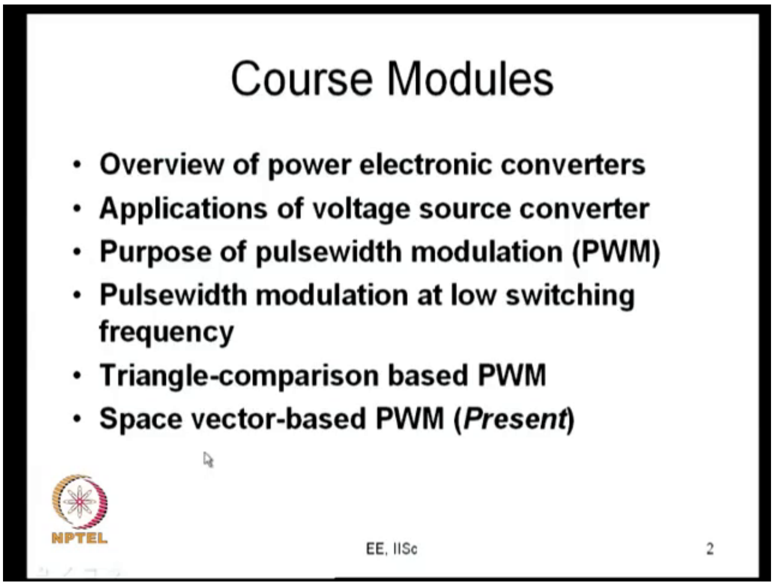
Pulsewidth Modulation for Power Electronic Converters
Prof. G. Narayanan
Department of Electrical Engineering
Indian Institute of Science, Bangalore

Lecture - 20
Conventional space vector PWM

Welcome back to this lecture series on pulsewidth modulation for power electronic converters and we have been looking at you know various modules in this course and here is a quick look at the various course modules.

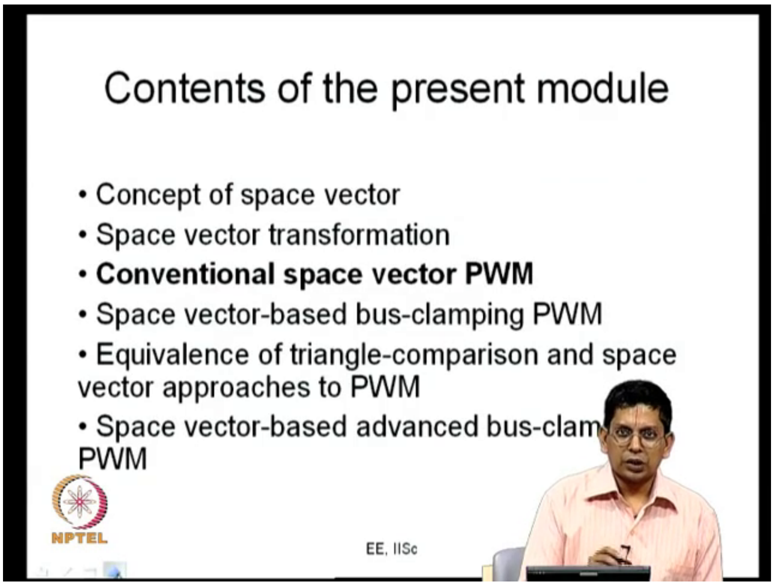
(Refer Slide Time: 00:29)



So, one of them is the you know initially as we have seen that we had looked at various power electronic converters and we looked at the applications of voltage source converters which is more to drive an active front end converters and so on. Then we looked at the purpose of pulse width modulation namely to control the fundamental voltage and to mitigate the harmonics, then we looked at pulse width modulation is very low switching frequencies and we covered topics such as selective harmonic elimination and all that.

Then we have been now focusing on I mean we subsequently covered all this triangle comparison base PWM where three-phase modulating signals are compared against a common triangular carrier and presently we have been discussing this space vector based pulse width modulation. So, this is what we did in the we started off in the last lecture and this is the second lecture in this module on space vector based pulse width modulation.

(Refer Slide Time: 01:21)



So, now as we you are aware the contents of the present module are as given here the concept of space vector is something we looked at last lecture, we also looked at space vector transformation how to transform a set of three-phase quantities into two-phase quantities that is something we looked at. And we just started off with space vector base pulse width modulation last time and so in this lecture we are going to cover this conventional space vector pulse width modulation in you know in greater detail, we are going to look at this particular method which is what I would say is the best I mean or the most popular method among the not the most popular method among various space vector based PWM.

So, one thing I would like to mention at this point of time generally the term space vector PWM is used by many authors and papers and all that are simply space vector modulation. When they say that they normally mean this conventional space vector PWM which we are going to discuss today I mean considering that this is so popular they you know that is what has generally done, but as far as this course is concerned space vector PWM would mean a class of pulse width modulation methods I mean, so where the idea of space vector is used in the you know generation of PWM and so on so forth and we would specifically use the name conventional space vector PWM and we will discussed what this method is shortly from now.

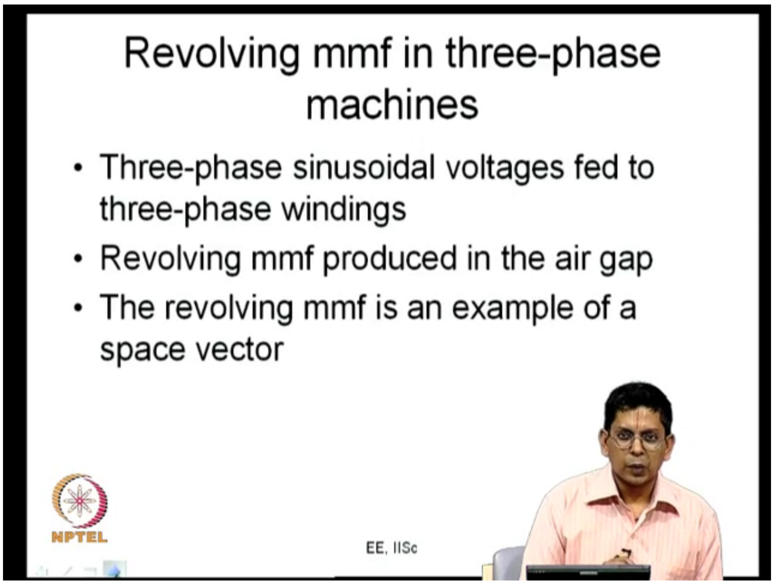
And in the next lecture we will be looking at space vector based bus clamping PWM. See we looked at their knowing the while dealing with triangle comparison based PWM we looked at discontinuous modulation methods which cause bus clamping, clamping of the phases to different buses. So, such bus clamping PWM how to generate such a bus clamping PWM from the space vector approach would voice or something that we will discuss in the next class.

And in this class what we will also do is you know like we are also going to look at the equivalence between triangle comparison and space vector approaches it is whatever you can produce PWM waveform you can produce during using triangle comparison method you can produce the same using space vector approach and we will be dealing with this you know and this one probably the next lecture we will deal this issue. So, some introductory part at least conventional space vector PWM how to look at it from the triangle comparison point of view is something we will look at in this lecture while the bus clamping PWM methods etcetera we would look at in the subsequent lecture now.

So, you know as I mentioned even in the last lecture this equivalence of triangle comparison and space vector approaches have been a stressed quite well in the literature, but what is relatively unknown or known to a lesser extent or these space vector based advanced bus clamping PWM methods, so what is there on the last line here. So, the space vector based advanced bus clamping PWM here there are you know what I would say this is probably what would be the last lecture in this module we would bring out certain pulsewidth modulation methods whose PWM waveforms can only be produced from the space vector approach and not using the triangle comparison approach. We will try and establish that space vector approach is more general than the triangle comparison approach and whatever is the followed and what advantages it gives etcetera and so on and so forth now.

So, now over to our present lecture where we are going to be focusing particularly on conventional space vector based pulsewidth modulation. So, as I said this is based on space vector and I am calling it conventional space vector PWM this is what is most commonly used among the space vector PWM methods right.

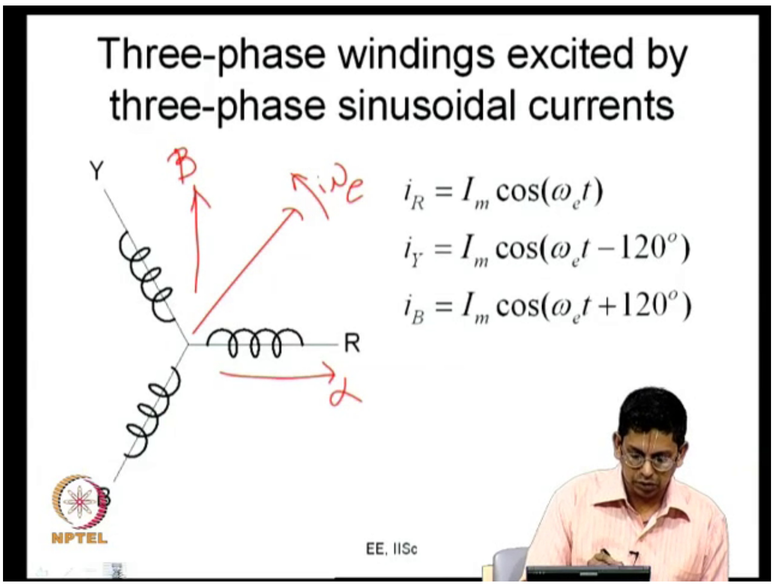
(Refer Slide Time: 04:45)



So, to start with it is a brief recap on the concept of space vector and space vector transformation.

So, we had these revolving mmf. So, you apply three-phase sinusoidal voltages and to three-phase windings of in an AC machine that produces the revolving mmf in the air gap. So, this revolving mmf is an example of a space vector. This is what we had seen in the last lecture 2.

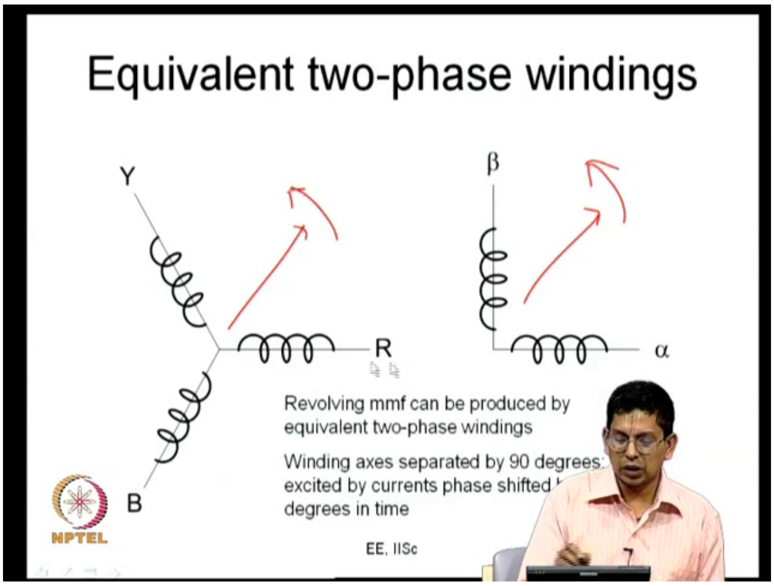
(Refer Slide Time: 05:07)



And now you have these three-phase windings and these three-phase windings you have you know you can inject currents i R is equal to I m cos omega e t i Y is equal to I m cos omega e t minus 120 and i B is equal to I m cos omega e t plus 120 degree when these are excited by that what happens is it produces an mmf which will revolves at an angular frequency omega e like this, this is what happens. So, this is what we would just see for that. So, now, this is an mmf produce by three-phase windings.

But the mmf itself is only on a single plane it is only on a single plane and you can say the that the mmfs 2 components one along this direction, excuse me. So, one along the horizontal direction and excuse me, this is one along this axis what you can call as alpha and you can look at it at the other one which is along this axis which we can call as beta. Now, let us move to the next one and we will see this.

