Pulsewidth Modulation for Power Electronic Converters

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Lecture - 13
Selective harmonic elimination

Welcome back to this lecture series, on Pulsewidth Modulation for Power Electronic Converters.

So today, we will be looking at, this is the thirteenth lecture and we will be looking at selective harmonic elimination. The last few lectures, we have been focusing on, you know low frequency pulsewidth modulation.

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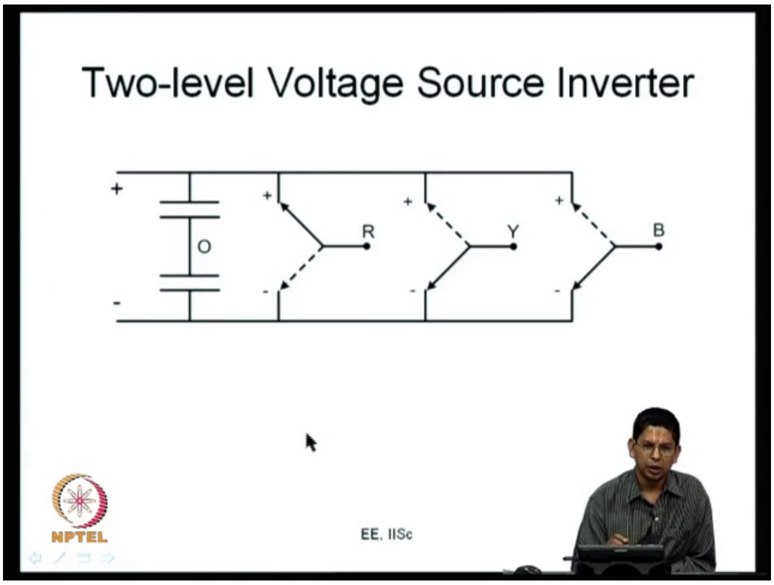

That is we have a voltage source inverter like this, this inverter, it switches at certain frequency. So, at what frequency, that is by enlarge, decided by the power rating and the nature of the devices that you have here. So, sometimes, these devices can switch at very high frequencies. If you consider certain MOSFET inverters, they might probably be able to switch at several 100 of kilohertz. So, it depends on what the devices and in case of IGBT inverters, you will find that many times, this can probably switch at the order of kilohertz, typically IGBT switch up to 20 kilohertz.

So, you may find about 5 or 10 or 20 kilo hertz kind of this thing, these all depends on, you know the choice of device depends on the application, the rating etcetera. So, you come to still higher power converters. You will use, you know, you will switch the devices at lower frequencies, as the rating goes on increasing. The switching frequency is low and at very high power levels, you will tend to switch at fairly low frequencies that are because every time the switch turns on turns off every time it turns on, turns off, there is certain amount of energy that is lost. So, the total switching loss is the product of this energy, loss per switching cycle, multiplied by the number of switching cycles per second. Therefore, to keep the switching loss low, within permissible limits, if your energy loss is very high, you have to keep your switching frequency low and so, therefore, at high power levels.

when the voltages and the currents that are being switched, at very high, you tend to switch at very low frequencies and so, we have been looking at, how to modulate inverters, at low frequencies, that is what we have been looking at on the last couple of lectures how do we go about doing it; so one of these methods that we have come upon. Now, is selective harmonic elimination.

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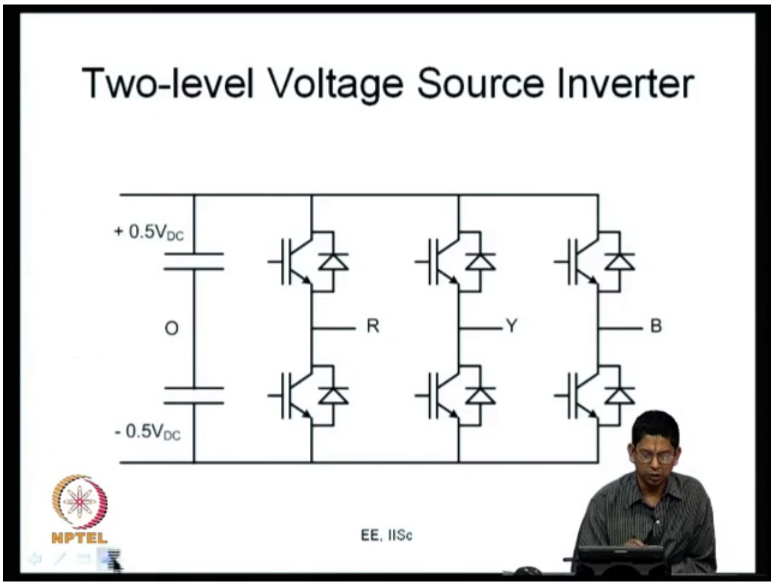

So, here we have looking at, and I have indicated it as, IGBT inverters as I mentioned, this could be any kind, but at a very higher power levels. It could certainly be high voltage and the high power IGBTs. So, there are IGBTs, which can handle, voltages of the order, of 3,000 volts or and currents are also going into 1,000 amperes and so on. It is possible these days to realize IGBT inverters up to power levels of even a few mega watts or so in the power level is high typically the switching frequency tends to be low.

