let us say if I have a parabola like this. This is y and this is x. And now what I want is this is some value of x 1, this is the corresponding value y 1. And I want the area under this parabola. I want this area under this parabola. This parabola is of the form y is equal to x square or kx square, or y is equal to ax square.

So, I want this area under the parabola. So, what is the area under the parabola? As I told you the other day, area that is shaded is 1 by 3 times area of rectangle. Which is the rectangle? Let us call is O A B C, of this rectangle O A B C. So, it is very easy to see. Y is equal to x square, if we integrate this x cube by 3. And x cube is the area of this, this is x 1 this is x 1 square. Or whatever let us take a simple case of y is equal to x square. This is x 1 square. So, the entire area of the rectangle is x 1 cube. And now the shaded area is x 1 cube by 3.

So, it is very easy for us to see that. That is true when you have the parabola somewhere else also. Like for example, if you take the x axis and the y axis being here, and let us say you have a parabola like this, and now the looking at this shaded area. This is the axis of the parabola, this is you are look at the shaded area. Once again you can constrict this rectangle. So, the rectangle one of the sides of the rectangle is the horizontal axis the other side of the rectangle is the axis of the parabola, and you know the other vertex is given by the point that you have interested.

So, this area of this rectangle, and the shaded area is once again the shaded area is one third of that area of this. So, it is very obvious right you know you are just shifting that. So, there is nothing as you can prove this also mathematically now. So, this is the idea which we are going to use now. And you get parabolic sections bk by integrating or squaring I am sorry squaring the linear sections, but what kinds of parabola? You are you are having a straight line that intersects at a particular value of x.

So, it has a particular root and you are squaring, the even you square you get a parabola. And the parabola is going to have real roots, and equal roots what is that root equal to; wherever it intersects the horizontal axis. So, these kinds of parabola have real and equal roots. If they have you know x 1 x 2 are the roots, you will have x 1 is equal to x 2 where there are these are the roots of the parabola. So, you will have real and equal roots for the 2 parabolas now. So, that is the kind of parabola that you will get whenever you square these kinds of straight line and that is what we will be dealing with now.

So, there are lots of things which are actually I can give you in a reference here, but before that let me just do a generalized you know a thing. Now you may be wondering what this question how do I formally prove that you know that; that shaded area is equal to one-third of the area of the rectangle and how I can use it now.

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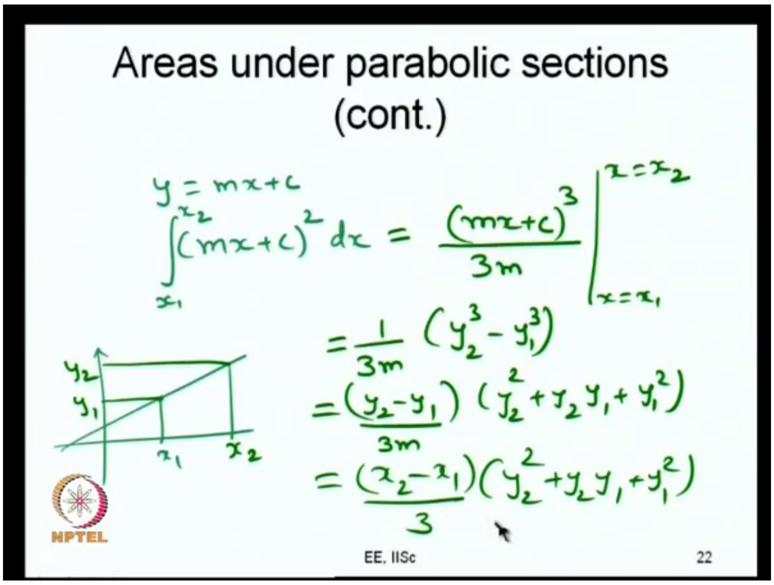


Let us say I take the straight-line y is equal to mx plus c. I take this straight-line y is equal to mx plus c. Now what I want to do this, I want to evaluate the area under the parabola I am first squaring that line mx plus c the whole square. Then I am going to integrate it with respect to xdx. I am going to do it between some values x 1 and x 2. And this is what I am planning on doing now. So, what do I get here? That is let us I have a straight line like this. So, some straight line, this is some straight line, I have points x 1 and I have a point y 1, sorry this is x 2 sorry. This is x 2 and the corresponding points are y 1 and y 2.