(Refer Slide Time: 06:16)

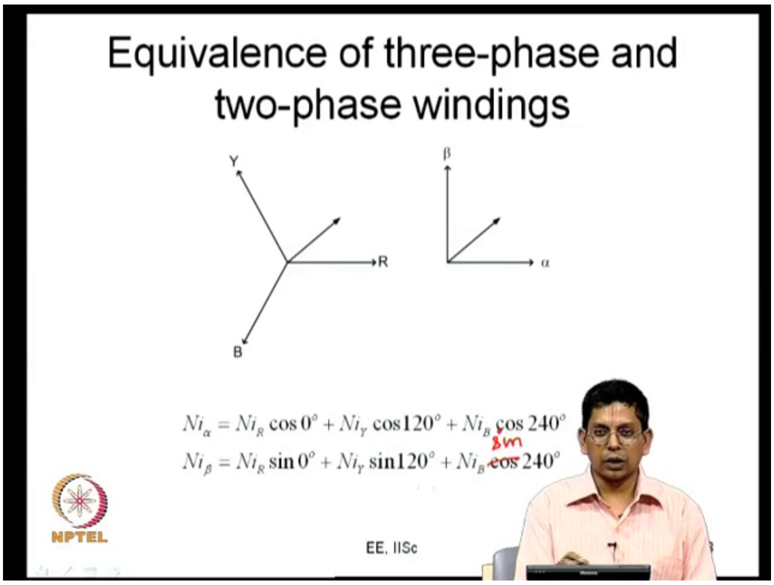


So, this is a three-phase winding this three-phase winding as I was mentioning produces a revolving mmf. The same revolving mmf can be produced by an equivalent two-phase winding also because this mmf that you have here it revolves in on this plane.

It has only two components. So, you know any two orthogonal axis you can considered one possibilities to consider the R phase axis itself as one axis which you can call as an alpha axis and the other one orthogonal to that which you can call as beta axis. So, whatever is produced instead of considering these three-phase windings which are shifted in space by 120 degree you can probably consider a two-phase winding which are shifted in space by 90 degrees, the angle between their axis is 90 degrees now all right. So, you are going to be the exciting this by three-phase currents shifted in phase by 120 degree rather what you can do is you can also you know you can a excite these two windings by currents which are shifted in time by 90 degrees and this way you will be able to produce the same kind of revolving mmf.

That can be produced by a three-phase winding with this two-phase winding and two-phase currents.

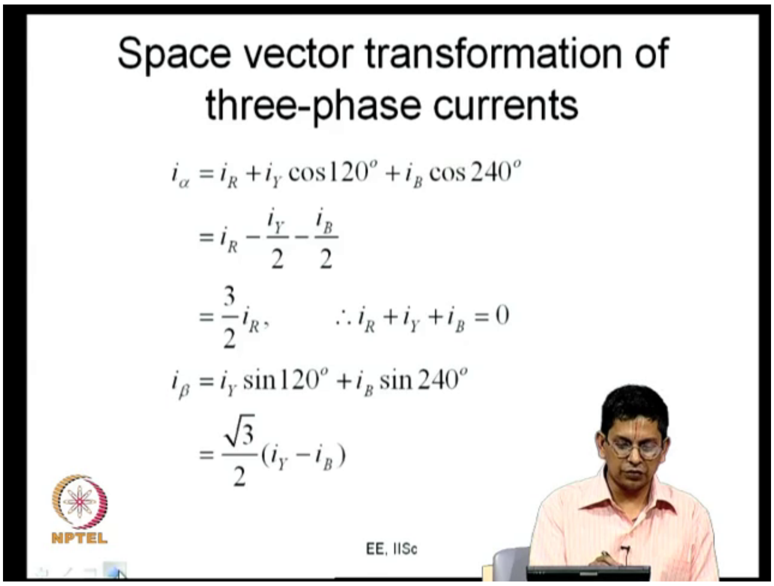
(Refer Slide Time: 07:28)



So, let us look at it little further now. So, these are the three-phase axis and these are the two-phase axis now, I said that the 2 mmfs are equal, when you say the 2 mmfs are equal one way to put that is you can say the alpha axis component of the mmf and the beta axis component of the mmf are equal in both the cases now. So, the alpha axis component of the mmf is Ni alpha right it is Ni alpha as produced by the two-phase winding. If you consider a three-phase winding the equivalent alpha axis mmf is n times i R cos 0 plus n times i Y cos 120 plus n times i B cos 240 right.

And about the beta axis component, so for the two-phase winding N times i beta is the beta axis component of the mmf whereas, for the three-phase winding it is Ni R sin 0 plus Ni Y sin 120 plus Ni B sin 240. I am sorry that must be sin it reads as cos there. So, please change this as sin. So, this is what because you know they are along the coordinate I mean along orthogonal axis. So, this is the cosine components and these are the sinusoidal components here now, right.

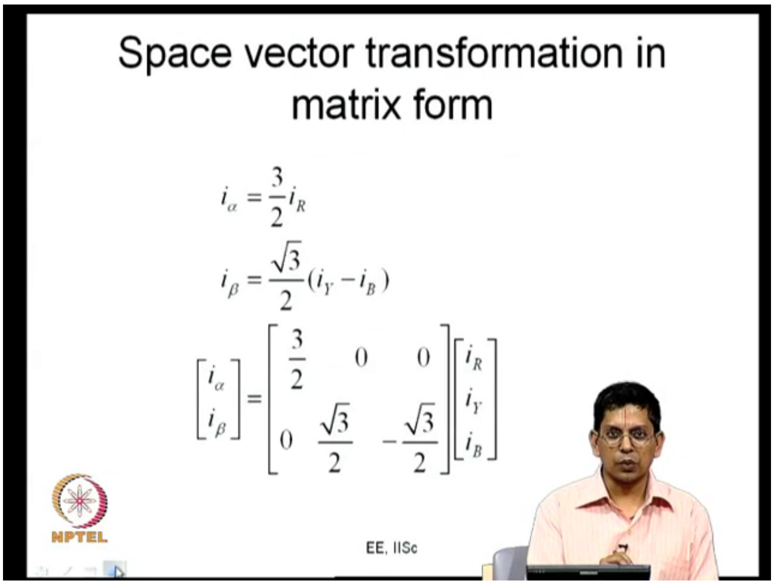
(Refer Slide Time: 08:34)



Now, let us move on to the next one. So, equating the 2 currents what you have is you have i alpha is equal to i R plus i Y cos 120 plus i B cos 240 which is nothing, but i R minus i Y by 2 minus i B by 2 and since i R plus i Y plus i B is equal to 0 or i Y plus i B is equal to minus i R you have i alpha is equal to 3 by 2 times i R is what we saw in the last class.

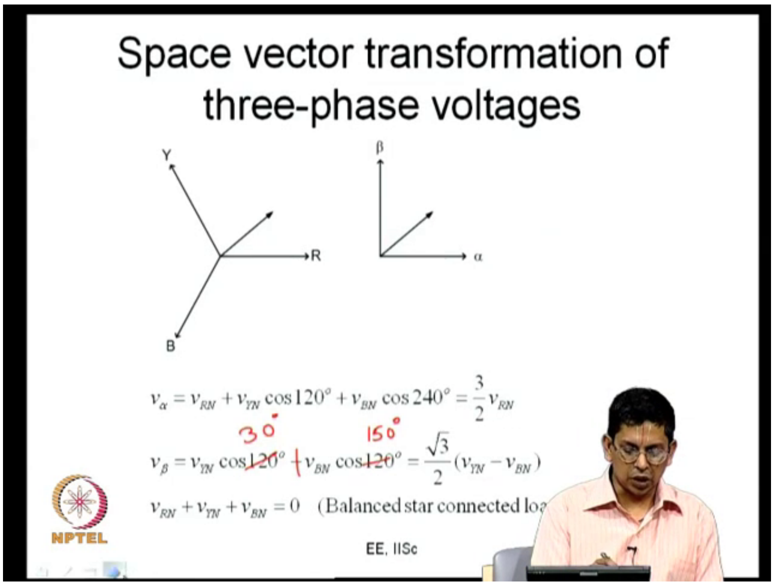
Similarly, you can look at i beta i beta is i R sin 0 which is 0 plus i Y sin 120 plus i B sin 240. So, sin 120 is root 3 by 2 and sin 240 is minus root 3 by 2. So, you get i beta is equal to root 3 by 2 times i Y minus i B. So, this gives a space vector transformation that is you start with currents i R, i Y and i B these are three-phase currents and they is they can be transformed into two-phase quantities namely i alpha and i beta that is what you going to happen now. These are i alpha i beta, so you know that is three-phase to two-phase transformation are what is space vector transformation of three-phase currents now.

(Refer Slide Time: 09:36)



So, the same thing as being reproduced here and it can be written in matrix form you know if it convenient to write. So, you can write it in a matrix form that you can i alpha i beta is this matrix multiplied by i R i Y i B this becomes the matrix for space vector transformation right.

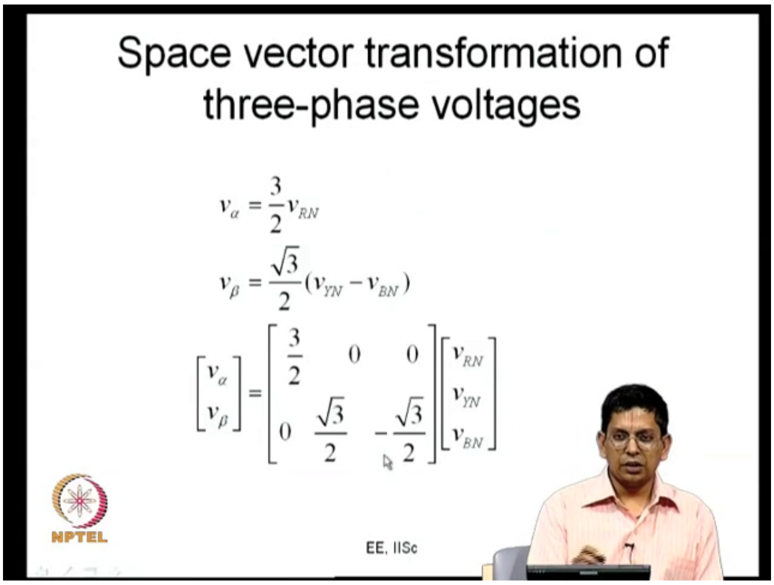
(Refer Slide Time: 09:54)



So, the same way you can handle also, you can handle the three-phase voltages as well not just three-phase currents. So, here again you have voltages v RN v YN and v BN which are the three-phase voltages applied on to balance with three-phase load R Y B with n being the load neutral. And your v alpha is nothing, but v RN cos 0 plus v YN cos 120 plus v BN cos 240 and as in the previous case this would add up to 3 by 2 times v RN. Then we have v beta v beta is equal to this again an error here. So, v beta would be v YN cos 30 and then let us say v BN cos 150 degrees just as we did last time or you can also call it as you know sin 120 and sin 240.

So, this is going to be root 3 by 2 times v YN minus v BN and since you know we have v RN plus v YN plus v BN is equal to 0 that is why this becomes 3 by 2 times v RN and this is just the same thing whatever is applicable to the three-phase currents now we are doing the same thing with three-phase voltages now.

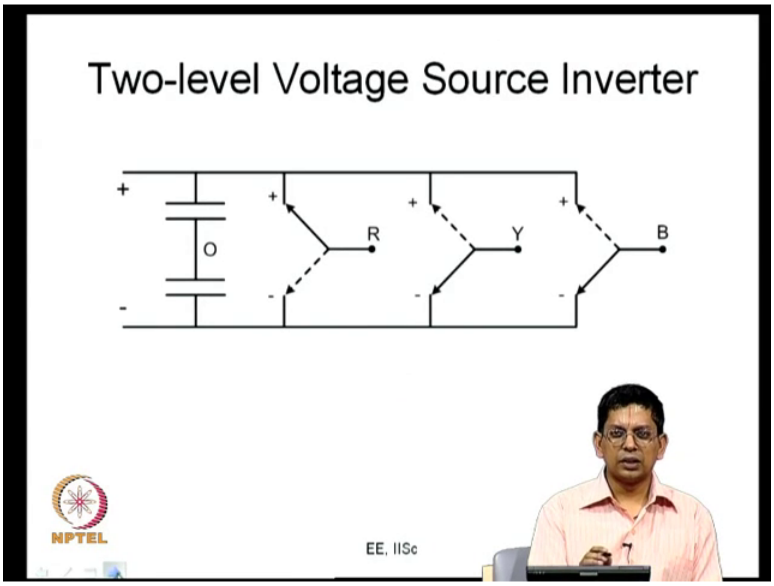
(Refer Slide Time: 11:02)



So, you have the transformation of 3 voltages into two-phase voltages. So, v alpha is 3 by 2 times v RN, v beta as equal to root 3 by 2 times v YN minus v BN. So, this is space vector transformation. So, as I just made a brief mention last time there are also certain other kinds of transformations you can do means it is a pretty similar, you know let us say you multiply both of them by certain constant k that is fine.

