Pulsewidth Modulation for Power Electronic Converters
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Lecture – 19
Concept of space vector

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters.

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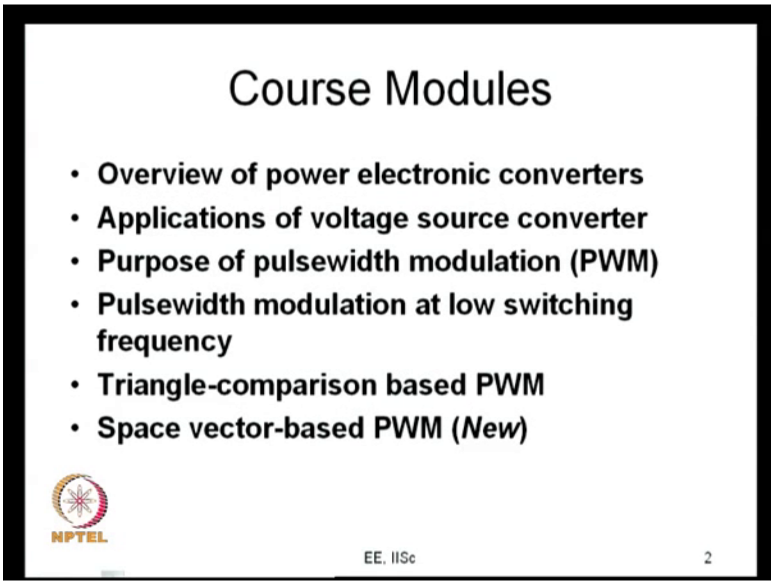

So, let us take a quick look at what are the various modules that we have covered so far in this course now. So, as you are aware initially we started with an overview of power electronic converters this is was probably one of the longest modules covered till now. We looked at a various kinds of power electronic converters started from switches and then we went on to DC convertors buck boost etcetera and we looked at voltage source and current source converters multilevel converters etcetera, then if we had a particular focus on three-phase voltage source convertor and we looked at applications of voltage source converters.

And applications of voltage source converters would be like you know the most important application that we are concerned about here is motor drive that is an induction motor drive. Other than an induction motor drive you know there are other things that you are look at for example, we are looking at active frontend converters and you are looking at power quality equipment such as you know static compensator which you can use for compensating supplying reactive power to the system and further you also use voltage source converters when you want to clean up the harmonics drawn by certain loads that is harmonic filtering active filters as they are called.

So, these are the various applications we had a quick review on these various applications and you may remember that each one of these applications could actually be a course by itself and we had a very quick review on this now. And once we had this you know we the role of a voltage source converter we looked at pulsewidth modulation for a voltage source converter and we first look at the purpose of pulsewidth modulation. And if you look at it the voltage source converter there are three things one is the DC side voltage then the AC side voltage then the modulation.

So, the purpose of pulsewidth modulation is to make sure that on the AC side you get the desired fundamental voltage, but in addition to the fundamental voltage you also get some harmonics. So, the trouble is you want to avoid those harmonics. So, the purpose of pulsewidth modulation as we saw was to control the fundamental voltage and to reduce the harmonics and their ill effects. So, these are the main purposes of a pulsewidth modulation and then with this we went into low switching frequency pulsewidth modulation where you barely have you know one or 2 switching angles very few switching angles in every quarter cycle I mean the inverter switching frequency is very low is just you know the you know a few times that of the fundamental modulation frequency like 5 times or 7 times if you are taking a modulation frequency 50 hertz, we could consider you know just a switching frequency of 250 350 hertz etcetera.

So, I talked about the practical relevance of this and why I mean why it is relevant from an academic point of view etcetera earlier and this is probably where the last 2 this is where we came to you know like these are what you would call as now very important PWM methods and the triangle comparison method is what is probably most alone from a practical point of view. If somebody is familiar with many of these material then this is probably one where one would particularly focus on and in many of these practical converters we do have this triangle comparison based PWM the best known of this category is the sine triangle PWM where you sinusoidal modulation signals and compare them with the triangle or carrier and you produce PWM signals and to the sign you can add some third harmonic or you can add many other triple frequency components common mode components you know those this is what we discussed in the previous module now.

So, today we are going to start up with a new module which is again about PWM generation. So, the earlier module was about generating PWM for a three-phase voltage source converter that is following what is called as a triangle comparison approach. Now once again you are talking of generating PWM for the voltage source convertor, but what happens is we are going to use a different approach and what is called space vector approach. So, we are going to deal with space vector base PWM in this module now.

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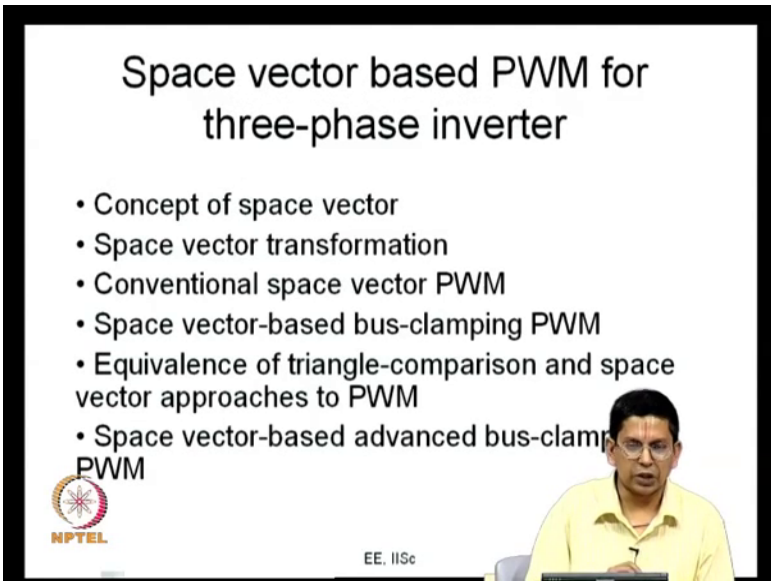

So, what is the space vector base PWM? So, these are the various things that we will be covering as part of this module.

So, first we would look at the concept of space vector that is what we will primarily be doing today. So, once you know what is the space vector we know we understand the term space and we understand the term vector individually. So, what we mean by the space vector something we need to look at and then we would look at what is called a space vector transformation that is actually three-phase quantities can be transformed into 2 phase vectors and that is what you call as space vector transformation you would look at this and from then on we would go into conventional space vector PWM method which is the most popular PWM method using this space vector approach.

So, that is conventional space vector PWM I would say is as popular as sine triangle PWM I mean if not more popular and in fact, it is this benefit is you know compared to sine triangle PWM it can give a higher amount of AC side voltage for the same DC bus voltage. I mean whatever advantage we got with third harmonic injection a similar advantage you can get with this conventional space vector PWM also. So, you will be discussing this conventional space vector PWM now. So, this is what I would call as the most important method using the space vector approach now.

And there are alternative space vector based approaches this bus clamping PWM you recollect that the discontinuous PWM or bus clamping PWM which we discussed in the earlier module triangle comparison methods we will now discuss in this module also how to produce this bus clamping PWM not by comparing triangular carriers and modulating signal, but from the space vector approach this is what we would also be looking at from this now. And then these 2 methods triangle comparison and space vector would look a little different from each other, but actually they have lots of similarities.

So, we would look at the equivalence of triangle comparison and space vector approaches to PWM and most of the literature stops here there are very you know if you read through good part of the literature you would get a feeling that both these approaches are pretty equivalent. But what we would like to stress in this course is space vector approach is a little more general than triangle comparison approach that is whatever can be done using triangle comparison approach, whatever PWM waveforms can be generated using triangle comparison approach can also be generated using space vector approach.

On the other hand everything that is possible using space vector approach is not necessarily possible using triangle comparison approach. I would call space vector approach as more general than triangle comparison approach and you know one category of space vector based PWM is this advanced bus clamping PWM these PWM signals can only be produced from the space vector approach and they cannot be produced from the triangle comparison method is what I am going to talk about towards the end of this module now.

So, that is just to give you an overview of what you would expect in this module which we are hoping to cover in four lectures I am just getting started with today’s lecture the beginning of this module that is the concept of space vector what do you mean by a space vector.

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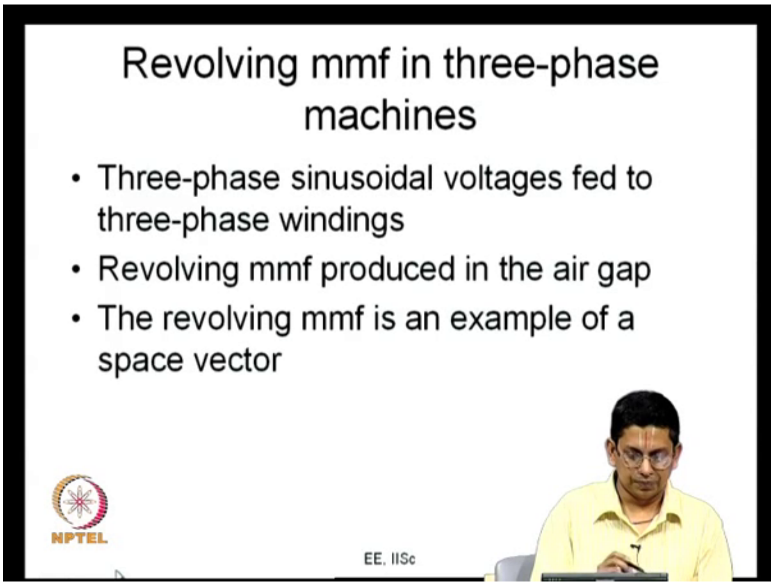

Let us start with an example this revolving mmf this is about an example of a space vector approach of what is the space vector. So, this revolving mmf in three-phase machines that is what I would like you to consider, most of you should be familiar with three-phase machines and even in this course we have been dealing with three-phase machines on and off.

Now, you consider a three-phase machine like a three-phase induction motor or a three-phase synchronous machine. So, what you have is you have a three-phase winding sitting on the stator and what you do to the three-phase winding you apply three-phase sinusoidal voltages usually and when you apply a three-phase sinusoidal voltages on to this three-phase winding it produces a revolving mmf there is a magneto motive force this field this magnetic field produced and that magnetic field revolves in air gap this is what happens now and this revolving mmf is what where does it revolve it revolves in space.

