Pulsewidth Modulation for Power Electronic Converters
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Lecture - 24
Analysis of RMS line current ripple using the nation of stator flux ripple

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters.

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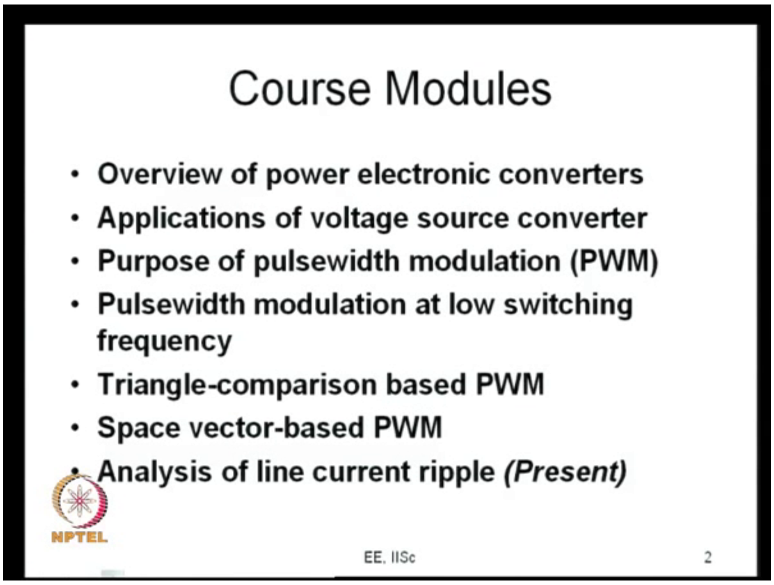

So, we have covered quite a few modules up to this point of time. We have had an overview of various power electronic converters and we will with the particular focus on the voltage source converter. We looked at different applications of voltage source converters such as motor drives active front end rectifiers you know active power filters and so on.

And then after is brief overview of the power converters in their applications we started off with the power basic purpose of pulse width modulation namely to control the fundamental voltage into control I mean mitigate the harmonics and their undesirable effects. So, we had a review of Fourier series and then the effect of different waveform symmetries etcetera, during that module. Then subsequently we went to pulsewidth modulation at low switching frequency.

So, we considered very low switching frequencies such as one switching angular 2 switching angles in every quarter. And then we tried to see how with just 2 switching angles we can control the harmonics and so on and so forth. And the from that we went on to selective harmonic elimination and a offline optimize PWM. And these portions and subsequently we move towards real time PWM real time generation of PWM which is what is most commonly done. And again, it is not at low frequency, but at frequencies much higher than the fundamental frequency.

So, something like you know if you are looking at a motor drive that would work up to 50 hertz fundamental frequency, we are looking at a switching frequency something like off a few kilo hertz like 5 kilo hertz or 3 kilohertz and 8 kilo hertz and so on and so forth.

So, these 2 modules we have been looking at you know generation of PWM at where is the switching frequencies are fairly higher than the fundamental modulating frequency. And here the generations is kind of based on triangle comparison that is you use 3 phase modulating signals and compare them with a common triangular carrier and you produce PWM waveform and this is an alternative approach which is space vector based PWM. And we showed that the space vector base PWM in is more general then triangle comparison PWM many continuous PWM and discontinuous PWM methods are there.

And those PWM ware forms can be generated either using the triangle comparison approach are their space vector approach there are certain advanced this space vector based bus clamping PWM techniques. Those advanced bus clamping PWM techniques you know those PWM wave forms pertaining to them can be generated only in the space vector approach and not using the triangle comparison approach. And this after dealing with the PWM generation at high frequencies, I mean at switching frequency is much higher than the fundamental frequency we have been looking at the analysis of the line current ripple we have start looking at the harmonic content of those waveforms which basically determine the waveform quality.

So, in the last lecture we had an overview of the harmonic spectra.

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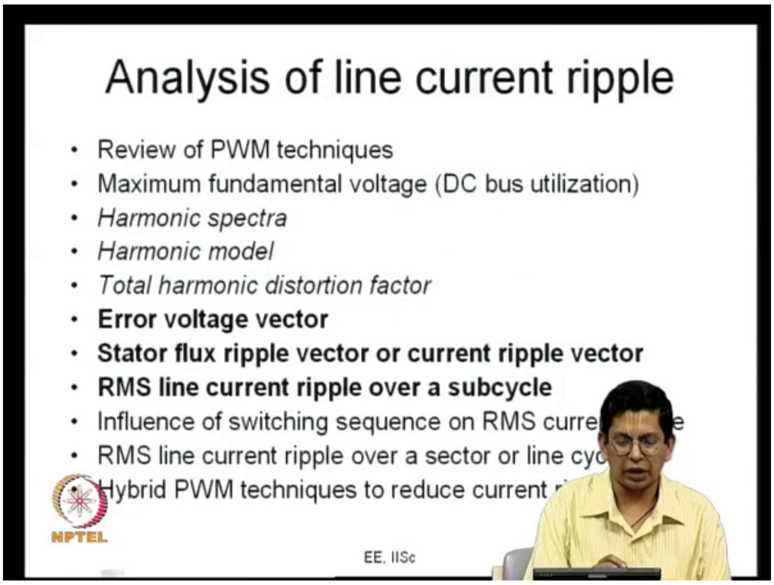

So, we have actually had a; we after review of PWM techniques. We looked at what is the maximum fundamental voltage are the DC bus utilization with the various methods. So, if the DC bus voltage is V DC the peak phase fundamentally can be 0.5 times V DC for sine triangle and with most of the other methods with common mode injection and so on. And the space vector bus PWM methods the DC bus utilization can be increased by 15 percent. Now and we also looked at the typical harmonic spectra corresponding to sine triangle PWM and so on and the harmonic model.

So, we would quickly revisit today, how the typical voltage harmonic spectra would look like for a PWM technique you know which is maybe based on triangle comparison or space vector, but where the switching frequency is much higher than the fundamental frequency and we would look at the harmonic, if we would quickly review the harmonic model how an induction motor is seen by a harmonic voltage source or the harmonic components in the update voltage. And we would look at the total harmonic distortion factor after a quick review of these we would go into the topic of the day namely the error voltage vector there is mentioning the other day that is the previous lecture when I was beginning with this.

So, this error voltage vector is nothing but all these harmonics that you have, other than the fundamental all the harmonics. So, these harmonics are some of the harmonics you can call it as voltage ripple. And the 3-phase voltage ripple translated transformed into the space vector domain is the error voltage vector. So, we would have a discussion on this error voltage vector. And from then on, we would move towards what is called a stator flux ripple vector, which would be our main focus of these things would be the main focus of our lecture today. And what is the stator flux ripple vector it is the integral of a error voltage vector. So, why do we have to integrate that you know because the motor model is a the harmonic model is simply leakage inductance.

So, the harmonic voltages are getting applied to the leakage inductance of the machine. And thus, are there corresponding harmonic current is proportional to the integral of the harmonic voltages. So, the current ripple is proportional to the integral of the error voltage vector. So, the integral of error voltage vector is what we call as the stator flux ripple vector which is a measure of the current ripple vector as we will see today. And we can use this to study the RMS line current ripple over a subcycle and we will try reaching this point. And so, further otherwise things will be done in the next couple of lectures from now so first to have a review of this. So, today is lecture focuses on analysis of line current ripple using the notion of stator flux ripple.

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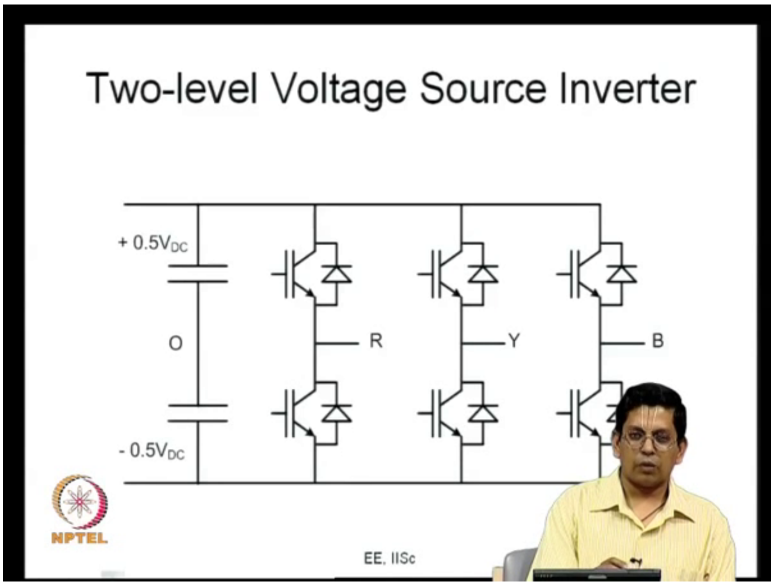

So, we would have a quick review. So, this is the voltage source inverter and you have 2 devices in every lug they are switched in a complementary fashion.

So, every lug is actually a single pole double throw switch this is the pole there are 2 throws. So, every lug is a single pole double throw switch and this terminal is commonly called as the pole. And pole voltages are usually measured with the respect to the DC midpoint. So, VRO for example, is called the pole voltage and V Ry is the line cline voltage.

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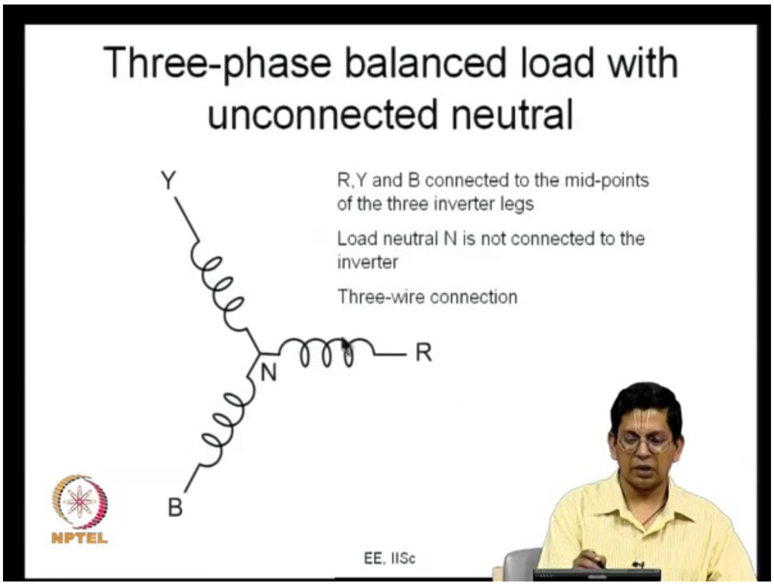

And the load usually that we see is connected is like this R Y B star connected load whose neutral N is not particularly connected anywhere and this R to N. For example, is V R N this we will regard as the output voltage that is the phase voltage output phase voltage of the inverter?