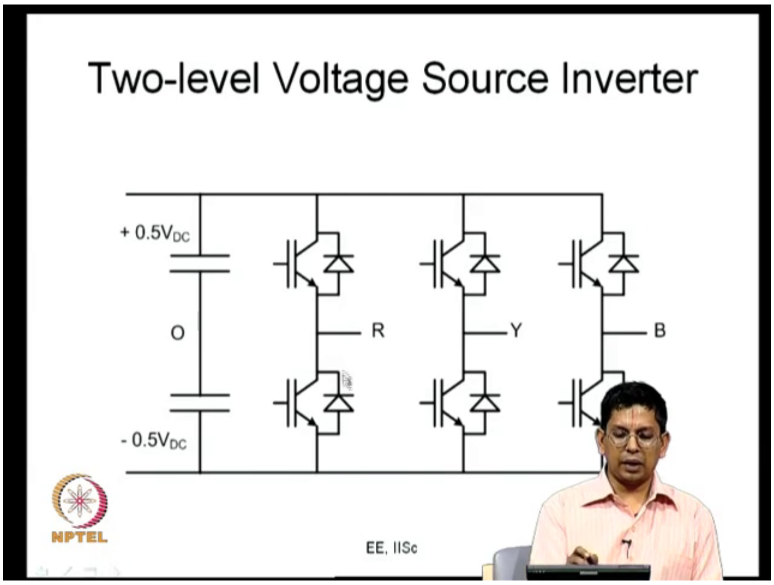
So, there are some reasons why somebody would want to do that etcetera, but this is what we will call as space vector transformations. So, if you find in a different papers, in different textbooks etcetera the definition of space vector transformation could slightly differ from one another this is the definition that we will use in this course. So, you have v alpha and v beta that is a written in terms of v RN, v YN, v BN, in a matrix form I shown here right. So, this is now a quick recap on space vector transformation which we did last time.

(Refer Slide Time: 11:48)

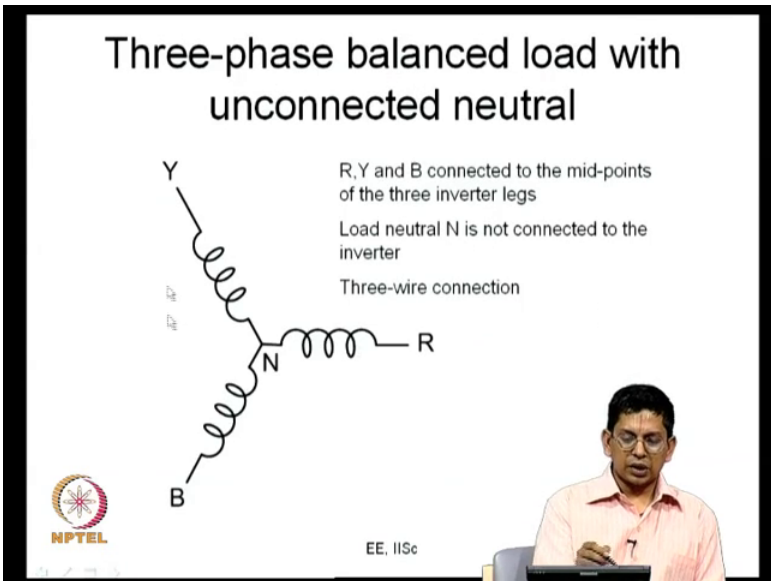


Now, the space vector transformation transforms these three-phase quantities into two-phase quantities now and now we are looking at the three-phase quantities which are produced by the inverter. Now the inverter has the 3 terminal voltage is R Y and B and these voltages can be measured with respect to O which are called the pole voltage v RO v YO and v BO, but what is applied on the winding is v RN, v YN and v BN. Thus just look at that little later. So, this is the actual inverter means it is the genetic switches have been replaced by igbts and anti parallel with anti parallel diodes.

(Refer Slide Time: 12:17)



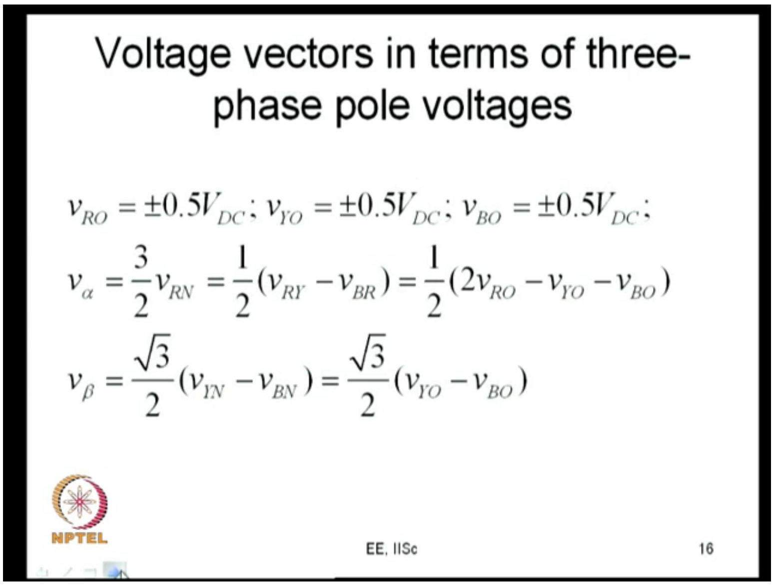
(Refer Slide Time: 12:25)



And now let us say the load is of the shape you know it is not you know in this some load it is not necessarily inductive or whatever on you know you can take it as a star connected winding on. So, for example, your R Y B are the load terminals and it has a neutral point N. So, R, Y and B are connected to the midpoints of R phase R phase leg Y phase leg and B phase leg in the inverter, but n is not particularly connected to any of the points in the inverter. So, is what you have is a three-phase connection is what I was as I was just telling you now.

So, if you go back here this v RO, v YO and v BO will have to be converted into v RN v YN; v RN, v YN and v BN as indicated here before you can do a space vector based transformation.

(Refer Slide Time: 13:08)

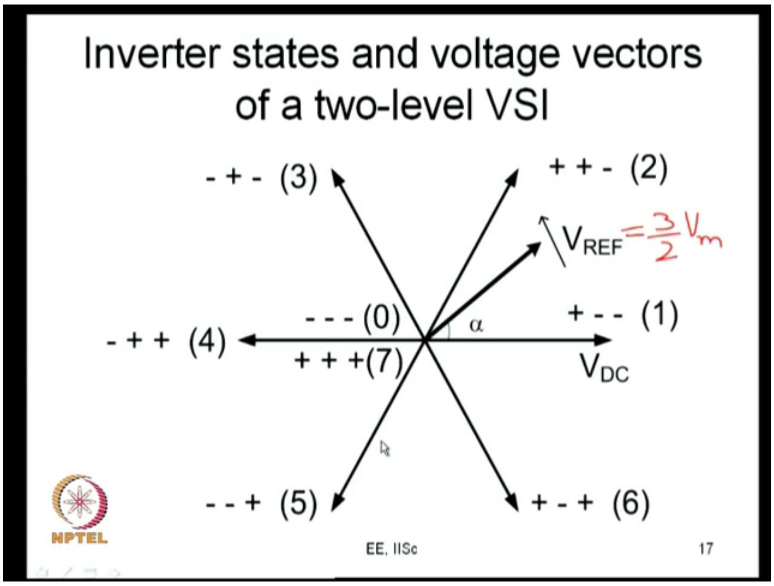


So, your inverter produces 8 sets of output voltages 8 sets of output voltages why 8 sets of output voltages you can say you know R phase if you take every phase in any phase R phase the top device can be on or the bottom device can be on and therefore, R phase pole voltage can be either plus 0.5 V DC or minus 0.5 V DC as we discuss even earlier. Similarly v YO the top can be on or the bottom can be on v BO also the top of the bottom can be on therefore, you have 2 multiplied by 2 multiplied by 2 that is 8 inverter states 8 combinations are possible.

So, the 8 sets of output voltages are possible. So, this 8 sets of output voltage is what we are trying to look at and you know you can want to say v alpha is 3 by 2 times v RN and v RN is it can be expressed in terms of the line to line voltages v RY and v BR is shown here, v RN is one third of v RY minus v BR one third of v RY minus v BR. So, that leads to this expression now. Here v RY and v BR can be replaced, can be written in terms of v RO and v YO here and v BO and v RO here, so you get half of 2 v RO minus v YO minus v BO. And the same way it v beta this is root 3 by 2 times v YN minus v BN this becomes root 3 by 2 times v YO minus v BO. So, what have we done here we earlier first defined space vector transformation, that is v alpha and v beta and this is defined in terms of v RN v YN and v BN we have now changed that you know or to you know we now we are expressing v alpha in terms of v RO v YO and v BO and v beta in terms of v YO and v BO. So, this is convenient because the pole voltages are the ones that are known to as readily many a times that is once you know the switching state of the inverter it is very easy to say what they have pole voltages are and the pole voltages can readily be transformed into these things now.

So, you have 8 sets of pole voltages and if you would expect 8 vectors, but as far as 0 states are concerned that is it can be 0.5 V DC in all the three-phases that is all the 3 top devices can be on or it can be minus 0.5 V DC in all the 3 bottom phases or all the bottom devices I mean all the three-phase all the bottom devices are on. In both these conditions v RY (Refer Time: 15:19) I mean and v BR are 0 and therefore, v alpha and v beta will be 0 in the remaining 6 cases v alpha and v beta are distinct. So, they produce 6 different vectors and that is what is shown here now.

(Refer Slide Time: 15:29)



So, when all that bottom device are on or all the top device is on the vector produces the null vector other than this you have 6 different states and these 6 states are called active states why, in these 0 states the load is shorted there is no transfer of power between the DC supply and the AC side in the DC side in the AC side whereas, in all the other cases there is some transfer of power between the DC side and the AC side. So, we call them active states and the corresponding voltage vectors can be called active vectors right. So, you know in this you would find that not all the phases are connected to the same DC bus.

So, some, one may be connected to 1 or 2 will be connected to positive bus and the remaining one or 2 will be connected to the negative bus that is what you find here. And for hours space vector transformation, these three-phase voltages sorry any of these would produce a voltage vector of magnitude V DC. And the angle law of this voltage vector is what changes from the you know it differs from inverter state inverter state and if you take the active state plus minus minus your R alone has the top device on and in the Y and B legs the bottom devices are on the voltage vector produces V DC and the angle is 0 I am measuring the angle with respect to the R phase axes or the alpha axis here. So, see this alpha may be slightly confusing this alpha you can also use a different terminology some theta, theta s something like that this alpha is the angle that it makes with respect to this particular horizontal axis that is what I am trying to write down here fine.

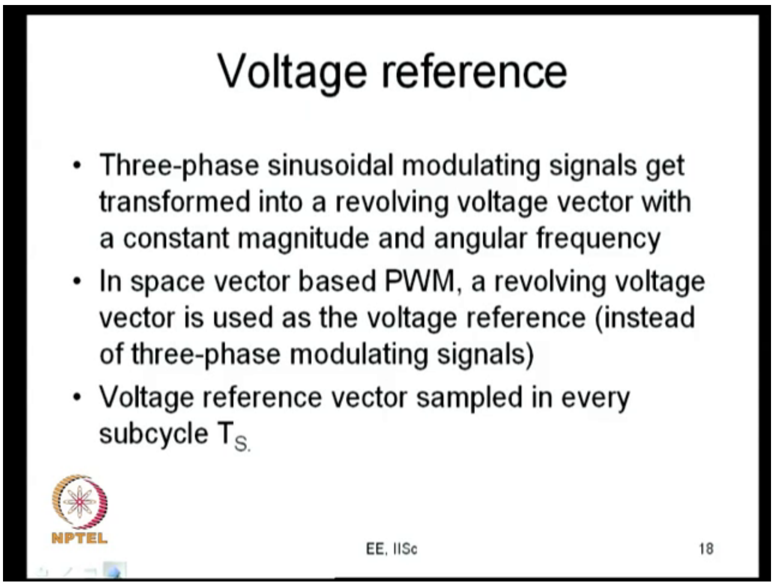
That is what the reference vector, we will come to it, will little later. So, there are these 6 active vectors as I was trying to say now. So, what the inverter produces are 8 inverter states and 6 inverters states produced is 6 vectors and the remaining 2 states produce this null vector this is what is now available for you correct. So, and if you consider the sinusoidal voltage what do what do you want the inverter to do we want the inverter to apply three-phase sinusoidal voltages on the induction motor and what do you what do you mean by three-phase sinusoidal voltages.