So, in some part of the air gap at some angle you may find the peak the magnetic field the flux density might be at its peak now it will gradually move around the space. So, you know this is a revolving mmf. So, that is it is actually a vector which really moves in space and. So, you can call that as an example of what is a space vector now.

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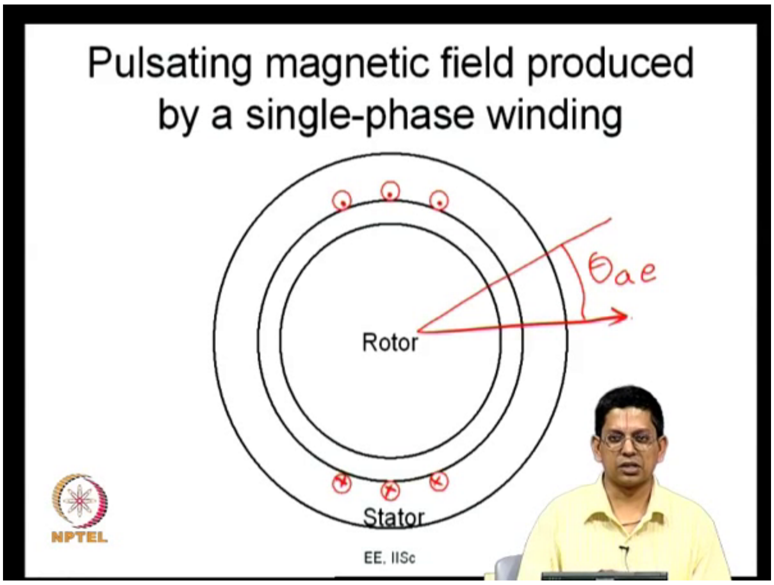

So, let us get a little more into this what we mean by a revolving magnetic field. So, a step towards understanding revolving magnetic field is a pulsating magnetic field. A revolving magnetic field is produced by three-phase winding a pulsating magnetic field is produced by a single phase winding.

So, first let us take our time and slightly review this pulsating magnetic field produced by single phase winding and from then on build that understanding to a three-phase winding. This is many of you would have done a course on machines, but it would be good certainly to review this here now. So, now, you have this rotor it is its clear. So, its marked rotor and this innermost circle stands for the rotor. Then this small ring that you have is the air gap and what you have here is the stator and it is also written in a stator here. And on the inner periphery of the stator you have a few conductors here and you have few conductors there they have a placed kind of diametrically opposite to one another which is the case in most machines you know sometimes the return path is not exactly 180 degree away.

So, what are called as you know short pitch coils and usually use full pitch coils. So, you know you can say that there are conductors going here and conductors going here for the conductor here the return path is here for the conductor here the return path is here for the conductor here the return path is here that is how we assume. So, it is anyway those details are not very important for this purpose now. Let us say you have a coil what does this coil do it produces a magnetic field what is the direction of the magnetic field is an important question now well that depends on the direction of current flowing.

So, let us consider some direction of current flowing what would be the direction, let us say the current is a dot and the current here is cross. I hope most of you would be familiar with this dot and cross dot basically means current is flowing out of the sheet it is like an arrow coming out of this you just see a point. So, it is like current coming out here and cross is like arrow penetrating into the sheet. So, arrow means current is flowing into this now. So, if you do this what is going to be the direction of this well you can hold on to your you know right hand and you can look at this and now for current flowing out through the top this is going to be the direction of magnetic field.

And this is what you would call as the axis of this magnet what is going to happen is this is going to produce magnetic field which really goes around the this things and this is what is really the axis of this magnet. And when the current is constant this magnetic field is a constant magnetic field when this current varies with time certainly the magnetic field also varies with time, but the axis is unchanged this is the axis of this magnetic field. So, if the current alternates then the magnetic field also alternates sometimes you know it is in this direction sometimes it is in the other direction and you know the amplitude goes on changing its magnitude goes on changing. So, you might get a pulsating magnetic field.

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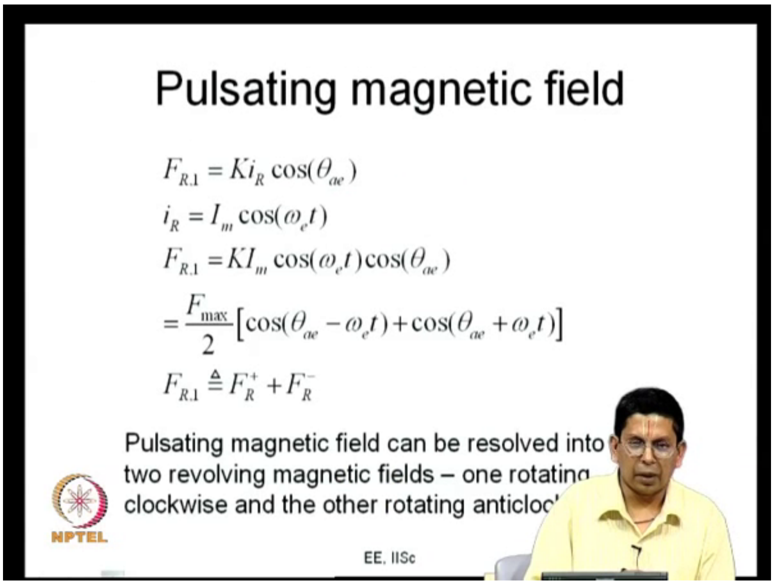

So, let us get a little clearer about the pulsating magnetic field let us do that mathematically. So, what are we going to do that to deal with it mathematically? So, let us say you have this mmf. So, what is this F? F stands for mmf, R stands for the winding its R phase winding we just considering a single phase and just let us call that simply R and 1 basically what our mmf is produced by a winding let us say like what you have here it would be an mm it would be some alternating waveform, it need not be sinusoidal it will not be sinusoidal. To make it closer to a sinusoid as why you do a few things you know you design this you distribute the windings and all that.

But you know, but still it is not sinusoidal, but most of the machine analysis you would ignore the special harmonic components and you will deal with fundamental component there now. So, when I say this a 1, the subscript 1 refers to the fundamental part of the mmf. So, this spatial harmonics of this mmf are ignored and you know this what you have is F R 1 that is the current passing through this R phase winding produces some mmf. So, the fundamental part of that mmf is what is taken as F R 1 now.

So, what is that? That mmf should obviously, be proportional to the current that is flowing through that right i R. So, it is i R there and k is some constant of proportionality and now this theta is the angle for example, if you consider here let me say you consider like this, this angle you can call as theta a e theta is because it is an angle a to say that its air gap and e is the electrical angle I am talking of the electrical angle here I am just considering it as a 2 pole machine for simplicity. So, you know you just call it take it as theta ae this is what you have now.

So, if the magnetic field is highest here as you go around that as you go around the strength of the magnetic field will go on reducing and the magnetic field will be 0 here and if you go here it will be negative peak here and then it will strength will reduce and will be 0 and then it will be back to positive. So, you will get a distribution which is alternative and which is actually you can presume it to be sinusoidal distribution now and therefore, you know what you will have is this is what you get here that is cos theta ae would be the mmf there at that angle theta ae now.

So, I am just not getting into the details of k that would for one thing it would involve the winding distribution factors etcetera. So, I am not getting into that. So, there is a constant k let us simply leave it at that F R 1 is K i R cos theta ae. Now what is that i R? That is not going to be a constant current you are going to be injecting an alternating current I m cos omega e t is there omega e t is the fundamental angular frequency, if you are talking of a fundamental frequency 20 or 50 herts this is 2 pi into 50 a 100 pi that is so radians per second that is omega e.

So, you and I m is the amplitude of that co sinusoidal component. So, you have i R is equal to I m cos omega e t therefore, you put one into the other you have F R 1 is equal to K I m cos omega e t cos theta ae therefore, you find that this mmf special mm it is a function of special angle theta ae and it is also a function of time t. So, it is a function of both space and time. Now, it is a product of 2 cosines you can always write it as sum of 2 cosines like this. So, it is F max by 2 this K I m let me give a term called F max for that let me just call that F max for convenience and this F max by 2, I can say that is it is cos theta ae minus omega e t which is basically the difference between these two arguments plus cos of theta ae plus omega e t which is the sum of these two that is what you have now.

So, I can write this as this F R 1 as 2 different components theta ae minus omega e t theta ae plus omega et. So, now, what really happens is this itself is one component which rotates this is another component which rotates now one rotates in the positive direction or the a anti clockwise direction the other one rotates in the negative direction or the clockwise direction. So, you can write this F R 1 as some F R plus and F R minus the clockwise component you can write it as F R plus and F R minus thus you can resolve a pulsating magnetic field into 2 I mean revolving magnetic fields as many of you might be aware. In fact, this is what is actually used in an index single phase induction motor also.

You produce two magnetic fields and you know this will not be able to start what you will do is you will do some trick you will slightly we can one magnetic field and strengthen the other one. So, strengthen one relative to the other and you will get the machine running. So, that is what you do in a single phase motor as many might be aware. So, you most of you should be aware of the fact that the pulsating magnetic field can be split into 2 revolving magnetic fields one rotating clockwise and there other rotating anticlockwise. So, and this is the math this behind what you can I mean behind this statement now right.

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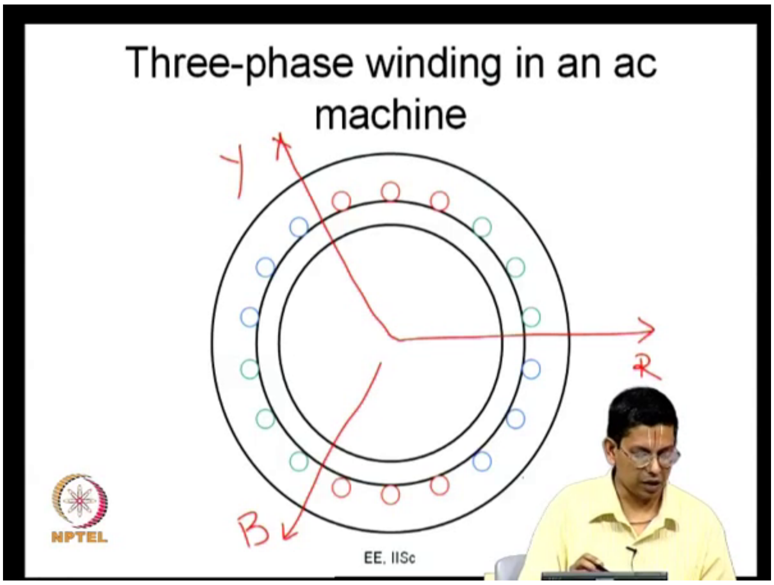

So, now instead of one winding what you have is you have three windings. So, you can say that this is the old R phase winding we are talking about, these are the corresponding written conductors now. Then about the Y, so for Y what I have do I have I have used green color here. So, this is the Y phase belt and the corresponding belt is Y here then the B phase I have used blue color here and this is one that is one now and if you look at the various axis what you can find is the R phase axis is like this, so you have these windings and you have these windings the R phase axis is like this as we saw earlier. This is the R phase axis.