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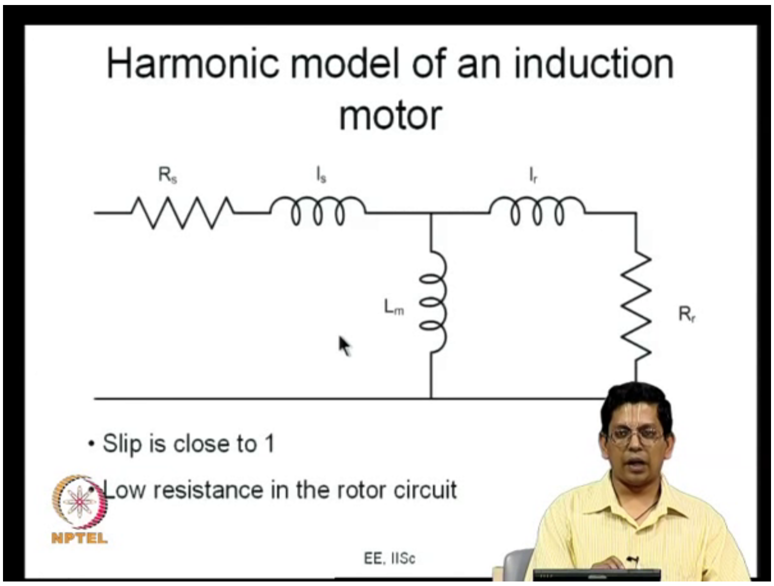

And we would look at the harmonic spectrum of this and a. Firstly, if V R N let us say has it has a certainly fundamental component, that is whatever gates applied here has a fundamental component and also has harmonic components. The fundamental component which we can call let us as V R N one sees the fundamental equal and circuit of the motor then there are harmonics. And how what do the harmonics see?

Now you can look at a quick review of this. So, the equivalent circuit looks like this the harmonic voltages you know produce the synchronous frequency corresponding to the harmonics is very, very high. And therefore, the difference between that synchronous frequency and the rotor speed is very, very high. And therefore, the slip is close to one what we have is Rr by S usually and this slip is close to one and when you have the slip is close to one the slip.

So, this is a very, very small resistance now in the case of fundamental component the slip is very small something like 3 percent or 5 percent. So, Rr by S is a big number here S being almost 1, Rr is a very small number in half. So, and there is low resistance in the rotor circuit and both these resistances Rs and Rr are negligible compared to the reactance's omega l S, sine omega l R respectively at harmonic frequencies.

So, their frequencies are very, very high.

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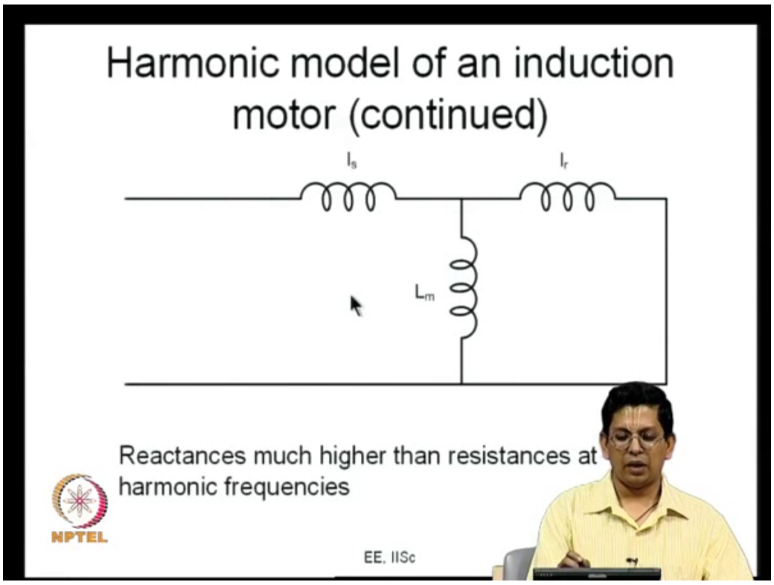

And therefore, you have the equivalent circuit reduces just to this is the magnetizing inductance this is the stator leakage inductance and this is the rotor leakage inductance. Now you go and step further this to this leakage reactance is much lower than the magnetizing inductance. So, this inductance is much lower than the magnetizing inductance.

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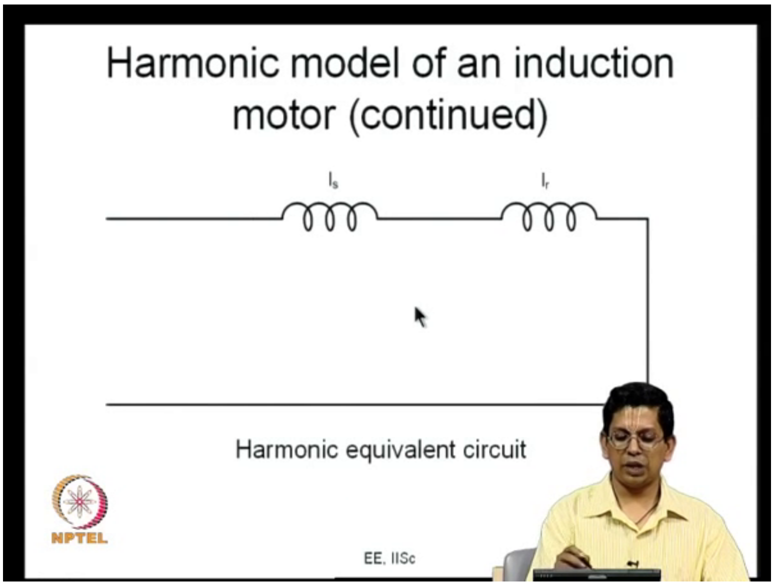

And therefore, the equivalent circuit can be further reduced here, and this is the harmonic equivalent circuit this is what we had seen before now.

So, every harmonic voltage sees basically this as the equivalent circuit. And therefore, the corresponding harmonic current is derived by dividing the harmonic voltage by the harmonic reactance. And the harmonic reactance is proportional to the order of this. So, 7th harmonic sees higher harmonic reactance and fifth harmonic. It is N times omega multi plug by l S plus l R. Therefore, higher harmonic voltages see greater harmonic reactances. And therefore, they you know for the same amplitude they produce lower amount of harmonic current that is the idea now.

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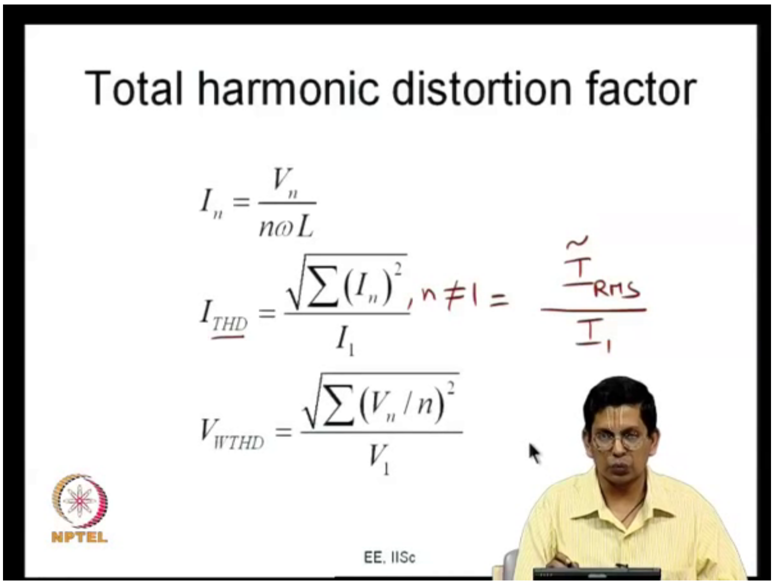

So, this is what is given here IN is the nth harmonic component and V N is the nth harmonic voltage component that divided by N omega l here that l is that some of l S plus l R that have taken the previous thing. So, this is the amount of harmonic current that would flow. Now if you a considering something like let us say the forty ninth harmonic then the corresponding forty ninth harmonic voltage that divided by 49 times this reactance, that would give you the forty ninth harmonic current. And similarly, you can sum up all of this that is all the harmonics etcetera that you can sum up.

This is sigma of IN squared for N not equal to 1, which I should probably indicate there that is with N not equal to 1. That is, you leave along the fundamental you consider all the other harmonics take their sum and divide by I1 that is the what is called as the total harmonic distortion factor of the current waveform the, THD stands for the total harmonic distortion and the ITHD is the total harmonic distortion of that current wave form. And here is you can see component by component if you can go IN this is what V N by N omega l. So, IN is proportional to V N by N as you can see here.

So, you can say that this is sigma V N by N the whole square is a measure of sigma IN square and that divided by V1 gives you the weighted total harmonic distortion factor of the voltage waveform which is equivalent to the current THD. Now what you can do is if you want to calculate to get a measure of the current THD, you can try and compute weighted THD, if you have to try and compute weighted THD what you can do you have to come up with V N are you know for ITHD you have to come up with every individual IN that requires every individual V n. So, it means you have to compute the harmonic spectrum. So, that is one way of doing it now.

So, you can compute individual harmonic voltages. And from there be individual harmonic currents and therefore, you can get ITHD are you can compute the individual harmonic voltages. And there by you can compute the weighted THD of the voltage waveform. Now if you look at it a little differently. Now let me say this I can write it as I tilde RMS prepares, what is that? This is the sum of all this is sum of all harmonic currents square of the harmonic currents now. And so, this is what you have here is a really the mean square value of the ripple current now. And if you take it under the square root it becomes a ripple.

So, there is the RMS ripple current where I tilde RMS is the RMS ripple current divided by I1 is the fundamental current. I can get this do this now. So, to calculate ITHD, one possibility for me is basically to try and evaluate I tilde RMS directly. So, can I evaluate I tilde RMS directly? Can I get a measure of I tilde RMS directly is a question now. That is what we will address we will start addressing in this class. I will and then we will probably complete in the next lecture and so on. And we will also see how this I tilde RMS is influenced by different PWM methods that is when you consider different PWM methods you are going to compare them at the same fundamental voltage.