Now, remember we are in this present vectors plane now on the space where the plane three-phase sinusoidal voltages mean a revolving vector as shown by this V REF here is a revolving vector, if you transform three-phase sinusoidal voltages into the space vector domain what you get is a revolving voltage vector of a constant magnitude and this magnitude will be equal to 3 by 2 times the peak value of the sin. If you which as sin value as a peak of V m then this magnitude v R will be equal to 3 by 2 times of this thing let me also write it down if necessary it might be useful some time. So, this is actually 3 by 2 times V m where V m is the peak phase voltage now and it rotates at this at certain angular velocity you know that there is some frequency with which rotates and that same as the fundamental frequency. So, in one fundamental cycle we will go around that that is what I wanted to said now.

And let us get a little clearer and how does it rotate etcetera. So, this position is some arbitrary position that is at an angle alpha with respect to this vector plus minus minus and this originally stands for your R phase axis. So, what I can say is this vector would have been along the R phase vector when the R phase voltage was at its peak of the three-phase voltages we are considering when R phase voltages at its peak on the other this vector would I have been like this. And when the Y phase voltage is at its peak the vector will be aligned along here and when the B phase voltage is at its peak it would have been like this and these are during the positive peaks similarly during the peaks, when the R phase voltage has a negative peak the vector would be in this position. So, you will get this now. So, it is actually a revolving vector.

So, when the R phase as a positive peak it is here by the time B phase as a negative peak it is here then it keeps rotating by the time Y phase has its positive peak the vector is here then it keeps rotating when R phase has a negative peak it comes here and so it kind of goes on. I think you know as we go on further and further the correlation between the position of this vector and the angle of the fundamental will become better.

(Refer Slide Time: 19:40)

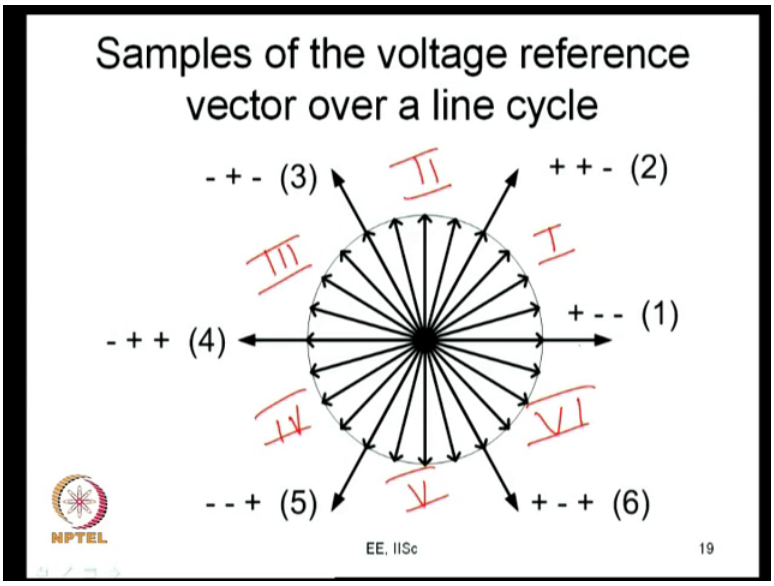


So, what we are trying to say is first is about the question of voltage reference earlier when you trying sin triangle PWM what we do is we use three-phase sinusoidal modulating signals and when you look at any other triangle comparison method to the three-phase sinusoidal signals. We were just adding some a common mode component and we are using some three-phase modulating signals though not necessarily sinusoidal.

Now, what are we doing we are taking this three-phase sinusoidal signals and we are transforming it into a revolving vector if you transform them into the space vector domain it becomes a revolving voltage vector and it is a constant magnitude and angular and this magnitude is it gives the amount of fundamental be a magnitude of fundamental voltage you want to produce and this angular frequency is your modulation frequency there is a modulation frequency right. So, what we do is you know the three-phase waves becomes a revolving will voltage vector in the space vector plane and in space vector based PWM you use a revolving voltage vector as the voltage reference you do not use three-phase modulating signals that is one of the differences we start with.

In sin triangle PWM or in a triangle comparison PWM you start with those three-phase modulating signals and you compare them with the triangular carrier instead what you do here is we use a revolving voltage vector. And this revolving voltage vector is sampled once in every sub cycle and the sampled voltage vector gives you the voltage command for the given sub cycle.

(Refer Slide Time: 20:57)

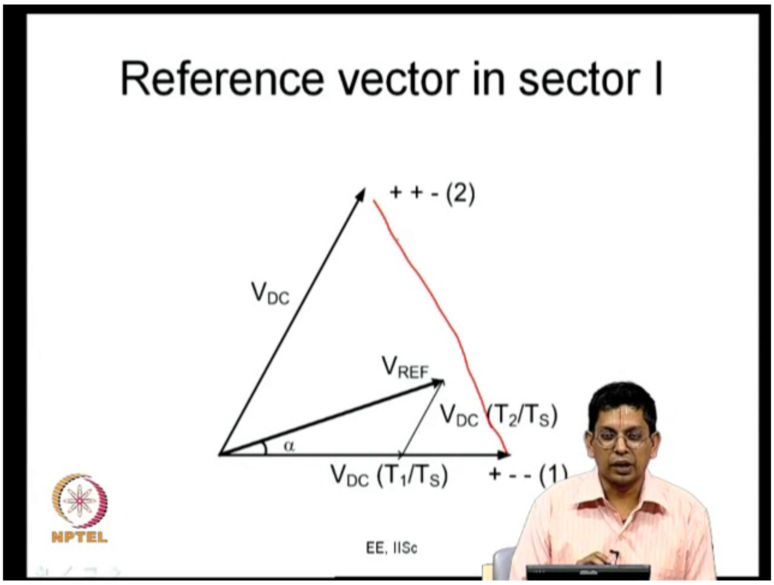


Now, let us say if you sample this, this is look you know it is dissolved it is going to look. So, it is a rotating and it is being sampled here I am I have shown as though in a just for illustration purpose it is getting sampled once in every 15 degrees. So, you find samples every here. So, I have considered that every 15 degrees of the fundamental cycle constitutes one sub cycle. So, I mean I am taking m Y sub cycle to be 4 times 6 is 24, 124th of the line cycle is what I am taking it to be.

It is generally higher you know if you if you look at inverters using motor switching motor drives and power levels of a few kilowatts or tens of kilowatts igbt inverters switch at something like 10 kilohertz or 15 kilo hertz this kind of frequencies are commonly and your modulating frequency could be 50 hertz. So, if it is 10000 hertz and 50 hertz you have a very very high switching frequency and you will really find several sub cycles within a half carry within one cycle now. So, this is anyway just for a purpose of illustration. So, once you have revolving vector and you sample them you are going to get samples all around right.

So, sometimes this some sample may fall here sometimes the sample fall in this region sometimes it may fall in this region here here here. So, it is convenient to give some names to these various spatial regions. So, this first spatial region I will call this as I am writing this as capital I mean the roman letter one I will use. So, roman number I. This is I would call as various sectors I would call this as sector I, I would call this region as sector II, I can call this region as sector III and this is sector IV and this is sector V and this is sector VI. it is just a definition you know you can start from somewhere else and whatever.

(Refer Slide Time: 22:47)



So, now we will just let us call them define them as 6 sectors as I have just mentioned here and let us go on go further. So, now, this is I have a particular reference vector which is in sector I.

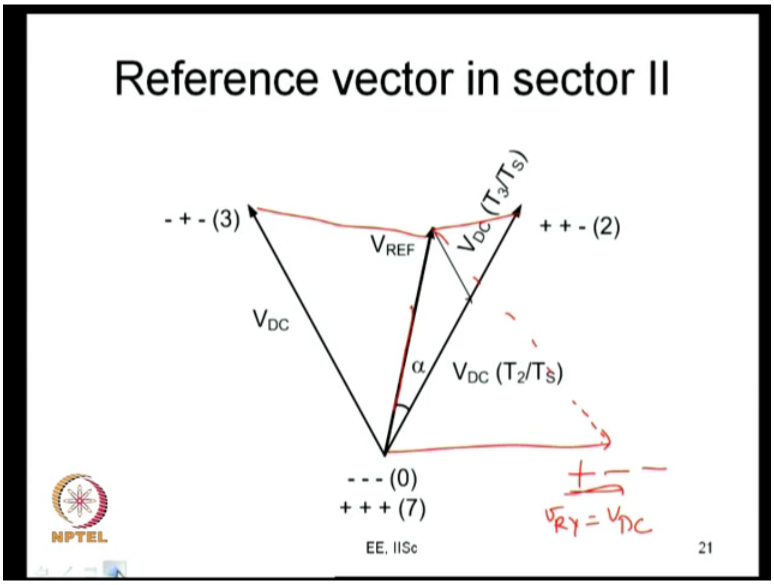
So, why is it in sector I because it is between active vector 1 and active vector 2 and this is where the reference vector is and it make some angle alpha with vector 1 now. So, how do I produce this is the reference vector that is this is the voltage that has been commanded you are supposed to produce so much vector that is a vector of magnitude V REF an angle alpha measured from the R phase axis vector one is same as R phase axis or alpha axis, so it is the same. So, how do you produce that now? So, you can see that you know you can resolve it for example, like that. Let us say from here you draw a line which is parallel to your vector 2 and the line comes on intersects here now. So, you can express this as a sum of this vector and another this vector the first vector is parallel to V 1 vector and it is a fraction of V 1 vector and the second vector is parallel to V 2 vector and is a fraction of V 2 vector.

So, this vector can be realized by adding a fraction of V 1 vector and another fraction of V 2 vector how can you do that you can do it in a you know you cannot apply V 1 vector and V 2 vector concurrently, but what you can do is you can apply V 1 vector for certain duration of time T 1 and you can apply V 2 vector for another duration of time T 2. If you do that what you would get is you will get V DC times T 1 by T s this vector plus this vector is what you will get and T 1 plus T 2 will be less than T S. So, T 1 by T S for example, let us say is 0.6, T 2 by T S could be 0.3 there is still 1.1 is there. So, for the remaining 0.1 T S you can apply this null vector which will not add which you know or you know effect the sum of these 2 vectors.

So, this is time averaging applying active vector 1 for T 1 seconds, active vector 2 42 seconds and the null vector or 0 vector for another T Z seconds would give you this kind of you will by doing this you will be able to produce this reference vector. As I mentioned in the last class you can actually synthesize any reference vector whose tip is within this let me just draw that current goes to here

So, let me just draw a line here joining these 2. So, you can synthesize any reference vector whose tip falls within this triangle formed by V 1 vector V 2 vector in this line. So, that is what I am trying to say you can produce anything there.

(Refer Slide Time: 25:18)



Now, you might have your vector falling in another sector now this is sector II it can be a sector III a sector four you just. You consider sector I now sector II. How do you synthesize the vector the same way, what you do now is instead of vector one and vector 2 you consider vector 2 and vector 3 these are the 2 closest vectors and the idea of the 2 closest vectors is also that you know intuitively you can see this is what is good. What do you mean be this is what is good? Well, this is what can give you lower harmonic distortion. How can you say this is going to give lower harmonic distortion? Now let us say I have taken this vector I have taken this vector. So, I want to produce this reference vector. Now while why want this vector I am applying here. So, there is a large error vector that is there between these two, that is what I am showing in dotted line is the error vector now.

So, this error vector is reasonably small when you are considering these two vectors, this error vector is reasonably small when you doing as we will see later this error vector is nothing, but so this reference vector is nothing, but the space vector transformation of the three-phase sinusoidal voltages three-phase fundamental voltages see we are producing three-phase voltages and the three-phase voltages have fundamental as well as harmonic components. If that fundamental components are transformed that is what you would get you get this reference vector. If you subtract the fundamental components from all the three-phase voltages you get ripple voltages three-phase ripple voltage. The voltage ripple on the 3 along three-phases or all a is transform in to phase vector domain you get an error vector and so this is the error vector now. You want the error vector to be as low as possible and in this case the error vector is very very high. So, you will find that this is very very large now.

Another thing that I can tell you which you can probably verify a little later when your reference vector is here you are supposed to apply these 2 active vectors on the null vector and if you are applying any other vector for example, like vector V 1 what will happen is you will see a pulse of opposite polarity. What do you mean by pulse of opposite polarity? Let us take v RY, let us take v RY when your vector is here right you know or you take for example, v YB if you take v YB when your when your reference vector is here v YB you can see from here is positive v YO is 0 and v BO is minus V DC by 2. So, the difference between the two, if you take v YB will be plus V DC in this case and here also V DC v YB will be equal to plus V DC.