How about Y phase axis? You have these windings in these phase belt here and Y phase is it goes like that this would be your Y phase axis in a machine and what it is the axis of the B phase winding, you have here and here this would be the B phase axis these are the axis of the three windings of this machine now.

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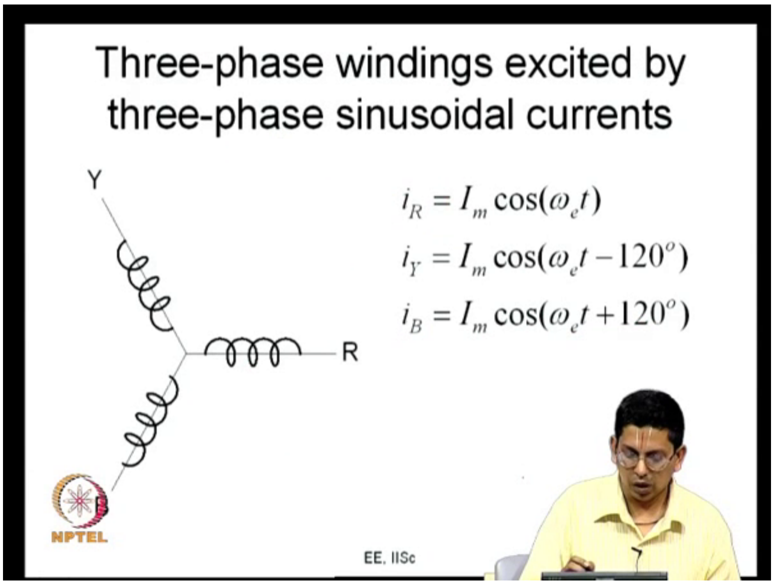

So, let us look at what happens when they are excited. These three-phases now I am just the same three-phase windings I am putting them in a different form I am just indicating the three axis here calling them R Y B as I said before and we pass certain current through this R phase winding what is that current i R is equal to I m cos omega et.

So, sinusoidal current I am taking that the value of sine they mean that is equal to 0 at time T equal to 0. So, I am writing this as a cosine function here i R is equal to I m cos omega e t if that is the current then the Y phase current is I m cos omega e t minus 120 and the current through the B phases I m cos omega e t plus 120 degree right. So, this is what we have. And from this we move on what we are doing is three-phase windings we are you know we are passing three-phase sinusoidal currents through that through three windings now.

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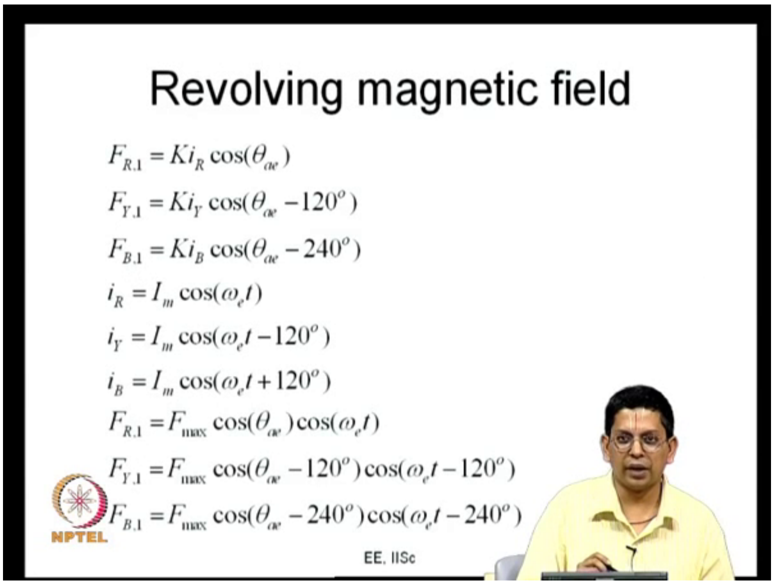

So, this produces a revolving magnetic field, how? First we had this earlier case F R 1 is K i R cos theta ae as we had said before, now if you had considered the Y phase axis Y phase mmf would also be the mmf produced by the Y phase winding will also be you know some proportional to the Y phase current. So, it is K i Y and cosine of now we are looking at the mmf at angle theta ae, but the Y phase itself is at an angle 120 degrees from the R phase and you know this is theta ae measured from the R phase axis. So, this is theta ae minus 120 cosine of that is what would be the component along Y phase axis. So, F Y 1 will be proportional to you know cos of theta ae minus 120.

Then you have the mmf at that angle produced by the B phase winding. So, the current flowing through the B phase winding is i B. So, this is F B 1 is equal to K i B cos of theta ae minus 240 that is because the B phase winding is 240 degrees away from the R phase winding and hence you have this now. And what are our i R, i Y and i B, I m cos omega e t, I m cos omega e t minus 120, I m cos omega e t plus 120 or this is also minus 240. So, these are the three-phase currents that you have now.

So, plug in i R i Y i B into the previous thing you get a similar expression as we found earlier each of them produce a pulsating torque I mean pulsating magnetic field as we discussed before. F R 1 is F max cos theta ae cos omega et, so you know F max is now this k into I m that is what is taken in that way. Similarly F Y 1 would be F max cos of theta ae minus 120 and i Y is substituted by you know for I mean i Y is proportional to cos omega e t minus 120. So, you get this cos omega e t minus 120. And if you take F B 1 that is F max times cos of theta ae minus 240 multiplied by cos of omega e t minus 240 you see, the difference between theta ae in omega e t and you again see the difference between this argument and this argument.

The difference between theta ae minus 120 and omega e t minus 120 is the same theta ae minus omega e t as before. Similarly theta ae minus 240 omega e t minus 240 is also the difference is still the same.

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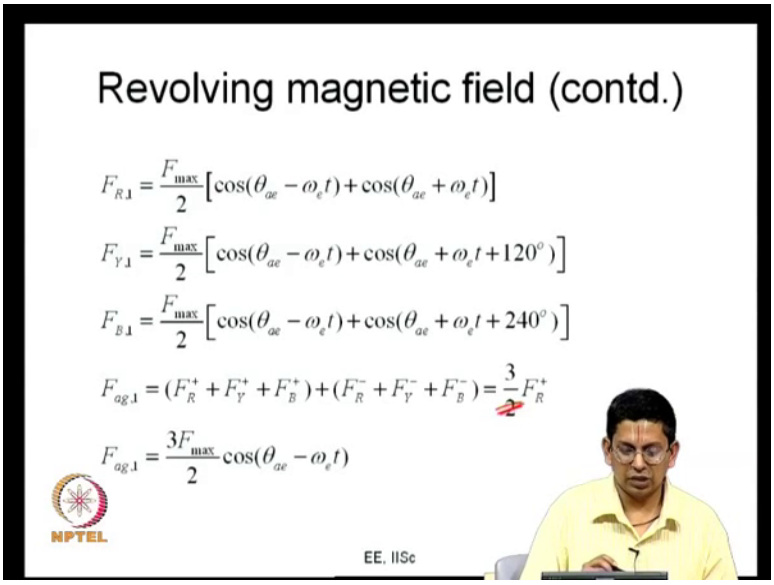

So, let us write down the equations further what you can say is F R 1, F Y 1, F B 1 they can also be written as F max by 2 multiplied by something sum of 2 cosine terms. The first term is cos of theta ae minus omega e t that is the case for all the three-phases. The second term would be cos of theta ae plus omega e t that is basically the sum of these two is basically the sum of these two terms and the sum of these 2 theta ae plus omega e t in the next cases it is theta ae plus omega e t minus 240 which is plus 120 and in the third case the sum of the 2 is theta ae plus omega e t you know plus 120. So, this would be plus 240 here. So, this is what is here now.

So, the R phase Y phase B phase all the three of them produce pulsating magnetic fields and each pulsating magnetic field can be split into one component revolving in the anticlockwise direction and another component revolving in the clockwise direction now. So, the ones revolving in the anticlockwise directions are adding up, now the ones revolving in the anticlockwise direction you add these three they will sum up to 0. So, here you can call this as a F R plus F Y plus and F B plus they add up they add up to what three into F max by 2 cos of theta ae minus omega e t.

On the other hand these three you know this is some cos of x this is x plus then there is a another 120 degree added here there is another 240 degree added. So, these three cosines add up to 0 F R minus plus F Y minus plus F B minus would add up to 0, means it essentially becomes equal to you know it should have been three times F R plus. So, you know just you make a change into that. So, this would be simply be three times F R plus is what would be your some of that and that is nothing, but F ag 1 is 3 by F max by 2 cos of theta ae minus omega et.

And now this you can see is a revolving magnetic field. So, thus you get a revolving magnetic field produced by three-phase windings excited with three-phase sinusoidal currents. So, this is a revolving magnetic field.

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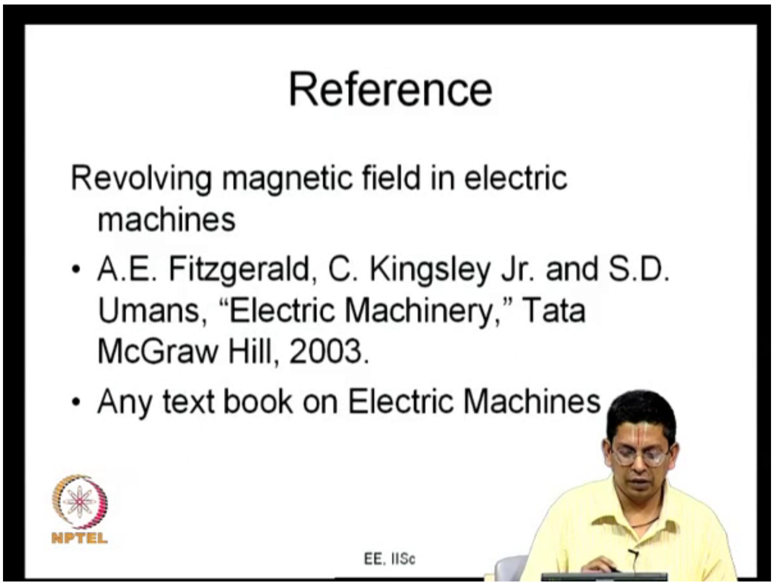

And now if you want more on this revolving magnetic field you can actually take any book on electric machines. So, one of the references I would suggest is A E FiT Zgerald Kingsley and Uman's "Electric Machinery," by Tata McGraw Hill 2003, well there are also different additions of this book available I mean there are also older additions this and there are several books in the subject of electric machinery.