So, this is going to be the same, but the nth harmonic component is going to be different. Therefore, the nth harmonic current is going to be different right. Therefore, what you have is this I ripple current is going to be different are this Vn by N the whole square is going to be different in that, let us say we are comparing conventional space vector PWM and the particular bus clamping PWM or a particular advanced bus clamping PWM method you would; obviously, compare them at the same V1. So, what will be different the harmonic voltages will be different. And therefore, the harmonic currents will be different and therefore, be occur current ripple are THD will be different now.

So, we would try computing the I tilde RMS directly going instead of going from the you know through the frequency domain we will approach the problem in the time domain are in the space vector domain. So, to say and we will try and evaluate this I tilde a RMS which is what we would look at today now.

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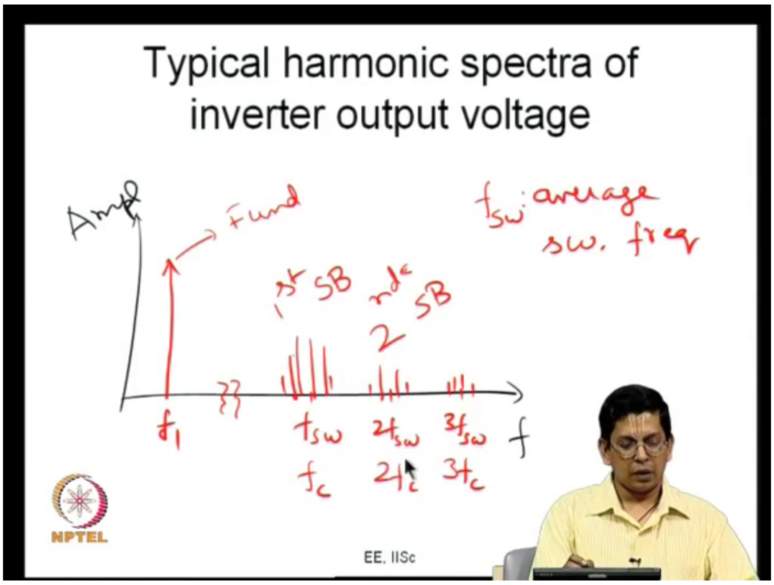

So, before that let us just review a typical harmonic spectra of the inverter output voltage which we looked at in the last class. So, let us say this is your frequency and this is the amplitude this is the frequency and amplitude now.

So, let me now draw a few different components. This is the fundamental component at f1. And the fundamental component is what I was trying to tell you it can there is a difference on the DC bus utilization. This can be a I if you talk in terms of peak phase fundamental voltage. This can be as Y as V DC by 2 or V DC by root 3. And it can be anywhere between 0 and V DC by root 3 right. Then we are looking at PWM methods where the switching frequencies are much much higher than the fundamental frequency. So, what we will have is we will have the switching frequency here. And then we will have 2 switch twice switching frequency and thrice switching frequency and so on.

So, you will have harmonic components here. We may have some harmonic components in this. You will have some harmonic components like this. So, this is the fundamental and this is first sideband SB standing for sideband. This is the second sideband and this is the third sideband and so on and so forth. And now this fsw is the average switching frequency. Fsw is the average switching frequency. So, what is that average switching frequency? If I talk of conventional space vector PWM are sine triangle PWM or any continuous modulation method.

Then that fsw is directly equal to the carrier frequency itself. It can be directly equal to fc itself, now if I talk of bus clamping PWM method then the average switching frequency is 2 thirds of the carrier frequency. That is what it is now and then if you look at let us say the advanced bus clamping PWM methods. So, what happens is in some regions it switches in some regions it does not switch.

So, in some regions it switches let us say at a nominal frequency and in some regions, are double the nominal frequency and some regions it is 0. And therefore, you have what is called as the average switch frequency. And you get for all the advanced bus clamping PWM methods you get them you get their side bands like this now. So, the other way you can also find out in some cases that, it is the carrier frequency here this is f c 2 f c 3 f c this may be a little confusing usually. So, let me say a few more words on this now.

So, let me say I take sine triangle PWM that is what we discussed first. So, the carrier frequency is fc. So, this is your carrier frequency, and then around the second and then the third harmonic it. So, on it produces them. So, you take any train instead of sign you had third harmonic component also. You have your side bands happening around fc 2 fc and 3 fc the amplitudes of these components will change. Now that is when you add third harmonic component for example, of one fourth third harmonic there is certain amount of variation that you get in this higher harmonics and it you know the harmonic properties improves now.

So, you get them around f c 2 f c 3 f c etcetera now. So, if let us say you use triangle comparison PWM, and you use something like bus clamping PWM in bus clamping PWM this carrier frequency. This carrier frequency if it is 3 kilo hertz the corresponding switching frequency is only 2 kilo hertz. So, in that case also you should actually take it as f c 2 f c and 3 f c. So, that was the advantage I was mentioning the other day, if you want compare let us say conventionally space vector PWM and bus clamping PWM at the same average switching frequency. Then what happens is for a bus clamping PWM your carrier frequency can be increased one and half times.

So, if you are talking of 2 kilohertz switching frequency. So, for conventional PWM the corresponding carrier frequency is 3 kilo hertz for the bus clamping PWM. So, there is a shift the this the first sideband shifts by 1, and half times, and therefore sees a higher harmonic reactances, and therefore better filtering of the components is what as I mentioned before now.

So, if you are talking of this kind of bus clamping methods conventional space vector are sign are sine triangle PWM, then you can say this f c 2 f c 3 f c are where you have all the side bands around now. And if you are talking of the conventional space vector are sine triangle PWM etcetera all the continuous PWM in those cases of c and fsw are equal; the switching frequency because it switches at uniform frequency throughout the line cycle. So, they are the same there is no confusion. And if you are talking about advanced bus clamping PWM. Then fsw is the average switching frequency 2 fsw is the twice the average switching frequency 3 fsw is the twice third 3 times the average switching frequency. So, this is what I would like to tell you now, but never the less you know this is the typical spectra.

So, you have fundamental component and there is a large separation here this can be 50 hertz and this can be something like 5 kilo hertz are. So, there is a large separation between these 2 frequencies.

And therefore, it is possible that you can develop separate analytical methods for this component and those components which is what we are doing now. So, one approach is to calculate all the individual harmonic components, which is which has been done before and from the individual harmonic components find the corresponding harmonic currents. And then calculate the ITHD you want, but as I mentioned what we are trying to do this we not trying to go by this root we are trying to see if we can come up with the I tilde RMS more directly.

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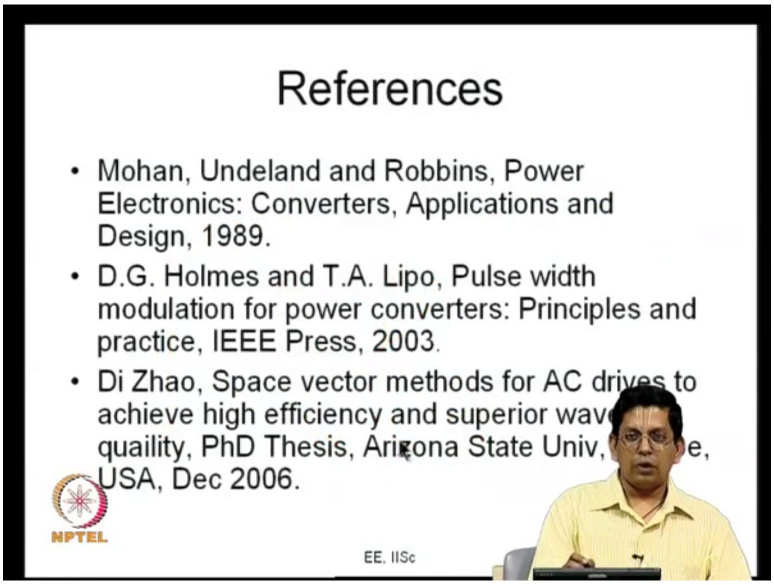

So, you have these references as I mentioned if you are looking at sine triangle PWM about the frequency spectrum you can find it in text books such as the book by Professor Mohan. And then as I mentioned if you are looking for the amplitude you know the frequency is spectrum and what are the various amplitudes of these components corresponding to conventional I mean are continuous and discontinuous PWM methods. Continues and discontinuous PWM methods a continuous such a sine triangle PWM or conventional space vector PWM discontinues PWM method such as 60-degree clamp 30-degree clamp etcetera.

You could probably look at this paper book order quite a few papers in this area where they use the idea of double Fourier series to calculate this. So, you could consider that now. And if you want to compute those harmonic components like this for advanced bus clamping PWM. Then the reference you should look at is the PhD this is by Dr. Di Zhao space vector methods for ac drives to achieve high efficiency in superior wave form quality and submitted to the Arizona State University. So, this is a PhD thesis which actually discusses the harmonic spectra of advanced bus clamping PWM methods and how you can calculate those amplitudes using what is known as the Fourier theory of jumps. So, these are these are useful references for you why if you want; if you are interested in the amplitudes of the individual harmonic components.

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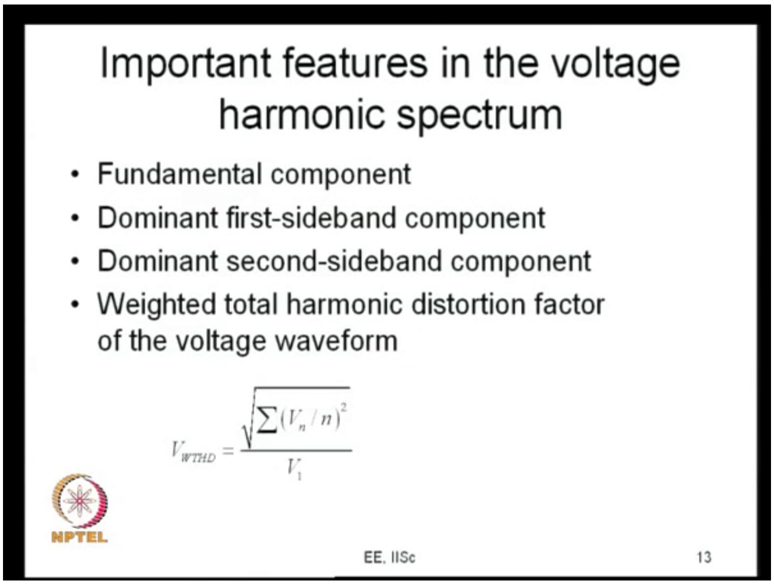

Now, let us move on now in the harmonic spectrum we have some important features as I mentioned that is what we would focus on the fundamental component, dominant first sideband, dominance second sideband component and the weighted total harmonic distortion factor of the voltage waveform. So, these are what we would call as. So, the first 3 are the important features. And the 4th one is actually at you know you saw what is computer doubt of all the harmonics all the; or that are relevant and significant that now right.