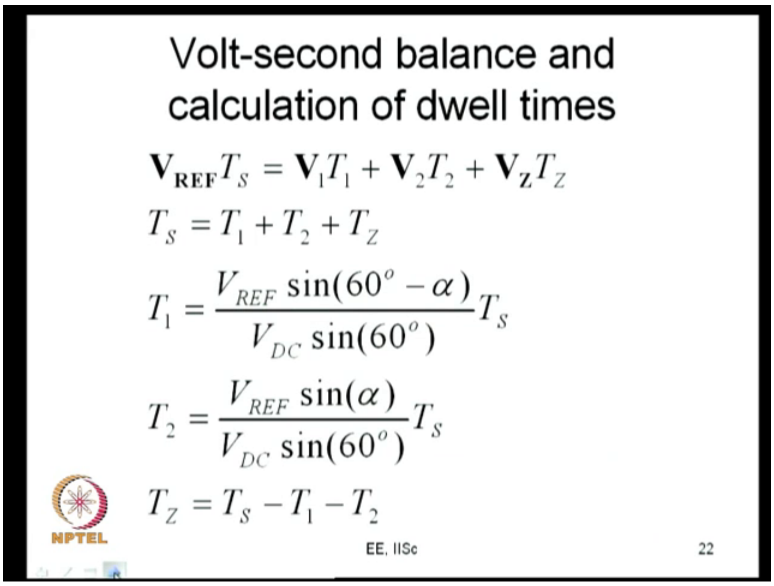
Then the null vectors are when the null vector is applied v YB is 0 now. And actually this region falls within the positive half carrier I mean half cycle of v YB. So, similarly you can look at v RY if you look at v RY you will find that this is in the positive half cycle of v RY sorry, this will come in the negative half cycle of v RY. On the other hand excuse me, this part comes in the when this V REF is here this position of V REF corresponds to certain instant within the negative half cycle of v RY during that time if you apply this vector this is v RY is V DC in this case.

So, what happens is you end up applying a positive pulse then v RY is having negative half cycle that is the three-phase line to line voltages are there considered any line to line voltage say for example, v RY correct and v RY should have only positive pulses in its positive half cycle and should have only negative pulses in the negative half cycle that is what we mean by pulses of appropriate polarity in the line to line voltage waveform. So, if in the positive half cycle of v RY you get negative pulses or vice versa then its pulses of opposite polarity; that means, a lot of harmonic distortion a very high harmonic distortion and whenever you apply a vector like this for example, you do not applied 2 vectors which are closest and you apply a vector which is somewhere farther what you will see is you will certainly see a pulse of opposite polarity in one or more of the line to line voltages.

So, if you want to avoid pulses of opposite polarity in any of the line to line voltage waveforms you should only apply the neighbouring two vectors now. So, if you are in earlier when you are in vector sector I we applied active vector one and active vector 2 similarly when you are moving to sector II we should be applying active vector 2 and active vector 3 now right. So, how do you do that? Once again as before you see that you can apply active vector V 2 for certain T 2 certain duration T 2. So, within the sub cycle time T S. So, it produces you know this is you know V DC times T 2 by T S.

Then you can apply this minus plus minus for that is the vector 3 for certain other duration T 3 and for the remaining duration in the sub cycle you can apply the null vector. So, in effect what you will get is you will get this average vector that something that you will get now. So, similarly if you are going into the next sector you will be using vectors 3 and 4 and so on and so forth. So, wherever is your reference vector you will use the 2 active vectors which are closest to them in terms of the spatial angle and you will use the null vector, you will use these 3 vectors to synthesise the fundament the required reference vector now.

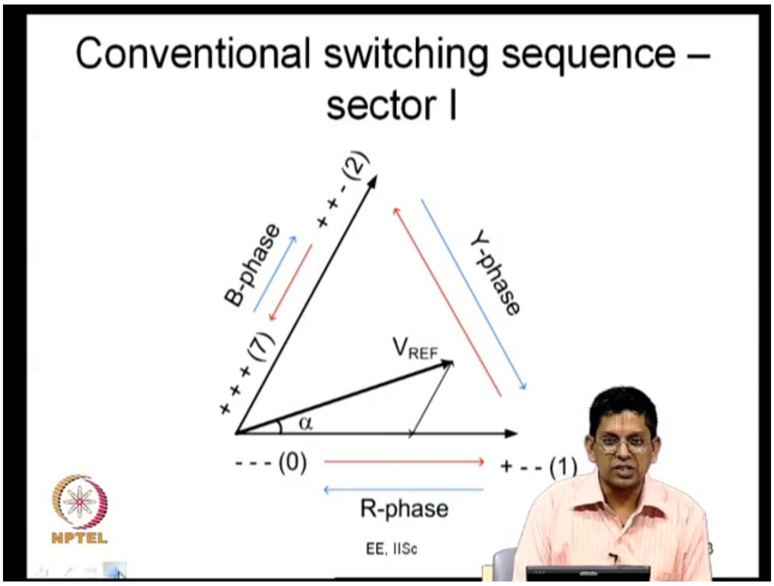
(Refer Slide Time: 30:35)



And this equation gives you the times for which the 3 vectors have to be applied the 2 active vectors and the null vector let us say you are in sector I, V REF vector is synthesized by time averaging of V 1 vector V 2 vector and V Z vector. So, V REF vector in to T S equals V 1 T 1 plus V 2 T 2 plus V Z T Z and as you can see that this is a vector equation and there are 2 dimensions to the vectors and there is actually 2 equations and there now if you separate them into the let us say in imaginary in real parts.

Then T S equals T 1 plus T 2 plus T Z is a third equation together you have 3 equations and there are 3 unknowns T 1 T 2 and T Z. So, you can solve for them. So, if you do that you will get T 1 is V REF sin 60 minus alpha by V DC sin 60 times T S and T 2 by T S will be equal to V REF sin alpha by V DC sin 60 and T Z is equal to T S minus T 1 minus T 2. Remember this V DC comes because that is the length of the voltage vectors any active voltage vector is of length V DC for this space vector transformation we have considered here. So, this is what now, this is how you calculate the times for which that 2 active vectors are applied and the null vector is applied. Now the question is there is a redundancy here now.

(Refer Slide Time: 31:44)



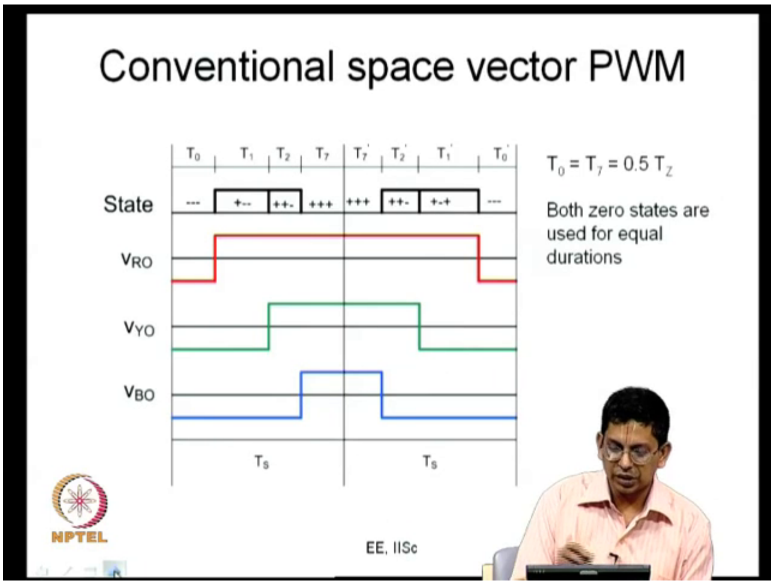
So, the null vector there are 2 different inverter states that is minus minus minus where all the bottom devices are on and plus plus plus where all the top devices are on. So, both of these produce the same null vector. The question is which one are you going to use, this is the redundancy in the inverter I have been mentioning to your time and again and this redundancy is responsible for several of the continuous modulation and the discontinuous modulation methods that we have in the literature right. So, what do we do which one do we use, conventional wisdom is it better use to both. So, let us say if you use only minis minus minus what will happen the lower devices are going to conduct for longer time and they might suffer with greater conduction loss.

Similarly, if you use plus plus plus alone then the top devices might suffer greater amount of conduction loss. So, you know it might be better to use both of them for example, just you know, it is a good option not necessarily the best option, but it is only a good option and this is kind of conventional wisdom what you one who can call. So, start from let us say minus minus minus. So, that is a null vector you are having all the bottom devices are on in all the 3 legs now. So, you have applied this for some duration of time let us call it T 0 and from here you switch R phase this is what is R phase indicated here you switch R phase go. So, that will take you as the red line indicates you, minus minus minus becomes plus minus minus now and you are applying active vector 1 earlier null vector was applied now active vector one is applied now.

So, what do you do next is you apply this active vector 1 for some this T 1 seconds now after applying this for the required T 1 seconds you stay here for T 1 and then you switch the Y phase the moment you switch Y phase plus minus minus becomes plus plus minus. So, that is applying the active vector 2 on the load now. So, active vector 2 is getting applied on the load now, stay there for another T 2 seconds and after that what you do is you switch the B phase from minus to plus the B phase bottom is on and you turn it off and turn the B phase top. So, it becomes plus plus plus you come back to the null vector this is 7 you can apply this for certain amount of time T 7.

So, in conventional sequence what you do is you start from 0 and you go to 1 2 and 7 this is what you do in one sub cycle. In the next sub cycle what you do is you start from here because you do not switch the inverter unnecessarily you start from here stay here for T 7 seconds and from here switch the B phase and go to vector 2, apply this for T 2 seconds and from here switch Y phase and apply vector 141 seconds and come back and apply this null state 0 state 0 for remaining T 0 seconds. This is the conventional sequence now. So, and what you do also do this you apply T 0 and T 7 for equal durations of time as indicated here.

(Refer Slide Time: 34:22)



So, T 0 and T 7 are equal to 0.5 T Z you are applying both the 0 states for equal durations of time. This is one picture which shows the how the inverter state varies with respect to time. So, the horizontal axis here is time and this duration is one sub cycle time and this is another sub cycle T S now.

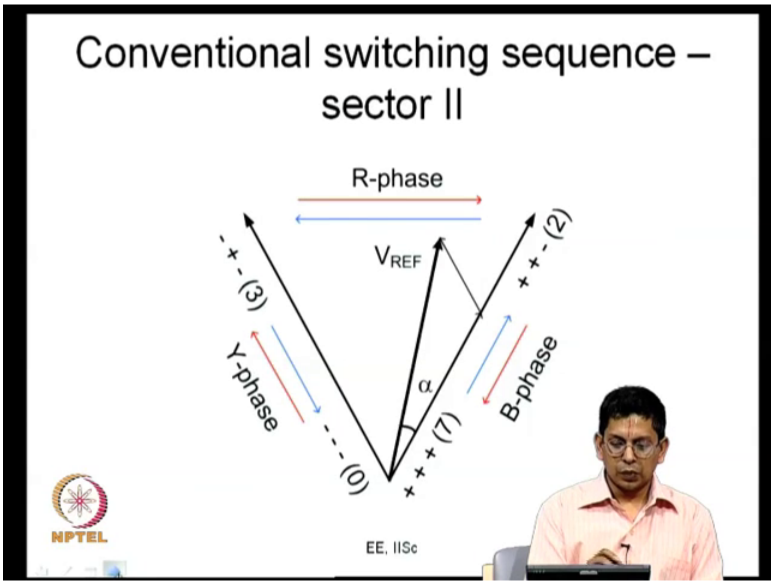
So, what you going to do is you have been given some reference vector and that reference vector you found that is falling in vector sector I and therefore, you have chosen to use vectors V 1 and V 2 and you have calculated the times T 1 T 2 and T Z it using the expressions that we are given just before that sin 60 minus alpha T 1 is proportional to sin 60 minutes alpha and T 2 is proportional to sin alpha etcetera right. So, you do the calculations and you are going to you are output in the inverter states is shown here minus minus minus plus minus minus and then you go to the plus plus minus plus plus plus this is what I show I showed here now. So, from minus minus minus you are going to plus minus minus and then plus plus minus and plus plus plus.