So, in you can actually take any textbook on electric machines and that would be able to give you a very good picture on the revolving mmf to greater extremes and particularly the design of windings and many things if you want you can actually look at those books there now. Our main interest in saying this is you know revolving mmf is one example of a space vector. So, we just first trying to understand that that is what we are trying to do now.

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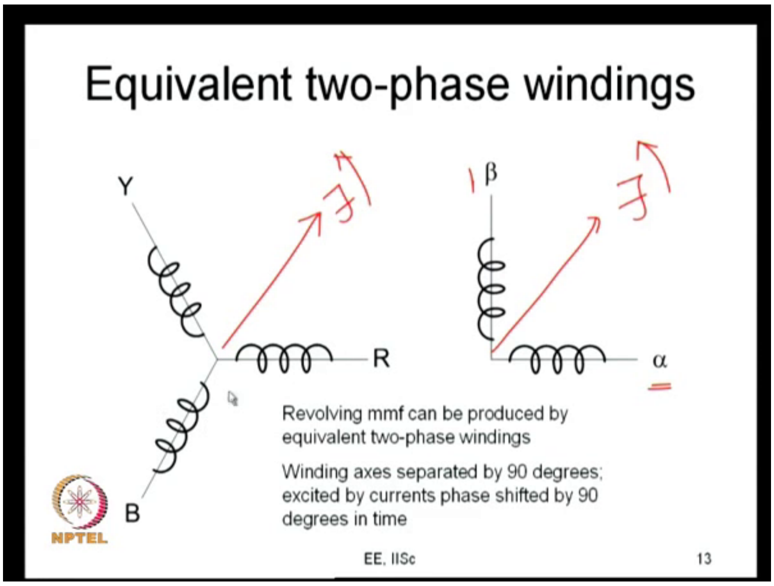

The next thing is from this we have to go into gradually understanding a space vector in a more general sense and coming up with what really would be space vector transformation now let just say one thing. Now you have three-phase windings and what does the three-phase winding do it essentially produces some mmf which rotates like this, this is what it does this is done by three-phase winding, now the question is can it not be done by 2 phase windings as shown here. That is I can also have a 2 phase winding you know that is when I can call as alpha the other one I can call as beta and this alpha and beta are 90 degrees away from one another they are separated in space by 90 degrees.

And I can also excite them with currents which are shifted 90 degrees in time and by doing this I might be able to produce the same kind of revolving mmf using this 2 phase winding. So, for three-phase winding you could consider an equivalent 2 phase winding which really can do this job of producing a revolving mmf.

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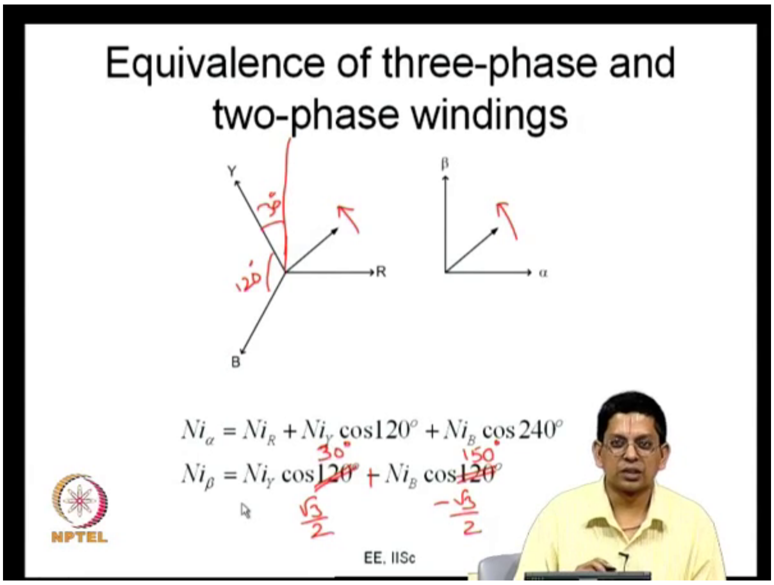

Let us go little further now I have shown the three-phase axis R Y B simply as a lines here and this is the revolving mmf that I had pointed out earlier. So, this is the revolving mmf. So, corresponding revolving mmf can be produced here.

Let us write down the equations if you really want to do that what should you be doing the two mmfs are supposed to be equal what is what are the two mmfs, one mmf is produced by this current i alpha excuse me. So, there is alpha winding and through this there is some current i alpha flowing and through the beta winding through which some i beta is flowing.

So, this N times i alpha should produce a component along the alpha direction and this i beta that N times i beta is a component along the beta direction now. So, N times i alpha should be equal to what now the R phase is the one that produces the mmf entirely along alpha axis. So, it is N times i R and a component of the mmf produced by Y phase is along the alpha axis that is Ni Y cos 120 again another component of the mmf produced by the B phase winding is along the R phase axis or the alpha axis. So, this is Ni B times cos 240. So, you need to satisfy one condition N i alpha equals Ni R plus Ni Y cos 120 plus Ni B cos 240 this is one of the conditions that you really have now right this is one condition and how about the current Ni beta.

So, Ni beta is the mmf along the beta direction now R phase winding does not produce any mmf along the beta axis it produces it is aligned along the F alpha axis therefore, it does not produce along that. So, what is the mmf produced by the this three-phase winding along the beta axis it is Ni Y cos of you know there is something you know this this should actually be Ni Y times you have cos of 30 degrees let me just change it here. So, if you look at it is going to be cos of 30 degrees some sorry there is a mistake here. So, it is cos of 30 degrees now.

And what is this again this is Ni B and that is going to be this B beta axis is 150 degrees away from here right. So, this is 30 degrees and you have another 120 degrees. So, this is going to be 150 degrees. So, this is going to be cos of 150 degrees. Now cos 30 degree is root 3 by 2 cos 150 degree is minus root 3 by 2 and this is actually it is a plus sign here you can say plus Ni B cos 150. So, this is what you really have.

So, you can produce whatever is the revolving mmf produced by a three-phase winding R Y B can also be produced by a 2 phase winding alpha and beta whose axes are separated by 90 degrees provided your i alpha and i beta satisfy the 2 equations given here.

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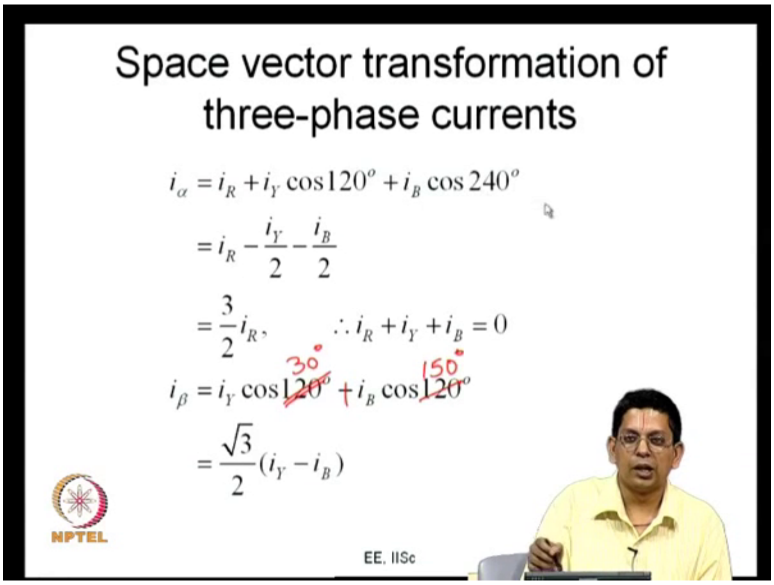

So, that is what I am writing down in the next slide, i alpha should be equal to i R times cos 120 plus i B times cos 240 and this should be i R minus i Y by 2 minus i B by 2 and further your i Y plus i B is equal to minus i R because your i R plus i Y plus i B is 0 therefore, i Y plus i B is minus i R therefore, you get this as 3 by 2 times i R.

So, this is what you have you have three by 2 times i R. So, i R plus i Y plus i B then how about your i beta it is a same mistake that has carried over here also. So, this is going to be cos 30 degree and you can say plus and this you can call this as cos 150 degree and this will be a root 3 by 2 times i Y minus i B. So, this you can call as. So, you you can you know the mmfs are equal that is the mmf produced by three-phase winding excited by currents i R i Y i B is equal to the mmf produced by 2 phase winding alpha and beta carrying currents i alpha and i beta, if i alpha is equal to 3 by 2 times i R and i beta is equal to root 3 by 2 times i Y minus i B and this is what is space vector transformation of three-phase currents that is what is reproduced here.