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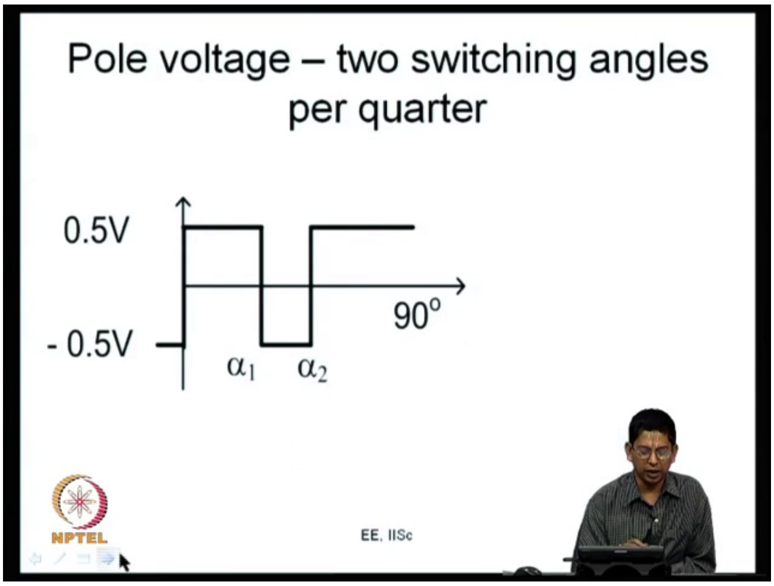

So, now it is, you know, we are even more interested in learning even, rather than you know it is the application, where the low switching frequency is useful, for high power drives, what we are interested in low switching frequency, gives us, a nice opportunity to learn, pulsewidth modulation. So, that is why, we have been focusing on, these two switching angle problem, that is the pole voltage, here has only two switching angles, here alpha 1 and alpha 2, excuse me, alpha 1 and alpha 2, that I have indicated here, the waveform is quarter or a symmetric.

So, the next quarter is a mirror reflection of this then it is half wave symmetric. So, it and then, there are three phase symmetries. So, it is only two switching angles, you need at least one switching angle to control the fundamental voltage, because we have two switching angles, it is possible for you, to influence the harmonics, to certain extent. So, that is what we are looking at now.

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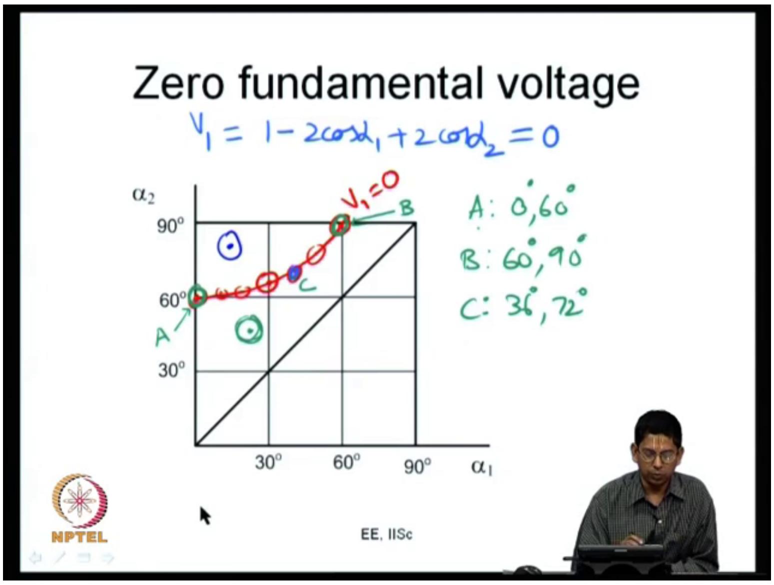

How do you go about influencing the harmonics now? So, we looked at the fundamental voltage control. So, what we saw was, is the switching angle problem, you can look at this waveform, as a point, on this plane, alpha 2 versus alpha 1, but alpha 1 is one of the switching angles, and alpha 2 is the other switching angle. So, you see these points, how they go about. So, we saw that in any point here like those are indicated, here they are in any point, there is, it represents a waveform of a particular value of alpha 1 and a particular value alpha 2 and this red curve is the 0 fundamental voltage curve.

So, along that, if you choose alpha 1 and alpha 2, the fundamental voltage is 0. So, below that, that is as indicated by this green point, here it is, it has some fundamental voltage and if you look at blue, that also I have some fundamental voltage and these two have a, you know phase reversal between these two. So, here the phase will be something here, the phase will be and a 180 degrees later. Normally, we will focus on this range, if you go beyond this, the phase is going to be reversed up, it is going to be reversed, it is going to be 180 degrees, different from what we would want right.

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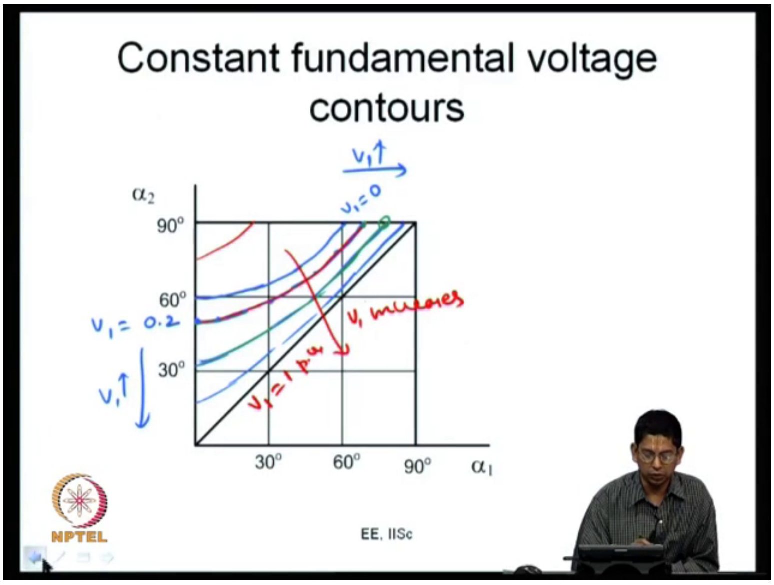

So, you can look at all this, there are several of these curves now. So, here that v 1 is equal to 0 curve, is shown by this blue line, then you have like 0.2, is indicated here as you go about increasing v 1, like this your curves keep shifting, like this.