So, that is what I have shown here minus minus minus and you are going to plus minus minus plus plus minus and plus plus plus. And the next time you are coming from plus plus plus is to plus plus minus and then plus minus minus and minus minus minus and that is what I am showing here now. So, we would call them as 0 1 2 7 and 7 2 1 0 right. So, when you are applying such an inverter state this is the output of your inverter when you are doing that what happens is state means you know all the 3 thus information and all the three-phases are included here - minus minus minus means all the bottom devices are on therefore, v RO v YO and v BO are all equal to minus V DC by 2 and from there when you switch to plus minus minus v RO becomes minus V DC becomes plus V DC by 2 it changes from minus V DC by 2 to plus V DC by 2, the other ones remain at minus V DC by 2 minus V DC by 2.

And movement you make a transition from plus minus minus to plus plus minus. So, now, Y phase is switched Y phase changes from minus V DC by 2 to plus V DC by 2 then B phase continues to be minus V DC by 2 and at the end of this time T 0 plus T 1 plus T 2 what you are doing is you are switching B phase. So, the moment you switch B phase this changes from minus V DC by 2 to plus V DC by 2 and you have this now. And in the next time it is going to switch in the reverse sequence. So, in all this v RO v YO v BO are the three-phase pole voltages this is the pole voltage that is the midpoint of R phase leg measured with respect to the midpoint of DC bus v RO. So, this is plus V DC by 2 here and this is minus V DC by 2 here I am not indicating them I hope that you can just follow what I am trying to say here.

So, this is plus V DC by 2 and this is minus V DC by 2. So, it switches on the sequence R Y B in this particular sector in a different sector it will be different. In this sub cycle it is R Y B and in the next sub cycle it switches in the sequence B R Y. So, R Y B, B R Y it keeps on going now.

(Refer Slide Time: 37:08)

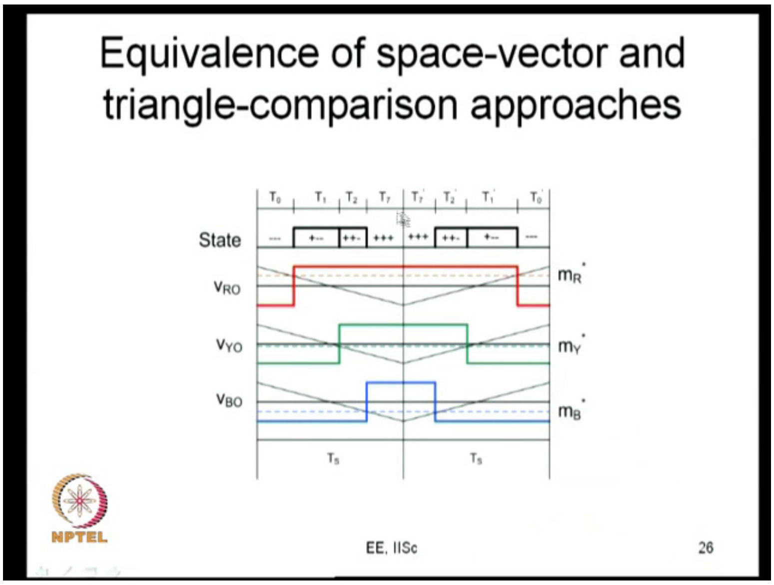


So, in sector II the switching sequence will be different as you will see now. So, if you are doing sector II what you would do you can similarly start from the 0 state it is plus plus plus and you can you can apply this for certain T 7 seconds. And from here you switch your B phase and moment you switch your B phase I mean B phase top is turned off and the bottom is turned on it becomes plus plus plus becomes plus plus minus and you are now applying this vector V 2 on your motor or on your three-phase load.

So, this vector V 2 is being applied. Let us apply this vector v 2 42 seconds at the end of this interval what you do is you switch your R phase R phase top is on now switch that off and switch the R phase bottom on that will take you to minus plus minus which is 3. So, vector 3 can be applied for T 3 seconds as calculated by those equations which we showed some time back and you know the similar equations can be used for all the sectors write the equations I showed you for sector one, but the same equation similar equations can be used for the other sectors also. After that when you switch Y from positive to 0 it becomes a minus plus minus it comes to minus minus minus. So, you can switch from plus plus plus that is 7 2 3 0 or you can switch back 0 3 2 7 as indicated by the red line.

So, the blue line no blue arrow marks indicate the sequence in one sub cycle and the red lines indicate the sequence in the other sub cycles. So, these to indicate the sequences in the alternate sub cycles now. So, as in conventional space vector PWM the null vector is applied using both the 0 states and both the 0 states are applied for equal durations of time though one of them is applied at the start of the sub cycle and the other is applied at the end of the sub cycle. The sequence starts from one 0 state and then moves on to the second one active state, the second active state and then the second 0 state here now.

(Refer Slide Time: 39:00)



So, this is what is now again I am showing back here as 0 1 2 7 etcetera is being outputted here right. So, you have been given a reference vector the reference vector is revolving and you are sampling it and it so happens in the sample falls in sector I. So, if it falls in sector I you are supposed to apply this active vector 1 and active vector 2 active vector one stands for plus minus minus which is given here that is R phase positive is on and ynb negatives are on similarly plus plus minus is in the active vector 2. So, you are you are supposed to synthesize the reference vector by using this active vector 1 active vector 2 and the null vector and in conventional space vector PWM you use both the 0 states the remaining time you have to apply the null vector on the null vector you can either apply minus minus minus R plus plus plus in conventional space vector PWM we use both of them for equal durations of time. So, you have your reference vector that is.

So, there is some magnitude V REF and it is certain angle alpha using this V REF and alpha you can calculate this time T 1 and you can calculate this time T 2 correct and the remaining time is T Z, and T 0 is equal to T 7 is equal to 0.5 T Z as I indicated to you earlier. So, all these switching instance can be identified and your output in this kind of a inverter sequence now. So, when you are outputting this is what you are doing in this sector sub cycle. In the next sub cycle what we are going to do you going to have a reference vector we are considering the steady state situation. So, in the steady state situation what do you have the reference vector will have the same magnitude.

Between here and here what would have happened is the reference vector would have rotated by a further angle of omega T S. So, it rotates way further angle omega T S. So, here you will have the same V REF, but your alpha would have changed it to something like alpha dash and consequently what will happen is this T 1 T 2 etcetera would slightly change. So, if you observe this now this I have given as a T 1 dash that is because alpha has changed to alpha dash. Similarly T 2 is T 2 dash because alpha is change to alpha dash something like alpha plus omega T S it is change to that and the that the remaining null vector time is also some T Z dash and that is equally divided between 7 and 0 is T 7 dash and T 0 dash. So, you have this kind of inverter state being outputted now.

So, you start in the space vector approach what you are doing is you are starting from the three-phase I mean you are start instead of three-phase sinusoidal voltages you are starting from revolving vector and you are sampling the revolving vector and from the sample of the revolving vector for that particular sector you are calculating the duel times T 1 T 2 and T 0 and T 7 and then your output in the inverter states like this. If you are outputting in what happens if you look at the voltages at the 3 load terminals v RO is like this as given by the red line and v YO is as given by this green line here I have been using green instead of yellow here, just for you know better visibility and again v BO for is given by this line and it is minus V DC by 2 here and plus V DC by 2 here this is the your output now right.

So, this is what we are now producing from the space vector domain. So, let us look at this now. So, this v RO is here and this v RO is sometimes minus V DC by 2 and sometimes it is plus V DC by 2 and it has some average value which is between the 2 minus V DC by 2 and plus V DC by 2 and considering that V D plus V DC by 2 is for longer duration and this average is indicated by this dash the line this is closer to plus V DC by 2 right its closer to plus V DC by 2 and the average voltage here. And the average voltage in the next sub cycle are going to be slightly different, but you know that is too different. So, I am just kind of ignoring it here. So, I am just drawing it as a horizontal dashed line and you are shown you some dashed line the dashed line is v RO average the average value of v RO in this sub cycle and in this sub cycle right ok.

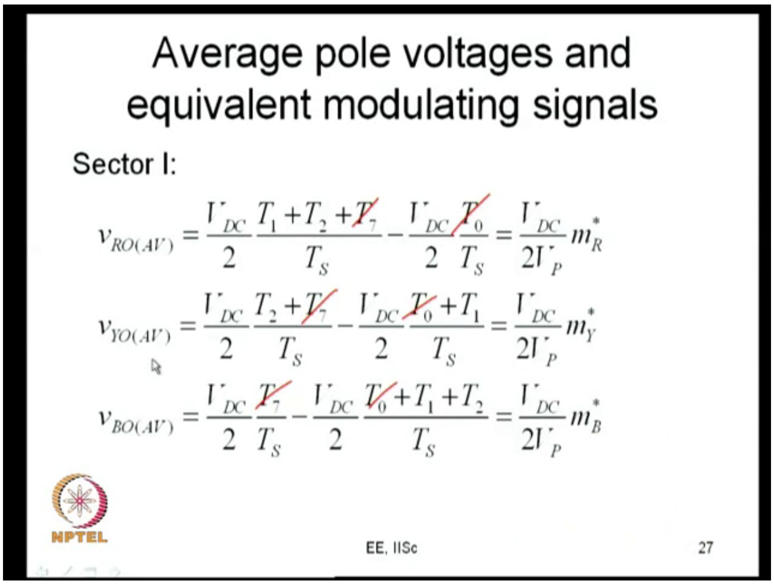
Now, that v RO average m R star is the equivalent modulating signal or it is a scaled version of v RO average. So, this v RO average is some k times V DC by 2 right you divide that V DC by 2. So, you will get this k that the m R star is the scaled version of this now. So, if that m R star is compared with the triangular carrier I shown here, compare this m R star with a falling ramp as shown here then the 2 will intersect at the switching instant similarly whatever is this m R star is the scaled version of v RO average you compile it with the rising carrier they will intersect here at the what is this switching instant. So, even this is the reverse direction instead of starting from your revolving reference vector and sampling and calculating T 1 T 2 and then outputting the inverter states and producing v RO v YO v BO one could have also started with this m R star and this m Y star and m B star.

And compare this m R star, m Y star and m B star with the triangular carriers as shown here and you could have produced you could have identified the same switching instance by such kind of comparison. So, that is why we say that this conventional space vector PWM can also be implemented from the triangle comparison approach. So, the same PWM waveform can also be produced from the triangle comparison approach. So, where you have to use this m R star m Y star m B star, we are only looking at 2 sub cycles now right and there is some small difference in m R star in this and here we are ignoring that right now. So, if you look at all the sub cycles, it will have some wave shape and this wave shape of m R star will not be sinusoidal.

It will be some sin plus some common mode added to that now similarly m Y star will be some sin plus common mode added will be a phase shifted version of m R star by 120 degree m B star will also be a phase shifted and so you can call this m R star m Y star and m B star as equivalent modulating signals. So, this v RO v YO v BO are the pole voltages and what you have seen in the dashed line are the average pole voltages v RO average, v YO average and v BO average these are pole voltages averaged over a sub cycle and m R star is the scaled version of v RO average and if this m R star is compared with this kind of triangular carrier you can identify the switching instance of each phase independently. So, this is what there is some similarity between the two.

So, the same PWM waveform can be produced either from the space vector approach or through the triangle compressive approach let us go and step further now.

(Refer Slide Time: 45:05)



So, I said average pole voltage right. I just need that kind of graphically there now I am doing it mathematically here. So, average pole voltage. So, what is average pole voltage? Sometimes the instantaneous pole voltage is plus V DC by 2 sometimes the instantaneous pole voltage is minus V DC by 2 for every phase now. You take R phase in sector I, so what happens in sector I when the 0 state minus minus is applied the average pole voltage is minus V DC by 2, other way I mean the pole voltage is minus V DC by 2, otherwise v RO is always plus V DC by 2. So, if you look at v RO average it is plus V DC by 2 multiplied by T 1 plus T 2 plus T 7 divided by T S and minus V DC by 2 multiplied by T 0 by T S. Of course T 0 is equal to T 7 is equal to 0.5 T Z in conventional space vector PWM. So, this is the average pole voltage for R phase now.

How about the average pole voltage for Y phase? So, Y phase top device is on during the inverter states 2 and 7 this is plus plus minus this is plus plus plus and during the inverter state 0 that is minus minus minus Y phase bottom device is on. Similarly in the inverter state one plus minus minus in the bottom device on. So, v YO is equal to minus V DC by 2 during the intervals T naught and T 1 and v YO is equal to plus V DC by 2 during the intervals T 2 and T 7 therefore, you have v YO average is V DC by 2 times T 2 plus T 7 upon T S minus V DC by 2 times T 0 plus T 1 by T S equals V DC. So, this is what is your average pole voltage for Y phase now is it ok.