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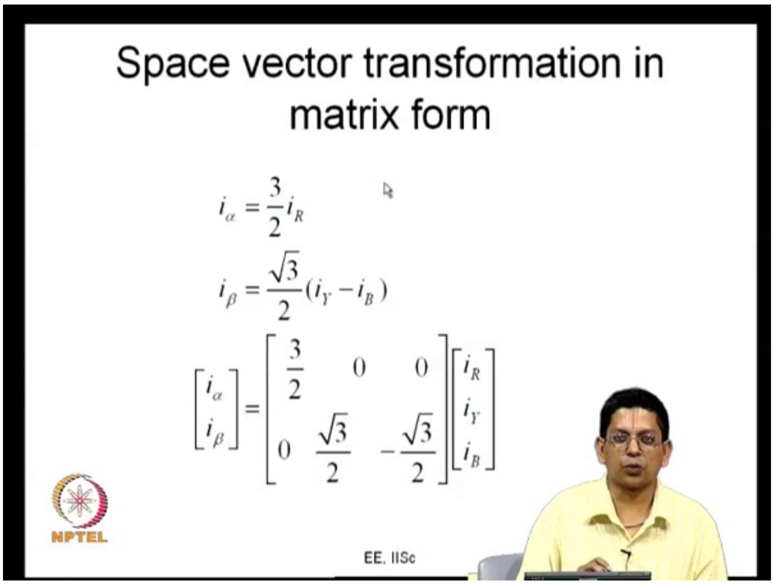

I alpha is equal to three by 2 times i R and i beta is equal to root three by 2 times i Y minus i B. So, we can just simply write this in a matrix form you can write i alpha i beta and this is 3 by 2 0 0 0, and the first 0 and 0 and first axis is 0 root 3 by 2 and minus root 3 by 2 you multiply by this by another matrix i R i Y i B. So, i alpha is three by 2 times i R i beta is equal to root 3 by 2 i Y minus root 3 by 2 i B. So, this is space vector transformation of three-phase currents I have just given this in a matrix form here, just for convenience sometimes you might find that convenient now.

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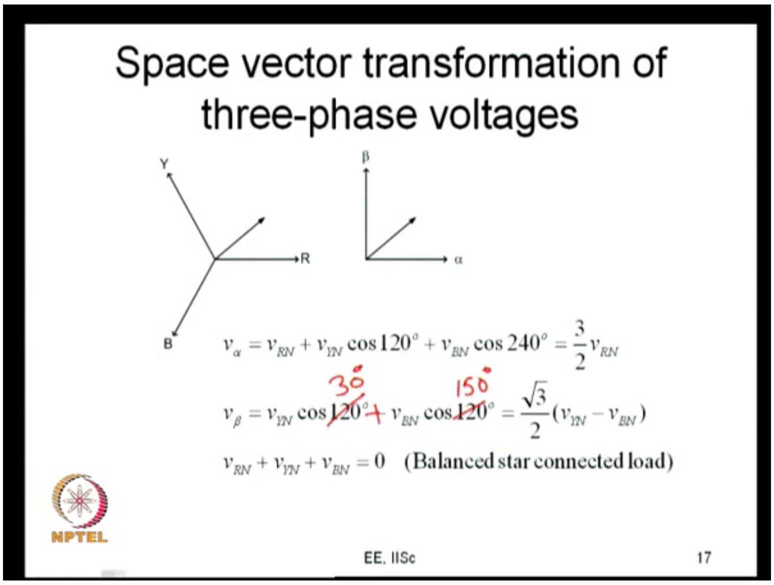

So, what is applicable for three-phase currents now we are looking at three-phase voltages. So, you can also extend this idea to three-phase voltage where also you have the same condition v RN plus v YN plus v BN is equal to 0 if you are considering a balance star connected load that is I am connect considering a star connected load. So, N is the load point neutral. So, now, v RN plus v YN plus v BN is equal to 0 into this condition what you can say is you go by your same thing as v alpha is equal to v RN plus v YN cos 120 plus v BN cos 240 and that goes to add up to root 3 by 2 vn the same mistake is perpetuated here also.

So, you can probably call this as cos 30 this is plus you can write and you can write this as cos of 150 degrees and you can see that this is going to be a root 3 by 2 times v YN minus v BN.

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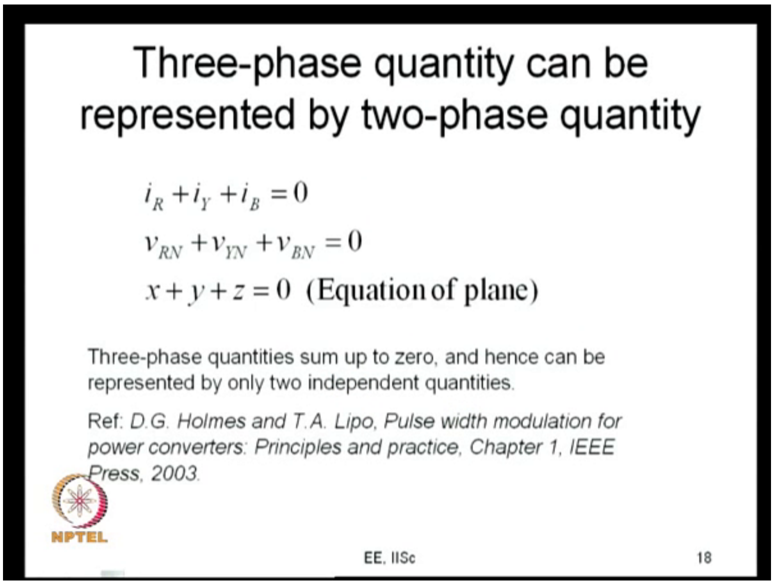

So, it is the same transformation that you are getting here also. So, you know that, so this is the transformation for three-phase voltages it its very similar to that v alpha is 3 by 2 v RN and v beta is equal to root 3 by 2 times v YN minus v BN. So, this is the three-phase transformation of I mean a transform a space vector transformation of three-phase voltages. So, three-phase voltage gives converted into an equivalent 2 phase voltage.

So, what you see here is you first have a situation let us say i R plus i Y plus i B is equal to 0 this is from a KCL. So, this is a three wire load and the sum of the three-phase currents is 0, similarly you can consider in the load neutral is not connected it is a three-phase three it is a three wire load and you have balance indeed you have v RN plus v YN plus v BN is equal to 0, this is the kind of three-phase voltage that you have now. So, both these equations if you see they are of the form x plus Y plus z is equal to 0 in the Cartesian coordinate system.

So, what does x plus Y plus z is equal to 0 represent in the Cartesian coordinate system it is the equation of a plane. So, it is a three-phase quantity, but you see that the three-phase quantities are they are not three independent quantities because they are equated you know there they have a relationship. So, a similarly here v RN plus v YN plus v BN that this three-phase quantity does not mean the three-phase they are three independent quantities they are actually they represent a plane and how many dimensions does a plain have only 2 dimensions. So, a 2 dimensional space can be spanned by simply 2 orthogonal vectors. So, a three-phase quantity like this can be represented by just a two-phase quantity i alpha and i beta.

Similarly, a three-phase quantity such as this can be represented by two-phase quantities namely v alpha and v beta that is the moral of this the three-phase quantities sum up to zero and they can be represented only I mean by only 2 independent quantities now. So, for further reference you can look at this book by Holmes and Lipo on Pulse width modulation for power converters principles and practices you can particularly look at chapter 1 of this book where there is a section on the concept of space vector where this is discussed further right.

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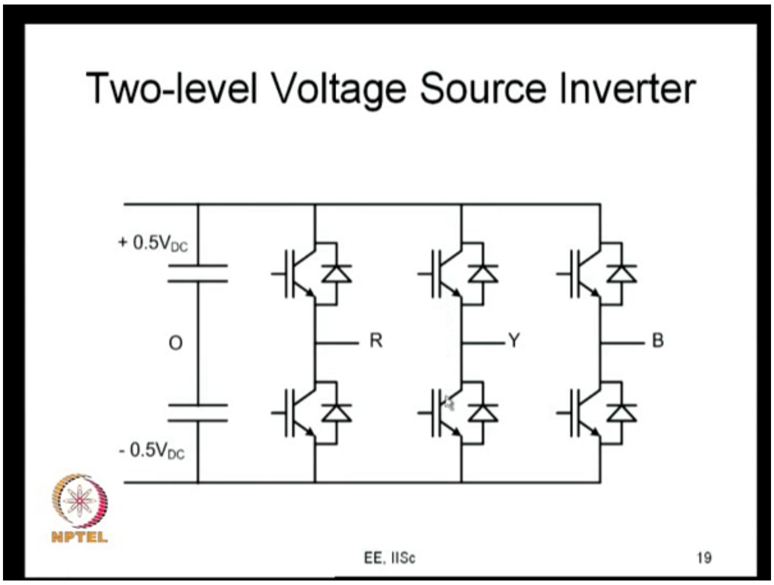

So, this is our idea of a space vector. So, now, we have come up with what is a space vector and you know a hard space vector transformation and we have got a transformations for three-phase voltages and three-phase current stuff. So, now, what we are going to look at is we are going to look at the specific case of a voltage source inverter. So, you have voltage source inverter it has a DC voltage V DC, it produces some AC side voltage. So, R Y B this AC side terminals these voltages can be measured with respect to some point or the other the preferred point is o this is what we said in the previous lectures also.

So, you can have V RO V YO and V BO that would be the output, what is the value of V RO, V YO, V BO, that would depend on the inverter state for example, if the R phase top device is on then V RO is plus V DC by 2 if R phase bottom device is R then V RO is minus V DC by 2, the same thing about Y phase and B phase. And you totally have 8 different combinations because this top or bottom can be ON, Y top or bottom can be ON, B top or bottom can be ON. So, 2 N multiplied by 2 multiplied by 2, so there are 8 possibilities which are called 8 different states of the inverter. And for this 8 different states of the inverter there are you know basically the inverter produces 8 sets of output voltages. How many sets of output voltages? 8 sets of output voltages it produces now. So, for these 8 sets of output voltages we have to find out what are the corresponding voltage vector. So, these are three-phase voltages right.

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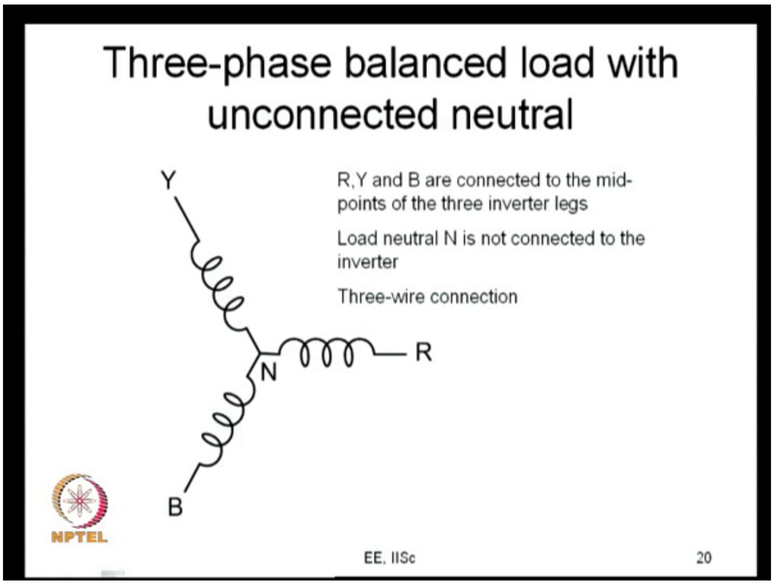

So, these three-phase voltages we should be able to transform into you know 2 phase equivalent 2 phase quantities let us come to that a little gradually. Let us first look at the load what is the nature of the load let us say the load is a star connected load, but it is a three wire load this R is connected to the midpoint of R phase, this Y is connected to the midpoint of Y phase leg, B is connected to the midpoint of B phase leg. So, R Y and B are connected to the midpoints of the three inverter legs. And this load neutral N is not connected anywhere particularly it is only R Y B are connected this is a three-phase three wire kind of a load that is what we are assuming is what is true in case of most induction motors. So, you go on with this.