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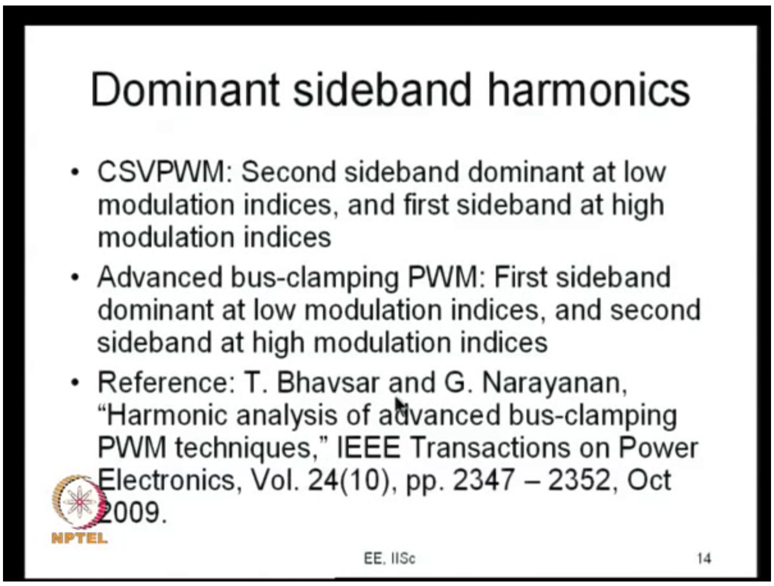

Then it is a let us we go down. So, if you look at the dominant sideband harmonics we had already discussed about the fundamental component that is the DC bus utilization.

So, PWM methods in general for given a Vda, DC bus voltage V DC can produce peak phase fundamental voltages up to V DC by root 3, and if you want to compare the first and second sidebands you see certain features, if you are talking of conventional space vector PWM that is what I abbreviate a CSV PWM now. You find that the second sideband is dominant at low modulation indices, if you look at low modulation indices that is when the fundamental voltage is low not really close to V DC by root 3, but something let us say V DC by 2 root 3 or something of that sort. And instead of 0.577 V DC you are looking at point 2 V DC. So, at such low modulation indices you will find to the second sideband is dominating.

So, what happens to the second sideband is dominating? So, the second sideband sees a leakage reactance roughly provides that as seen by the first sideband and therefore, the harmonic currents are much lower for the same amount of harmonic voltage amplitudes. And therefore, conventional space vector PWM has a low harmonic distortion at low modulation indices where as at high modulation indices you find that it is first sideband dominates.

So, the leakage reactance seen by the first sideband is lower and therefore, the harmonic currents are higher, the ripple current is higher. Now the contrary I mean the other situation is converse situation is seen in case of advance bus clamping PWM there you find that the first sideband dominates at low modulation indices and the second sideband dominates at high modulation indices. Therefore, what happens at high modulation indices second sideband dominates and they get filter better the harmonic amplitudes may be substantial voltage harmonic amplitudes may be substantial the corresponding current amplitude are low and therefore, your RMS current ripple is low. Therefore, you will find when you have compared CSV PWM in advanced bus clamping PWM you will find that the CSV PWM is better at low modulation indices, whereas, advanced bus clamping PWM you will find that they are better at high modulation induces now.

So, I am giving you a particular reference that T bhavsar and g narayanan harmonic analysis of advanced bus clamping PWM techniques. So, this paper actually gives a comparison of the dominant sideband components of conventional space vector PWM and advanced bus clamping PWM. It also gives some dominant components of are the bus clamping PWM methods right.

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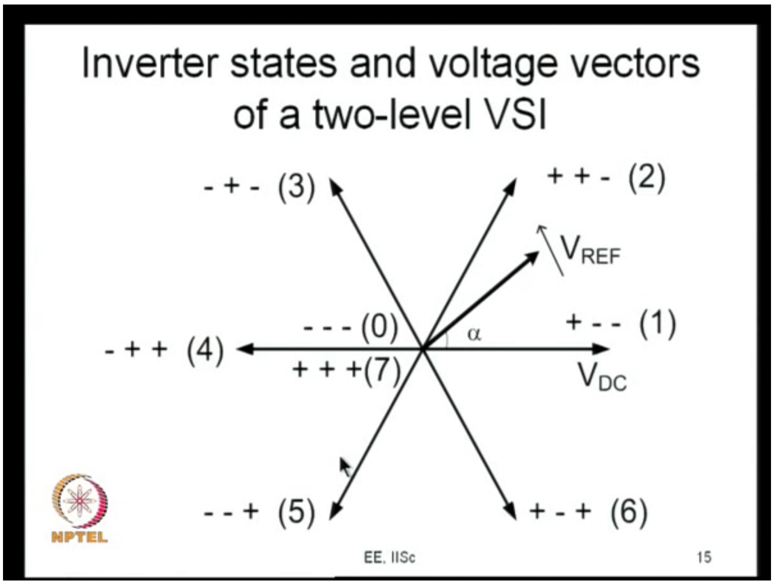

So, after this review; so we had just looked at the all the Fourier in the harmonic spectrum. Now let us try and see how we can do the same job in these space vector domains. So these, is knowing the space vector plane and do you thus sine angle and we are going about that. And this is a revolving vector and we are going to sample it every time, and if the sample falls in active vector in the sector 1, we will use active vector 1 or active vector to a null vector to produce this average vector.

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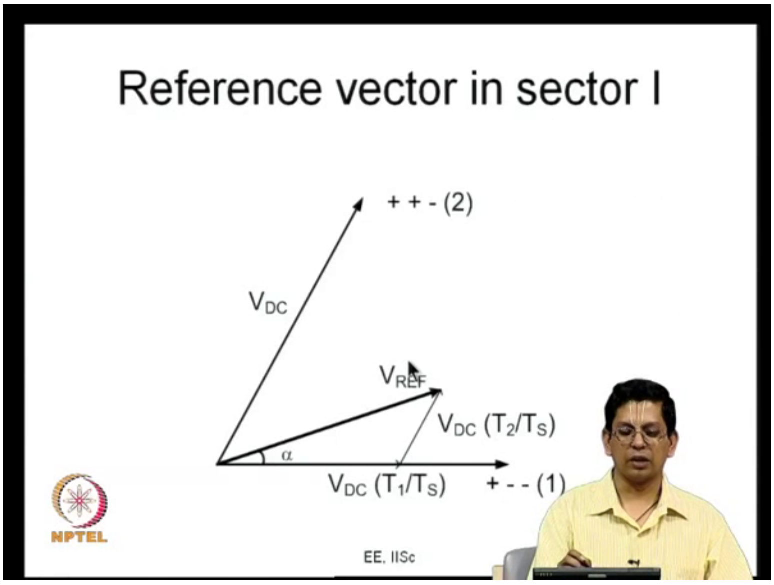

So, this is what you get; active vector 1 active vector to a null vector now. And they have to be applied for times T 1 T 2 and T z. How do you calculate T 1, T 2 and T z? Like, what I am showing here.

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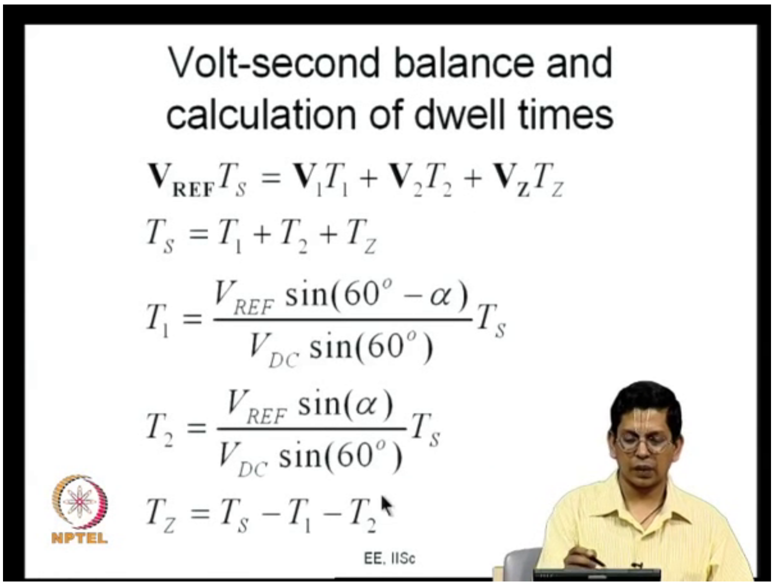

V 1 T 1 V 2 T 2; so this is the volt second balance and T 1 is equal to V ref sine 60 minus alpha T S divided by V DC sine 60. Similarly, T 2 is proportional to V ref sine alpha. So, it is V ref sine alpha by V DC sine 60 T s and T z is T S minus T 1 minus T 2. This is how you calculate the (Refer Time: 24:26) times for activator one activator 2, on the null vector as I have said in the previous lecture is as well.

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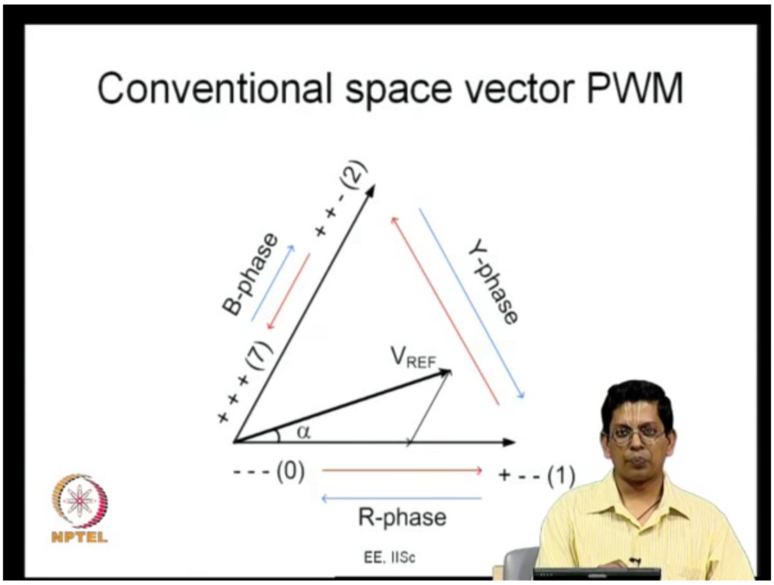

So now when you do this in conventional space vector PWM what do you do? Then null vector you use both the null states. A null vector can be applied using 2 different 0 stage; minus, minus, minus where it all the bottom devices are on an plus, plus, plus, plus where in all the top devices are on.