You can see there is a clearer difference between v RO average in v YO average here this T 1 is here in the positive and here that T 1 is shifted to negative one therefore, you can certainly see that v RO average is quite positive here whereas, this v RO average is lower here. And here you can see that v RO average should certainly with positive Y T 0 and T 7 are equal. So, you can be just cancel the two terms now let me just do that cancellation for you here. So, these two terms can just be cancelled off. So, the average pole voltage for R phase is V DC by 2 times T 1 plus T 2 upon T S which is by the positive Y is a positive because we are in sector one and sector one is very close to the R phase axis. So, that is R phase axis I mean the when R phase has its peak voltage the voltage vector is really along positive this is along the R phase axis now.

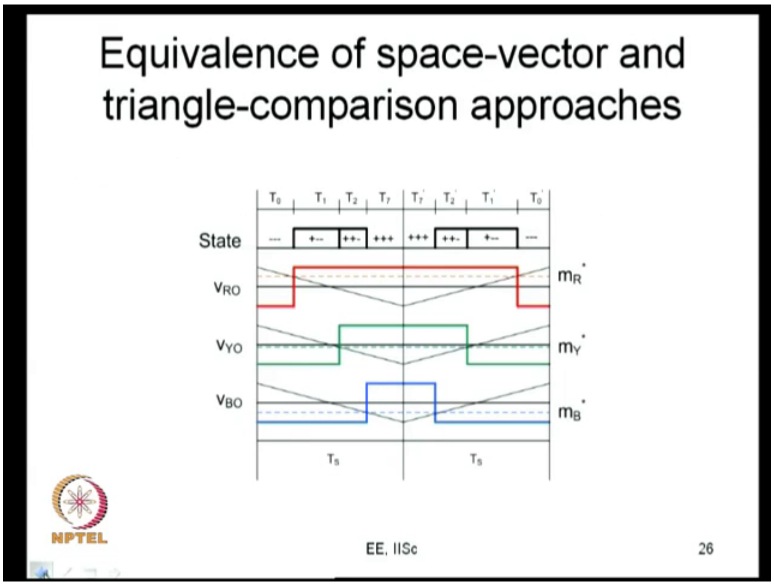
So, v RO is the R phase is going close to it is just crossed its peak actually. So, that is Y you know it is still very positive now and you take v YO you take v YO see this term is positive and this term is negative and here again you have T 0 and T 7 are equivalent there can be cancelled off in case of conventional space vector PWM let me do that cancellation here also let me do that cancellation here also. So, that things become clear now, v BO average you can very clearly see that is negative now.

Why should it be negative? Because we are in sector one and sector one on one side is bounded by the positive R phase axis and the other side it is bounded by the negative B phase axis. So, in sector I, R phase is close to its positive peak and B phase is closed with negative peak and this is because B phase is close to its negative peak we have v BO average is negative now. And how about v YO average when R is going close to positive peak B is close to negative peak why should naturally be closed it 0 crossing. In fact, the center of the sector one is what would correspond to the 0 crossing of Y phase.

Now let us say you have this, this is V DC by 2 times T 2 by T S this is minus V DC by 2 into T 1 by T S you have a special case let us say particular case where T 1 is equal to T 2 when we will be have T 1 is equal to T 2 when alpha is equal to 30 degrees you will sin 60 minus alpha is equal to sin alpha is equal to sin 30. So, when alpha is equal to 30 you will have T 1 is equal to T 2 and v YO average will be 0. The centre of R phase I mean the sector one corresponds to 0 crossing of Y phase. So, at that point you know your average pole voltage is 0.

But let us remember these average pole voltages are not sinusoidal quantities they have sin plus some common mode added to that and one can consider the scaled version of this I can write this v RO average as some V DC by 2 into m R star by V P. So, this V DC by 2 V P can be taken as the inverter gain and v RO average is proportional to m R star which is the equivalent modulating signal of R phase v YO average is proportional to m Y star the equivalent modulating signal of Y phase and similarly v BO average in m B star. So, these terms V DC by 2 V P this term can be regarded as the gain of the inverter now.

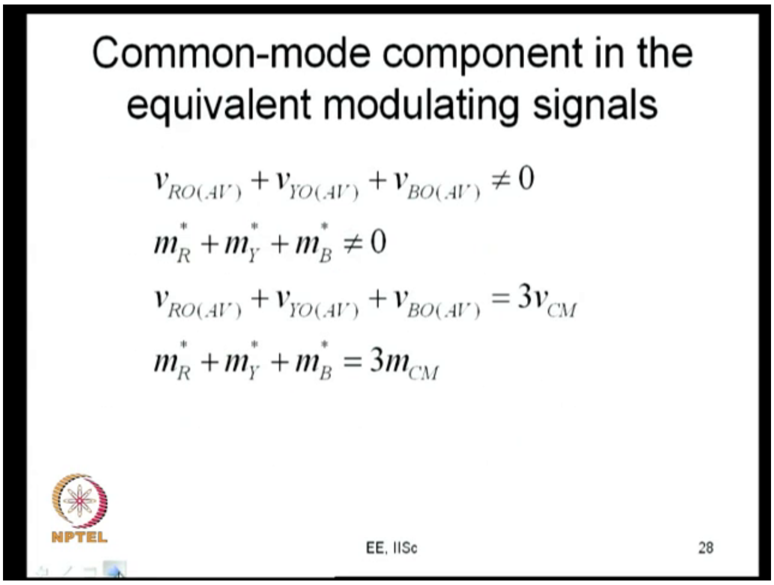
(Refer Slide Time: 49:42)



So, this is about our average pole voltages and be equivalent modulating signals that I derived here I showed you some average pole voltage I just derived an expression for this average pole voltage is there and the scale duration of the average pole voltage is m R star. And if there is m R star m Y star and m B star are compared with these kind of triangular carriers you can get the same switching instance.

So, what you can do is this can also be produced by doing a you know from the triangle comparison approach you can start from m R m Y and m B at some common mode, what does that common mode we still do not know if you add suitable common mode you can get m R star m Y star and m B star. Once you get that common mode component we can compare with triangle carrier and you can produce the same PWM waveforms that is conventional space vector PWM produces right.

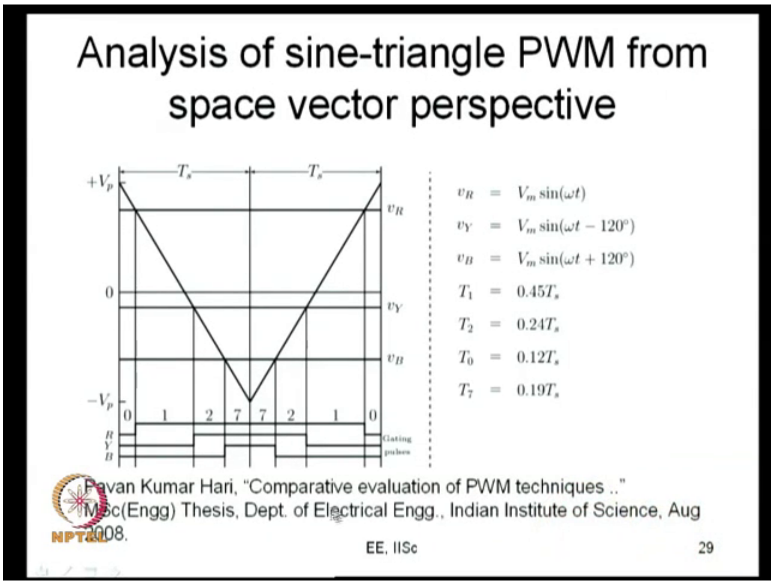
(Refer Slide Time: 50:21)



So, let before going there let me just underline the fact that v RO average plus v YO plus v BO average is not equal to 0; that means, there is some common mode component there are they are scaled versions m R star plus m Y star plus m B stars again not equal to 0 that is some scaled version is there. So, the sum of these 3 is 3 times the common mode voltage again the sum of these 3 is 3 times what you can call as m CM. So, let us you know take it from here.

So, what is that m CM we have to find out. So, this v RO average v YO average and v BO average I have discussed them only for one sector this is in sector II and sector III etcetera also you can write expressions for v RO average v YO average v BO average etcetera I would leave that as an exercise to you. In fact, if you sketch v RO average about the entire thing you can subtract it from the sinusoidal component and you can find out what the common mode component is, that is to sketch v RO averaged over the entire a cycle for example, and similarly v YO average and v BO average I would leave that as an exercise and from that the determination of the common mode component can also be an exercise.

(Refer Slide Time: 51:21)



But anyway we will determine the common mode components starting from the other side. What did we do now? We started with sin triangle PWM here I am sorry we started with space vector PWM and we calculated the duel times of the inverters inverter states and we outputted the states and we came up to this pole voltage then we considered the average pole voltage and then you know we can be sure that if you compare the equivalent modulating signals with triangular carrier also you can produce this now.

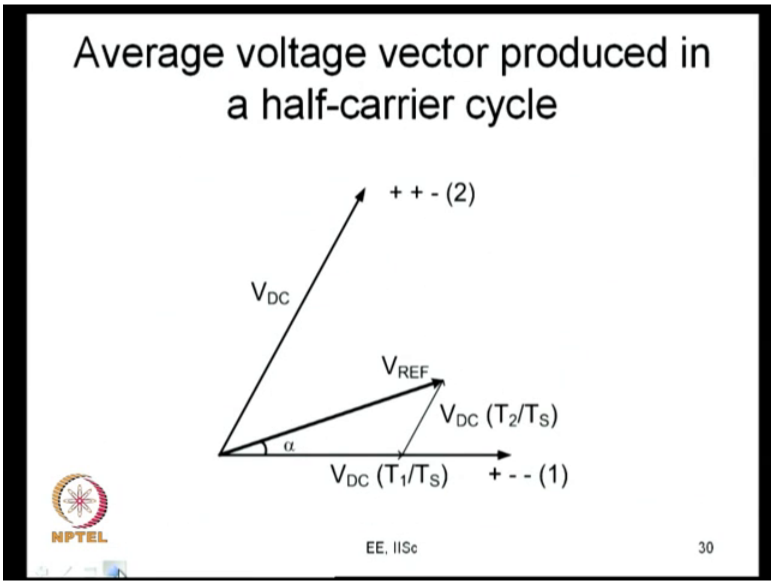
Let us start from the other side instead of starting from space vector PWM let us start some sin triangle PWM. So, this is a figure which is actually produced from you know Pavan Kumar Haris Msc engineering thesis on comparative evaluation of PWM which is on comparative evaluation of different PWM techniques and this evaluation is in terms of harmonic distortion in switching losses some work of Pavan Kumar Haris thesis will we will be discussing here in the later lectures when we are into you know going in to evaluation of line current and switching glass. So, now, let us look at the sin triangle PWM. What we are going do is looking at sin triangle PWM from a space vector point of view.

So, the sin triangle PWM what we have is you have three-phase sinusoids and you compare them with the triangular carrier I am just considering one carrier here and for simplicity you know the value of the triangle will I mean the modulating signal will change from one half carrier to another half carrier I am just ignoring that here for simplicity. So, sometimes this is called you know regular sampling at every carrier interval or it is called symmetric sampling because you know the switching instances are equidistant from this peak and that peak. That is let us say at this peak you have now R phase will switch here and R phase will switch there this distance and that distance are equal so you called as a symmetric sampling sometimes all right.

So, you have this carrier as shown here falling carrier and raising carrier one carrier cycle and you have taken some v R is equal to V m sin omega T and v Y V m again v Y some value of v R v Y and v B. So, on their actual values, you have considered some numbers for V m and omega T and you get these values now. And you compare them you see that R phase switch is here, Y phase switch is here, B switch is here similarly B Y and R and you look at the inverter states this is the inverter state 0 that is minus minus minus this is inverter state 1 that is plus minus minus this is inverter state 2 that is plus plus minus and here it is plus plus plus. So, the 0 1 2 7 and 7 2 1 0 is almost same as what you got here 0 1 2 7 and 7 2 1 0 is very similar to what you got there is very very similar to what you got there right.