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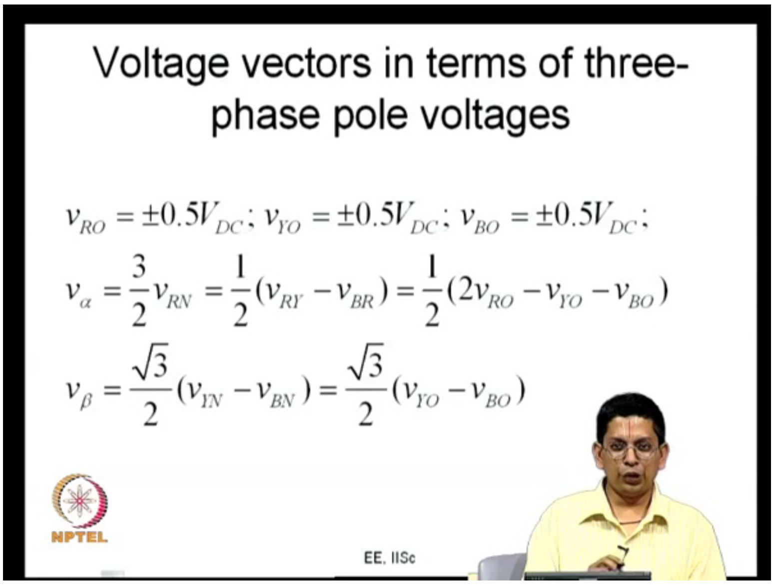

Now the load neutral N and the DC bus midpoint o are different that is what we need to bear in mind. So, your V RO is the pole voltage what we mean by the pole, by pole we mean the midpoint of a leg why because every leg is like a single pole double throw switch and its midpoint is the pole and on the 2 ends of the legs are the throws. So, we saw that it is a single pole double throw switch the midpoint is a pole. So, this is the midpoint potential of a leg measured with respect to the midpoint of the DC bus.

So, V RO can be plus or minus 0.5 V DC depends on whether the top device is on or bottom device is on in the R phase leg; this is what we just discussed a while back. Similarly V YO can be plus or minus 0.5 V DC depending on whether Y phase top devices on or Y phase bottom device is on, similarly B phase also if the B phase top device is on it is plus V DC 0.5 V DC bottom device is on it is minus 0.5 V DC. So, these are the three possible values of that and you have 8 sets of a three-phase outputs possible now.

Now, what is your space vector transformation? v alpha is equal to 3 by 2 times v RN this R N, this is the R phase voltage at the midpoint of the leg which is the load terminal, but measured with respect to N the load neutral and this is not the DC midpoint this N is different from O. So, you have v RN plus v YN plus v BN is 0, but your v RO plus v YO plus v BO is not equal to 0 that is an important point you have to bear in mind you take any of these values here say 0.5 V DC, 0.5 V DC, 0.5 V DC why how do they sum up they sum up to 1.5 V DC that is not equal to 0.

Similarly, you take any arbitrary values here here here they will never almost never sum up to 0 whereas, v RN plus v YN plus v BN always sums up to 0. So, now, the space vector transformation is v alpha is equal to 3 by 2 times v RN. What is this v RN? You can express this in terms of v RY and v BR v RN can be taken as one third of v RY minus v BR when you talk of balanced loads. So, you your 3 by 2 v RN becomes half of v RY minus v BR. So, once you have this v RY is v RO minus v YO.

Similarly, v BR is v BO minus v RO. So, if you do that if you plug in those you get it as half of 2 v RO minus v YO minus v BO this is your v alpha. So, this is now v alpha the original space vector transformation is in terms of v RN now we are trying to express that in terms of v RO v YO v BO. Now you see that your v alpha does not depend on v RO alone it depends on v YO and v BO also. Then how about v beta? v beta is root three by 2 times v YN minus v BN and v YN minus v BN is same as v YO minus v BO, it is basically the difference between the potential at Y the midpoint of Y phase leg and the midpoint of B phase leg you know whether the potential is measured with respect to N or o the difference is the same. So, it is v YO minus v BO.

So, here you have v beta this first part of the space vector transformation here you know the voltages which are the load phase to neutral voltages have been converted into the pole voltages measured with respect to the DC midpoints. So, now, you have v alpha and v beta available in terms of v RO, v YO, v BO. So, that is what we have now. So, for the 8 inverter states you know the values of v RO, v YO, v BO you can plug them in to get your v alpha again you know the values of v YO and v BO you can plug them in to get your v beta values.

Thus for all the 8 inverter states you can get the corresponding voltage vectors that is an exercise which we would do you can actually plug in the numbers and go and play income things related now.

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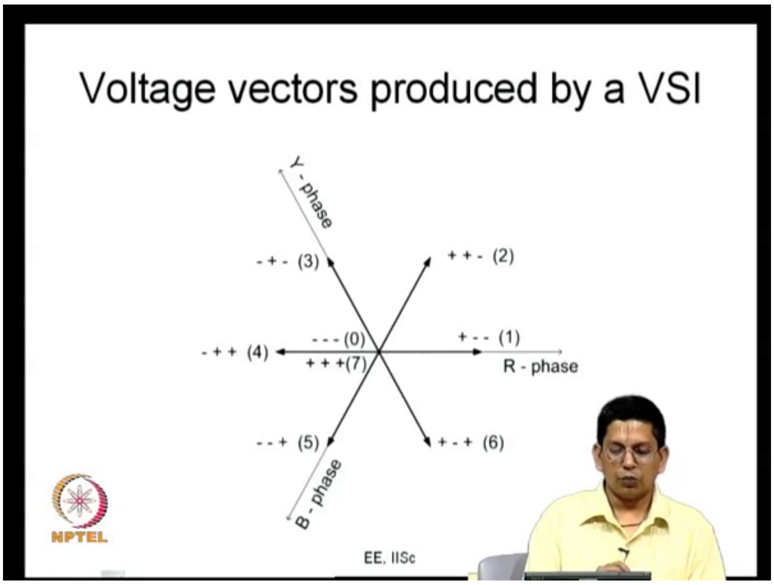

So, what I would like to do is I do not want to plug in the values here and show what is what I can very easily tell you that first consider like something like plus minus minus. So, this is v RO that what I mean by plus minus minus this is an inverter state which represents to a situation where the R phase top leg is on and Y phase bottom leg is on and the B phase bottom leg is on.

So, there are three signs given they correspond to R Y and B respectively in that order and R is positive meaning the top device is on Y is negative meaning the Y bottom device is on B is negative meaning B bottom device is on. So, v RO is plus V DC by 2 and v YO and v BO are equal to minus V DC by 2 since v YO and v BO are equal to minus V DC by 2 you can see from the previous slide that v beta is 0. So, it is only v alpha and you have v RO is equal to V DC by 2 therefore, this becomes V DC here it is all minus V DC by 2 and minus V DC by 2. So, it adds to 2 V DC and half of 2 V DC simply equal to V DC. So, it becomes a vector like this and this vector is as got a magnitude V DC I have not indicated this here, but I have indicated this in a later slide.

So, this inverter state plus minus minus leads to a voltage vector of a magnitude V DC at an angle 0 that is v alpha is equal to V DC and v beta is equal to 0 is what it leads to. Similarly you can do for all the other states one after the other now let us understand this a little better and then now thereby we can see things ourselves. So, what do we see here we see that when the R phase alone is connected to the positive bus and Y phase and B phase are connected to the negative bus you produce a voltage vector of magnitude V DC along the R phase axis.

Now, everything is symmetric the machine is symmetric you know the inverter is all symmetric now. So, instead of R phase alone connected to positive you have Y phase alone connected to positive on the other two are negative see that is the case here your Y phase alone is positive and the other 2 are connected to negative. So, what you should say by symmetry you should be able to say now that you know the resultant voltage vector will be along the Y phase axis just as it was along the R phase axis it should be along Y phase axis and the magnitude of the vector should be same like V DC for this transformation that we are considering here. So, you will get this now.

Similarly you can look at a situation where B phase alone is connected to the positive bus and Y R and Y are connected to the negative bus this will produce a vector of magnitude V DC aligned along the B phase axis. So, that is how you can say that these are easy you can plug in the values you know we had the expressions of v alpha beta in terms of v RO, v YO, v BO you can plug them in and come up to this now I am just trying to give a feel for that excuse me.

Now, if this is your v RO, v YO, v BO then you invert you say plus minus minus you go to minus plus plus excuse me yeah. So, instead of plus minus minus you have minus plus plus where all the three are inverted. So, v RO, v YO and v BO are inverted that naturally means v alpha and v beta should be inverted in sign it is going to produce a vector aligned the negative R phase axis. Here the R phase terminal is alone connected to the negative DC bus and Y and B are connected to the positive DC bus and the resultant vector is along the negative R phase axis the same arguments are valid for Y phase and B phase.

So, you can consider there are 6 possibilities for each of this possibility you will have a vector aligned along the positive R phase axis or negative R phase axis or positive Y phase axis and negative Y phase axis and so on one or the other axises you get all the vectors are of magnitude V DC. Now this is accounted for 6, there are two other states which are indicated here minus minus minus and plus plus plus minus minus minus means all the bottom devices are on; that means, the load is shorted here plus plus plus means all the bottom devices on the load is shorted we have discussed this earlier.