So, let us say you start from minus, minus, minus. So, all the bottom devices are on. You switch R phase, after T z by 2 seconds and you reach here plus minus, minus. This is active state 1. You stay here 41 seconds, and then switch Y phase and reach here; that is, plus plus minus active state 2. You stay here 42 seconds, and after that you switch the B phase and you reach here, and you stay here for remaining T z by 2 seconds. So, add plus plus plus now, and you do the reverse in the next subcycle. This is conventional space vector PWM. We are now going to take this convention space vector PWM as a specific example for trying to evaluate the RMS current ripple, in the motor when we know for the for evaluation of RMS current ripple.

So, we are going to look at the content look at an induction motor drive. And, an induction motor drive operated using a voltage source inverter switched using conventional space vector PWM, and try and understand how it is RMS current ripple would vary, right. So, here we go.

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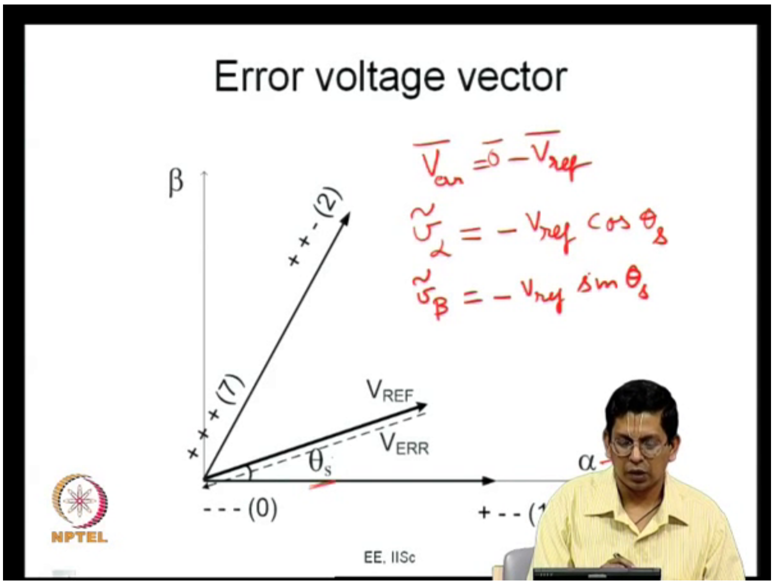

So now what is to be done first? There is this error voltage vector. Why is there a ripple current? Firstly, that is because there is a difference between what you need, and what you apply. There are harmonic currents. There is a ripple current. The ripple current you can say is the sigma of all the harmonic currents. Why is there a harmonic current? Because there is certain harmonic voltage.

So, you have there is a difference. So, the voltage that you need is sinusoidal, but inverters output is not exactly sinusoidal. Therefore, there are ripple you know there are harmonic voltages and correspondingly harmonic currents. So, what you can now look at it the same thing you can look at it in the space vector domain now. This V ref is what you want to have.

So, it is your trying to instead of 3 phase sinusoidal waveform, you are trying to apply a revolving voltage vector of magnitude V ref and the revolving at a frequency omega. So, this is V ref at a particular instant, here it is V ref now. At this instant you can not apply this V ref vector, because a inverter does not produce this V ref vector. What are the vectors it produces? The inverter produces a null vector, and around 6 active vectors here. And the those 4 active vectors are not very useful here. These 2 active vectors are the ones which are the closest to that now.

So, what you are trying to do is; in this for us over as small subcycle duration you are trying to apply this null vector and these 2 active vectors. And by time averaging you are trying to produce an average vector which is equal to V ref. The instantaneous vector is at any instant of time either null vector, or active vector 1 or active vector 2. And it is never equal to the reference vector itself and it cannot be.

So now let us say you have the required vector is V ref vector, but the applied vector is null vector. So, what do you have? What is getting, so there is an error vector. What is the error vector equal to? That error vector is equal to minus V reference vector. So, let me just write it down. So, let me call this as V error vector. This is V error vector that is equal to minus V reference vector.

So, this is in a particular subcycle in during whenever the null vector has been applied now. So, you can also resolve this vector along the alpha axis and the beta axis now. Let me say I would say V alpha tilde. Let me call this is the error along that. So, what is the voltage applied along the alpha axis? Nothing, because the null vector. So, 0 and what is the reference value? It is V ref cos theta S just note them just using theta S here instead of theta S here instead of alpha, just so as not to con confuse with this alpha axis it beta axis ok.

So, what we have here is V alpha tilde is equal to minus V ref cos of theta S. There is an error there is an error between the applied voltage and the reference voltage. The applied voltage is 0 vector and this is 0 vector minus V reference vector; this is actually 0 vector minus V reference vector it is basically minus V reference vector. So, what is V alpha tilde it is 0 minus V ref cos theta. Similarly, what is V beta tilde? That is the error voltage along the beta axis. This is minus V ref sine of theta S, and you can see that it will vary. When you go to different sectors. So, V alpha tilde and we beta tilde are very and they will vary from sector to sector now right.

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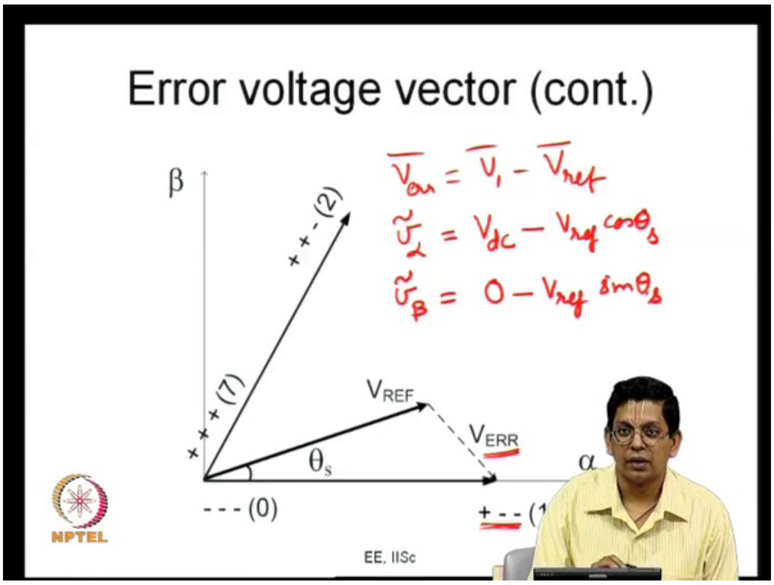

So now let us say we look at the other error voltage vector, when active vector 1 is apply. You have applied active vector 1. So now, the V error vector is V1 vector minus V reference vector. It is V1 V1 vector minus V reference vector. And that is what is given as this V error vector here now. This can be resolved along alpha and beta. So, let us say I take V alpha tilde. So, what is the applied voltage along with alpha axis? The applied voltage is V1.

So, that is V DC it is entirely aligned along the alpha axis therefore, it is V DC. And minus V reference cos of theta S. This is the error voltage along the alpha axis. Similarly, you can consider the error voltage. Along the beta axis and the what is the applied voltage vector along the beat axis? That is 0 because the applied voltage vector 1 has no component along the beta axis. This minus V reference multiplied by sine of theta s. So, this is what it is in the alpha axis, and this is what it is in the beta axis now.

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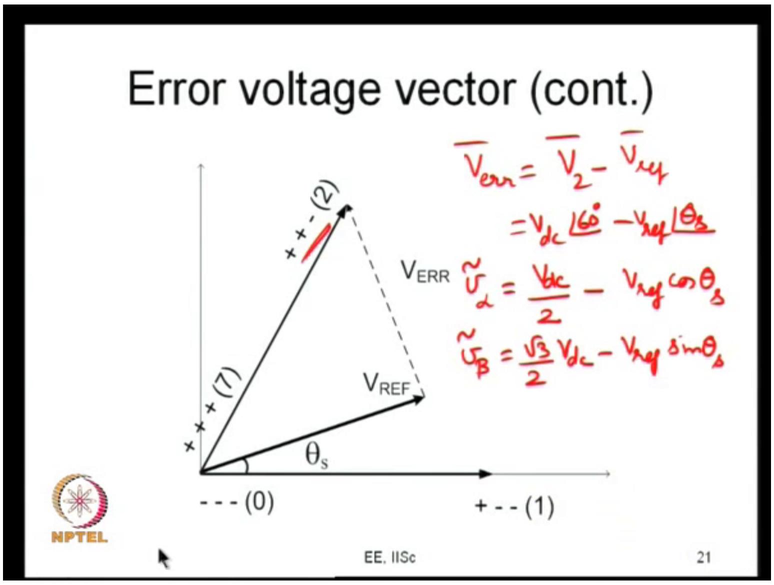

So, when the applied voltage vector is active vector 2. You once again have an error voltage vector. What is that error voltage vector? V error vector is equal to V 2 vector minus V reference vector. And this V 2 vector is nothing but V DC angle 60 degrees. This is the same angle V DC angle 60 minus V reference, and you can say this V reference angle is theta S. And for just for clarity and writing V 2 and V ref in term in the polar coordinates. So, that we are now a little clearer now. Then your what is a V alpha tilde? V alpha tilde is whatever is the voltage applied along the alpha axis; that is V DC cos 60.

So, that is going to be V DC by 2 minus V ref cos of theta S. And the error voltage along the beta axis is the applied voltage along the beta axis, which is root 3 by 2 times V DC minus V reference sine of theta s. So, these are the error voltages that you get along the alpha axis and the beta axis. So, you can integrate these error voltage is to get the measure of the current ripple along alpha axis and beta axis.