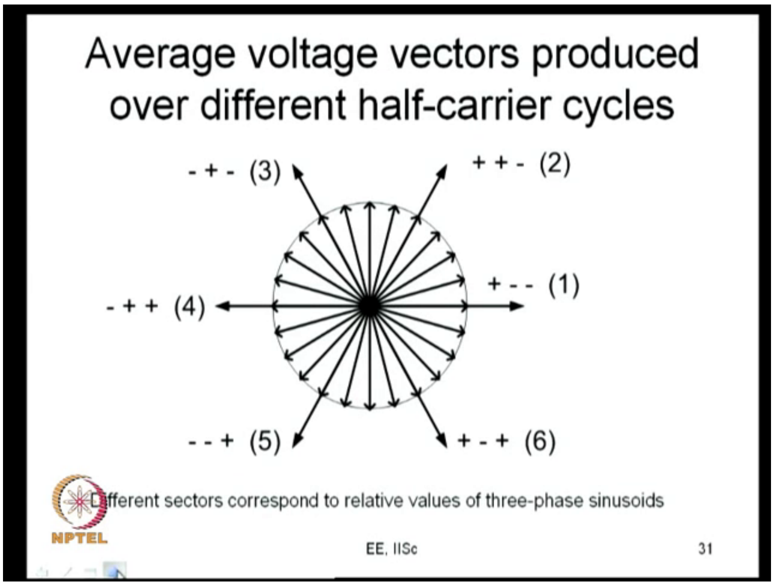
Now, what is the difference that is what we need to focus on now. The difference is right now I would point out that you can see that this 0 and this 7 are not applied for equal durations of time this 0 for how long is it getting applied it is applied for a duration proportional to this difference between plus V P and the R phase modulating signal. Similarly this 0 state 7 is applied for a duration proportional to the difference between v B and minus V P and these are not equal this vertical distance and this vertical distance are not equal and therefore, these two horizontal durations are not equal T 0 and T 7 are not equal. That is the main difference between sin triangle PWM and space vector PWM now.

(Refer Slide Time: 54:24)



So, you once again you start from 0 1 2 7 you can go around and you know you have active vector one applied and active vector 2 applied and null vector for the remaining time. And therefore, once again there is time averaging and you produce some average vector V REF now.

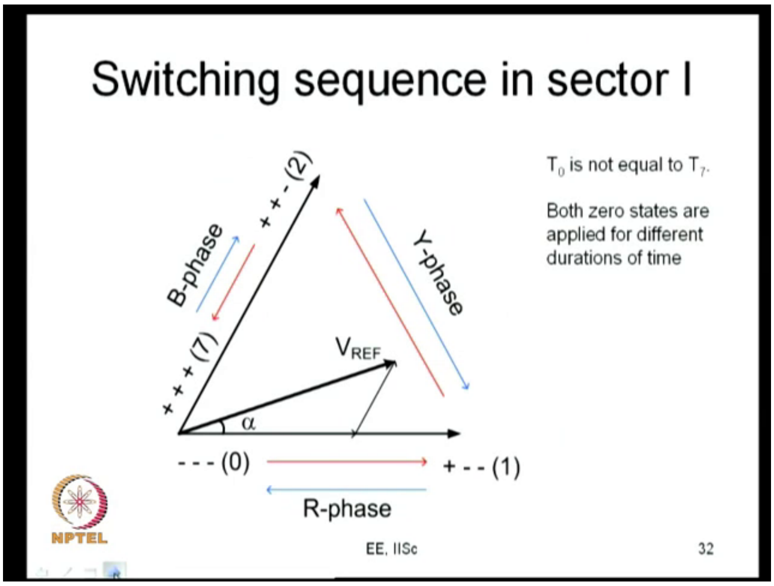
(Refer Slide Time: 54:41)



And this is in one sub cycle you look at all the half carrier cycles over the entire cycle you know you will have average voltage vectors is applied like this that is one once you will have this in the next half carrier cycle you see you may have another average voltage vector whose magnitude is the same, but the angle is changed by omega T S. So, if you look at the average voltage vectors produced over different sub cycles this is what sin triangle PWM is does, this is what sin triangle PWM does very similar to space vector PWM. In space vector PWM you will talk in terms of the sectors. So, you know your reference vector is between vector one and vector 2.

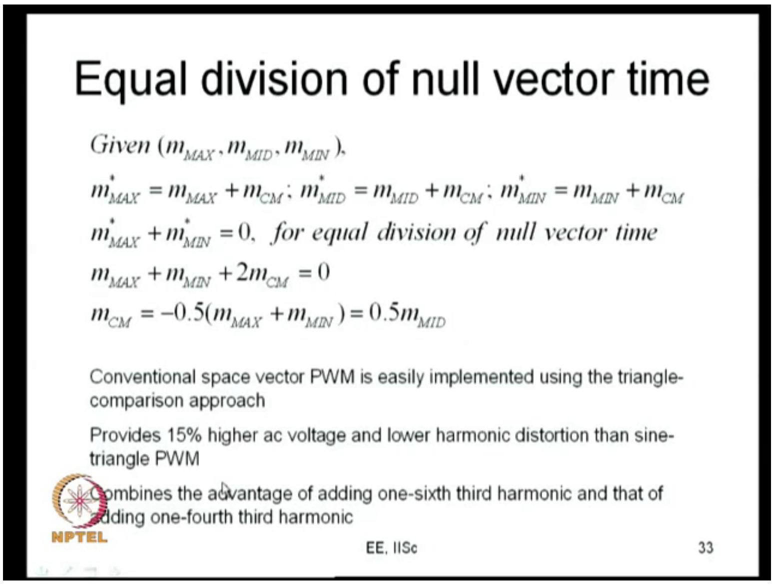
Here in sin triangle PWM relative values of three-phase sinusoids, if m R is greater than m Y the modulating signal of R phase is greater than that of Y and which m Y is again greater than m B then it would correspond to this sector. Similarly every sector actually corresponds to some particular kind of relative values. So, m Y is the middle value here, m R is the middle value here, m B is the middle value here and so on and so forth the 6 sectors corresponds to you know can be determined based on the relative values of three-phase sinusoids I will we will deal with this better when we do this bus clamping PWM in the next class.

(Refer Slide Time: 55:49)



And you go through the same switching sequence pretty much the same switching sequence as in conventional space vector PWM the only cache is T 0 and T 7 are not equal that is the only difference now.

(Refer Slide Time: 56:01)

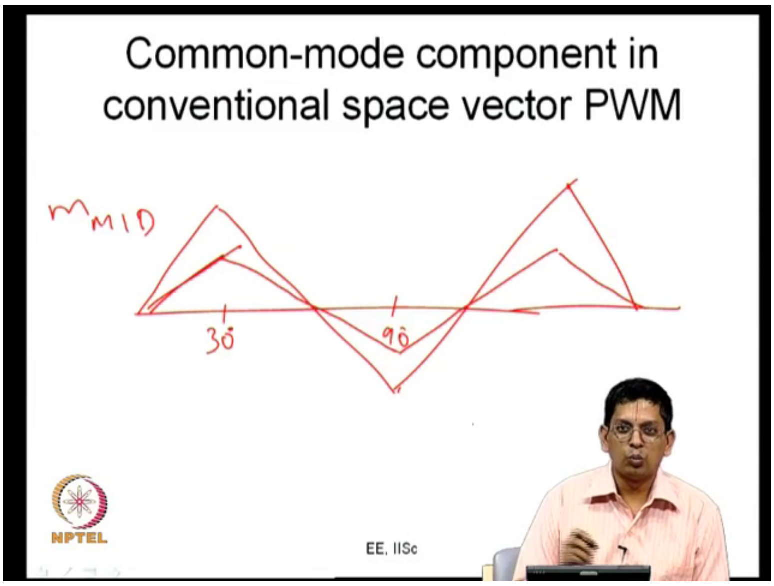


The simple question is how do you make them equal that is all that we are going deal with very quickly. So, you have 3 signals you call them m MAX and m MID and m MIN you call them m MAX m MID and m MIN all right. So, now, you want to add some common mode. So, m MAX star m MID star and m MIN star you know all common mode added you get this now what you want is you want the null vector time to be equal for the null vector time have to be equal then m MAX star should be equal to minus m MIN star both should be equal in amplitude, but opposite in sin or m MAX star plus m MIN star is equal to 0 this leads to the condition m MAX plus m MIN plus 2 m same is equal to 0 or m CM is simply equal to minus 0.5 times, m MAX plus m MIN or 0.5 m MID.

So, the common mode component should be equal to 0.5 times m MID this is what you have to do. In every half carrier cycle you will have m MAX m MID and m MIN you take 50 percent of m MID and add it as common mode component and that will give you equal division of null vector time. So, sin triangle PWM would get effectively change to conventional space vector PWM. This is very easy for you have to implement once you have implemented sin triangle PWM this is a very very small change and by doing this you can get 15 percent higher voltage as we got in the third harmonic injection case we will discuss probably this in a little greater detail in the next class. So, it gives you 15 percent voltage and also it gives equal null vector time and if 15 percent higher voltage means you get the advantage of one-sixth third harmonic injection now.

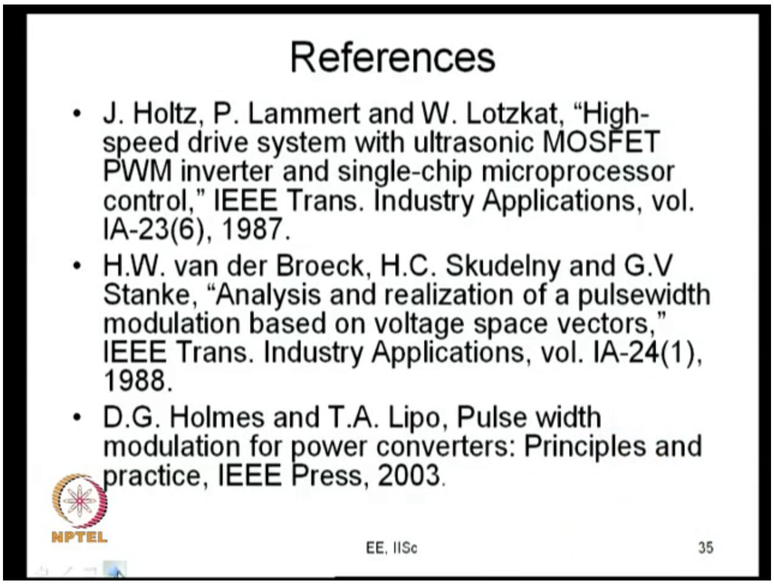
And in terms of harmonic distortion it gives you whatever benefit that one-fourth that harmonic gives you in the third harmonic injection there we will discuss this in some detail when we go into the line current ripple analysis now.

(Refer Slide Time: 57:37)



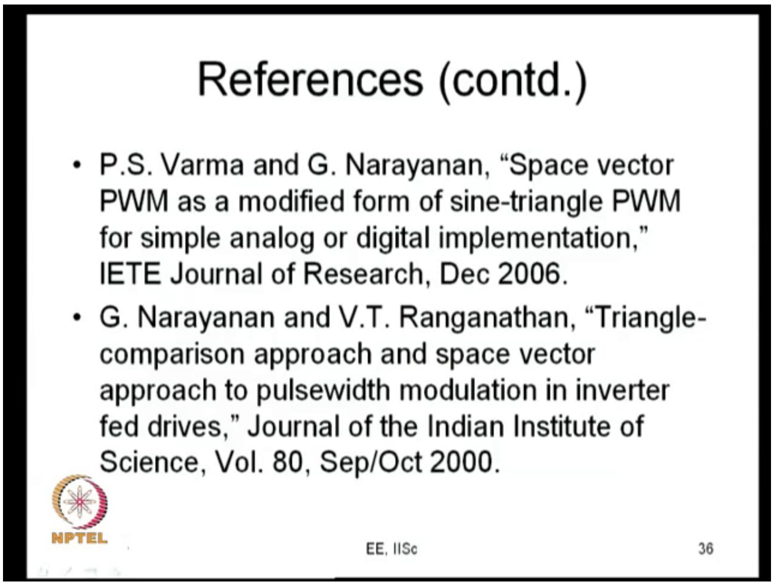
So, this is the common mode component that I told you. So, you know what kind of common mode component gets added now. So, what you will you will exactly find this you will find that a common mode component will go like this, this is m MID if you simply look at m MID, your R phase will be the m MID up to 0 to 30 degrees now. So, then the next one will be 90 degrees. So, your m MID if you draw it will be like this and your m CM will be half of this your m CM will be simply half of this. This will be a common mode component you will do more of this in the following class.

(Refer Slide Time: 58:18)



So, thank you very much for being with me and these are some references which I indicated on last time.

(Refer Slide Time: 58:23)



These are some additional references which will be very useful for you. So, thank you very much and hoping to meet you again in the next class.

Thank you.