We know that this is a redundancy that is there in a voltage source inverter because of both of this what happens the resultant voltage vector is equal to 0, you can say that here all v RO, v YO and v BO are equal and you know the line to line voltage is v RY, v YB, v BR are all 0 therefore, the line to neutral voltage v RN v YN v BN are also 0 v alpha and v beta are 0. These two are called 0 states because there is no transfer of power during these 0 states between the DC bus and the AC side. So, these are called 0 states. The other 6 states are called active states because during this process there is transfer of power between the DC side and the AC side, whether the power flows in this direction at that direction depends on the operating condition.

We will look at that at a later stage, but there is a power flow whenever active states are applied whenever the 0 states are applied the load is shorted its simply freewheeling you know the DC bus is opened out and the load circuit is opened out. So, there is no power really transferring between the two now. So, that is what is all. So there are 8 different inverter states there are 8 sets of output voltages, 6 sets of output voltages produce 6 voltage vectors of magnitude V DC we call them as active vectors and here we can call them as a 0 vector other 2 states produces 0 vector now, this is what we have and go here now you go to the next level.

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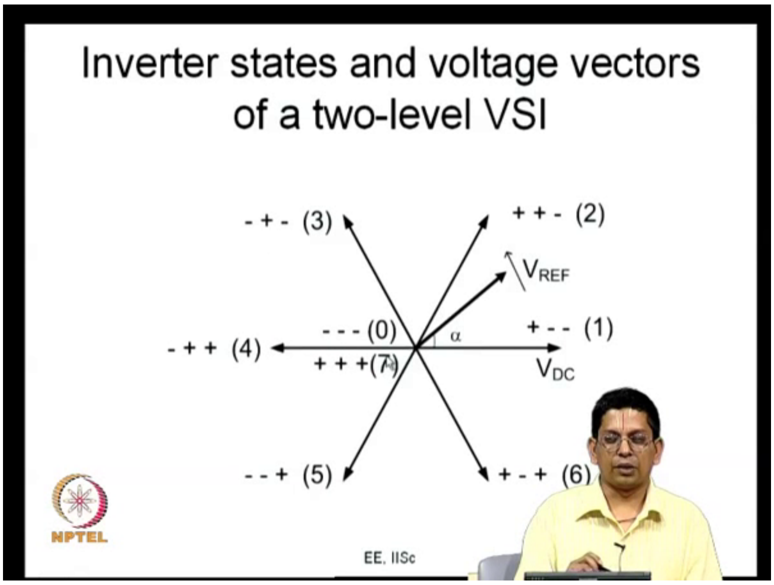

So, what do you have here? 1 2 3 4 5 6 I am just reproduce the same thing now. So, I indicate with this magnitude V DC here just for clarity. So, this is our for this space vector transformation that we have defined you come to this now. Sometimes you should remember that there are slightly different transformations are used basically they use a scale factor you can use some scale factor some you know root of 2 by 3 certain other factors can be used for some reasons and this may not be V DC it may be some k times V DC. So, that depends on the scale factor used.

But for the transformation that we defined a while back these are all going to be V DC now. So, now, what is going to happen? If this is only to show that there are 8 of them like this and the next thing is about the reference vector. We will come to this anyway little later. What we do here in space vector based PWM is we use a revolving voltage vector as the reference, what we do in sine triangle PWM we use three-phase sinusoidal signals. If you are transform three-phase sinusoidal signals into the space vector domain it is basically a revolving voltage vector and therefore, you have a revolving voltage vector here which is shown like this during some time it will be here later on it will cross here and go into this it will go here and it will circulate around all this now is what we will do ok.

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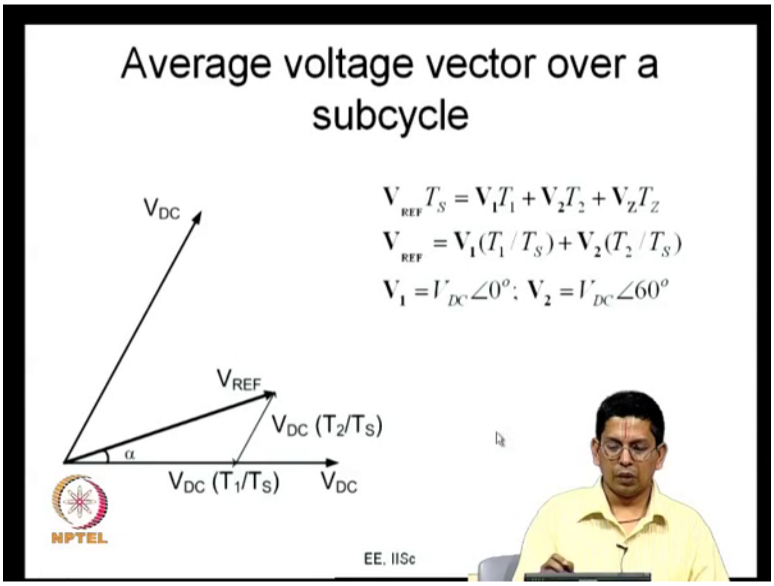

So, now let us say you consider a particular instant or you know in interval a small interval of time T S during which the vector is here now can we produce this vector is the question now what we did average pole voltage is recall the concept of average pole voltage your pole voltage can be as high as plus V DC by 2 or minus V DC by 2 and therefore, you can produce a an average pole voltage anywhere between plus V DC by minus V DC by 2 the same way here is the concept of average voltage vector over a sub cycle T S now. What you try doing here is now this is what I have to produce a V REF vector.

So, if I apply this active vector 1 for example, for certain interval of time T 1 of the sub cycle time T S, so it is applied for a fraction T 1 upon T S, then I am applying this vector which I can call as V 2 vector for some fraction of time T 2 upon T S now and for the remaining time I am applying the null vector. So, what is going to happen is I am going to get a vector like this another component this is parallel to V 1 vector and it is T 1 by T S times V 1 vector and this component is parallel to V 2 vector it is equal to T 2 by T S times V 2 vector, these two are added up to get your V reference vector this is the average voltage vector now.

Thus you can produce an average voltage vector whose tip can lie anywhere within this triangle. So, you consider that these are two sides of triangle you join these two tips by another straight line, so that will give you a triangle. So, you can produce any v reference any average voltage vector whose tip is anywhere within that that is what is possible by averaging. So, earlier you are doing an averaging of the pole voltages 2 instantaneous values plus V DC by 2 and minus V DC by 2. Here now we are not looking at three individual phases we are looking in them as collectively as three all the three together as inverter states or the voltage vectors.

Now, we use these two voltage vectors on the null vector to produce any average voltage vector within this triangle that is what is written here now if you want a vector v ref T S that v ref into T S V REF vector into T S will be equal to V 1 vector into T 1 plus V 2 vector into T 2 plus V Z vector times T Z. So, this is what is called as volt second balance now. This is the reference volt seconds this is the applied volt seconds over a sub cycle reference volt second should equal the applied volt seconds and this is given in an alternate form this V Z vector is a null vector. So, this term is a null vector term.

So, effectively you have v reflector is equal to V 1 times T 1 plus T S plus V 2 vector times T 2 plus T S that is what is illustrated in the figure now. And what is V 1 vector? V 1 vector is this vector which is V DC angle 0 and what is V 2 vector is the same magnitude V DC, but the angle is 60 degrees now. So, this is what you have as an average voltage vector in a sub cycle that is by applying this vector for T 1 this vector for T 2 and this null vector for T Z you can produce some reference vector like this that is what we are trying to say right.

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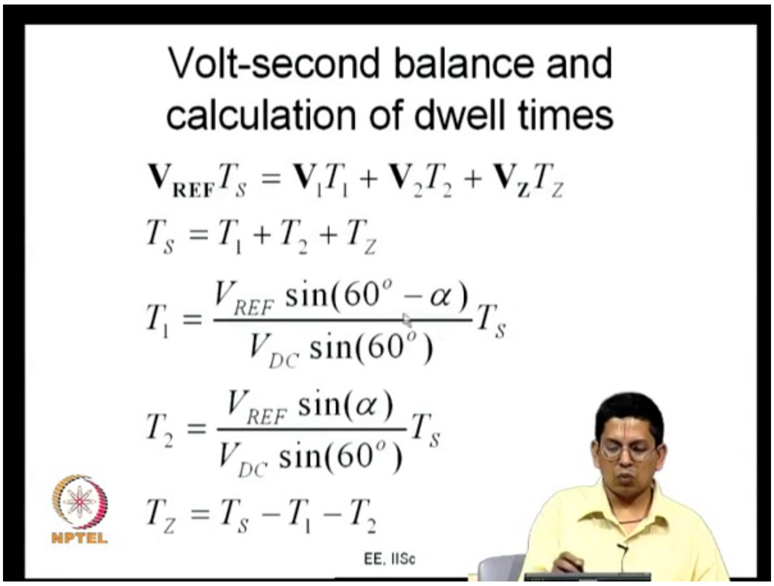

So, this is what the volt second balance equation is given here. So, for a particular value of V reference vector which is basically some v ref an angle alpha which basically is a magnitude V REF and angle alpha what should be the values of T 1 T 2 T Z you can just do this you can decompose them along the alpha axis and beta axis and you can derive these expressions. What we can call as dwell time T 1 is the time for which vector V 1 should be applied, T 2 is the time for which vector V 2 it should be applied, T Z is the time for which the null vector should be applied now.

So, this T 1 T 2 and T Z T 1 and T 2 can be calculated like this they depend upon the magnitude v ref and angle alpha of the reference vector. So, from there you can just derive this that is from the previous page you know you can for example, these are is a vector equation you can decompose it along the alpha axis and beta axis for example, and you can come up with these values for T 1 and T 2 you will get that T 1 is V REF sin 60 minus alpha times T S divided by V DC sin 60.

Similarly, you will get T 2 as V REF times sin alpha multiplied by T S divided by V DC time V DC into 60 degrees and T Z is the remaining part that is T S minus T 1 minus T 2 for the remaining portion of the time I you apply the null vector T Z. So, this is how you calculate the dwell times. We will be doing more on this in the next few lectures this is just to show that an average voltage vector can be synthesized using these two active vectors and one null vector now.