And finally this curve is your v 1 is equal to 1 per unit that is square wave. In square wave alpha 1 is equal to alpha 2, twist two switching angles come together. So, that is basically, there is no switching in a, in any quarter, there are only switching, is at 0 on 180 degrees. Here, that is, you go back to this waveform, if there is only switching at 0, these two come together.

So, there will be only other switching at 180 degree. So, a phase will switch only twice in a cycle. So, it is in this square wave operation, the whole pole voltage will be a square wave now. So, this is how the fundamental voltage goes on and we also saw that you know, you can think of a PWM technique, as a locus of a point which starts from one point, on this curve v 1 is equal to 0 and most towards v 1 is equal to 1 or some v 1 is equal to maximum that we would want.

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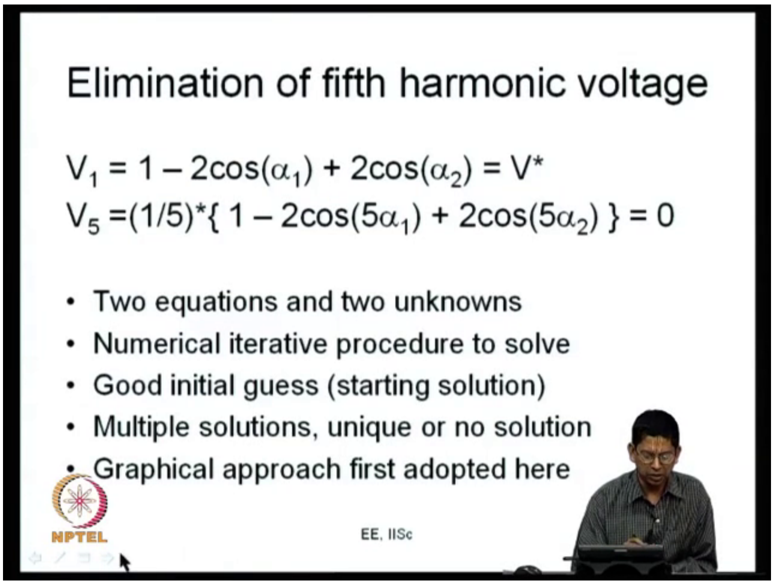

So, we are now looking at specifically, the problem of eliminating, a particular harmonic voltage. Since, we have two switching angles, there are two independent variables available, we can try to control the fundamental voltage and also try to eliminate one of the harmonics. In this case, we will try and eliminate the fifth harmonic, as I mentioned in the last lecture, fifth harmonic is what is the lowest harmonic and that is, you know, that is what phases, the lowest amount of leakage reactance, among the harmonic voltages.

Seventh harmonic will have. So, you know will phase, much you know higher than, that the harmonic voltages. They see the machine as it is leakage inductance . So, the reactance seen by the fifth harmonic voltages, is the 5 omega l and the reactance seen by the seventh harmonic, is 7 omega l. So, the reactance seen by something like the 31st harmonic is 31 omega l. So, the higher harmonics see a greater amount of harmonic reactance and therefore, their corresponding harmonic current amplitudes are going to be lower. So, the one that is going to cause maximum problem, is the lowest harmonic content, that is fifth, we will not have third harmonic here, because that will get cancelled you know. So, we have seen that you know.

You have two equations and two unknowns and therefore, you can go about solving them and you have to solve them, you cannot solve straight away, you need a numerical iterative procedures and to solve that you need a starting solution. If you are able to make a good initial, guess it can converge much faster and you know sometimes, it may lead to a particular solution, a sometimes it may lead to another solution, depending on what is your starting guess. So, sometimes there could be a multiple solutions as indicated here, but sometimes, the solution could be unique and sometime you know, your may not be any solution at all. So, what will be generally, do is you just try well, if there is no solution, after a long time, you kind of give up or you change your initial guess and try. So, what we are trying to see is try to understand this better.

When does this harmonic elimination problem; when is it likely to have no solution; when is it likely to have fewer number of solutions are just one solution, when is it likely to have more number of solutions; we want to get a grasp over these things and that is why we are going in for a graphical approach first, because it is possible to look at this graphically, since we have only two switching angles here. So, it develops some understanding and some inside as I was mentioning earlier.

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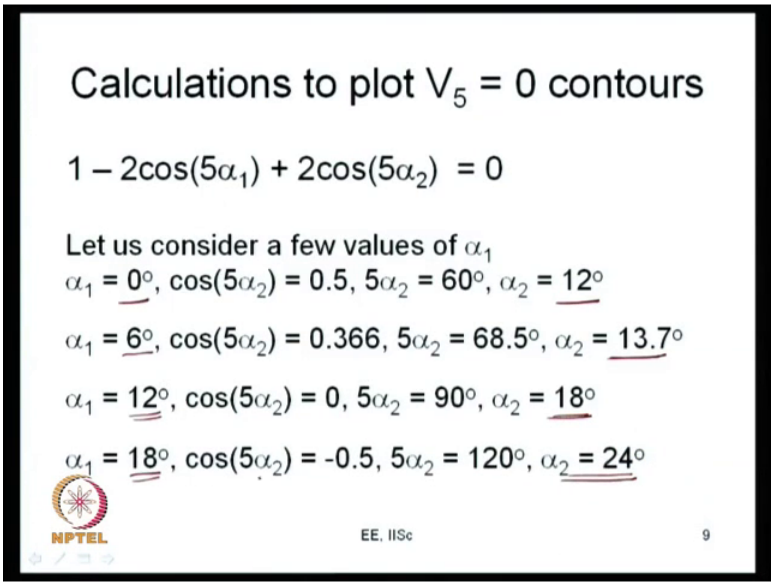

So, if I want to do that I want to plot let us say v 5 is equal to 0 contour. So, 1 minus 2 cos 5 alpha 1 plus 2 cos alpha phi alpha 2 is 0 that is the equation. Basically, I want to plot this. So, what do I do, simple go on substituting different values of alpha 1, different values of alpha 1 and find out what the corresponding values of alpha 2 or.