Now, I am going to square this to get a parabola. So, this is what I am trying to do. So, mx plus c the whole square, I am doing this is what I am with respect to dx. And I am trying to evaluate them with respect to u. So, what do I get here if I integrate this function, it is going to be mx plus c which is the equation of the straight line that I have squared. So, that I am integrating this it is going to be mx plus c the whole cube, divided by 3 m. This is evaluated you know, in the interval x is equal to x 1, and the x is equal to x 2. So, what is mx plus c if you look at? What is mx plus c if you look at? Mx plus c is nothing but y 1 at x is equal to x 1 is y 1, the other one is y 2. So, this is effectively it is 1 upon 3 m multiplied by y 2 cube minus y 1 cube. So, what you can do this? You can write this as y 2 minus y 1 divided by 3 m multiplied by y 2 square plus y 2 y 1 plus y 1 square.

So, this is what is y 2 minus y 1 it is actually the there is mx 2 minus mx 1 actually. So, this itself reduces to x 2 minus x 1 divided by 3 multiplied by y 2 square plus y 2 y 1 plus y 1 square. So, this is the expression we will use. When we are going to have any linear section like this, we are talking of any linear section, like a straight line between x 1 and x 2. We want to square that and evaluate that, this is the expression that we will use now.

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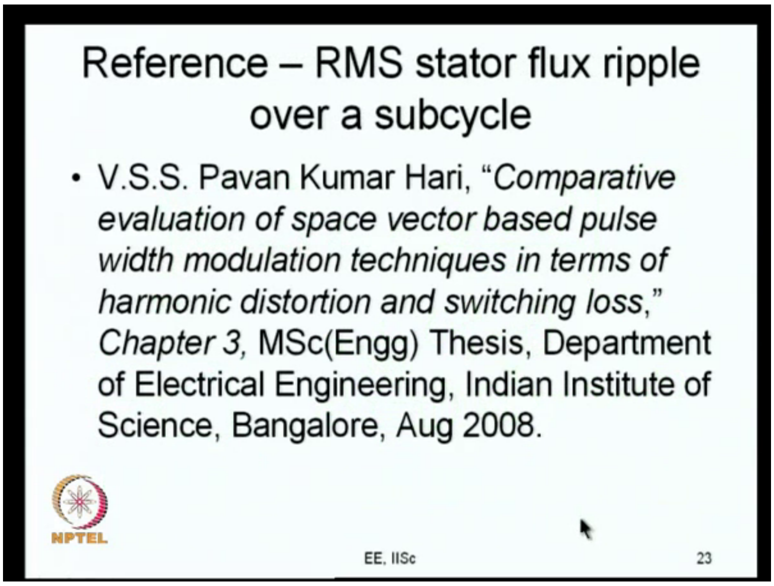


So, you can refer to this ms engineering thesis by mister Pavan Kumar Hari, on comparative evaluation of space vector based Pulsewidth modulation techniques in terms of harmonic destruction switching loss. You can particularly look at chapter 3 for this reference, where you know this where this question of RMS state of flux ripple over a sub cycle is discussed. And particularly a discussion on these kind of areas under the calculations, and whether the details of the calculations that I am presenting here are available here with regard to RMS state of flux ripple over a sub cycle now.

So, if you want to look at the d axis ripple over a sub cycle, it is fairly easy process now, whatever we have derived.