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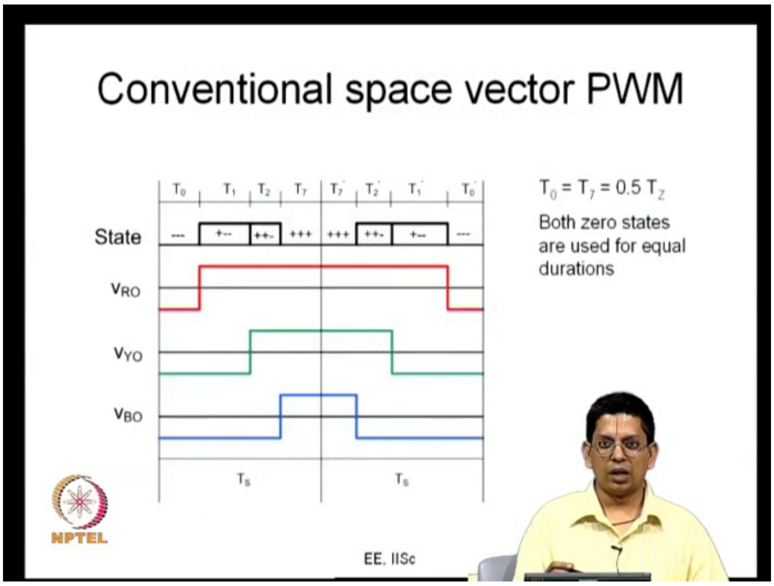

So, what we do in conventional space vector PWM is we try to apply these vectors now for example, this is a null vector and this is a active vector 1, this is a active vector 2 and again this is null vector. How exactly we apply here we go to the inverter states minus minus minus and plus minus minus and so on and do this here now if you start here you can apply this minus minus minus to produce null vector and from here you can switch the R phase and go here and that would be plus minus minus and then from there you can switch Y phase and go here to go to plus plus minus and from there you can switch B phase and go to plus plus plus. So, that is what is shown here you apply the different vectors.

And here you are applying the same vectors in the reverse direction plus plus plus all all the legs are positive then here B phase is switched then Y phase is switched and finally, R phase is switched you can see that you know R phase switches like this Y phase switches like this and B phase switches like this now. So, what we have is you have two active vectors applied in the middle and the 2 zero states are applied at the end. So, the 2 zero states in conventional space vector PWM what is done is this is 2 zero states what is called as T is 0 and 7 they are applied for a duration is T 0 and T 7 T 0 equals T 7 is equal to 0.5 T Z they have to be totally applied for a duration equal to T Z.

So, you are dividing that equally between the 2 and that is what is done in what is called as conventional space vector PWM which I mentioned while earlier that it is the most popular method. We will be discussing this in much greater detail in the following lecture. So, what you are doing is both the zero states are you applied for equal durations of time that is kind of conventional wisdom. You can apply a null vector I using either this option or that option by keeping all the bottom devices are on or keeping all the top device on what do I do, do it both equally you know apply this for half the time apply that for half the time is the is a simple logic and you just do this and that is conventional space vector PWM as I mentioned we will discuss more of this later now.

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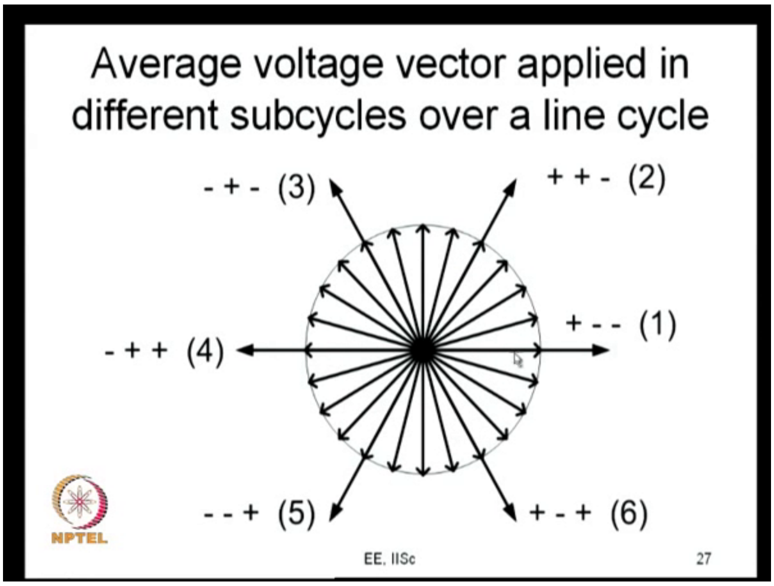

So, what you can do is you can apply an average voltage vector like this for some time and next you can go out to the next value if you have applied this over some T S now and over the next interval T is excuse me you can apply another voltage vector like this. Then over the next sub interval a sub cycle you can apply this now. Here the angles I have shown is 15 degrees you know between these 2 I am taking my sub cycle time to be equivalent to 15 degrees or omega T S where omega is the fundamental angular frequency multiplied by T S I am taking it as 15 degrees here which is actually big in many practical inverters of you know the power level of kilowatts or so this sub cycle duration could be much lower than that it can be just a few degrees now.

It is this is taken for an illustrative purpose now. So, what I am doing is I am let us say I am applying this average vector for 15 degrees and after that 15 the next 15 degree interval I am applying this average voltage vector, how I am doing by applying vector V 1 vector V 2 a null vector for appropriate durations of time T 1, T 2 and T Z. The next time I will go here, so what do I do here? I will apply the same V 1, V 2 and null vector for different durations of times such that I produce this average. The next sub cycle I will apply T 1 T 2 T Z for another you know appropriate set of appropriate values that I produce this average vector, thus in every sub cycle I produce an average vector like shown here like shown here around that now.

So, what I am doing is I am going close to a rotating voltage vector, what am I supposed to produce? I am supposed to produce three-phase sinusoidal voltages and in space vector domain three-phase sinusoidal voltage is nothing, but a revolving voltage vector I am supposed to produce a revolving voltage vector. So, that is what I am doing. I am producing a vector like this which stays there and after short interval it just moves by a small step it stays there after a short interval it moves by that. So, that the jump you mean the step by which it moves is equal to the you know is equivalent to the interval of time for which it stays there.

So, actually what is happening is circular motion is approximated by you know sampled circular motion just as you know sign wave is approximated by a sampled sign wave and a sampled sign wave is not very different sampled and held sign wave you know that is not very different from an actual sign wave provided the sampling intervals are close enough is the similarly these you provided these you know the sub cycle durations are substantially smaller than the fundamental cycle. You know this is a very valid approximation of a revolving voltage vector, that is you can thus you can produce a revolving voltage vector using a three-phase inverter and this can be applied to a three-phase induction machine.

So, you know when you are able to produce a three-phase voltage vector it simply means that you are able to produce three-phase sinusoidal voltages. So, this is about space vector PWM. So, what are we doing, what are the essential differences now? We are using this idea of space vector and what is the space vector three-phase quantities can be transformed into two-phase quantities you can. So, now, you can see this is only a 2 phase quantity now and there is a three-phase sign and a three-phase sign becomes a revolving voltage vector of constant magnitude and a uniform angular velocity. So, if its uniform angular velocity you sample it at equal intervals of time you are going to get samples at equal intervals as shown in this particular figure. So, this is what you go by you know doing this.

So, now, what you can do is now this is how can you produce this. So, the circular motion is being approximated by like this a vector which is stationary here for certain amount of in for an interval of time omega T S. After that it jumps by omega T S and it stays there for that and it goes on doing like this now. And how can you produce this vector for example? This vector can be produced by time averaging of this V 1 vector V 2 vector and null vector and if you are talking about this vector for example, this can be produced by excuse me, time averaging of this vector this vector and the null vector.

If you are talking about this vector it can be produced by time averaging of vector V 4 vector V 5 and null vector thus you know wherever you are you can call all these as 6 sectors and in each sector you use two of the neighboring active vectors on the null vector to produce the average voltage vector.

I am going to deal with more of space vector base PWM particularly conventional space vector PWM in the next class. This is only to give an idea of what you could call as you know the concept of space vector and space vector transformation and some introduction into space vector based PWM and a more detailed discussion on that would is reserved for next class.

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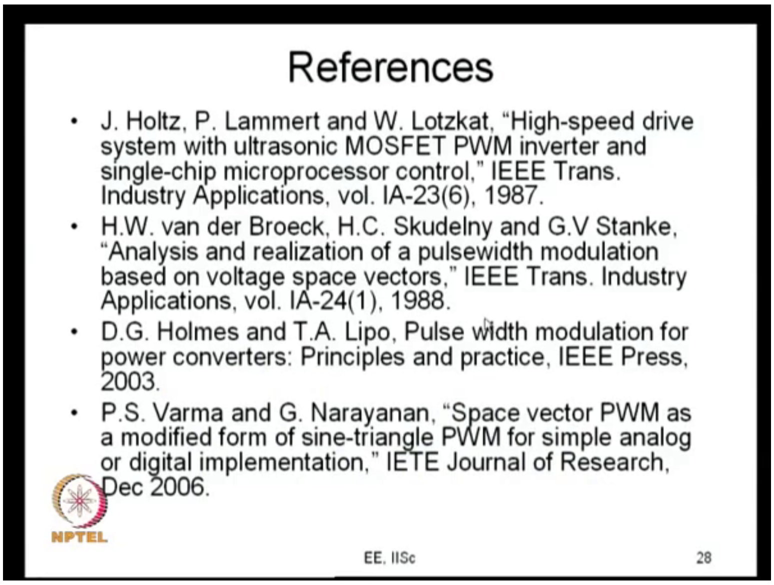

Here I have a few references for you please have a look at this. The first two references one is Holtl you know (Refer Time: 55:35) that is and on the other one by you know van der Broeck, Skudelny and Stanke these two I would call as you know probably the seminal work in the area of a space vector based PWM. So, there are other there were a few works in the middle mid 80s these two I would say are probably in the second half of a 80s. So, these two papers reported a space vector base PWM you know they are early papers in this area of what you would call as space vector PWM.

So, for further you know your knowledge and understanding you can probably look at these things. And another one is you know this book this D G Homes and T A Lipo is a book it covers many things about pulsewidth modulation and all that. So, it also takes about talks about space vector concept transformation and a space vector base PWM. So, that is a book which can also be used as a reference now. Then the last one indicated here is Varma and Narayanan that is space vector PWM has a modified form of sine-triangle PWM for simple analog or digital implementation. So, this is published in IETE institution of electronic and telecommunication engineer's journal of research. This is a tutorial paper this is particularly written from the point of view of understanding them easily.

So, these are some references I would like to give and I would give you more and more references as I would go on relevant references and you know I hope that you found this lecture interesting and we would continue this in the next lecture. And we will discuss conventional space vector based PWM in our next lecture.

Thank you very much.