So, here I have started with alpha 1 is equal to 0, why? Very simple thing, the alpha 1 itself can vary from 0 to 90 degrees, you know. So, I start from 0. So, I have cos 5 alpha

2 is 0.5 and corresponding alpha 2, I am getting is 12 degrees. So, let me just choose some ink here. So, for alpha 1 is 0, I am getting alpha corresponding, alpha 2 is 12 degrees. So, I know one point then I am going into the next point, what is this 6 degrees? I can choose one degree, I can choose two degrees, whatever I am trying to keep it a little higher six degrees. Why six degrees? You say, I have 5 alpha 1.

So, you know 5 alpha, one meaning 5 into 6 is 30 degrees. It is a little easier for us to handle such numbers now. So, I choose 6 degrees now. So, I am getting corresponding alpha 2 as 13.7 degrees, there is some small increase there, then I go to 12 degrees, I go to 12 degrees, add another 6 degrees more and I get a corresponding alpha 2 that is 18 degrees. So, my alpha 1 is increasing considerably in big steps of 6 degrees. Here, I see that the first step has increased by about 1.7, the next step is more than that, it has increased by something like 4.3 degrees, now all right. I go further up say alpha 1 is equal to 18 degrees.

I get my solution as alpha 2 equals 24 degrees. So, if a alpha 2 is 24; so 18 cos phi alpha 1 is basically, cos 90 that is 0 and therefore, you are going to get as cos 5 alpha 2 as minus 0.5. So, 5 alpha 2 is 120 and alpha 2 is 24 degrees. So, you get this point now. So, I have one more point.

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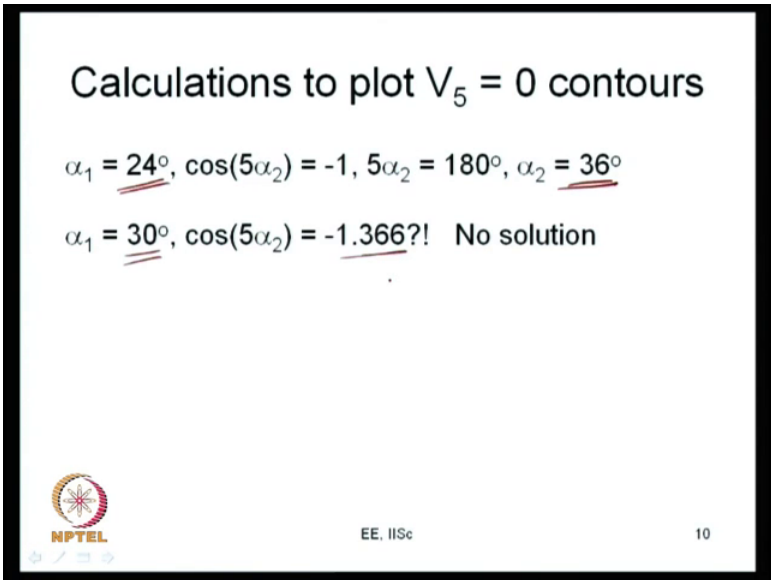

Similarly, you go about getting 24 degrees, you get minus 1 and 5 alpha, 2 is 180 degrees and alpha 2 is 36 degrees, then if I substitute 30 degrees, I get cos 5 alpha 1 is equal to minus 1.366, which is beyond that any cosine function can have values, only ranging between minus 1 and plus 1 and this is lower than minus 1.

So, if that there is no solution here. So, for some, for any value of alpha 1, there is no corresponding value of alpha 2, that is available here now. So, what we do, let us just start plotting this. So, if you plot this, you started with alpha 1 is 0 and alpha 2 equals 12 degrees.

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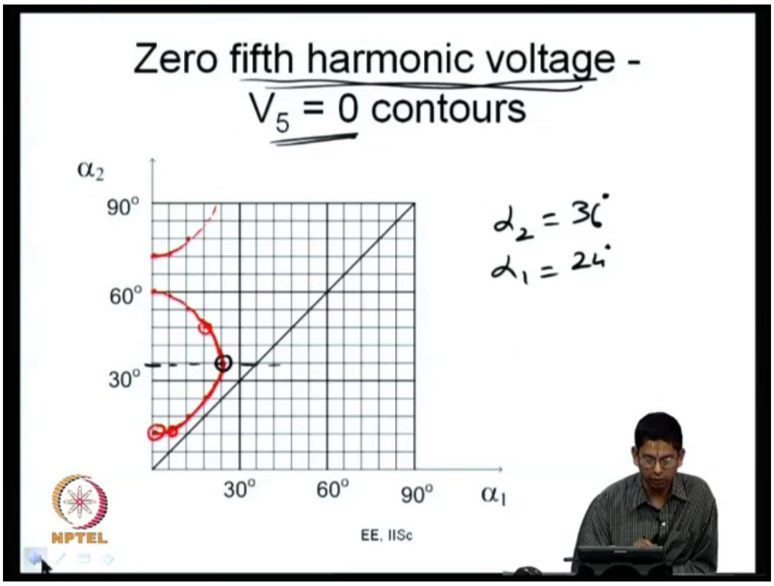

So, let me choose, you know alpha 1 is 12 and alpha 2 is. Sorry alpha 1 is 0 and alpha 2 is 12; so alpha. So, this is my point that is the point that I have indicated here. Now, the next point, a I have start considered is 6 degrees and 13.7 degrees. So, 6 is here and 13.7 is slightly above, that is slightly above, that you see, that I have formed a grid, once in every 6 degrees, which is basically 30 degrees is divided by 5, it is convenient to draw this. So, I have just taken a grid like this. Well, you can do it with any grid or you can do it on a graph sheet, you can do it wherever you want.