So now you can see that these quantities kind of go on varying; that is, you will have to do this over the entire all the 6 sectors now. And we will see that we will go into instead of alpha and beta we look at the dq reference frame. And in the dq reference frame we can come up with the procedure in which in where there is symmetry about every sector. It is enough if you do the calculations or every sector.

So, it is actually from alpha beta we will looking at the space vector plane here; that is in the stationary reference frame. That is this is the alpha axis and this is the beta axis though alpha and beta are missed out here. So, you have alpha and beta this is the stationary reference frame now.

So, the reference axis are fixed at now one can also consider reference frames; I mean where the reference axis are rotating. For example, one can consider a frame where the reference axis is rotate is attached with this V refractor, and it keeps a rotating and (Refer Time: 32:47) it keeps rotating like this now.

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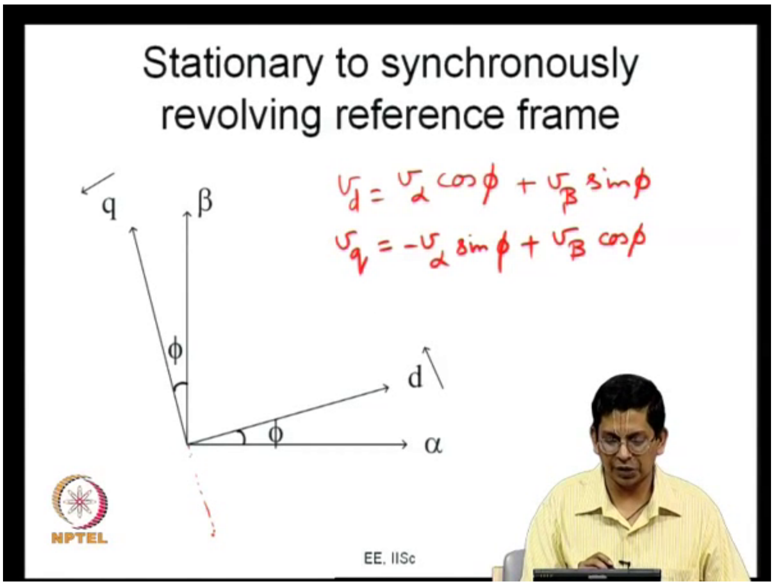

So, that is a stationary we can consider a synchronously revolving reference frame now. So, let us say you have the alpha axis and we have beta axis, and then you have what is called as a dq reference frame. So, you have a synchronously the revolving reference frame. This axis is called d axis or direct axis, and this can be called q axis or the quadrature axis. And let us say you have d and q and there is an angle phi coming between these 2.

So, you can consider are different thing and you can go about defining a transformation now. So, what you can do this let us say you have V alpha V beta, and you need V d, let us V d can be written in terms a V alpha and we beta. So, this is V d is equal to V alpha times cos V is it alright. So, then lecture there is some V beta plus V beta times cos of 90 minus phi or sine phi.

So, if you know V alpha V beta and the angle phi between the 2, you can compute what is V d. So, then you can also compute the voltage along the q axis. So, what is that? If it is V beta is very clear. That is going to be V beta times cos V. And what is V alpha? So, here you have V alpha, and V alpha sine phi is going to be something in this direction and it is opposite to the q axis. So, this is minus V alpha sine phi plus V beta cos V.

So, this is what you can call as a transformation. Transformation from is stationary reference frame to synchronously revolving dq reference frame. Where if you have the components alpha and beta. So, the 3 phase quantities V r and V y and V b. Let us say has already been transformed into V alpha and V beta. These can be further transformed into V d and V q by doing the same that is what we are trying to here know. Or let us say you have 3 phase currents ir i y and ib, that have been transformed into i alpha an i beta. This can be further transformed into id and iq as shown here now. So, that is what you can do that is using this transformation now. You can write this transformation in the matrix form V d V q is in ra and then cos phi sine phi minus sine phi cos phi is the transformation matrix. And multiply it by in ra V alpha V beta. It is what you can do and this matrix cos phi sine phi minus sine phi cos phi is a unit matrix right.

So, I mean it is determinant is 0 T it is easily invertible. So, it is cos phi minus sine phi sine is sine phi cos phi would be the inverse of the matrix. If you use the inverse of the matrix, you can compute V alpha and we beta using V d and V q. So, this transformation is reversible. So, you can have V alpha V beta V alpha V beta and the angle phi and you can get V d and V q. On the other end you can also an V d and V q, and you can have phi is given to you, you can compute V alpha and V beta.

So, this is the stationary to synchronously revolving reference frame, transformation and it is inverse that you can really see from the (Refer Time: 35:55), I am not writing down the inverse, but it is very, very easy for you to write it down yourself now. So, I am doing this primer now for example, we have been doing this. With this definition of d N q axis, let us go and continue with our study on the error voltage vector; which is what we have been looking at today now.

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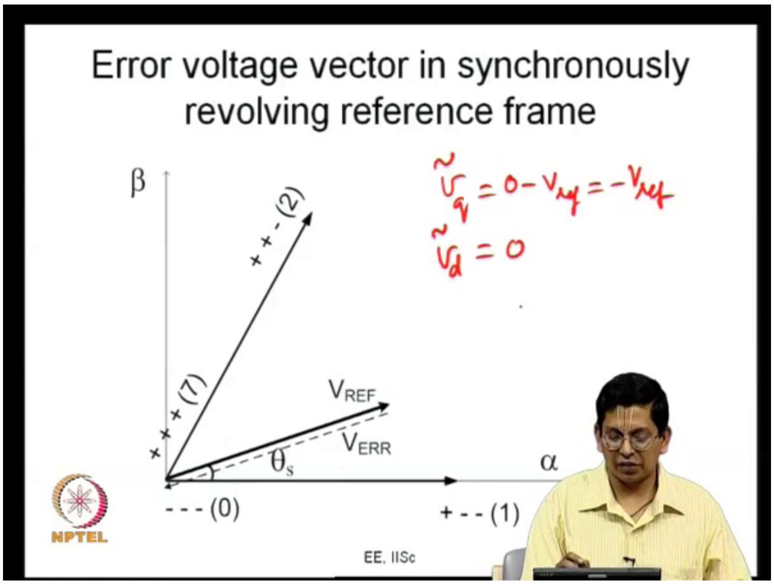

So now I would want the d and q axis. Now I am going to do the analysis the same error voltage vector, but I am going to try it resolve them in the synchronously revolving reference frame now. So, what I can first do is; I can take this as one of the coordinate axis.

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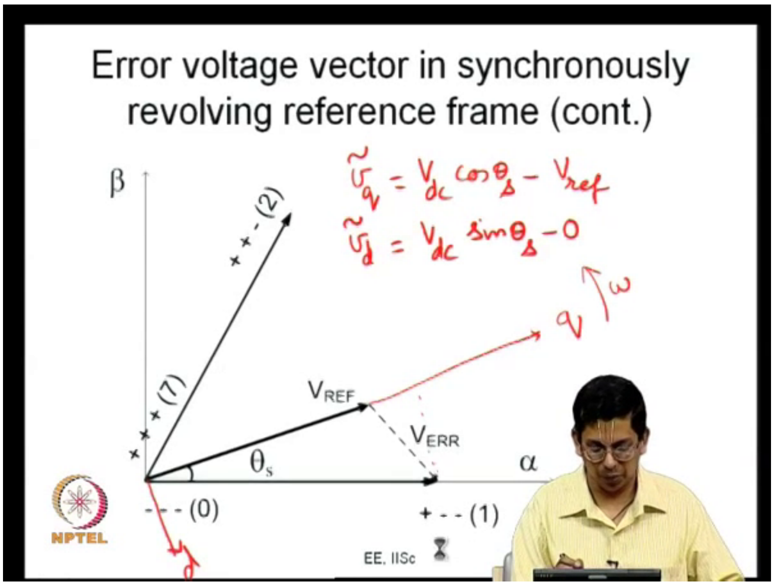

This reference vector that I have here as one of the coordinate axis, one of the revolving axis now. And since this is the applied voltage vector. And so, I can call this as q, and it revolves at this synchronous frequency omega. And the d axis is going to be 90 degrees behind that.

So, you take consider an induction motor. So, is the this is the voltage applied along the q axis. The voltage the stator voltage vector allied applied along the q axis, then the flux is along the d axis. So, that is also the reason for our convention calling this as q and calling this as d right. So, in this defining this as q axis and d axis alright. So now, I can write the same error voltage vectors the error voltage vector are still same right. For example, here you have V error, this is your V error this is V error corresponding to active vector 1 when active vector 1 is this is applied now.

So, if I can write down instead of V alpha and V beta, what I can do is I can write down this as V q tilde. This is the error voltage along the q axis. What is the error voltage along the q axis? That is the component of this applied along the q axis. So, what is the applied vector is V DC V DC angle 0. So, along the q axis it is V dc times cause of theta S; that is what it is, and this is the applied vector minus the actual vector. The actual wave thing is V reference.

So, this is the error along the q axis now. Similarly, you can define the error voltage component along the d axis. How is that? The applied vector. How much is the applied vector? It is V DC times sine of theta S minus 0, just to make it obvious, there is no meaning if the reference vector is entirely along the q axis and it is not along the d axis. So, your V d tilde is actually V DC sine theta S minus 0. This is the error voltage vector when you consider an active vector 1 to be applied here now.

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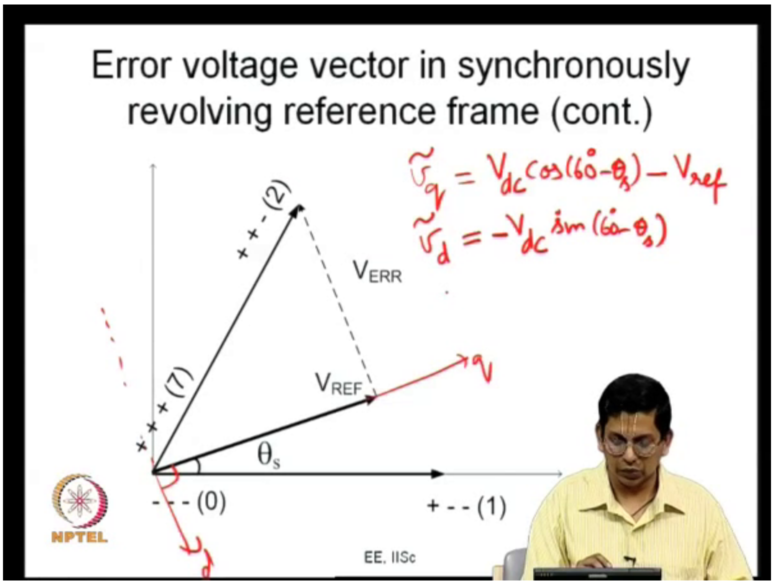

So, similarly if you consider the situation, where you are considering the active vector 2 to be applied right. When you consider the active vector 2 to be applied. So, you can once again come up you this is the error voltage vector. This error voltage vector we had earlier expressed in terms of the alpha axis on the beta axis. No we can express that in terms of the d axis and the q axis now ok.