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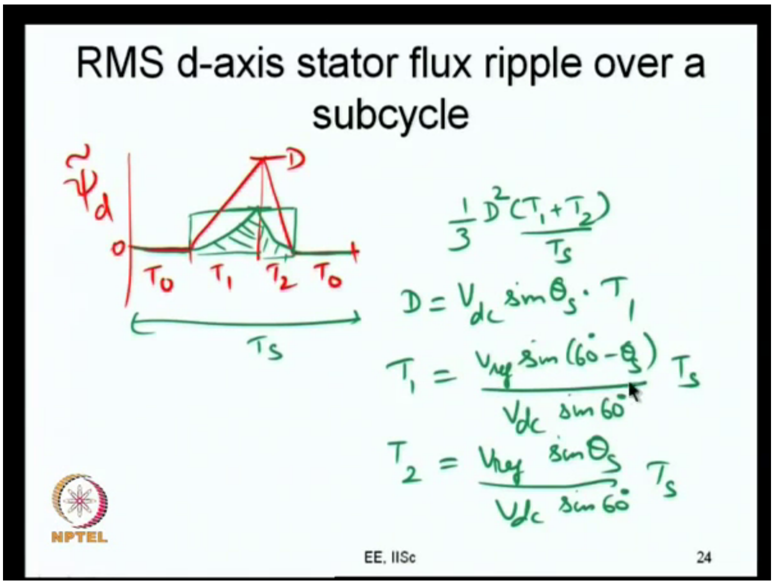


So, the d axis ripple is of this nature, it is like this it rises and it falls and it comes back. This is the whole time T s. So, this is I mean let me just draw that a little symmetrically here. So, this is T 0, this is T 1, T 2 and T 0. Here it reaches the value d, this is psi d tilde. So, when I get the RMS value here it is 0. So, this RMS value what do I do? I am just going to square. So, when I am squaring, it is 0 here, and it is parabolic here, and again it is a parabolic thing here and it is like this now. So, this area under this parabola, is one third of this area of the rectangle. So, this whatever is I can use the same this shaded area is one third of that, the same way this shaded area is one third of this area of this rectangle. So, I can write this value as 1 by 3 times d square multiplied by t 1 plus T 2 divided by, this is the total period of time T s.

This is like my mean square value of that, and you can see it is fairly simple. And if I take the square root of this I am going to get the RMS value. So, it is very easy, T 1 T 2 are all functions of V reference and alpha, V reference and theta s and so on. And d is also a function of all of them now. So, you can see that d is equal to V dc sin of theta s multiplied by T 1. And what is T 1? T 1 is V reference sin of 60 degree minus theta s divided by V dc sin 60 times T s.

Similarly, you have T 2 equals V reference sin of theta s divided by V dc sin 60 theta. These are things which we did earlier in the earlier classes. So, you can see that everything is a function of just 3 quantities, V reference just 3 quantities, V reference theta s and T s. So, the entire thing is a function of V reference theta s and T s. So, this is the RMS d axis stator flux ripple. The RMS q axes current ripple is a little more complex, but let us not worry we will do that. You will see that once again what you will see in the RMS q axes current ripple also, would turn out to be a function of V reference theta s and T s.

So, let us just get started now. So, this is the calculation of RMS q axes ripple over the first sub interval, I am just doing at sub interval by sub interval.

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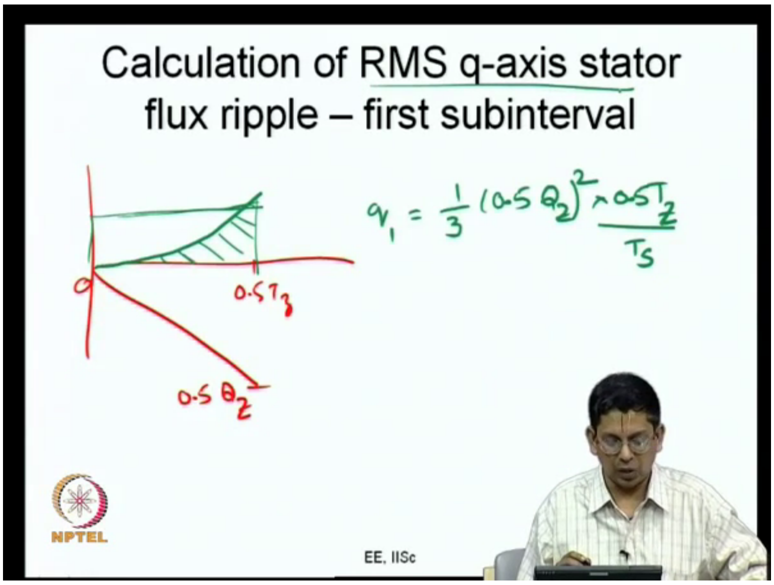


So, let me take something like this. Now what is the first sub interval this is 0, and this is 0.5 T 2. It is changing like this and this is going to a value 0.5 times Q z. So, when I am squaring it. When I am squaring it becomes a parabola like this. And the area under this parabola is that is the shaded area is one third of the area of this rectangle. So, what I can say here is; it is going to be 1 by 3 times 0.5 Q z the whole square 0.5 Q z the whole square multiplied by 0.5 T 2, that would be the area of the rectangle and this is going to be one third of that now, this is what you are going to get now. So, let us call this by some value. What thing shall I give, let me say this is some small Q 1 is this. So, this is the mean square value we are trying to calculate now. So, this 0.5 T 2 I can divide it by the overall time T s in an effort to find out the mean square value now.