There is a tendency and the part of students, generally, to plot all this, using a computer or package and plot them, that is very efficient in terms of time, but I suspect that, you might miss out on certain things. So, I would generally recommend students to plot this, at least, when you are doing it for the very first time or. So, to plot it manually so that you understand certain things, you will understand the trends of the curves and some impor, some inside, you know which is difficult, to even describe or whatever you will get that you know. So, I would generally recommend students that, you plot it first. So, you will give a good experience of good knowledge, of what this or the nature of variation is. So, at alpha 1 is equal to 6 degrees, you get something like 12.0; so degrees now.

So, if I go to 12 degrees, if I go to 12 degrees, it is 18. So, 12, it is 18, like this I will go to the next point. So, it is 18 is 24 alpha 1 is 18, then this is 24, then I am looking at alpha 1 is 24 alpha 1 is 24, it is 180 degrees. I am sorry, 24 alpha phi alpha 2 is 180 alpha 2 is 36 degrees right. So, I am getting here. So, this is what we have come let me join this together. So, here it is a slope, it looks like 0 slope, you can, you can actually verify that the slope is 0, when you are close to alpha 1 is equal to 0 degrees and you do this now .

So, this is how the curves, this is how the curves and many a times you know I, whenever, I asks students to plot this V 5 is equal to 0 contours, the curve stops here, but is this complete? No, there are more solutions available for the equation V 5 is equal to 0, more solutions within the region of our interest, what is the region of our interest, we are looking at only 0 less than or equal to alpha 1, which is less than or equal to alpha 2, which is again less than or equal to 90 degree, we are looking at this upper triangle, that I show here, in this figure. So, within that there are more. So, why have I missed out, one thing is very simple, when I say alpha 1 is equal to 0 and cos 5 alpha 2 is 0.5. It is not 5 alpha to 60, it is only one of the principal solutions there.

There are also other solutions, here 5 alpha 2 is equal to 60 is not the only solution, 5 alpha 2 is equal to 300 is also a solution. So, 5 alpha 2 is equal to 420 is also a solution, I have mentioned this before, but I just want to state this again. So, you get these things here again. So, when you say 5 alpha 2 is instead of 60, it is 300 degrees. So, if it is 300 by 5 is 60 degrees. So, at this point, you also have one more point like this. In fact, if you consider all of them, there is not just 300, it is also 400 and 20 degrees is also a solution 5 alpha 2 is equal to 420. So, that is 72 degrees. So, if you look at these solutions and the next solution might really go beyond that.

The next solution may really go beyond the range of our interest now. So, let us stop there. So, if you really look at this, if you considered all those points, you will get these points also, you will get these points also here and what you will have is really a curve going around like this, these 2 will be symmetric about this line and similarly, you will have another curve, which is just going up the same way like this, this would be basically, oh excuse me. So, this curve that I am showing this curve, you reflect it, you are going to get this curve and this curve is basically, shifted up to this curve, this is something that we plotted in the previous lecture 2.

So, but this is not all you need to go further from here. So, this is what we, you know you get here now. So, there are multiple solutions here . So, it is you first, you know it. Let us look at what those angles are? So, there are curves, we have only looked at the range alpha 1 is 0 to 24 degrees and at 30 degrees. We did not get a solution. So, we stopped there, but maybe for higher values, you will get solutions is that right. So why; what values? Will I get solution? Now, let us say for alpha 1 is equal to 0, I got some solution, which is alpha 2, is equal to 12 degrees, for this fifth harmonic elimination or V 5 is equal to 0 contour. Now, if I increase, you know the, it has a equation as a cos 5 alpha 1 term; so the cos 5 alpha 1 value is not going to change.

If I am going to change my 5 alpha 1 by 300 and 60 degrees or change my alpha 1 by 72 degrees . So, if I go here by 72 degrees is I can go here, to another 72 degrees. Let me go back, to the previous one to look at that r maybe, you can even look at a fresh plot r will come to this, here excuse me.

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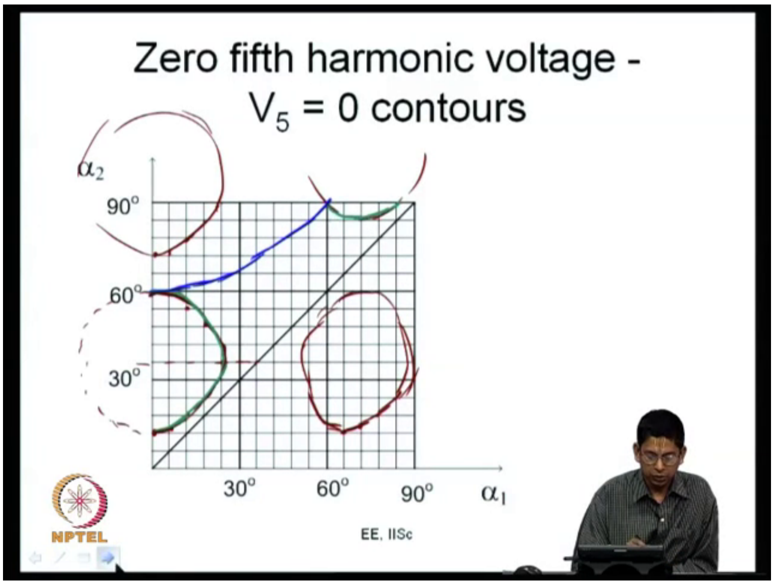

So, if you have a point here, 72 degrees, later you will also have a point here. So, 72 degrees, you will have a point here, why? Alpha 1 has changed by 72 degrees. So, 5 alpha 1 was changed by 360 degrees. So, cos 5 alpha ones values unchanged now; so the same way, you should remember actually that, this curve has a negative side also for negative values of alpha 1 also you have this curve going symmetrically, on the other side.