So, this is the q axis and this is the d axis, and there is 90 degrees cl. And what is the error voltage along the q axis? That is V q tilde is equal to the error the voltage applied voltage along the q axis. Now the applied voltage has a magnitude V DC. And it is component along the q axis is V DC cos of 60 minus theta S. This is the applied voltage. Minus V reference. So, V q tilde is V DC cos 60 minus theta S minus V reference. And there is also error voltage component along the d axis. What is that? The error voltage the applied voltage magnitude is V DC right. It has actually a component like this. So, that is V DC sine of 60 minus theta S. This is in the direction opposite to d axis and therefore, we have minus V DC sine of 60 minus theta s. So, this is V q tilde and this is so V d tilde right.

So, we early saw how you can do it in the when, when you have you with the error voltage vector we are trying to express it the dq reference frame when active vector 1 is applied, active vector 1 is applied. Instead when active vector 2 is applied you can do this. Similarly, when null vector is applied also it is much simpler. When null vector is applied it is simply equal to V q tilde as minus V ref, that is all. And V d tilde I is 0. That is what you (Refer Time: 40:36) which I have not stated here right.

So, let us say we go further, excuse me. So, this is a previous case very we can probably look at that. So, we just for completeness let us finish that also so that here what you have is V q tilde yes, what is applied is 0 minus V reference. So, V q tilde is minus V reference. And V d tilde the applied voltage vector is null vector. So, it has no component along either d axis are q axis. So, it is 0 and the reference vector also has no component along the d axis. So, V d tilde is simply 0. So, this completes our error voltage vector study in the alpha beta domain and also in the dq domain. So, this is the error voltage vector corresponding to null vector, and this is the error voltage vector corresponding to active vector 1. This is the vector corresponding to active vector 2 right.

So, what is the next step? We should go about integrating this error voltage vectors. That is what we have to go we have to go about integrating this error voltage. Vectors to get a measure of the line current ripple, because as far as ripple voltage is concerned the machine is seen as a leakage inductance. Therefore, (Refer Time: 41:47) integral of the voltage is proportional to the current right.

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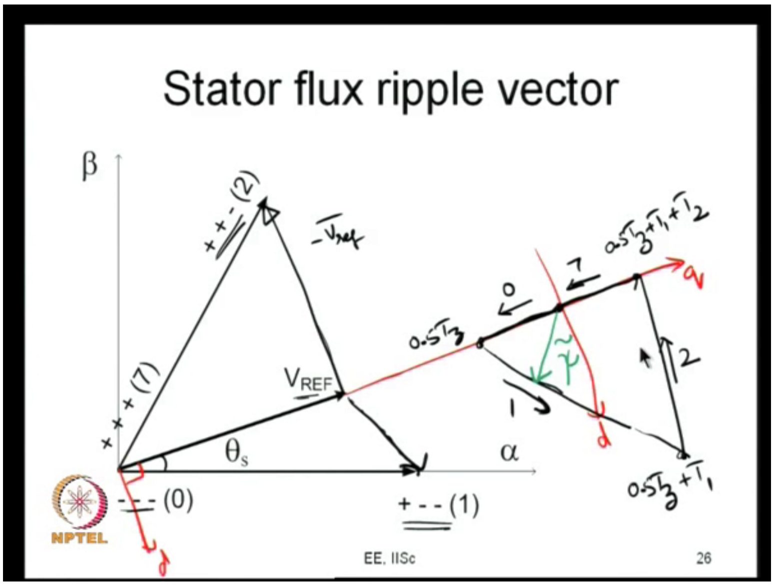

So, this is stator flux ripple vector. What is stator flux ripple vector? As I mentioned to you this is the integral of the voltage error vector now. I would not draw anything. I have just given the same vectors this is you, you know you have your V ref vector of magnitude V ref and angle theta S measured from the alpha axis. This is your desired reference vector to be produced in a sub cycle. You are going to be doing this by applying active vector 141 seconds active vector T 2 42 seconds and null vector for T z by 2 seconds. And your going to do this by employing the conventional switching sequence, where you would apply this 0-state minus minus minus for T z by 2 seconds and they active state plus minus minus 41 seconds, then plus minus minus 42 seconds and finally, plus plus plus for T z by 2 seconds now.

So, let us go into let us do the job now. So, what we do this now let us do one integration of the error voltage vector. We looked at the error voltage vector when a particular thing is applied, now we are going to look at the error voltage vector and it is integral over a cycle or a sub cycle. Let us just start with this now. So, let me call this as q axis. Let me just draw that ignore dry inaccuracies as just straight line; the same which is committed to the reference vector and this is the d axis and this angle is 90 degrees.

Right now, this is here. What I can say is for convenience let me say, let me not draw the d axis here, let me draw the d axis here correct. So, I am just drawing the dq reference frame, and this is the origin point now. So, let me now go into the next step. So, what I have is; I have this kind of reference vector, I am going to produce an average voltage vector equal to the reference. So, what if us do is I apply the null vector minus minus minus for certain seconds T z by 2.

So, I start from here. The error voltage vector is like this. And therefore, the integral of the error voltage vector is going to grow like this, it will grow like this. So, the error voltage vector is constant and what is it is magnitude? It is equal to minus V reference vector. It is equal to minus V reference vector. And what we are trying to do is we are integrating minus V reference vector here? When you have applied here. And at the end of 0.5 T z seconds you have reached something like this. You have reached something like this. Now 0.5 T z is over. What is the next state to be applied? Plus, minus minus. What is the reference vector? It is the same V reference vector. What is the error voltage vector? Now this is the error voltage vector.

So, what is going happen is you will have something which is parallel to this line. We will have something parallel to this line. And so, this will go on. You will have something parallel to this line. So, this is the error voltage vector. And this is the error voltage vector we are integrating that. See it will move along this direction, this direction is parallel to the error voltage vector corresponding to active vector 1. So, you have you have reached here now. So, here at the end of this. This is 0.5 T z plus T 1 you have reached here now.

At the end of this you are apply the active state 2, and the corresponding error vector is like here now. So, it is going to move like this. And in this direction, it is going to move in this direction; when active vector 2 is applied it will start from here and it will move here. And finally, it will reach here at what instant? 0.5 T z plus T 1 plus T 2. Then from here it will move back when you are applying the 0 state 7, I did go back to the origin. It has to go back to the origin, why? Because of volt second balance.

So, V reflector into T s is equal to the average voltage vector multiplied by to T s. What you are trying to integrate is the error voltage. So, the error the average error voltage over a sub cycle is equal to 0. And therefore, whatever you start from you get back there. So, this is what this we stator flux ripple vector. Now this is actually a vector. This is actually a vector, let me use some different color here. And let me call this as vector now. And this has certain (Refer Time: 46:20) we can call this by some psi and tilde. So, the psi tilde vector has it is magnitude go some changing. So, initially if you look at it. It is magnitude 0. And it is magnitude is increasing, and the angle is not changing now. When the null vector is applied. It is increasing along the q axis on to the negative q axis to be precise now.

From that particular point what happens? When one is applied it is both it is magnitude and angle start changing, it is magnitude and angle start changing and go up to this point here. That it is tip is excuse me, but it is tip is moving at a pace. What is that pace at which it moves? That is equal to the magnitude of this error voltage vector, whatever is this V on vector it is it is magnitude at which we draw, find it was moving here the speed at which it is moving is proportional to this magnitude of the error voltage vector; that is minus V ref, and here this speed is proportional to this error voltage vectors. And when you reach here it is 0.5 T z plus 1, and from 0.5 T z plus T 1 T 2 vector is applied you know mean vector V 2 is applied for time T 2 seconds and it moves here. It moves at a constant speed. And this speed is proportional to this magnitude of the error voltage vector. Which is then it goes and reaches here and finally, it goes back here, and it is this now.

So, this sigh tilde a vector is a measure of the current triple vector; that is this error vector that we are considering here, you can consider that as the 3-phase transformation mean the 3 phase to 2 phase transformation of the space vector, I mean of the ripple. That is in every phase we have harmonics sum of all the harmonics is ripple, and the 3-phase voltage ripple can be transformed into the space vector domain and that is the voltage vector now. In a integrate that voltage vector you get the stator flux ripple vector, which is a measure of actually the current ripple vector now.

So, this is actually a measure of the 3 phase ripple currents transformed into the space vector domain. So, this is what it is a measure of now. It is cut a triangular trajectory; this vector has got a triangular trajectory. And you can see that you know every side of the triangle is proportional to the error voltage vector in the base of the triangle which is along the q axis is parallel to the error vector corresponding to null vector. And this side is parallel to the error vector corresponding to vector 1, and this side is parallel to the error vector corresponding to vector that V 2.

So, next thing that you can do you (Refer Time: 48:43) further analysis we can try and resolve this along the d axis and the q axis components, and see how it goes about here now.

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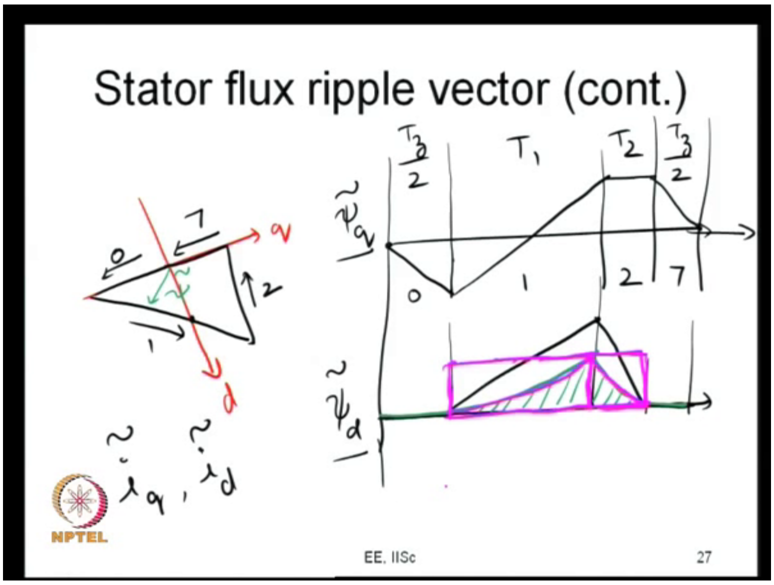

So, let me just try reproducing that figure roughly. Now let us say this is the q axis, and this is the d axis we had in the previous graph. And let me just draw the vectors that you really had there. So, this is when 0 state 0 is applied. This is when one is applied. This is when active state 2 is applied. This is when 7 is applied now.