So, I am just try I am finding the contribution of the first sub interval alone towards the RMS q axis stator flux ripple. If I considered the second one, how is the second one if you look at the second sub interval?

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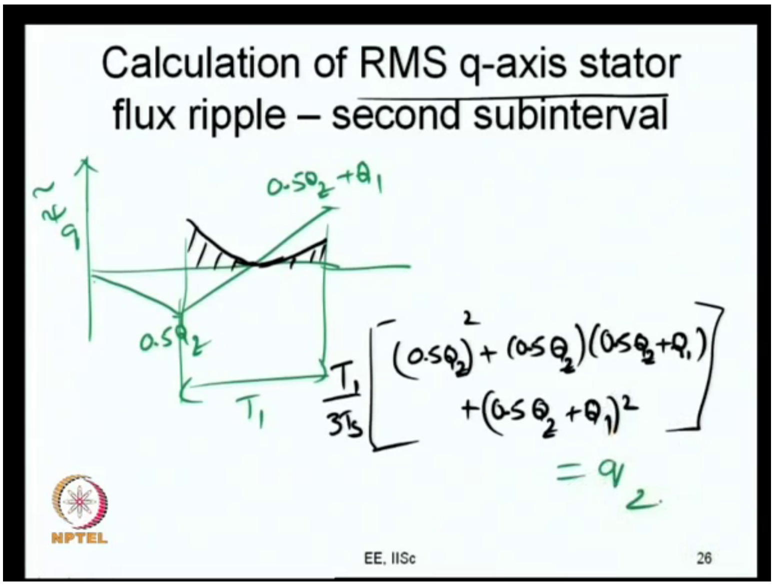


So, this is the first sub interval, where you minus point when the second sub interval it increased like this. You just recollect the psi q tilde graph that I had drawn before, and this is 0.5 Q z and this is 0.5 Q z plus Q 1. And this is the time interval is T 1. So, if I am going to square that now. So, what I am going to do? I am going to square this. When I square this, I will get a parabola and that parabola will be sitting here. I am going to get a parabola like this, like this now, and there is this area under this parabola. This we had just found out the earlier expression, how to get this now? That is what you do is you have this 0.5 Q z, you square it then plus 0.5 Q z multiplied by 0.5 Q z plus Q 1. Then what do you do? Plus 0.5 Q z it plus Q 1 the whole square.

So, this is I am just following what we did before, you know by squaring the linear section. I am going to take 1 by 3 T s of this, and multiply this by T 1. Now this is going to be the contribution of the second sub interval to the mean square q axes state of flux ripple. These are the mean square values I am just trying to compute.

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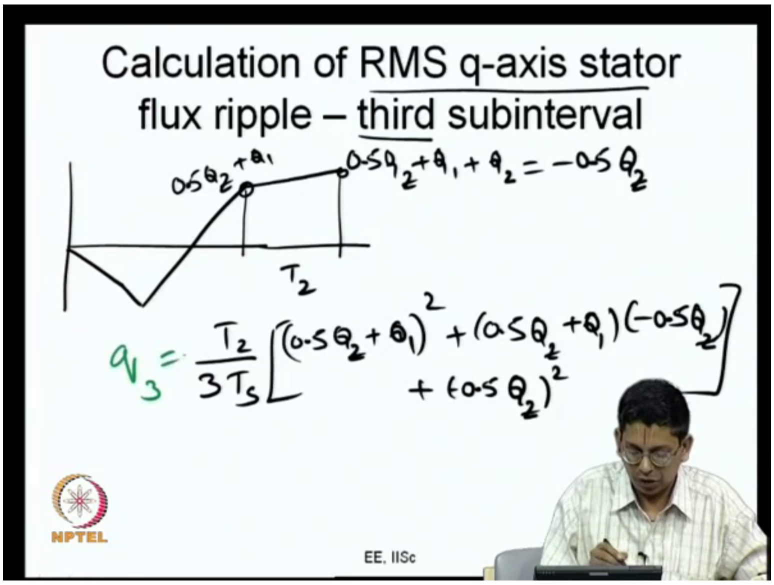