You have it symmetrically, on the other side. So, this will also shift up here. So, what is going to happen; is if this is alpha 1 and here if we have a point, this point is about 24 degrees. So, this will also shift here at 24, meaning 24 plus 72 is 96 degrees, which is a, it goes a little outside here now. So, if I take this 0 instead of 0, I mean, I have got a point here, let 0 next is 6 degrees. So, 6 plus 72 is 78 degrees here. So, I will get a curve like this, I will get this curve, going like this another point like this. So, I may get certain points going an out. So, I will actually have another curve like this.

I will have one other curve like this; which is basically, the first curve, shifted by 72 degrees. Now similarly, this curve that you see in the top is again this bottom curve, shifted up by 72 degrees. So, if alpha 2 is changed by 72 degrees, 5 alpha 2 changes by 360 degrees and cos 5 alpha twos value is unchanged . So, you can shift this entire curve. So, it is basically, it is part of another curve, there this curves been shifted up. So, you can shift it by right or left by 72 degrees or you can also shift it to top or bottom, you will get the same curve, once you get one V 5 is equal to 0 contour, you can get as many of them as you would want now.

In fact, if you shift it, further this one to the right or this one up, you are you. In fact, going to get one more curve like this and if you see certain curves are in this, in our region of interest. Let me just change the ink colour are. So, this part of the curve is in our region of interest, what I am drawing with the green ink. Now, again this part of the curve is also in our region of interest. So, let me indicate with a different colour, V 1 is equal to 0 contour, the V 1 is equal to 0 contours, we looking at, goes something like this.

So, these 2 curves are solutions, what I have been indicated here by the green lines, this curve and this curve our solutions here now. So, one other way is, to look at the slopes.

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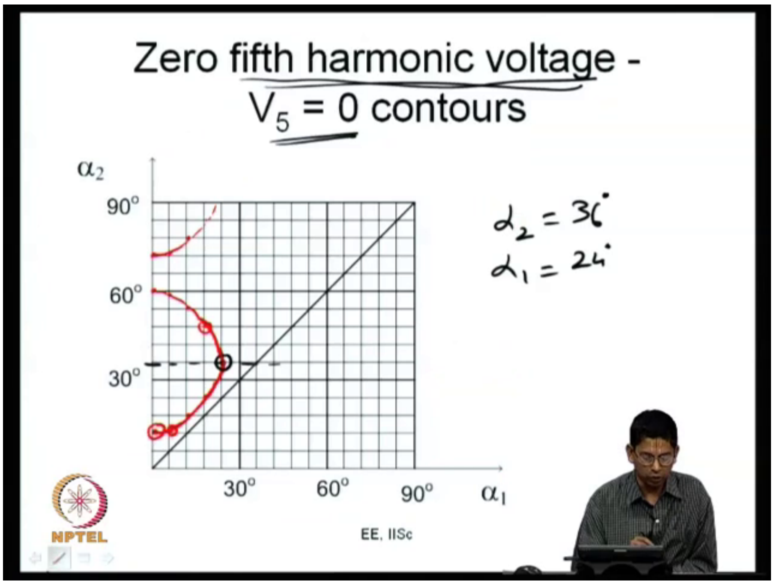

If you look at the slopes, what is that 1 minus 2 cos 5 alpha 1 plus 2 cos 5 alpha 2 0; if you want to get the slope of V 5 is equal to 0 contour, you differentiate this equation, both sides of the equation with respect to alpha 1. So, you will end up getting this is you know 10 sin 5 alpha 1 minus 10 sin 5 alpha 2 is times d alpha 2 by d alpha 1 is 0, then you have d alpha 2 by d alpha 1 and take the other terms to the other side, you get an expression d alpha 2 by d alpha 1, is equal to sin 5 alpha 1 by sin 5 alpha 2. So, this is how the nature of the slope is.

So, sometimes your slope is 0, whenever the numerator is 0, when is the numerator is 0, the numerator is 0, whenever sin 5 alpha 1 is 0 and the denominator is 0. Whenever sin 5 alpha 2 is 0 that is the slope is infinity now. So, you can see that, there are points at alpha 1 is equal to 0, the slope is 0 again, when alpha 1 is equal to 36 degrees 5 alpha 1 is 180 and sin 180 is 0. So, at alpha 1 is equal to 0 at alpha 1 is equal to 36 at alpha 1 is equal to 72 etcetera, it is possible to see one such point. So, you say alpha 1 is equal to 0 and you look at what are the corresponding values of alpha 2; if you take alpha 1 is equal to 0, the other corresponding values of alpha 2 could be 12 degrees, 60 degrees, 72 degrees, etcetera in every one of this point, at every one of this point, the slope of V 5 is equal to 0, contour is 0, it is 0. Here, the slope is 0 here, the slope is also 0 here.

So, again infinity, when sin 5 alpha 2 is 0, it is infinity sin 5 alpha 2 is 0. When alpha 2 is equal to 36 degrees, this is alpha 2, is 36 degrees. So, when alpha 2 is equal to 36 degrees, your alpha 1, your alpha 1, sorry it is 24, alpha 2 is 36 degrees, the corresponding alpha 1 is 24 degrees. So, these points you can very quickly identify, the points where this curve has 0 slope and infinite slope, it has 0 slope and infinite slope and you can also plot them very quickly.