So, you can resolve this vector into the d and q axis components now. So, let me once again write the psi tilde vector also. So, this is this called as psi tilde vector now. So, let me draw the axis. This is let say the time axis. And let us say this is my psi q tilde, the q axis component. What happens this is 0, and the q axis component changes like this. It becomes negative on negative in 0 states applied this is when the 0 state is getting applied. So, it reaches some value like this now.

Subsequently the active state one is applied during that time you find that this is the line when you go around this line, you have seen that the it becomes less negative; that is, it start knowing towards the positive direction. And at this point it is crossing, the in it is becoming positive and like this. So, I can say it increases like this. And when I am applying the active state to you can say that the psi q is more or less equal. I am sorry about that. This, this is not there this no (Refer Time: 50:41) here let me redraw that. So, it is more or less flat here. And finally, when I applied the active state 7 it goes back to 0. It goes back to 0, let me extend this (Refer Time: 50:56).

So, this is the entire sub cycle time T s. So, this region is T z by 2. This duration is T 1. This duration is T 2, when this duration is T z by 2. And the states applied are 0 1 2 and 7. So, this is one component of this is how one component, namely the q axis component of the current ripple vector are the stator flux ripple vector change now, right. Then I can plot the psi d tilde vector. And the psi d tilde vector is during the initial time it is 0. When 0 is applied it does not change at all. Again when 7 is applied it does not change at all. In between that to what happens, it goes about increasing it reaches a peak, when at the end of this interval T 1 and the start of interval T 2. And then it falls linearly here, this is what it is now.

So, this is how the d axis ripple varies. So, there is the q axis ripple. Then there is the d axis ripple. And you can also equal these are also equivalent to i q tilde and i d tilde, it is just a question of scaling them iq tilde, and id tilde. So, if you can calculate this iq tilde and id tilde, now what you have them us, you have the ripple quantities in the q axis and d axis. What you need to do is; you use the inverse of this you know stationary to synchronize reference frame transformation. That is from stay synchronous to reference frame to stationary reference frame you can go to i alpha tilde and i beta tilde and from there you can actually move on to i r tilde i y tilde and i b tilde.

So, you can thus it is possible for you to compute ripple, that is what I am trying to reevaluate. So, I am just driving you there it is kind of trying to take you through the steps now. So, this is the initial step I am just trying to give an indication of how exactly we can reach there. So, it is possible for you to get iq tilde an id tilde, that is the very. And from iq tilde an id tilde we you can use transmissions i alpha tilde i beta tilde I know ir tilde etcetera it is possible to come up with, all right.

So, this is how it varies. And iq tilde and id tilde have their whole significances to; that is both of them contribute to the current ripple. As we will see later in the module on, pulsating torque iq tilde effects the torque pulsation, much more than id tilde thus.

So, we will that is something that you look at in the next lecture. So, we will just try and do something more now. So, what is that? So, we just trying to understand this ripple. So, you can see that the ripple starts from 0, and ends from 0. You look at both the components both the components are 0 at the start as well as the ending. And the d axis ripple does not change whenever the null vector is applied. So, that is actually one of the benefits by aligning, this see the q and d axis should be revolving. But we made sure that the q axis is aligned with the reference vector, and the d axis is in quadrature with that by making the q axis aligned with the reference vector, we have made sure that it has no component along the d axis, that is what is given this advantage. V d tilde is 0 use in these intervals. And so, it is a kind of easier to calculate now.

So, this you can say is the instantaneous stator flux ripple along the d axis and q axis, and this also gives you the instantaneous q axis and the d axis ripple now. So, for it now let us say if we want to go to the RMS value, how can you compute the RMS value is an interesting question now. I will just try to look at that question, no I will just start off with one of them. Namely let me just take a different color now.

So, I want to calculate psi d tilde a RMS value of psi d tilde. What can I do? I can just square this. So, if I square this, 0 squared is 0. Again if 0 is query it is 0. Now this linear portion if I square, it is going to become a parabola like this. It is going to become a parabola like this again this. Liner portion if I squat it is going to become a parabola I like this. And what you need to do is I need to compute the areas under this parabola. We need to compute the areas under these parabolas. How can I compute the area under this parabola? There some simple ways are available.

So, let me just choose it another color here. Let us say, now let me say there is a rectangle. This is a rectangle. Again, this is another rectangle. So, the area under the parabola, like here is one third of the area under this rectangle. That is right? This is the property, and the parabolas. Now we will discuss about this a little more probably in the next class in a slightly more relaxed fashion. And again, if you look at the area under this parabola is equal to 1 third of the area of this rectangle. It is not very difficult, because you can see that in the parabola, if you look at this Y is equal to x squared is what you have the equation for a parabola.

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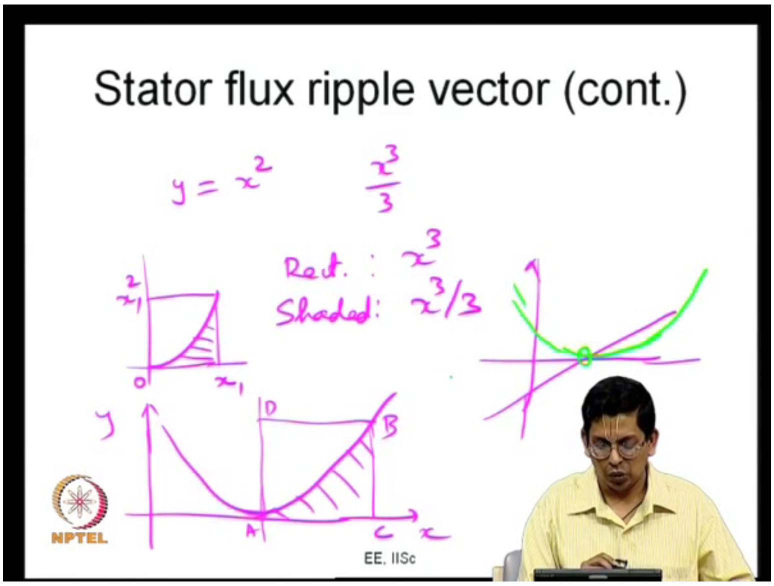

So, if you are trying to integrated from 0 to x like this. Y is equal to x cube, and you are trying to integrate in this area. So, you will see that the area is x cube by 3. This area is x cube by 3. And what is the total area? So, this is let us say x, this is x squared. So, the total area is x cube. So, the area of rectangle is x cube, and a rectangle is has an area equal to x cube and the shaded area is x cube by 3. This you can see that the area under that is one third of that.

So, I am just trying to use that idea to calculate this now. So, this is not necessarily when it is 0. This may be some x one this may be x 1 squared and you can come up with that now. Let us say you are not a you are having a parabola which is sitting not not really there, but it is sitting somewhere like this. This is the axis. This is the horizontal axis x.

This is the vertical axis Y. The parabola as touching here I mean just there is a drying in accuracy parabola value is equal to 0 at that point now. So now, let us say I want to compute the area here now. This area is equal to 1 third of the area of this rectangle. If I call these points, let us say A and B and call this as C and D.

So, the shaded area ACB is one third of the area of the rectangle AC BD; that is, I am the time I am trying to saying out. So, that you know the parabola may be wherever it is. So, why are we interested in this kind of a parabola? If you actually multiply any straight line, this unsquare that straight line this is the kind of parabola that you are going to get. So, what you are actually having is you are having is straight line. And that say you have a straight line like this. If the straight line is squared, you will get a parabola like this. That is a parabola which will have equal roots, real equal roots; that is how it is going to be.

So, for those kinds of parabola you can calculate the area like this. And so, we would go into this and it is possible to do the calculations over a you know sub cycle, and try to compute the RMS value over a sub cycle. So, we will do this kind of excise greater detail in the next class. Before the time anyway I just indicate some references where these kind of calculations have already been done for you.

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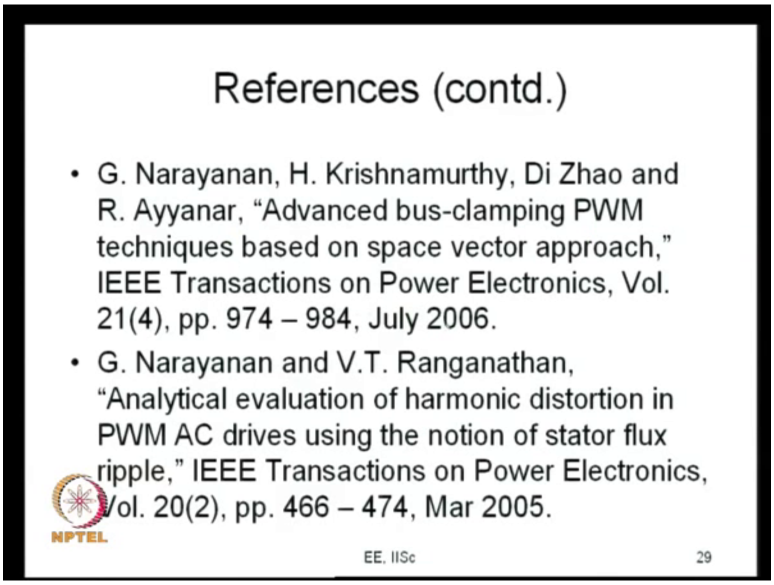

So, these are the 2 papers where you can probably you know there are some one other 2 papers also, where these kinds of calculations have been done. The next lecture we will do this calculation of RMS current ripple over a sub cycle in a greater detail. And we will also look at how this RMS current ripple is influenced by the PWM method of the switching sequences.

Thank you very much for your interest. I will look forward to your participation in the next lecture.

Thank you very much.