Now, the same way if you go to the third sub interval and the third sub interval it would look like this now. It changes like this and then it is going up then the third sub interval is what we found in the specific example was more or less flat. This is T 1, here you have the value 0.5 Q z plus Q 1. Here you have the value 0.5 Q z plus q 1 plus q 2. You will interestingly find that this whole thing is equal to minus 0.5 Q z minus 0.5 Q z, because Q z plus q 1 plus q 2 should add up to 0.

So, you will find that is that. So, if I could do the same kind of an expression as before what I will get is I will get T 2 upon 3 times T s multiplied by 0.5 Q z plus Q 1 the whole square. I am trying to evaluate the area under the parabolic section using these values, using these 2 values now. Plus 0.5 Q z plus Q 1, I am going to multiply this by minus 0.5 Q z then plus minus 0.5 Q z the whole square.

This is going to be the contribution of the third sub interval to the mean square q axes stator flux ripple.

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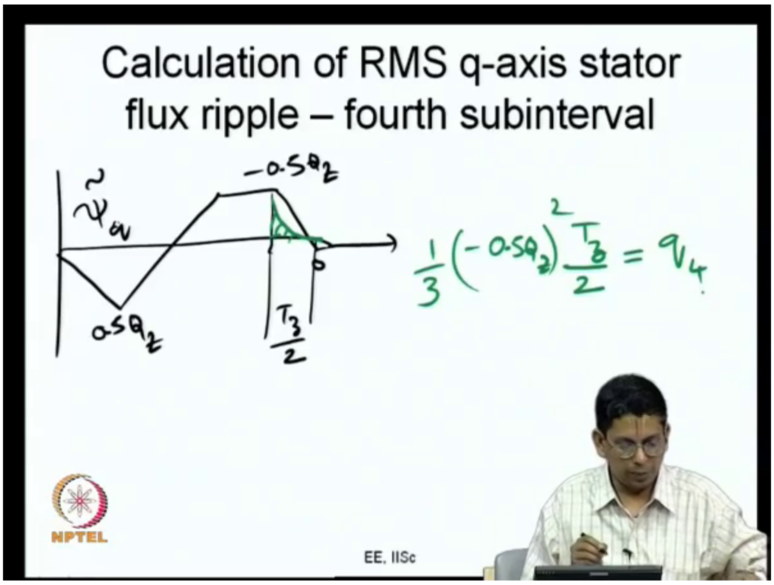


Now, how about the last sub interval? The last sub interval is kind of very similar to the first sub interval. So, this is 0.5 Q z the last sub interval it is minus 0.5 Q z, from here it is 0, this value is 0. So, this duration is again T 2 by 2. And this is psi q tilde I am talking about, I am now going to integrate psi q tilde. So, if I integrate here I am going to get a parabola like this. I am going to get a parabola like this. Now I am going to evaluate the area under this parabola. So, what is that going to be? It is 0.5 Q z. You can even take the minus 0.5 Q z, and square that multiply this by T 2 by 2 or 0.5 T 2, correct? That is the area under that rectangle and one third of this. And this I would call as q 4. The previous 1 I can call this as q 3. This previous 1 I can call this as q 2. And this previous 1 is Q 1 now.

So, I have got the mean square values in the various sub intervals now.