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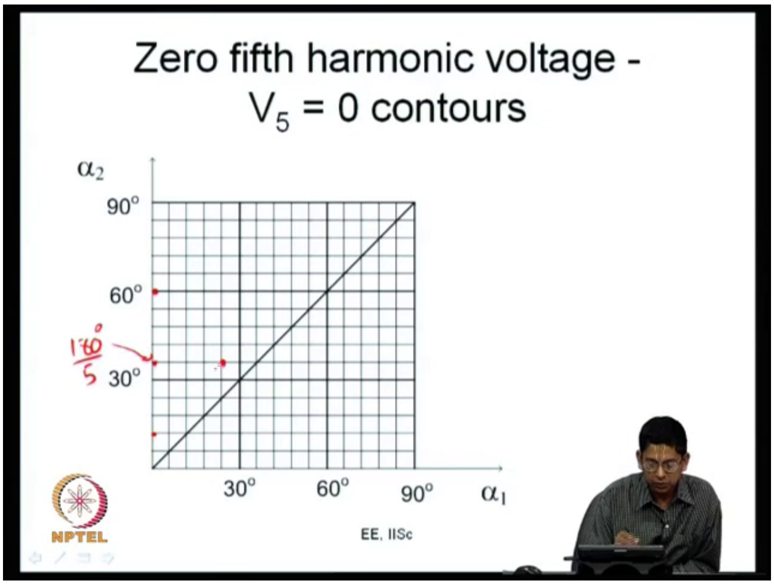

If you start from here, let us say I choose some ink, there is something centered at 36 degrees, this 36 degrees are basically 180 divided by 5, let me write 180 divided by 5, that is this point and from here, it is 24 degrees away. So, a this is a point, where you have 0 slope again, this is a point where you have 0 slope. So, if you look at another point, where in the slope is infinity, you will find that the slope as infinity here and similarly, on the other side maybe it is good, if I erase this. Let me erase this point.

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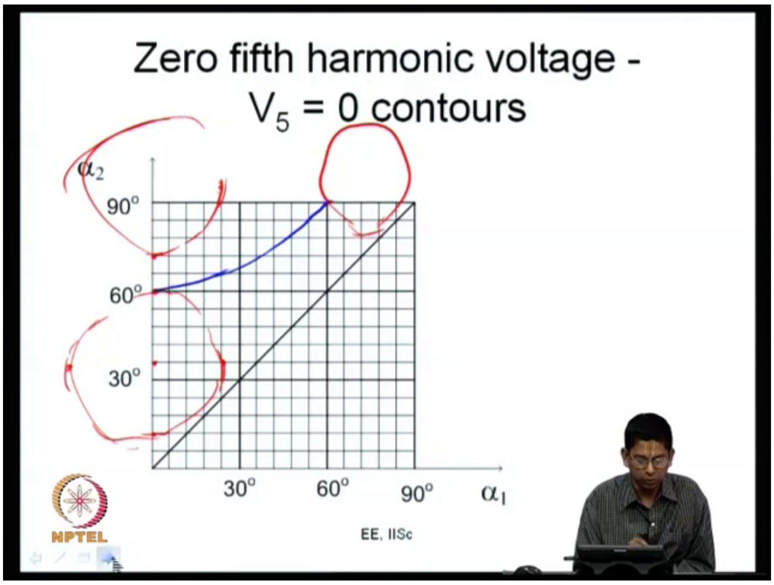

So, there is another one on this side which is basically, minus 24 degrees. So, you can very quickly come up with this contour, it looks like a circle, it is not exactly circle, it is more curved in this sides, I mean it is and it is kind of straight line towards the centre now. So, what you really have is that curve, which is going like this now. So, you can also identify the next set of points. Similarly, at 72 degrees, you have this and again is another point here.

You this is plus 24 72 plus 24 is 96 degrees. So, at 96 degrees, you will have again infinite slope. So, you get this like that. So, you can identify these points, where the slope is 0 and infinity, and you can go about getting these curves much more quickly, this is what I have been trying to put a cross. So, now these three covers, three cover practically, there, our region of our interest, there is no fourth curve that is possible, within our region of interest. Now, our region of interest, being alpha 1 greater than 0 and alpha 1 should be less than or equal to alpha 2 and again alpha 2 has to be less than or equal to 90 degree, they are upper triangle that is shown here.

So, there are now only these three curves and once again as I mentioned at little earlier, you take V 1 is equal to 0 and you are looking at only one face, you are not looking at the other version, like V 1 is equal to 0.5 or minus 0.5. So, your bounds are only within this. So, you have these two solutions available now. So, this gives us a clearer picture of what we are trying to do. So, let us see, what are those two curves we are getting?

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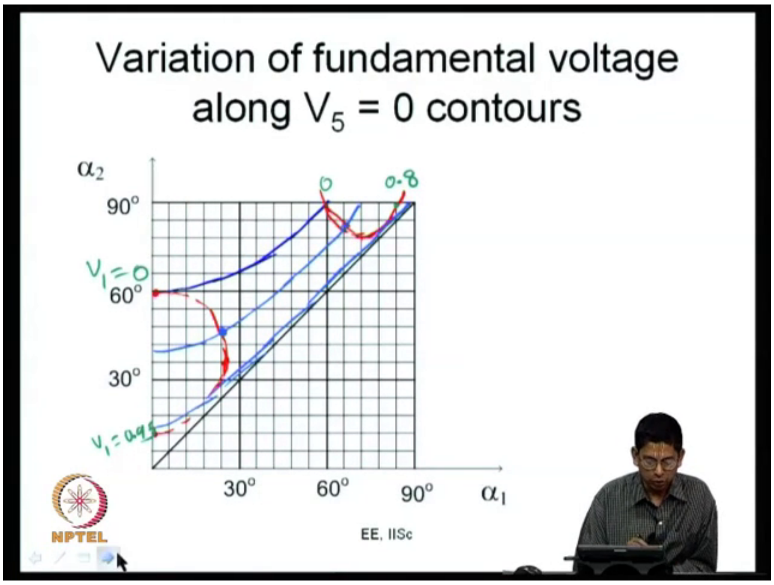

Let me just sketch this part, this is, I am drawing it very roughly, I am not doing very this thing. So, let me say; what are the two curves that we got, one curve started from here, one curve was like this and the other part of the curve is like this, like this, like this. So, we looked at this question, about the solution for fifth harmonic before that let us start looking at the fundamental voltage, how does the fundamental voltage vary along this now.