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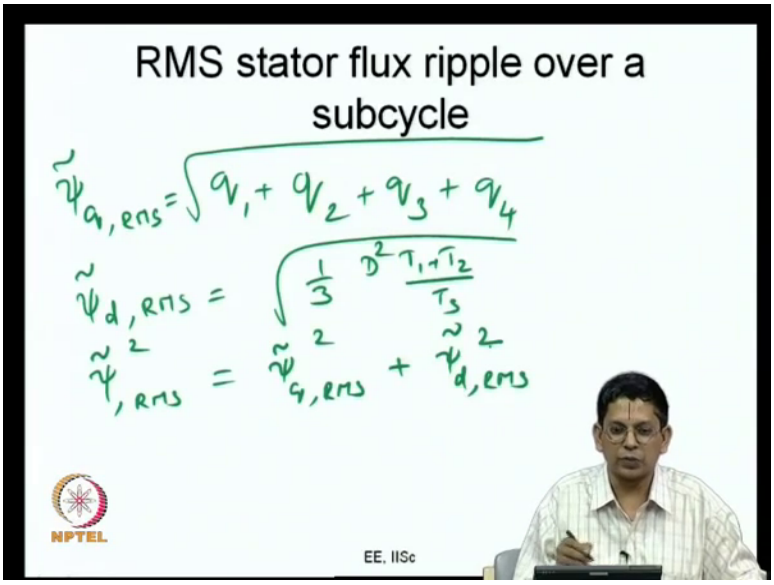


So, what I just have to do is; I will have to add q 1 plus q 2 plus q 3, plus q 4 to get what the mean square value to get the mean square q axes value. This I can always take it under a square root, and I am going to get the mean square value under that. So, let me just call this as you know psi q RMS if you wish. So, psi q tilde RMS I am going to get like this now. The psi d t tilde RMS is something which we got even earlier, that that was more or less a straight thing that we had found before which is basically one third of d square t 1 plus T 2 by T s.

So, this is my psi d tilde RMS. So, psi d tilde RMS is as we calculated before. So, this is 1 by 3 d square t 1 plus T 2 put 2 s the whole thing under square root now. So, what is your psi tilde RMS square? That is psi q tilde RMS square plus psi d tilde RMS square. You can just add the 2, and you will get the RMS stator flex ripple over a sub cycle you can look at the thesis there are also other references this thesis is one good reference where this is all been explained in a fairly clear manner now.

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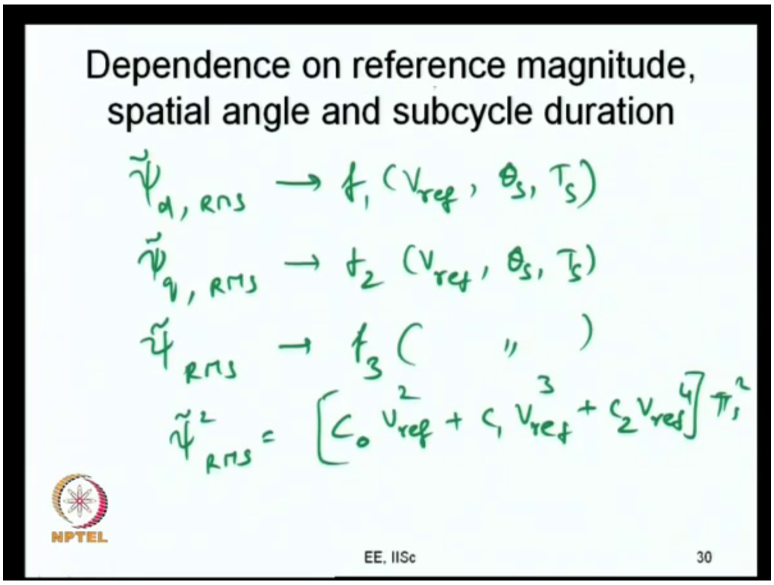


So, you will find that the dx is flux ripple the psi d tilde RMS we found is a function of V reference theta s and T s. Similarly, you can also see that psi q tilde RMS is also a function though a different function of V reference theta s and T s. And therefore, the whole of psi tilde RMS is also some function of the same 3. You will see that and this is been evaluated, and now you will see that this is it will actually interestingly it is of the form c 0 V ref square plus c 1 V ref cube plus c 2 V ref power 4, the whole thing multiplied by T s square. So, this is your psi tilde RMS square is generally of this form where you see that this is the function at dependence on T s and so on.

So, let us look at the remaining things in the following lecture. So, this is just to say how this dependence on all the 3 things now. So, I look forward to your this thing in the next lecture; where we will discuss further details on this calculation and do the other PWM methods.

Thank you very much.