Let me choose a different in colour, what is the fundamental voltage? Here, this fundamental voltage is 0 V 1 is equal to 0 along this line is it. So, it is also 0 here, V 1 is 0 here too and as you move along this, as you move along this V 1, actually goes on increasing and you will, you see here V 1 will be something like 0.95, when you come to this point V 1, will be something like 0.95. Similarly, along this point, if you start and you go moving here, at this point V 1 will be something like 0.8. So, this is how it varies, as you move along this curve, you are coming within this area. So, your V 1 is actually increasing little and you are going all the way up to V 1 is equal to 0.95 and here you are going up to 0.8 now.

So, what does it say, if your this V 1 is equal to 0.95 is the highest value, is the highest value along of fundamental voltage, along this particular curve and again this V 1 is equal to 0.8 is the highest value of the fundamental voltage, along this particular curve. The other red curve that I have been showing here. So, in this case, the fundamental voltage varies anywhere is between 0 and 0.8, and along this curve it is anywhere between 0 and 0.95. So, can there be no solution to the fundamental equation that was a question, that we were asking yes, the fifth harmonic elimination will it have a no solution. Of course, if you have a fundamental voltage that is greater than 0.95 I mean this.

Please remember this 0.95 is a, it is not a very precise number I am giving it, you can just work out what that number exactly is, it is after all alpha 1 is 0, and alpha 2 is equal to 12 degrees now. So, if you take anything be above that, there is going to be no solution for your fifth harmonic elimination. Can I solve fifth harmonic elimination for example, when I want V 1 is equal to 1 per unit, if I want V 1 is equal to 1 per unit, it is a square wave and there is no switching. So, you cannot eliminate any harmonic. So, if a fifth harmonic elimination is not possible for V 1 is equal to 1, it will not be possible for some values, very close to one. How close is that, is the question that you have coming to determine, you have. I mean that is what we have come to determine now.

As far as the fifth harmonic elimination is concerned something like 0.95 is, what the number is, if you are really above this, if you are like 0.98 or 0.99, there is going to be no solution for your fifth harmonic elimination problem. Now then how about at they are going to be multiple solutions, yes how, when if you are looking at a fundamental voltage. If you are looking at some fundamental voltage, let me once again draw it with, let us say a blue ink looking at something like 0.4. So, I am going to get a curve which is like this. This curve may be like V 1 is equal to 0.4 or whatever 0.3 intersects. This V 5 is equal to 0 contour, and it also intersects the other V 5 is equal to 0 contour.

So, it has two different solutions now, up to what range, up to 0 to 0.8. There are two different solutions possible, because any V 1 is equal to 0. V 1 is equal to constant contour, where V 1 is anywhere between 0, and 0.8 will intersect both this curve, as well as that curve. Whereas, if you choose something like V 1, is equal to 0.93 or. So, it may intersect only this curve, but it may not intersect the other curve at the top, the top red corner. So, this has only one solution.

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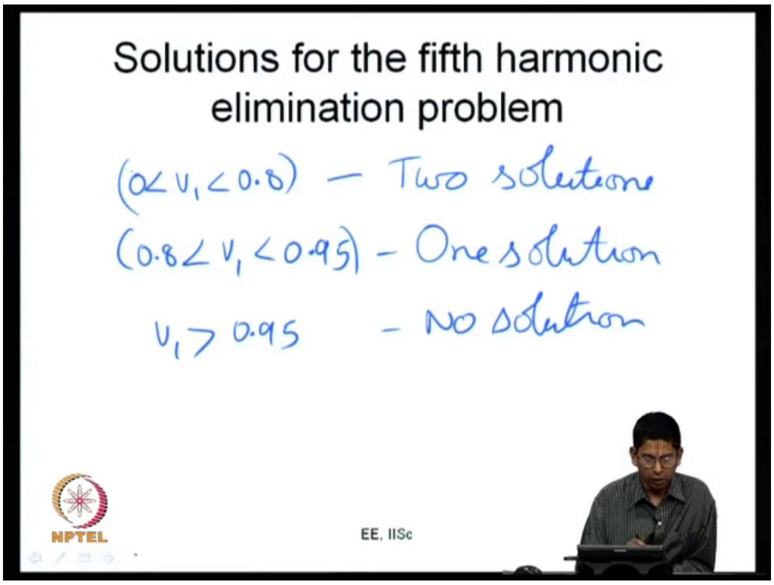

So, we can say the solutions for fifth harmonic elimination problem in the range of V 1, something like 0 to V 1 to 0.8.

In this range, you have two solutions are possible, you have two solutions possible. So, if you look at the range 0.82, something like 0.95 or 0.96, we will have only one solution is possible. And if you are talking of V 1 greater than 0.95, there is no solution. So, whenever you are very close to a six step, you know there is a chance that there may be no solutions; that is something that we have to be, you have to remember. And again it is not necessary that as you go down to lower and lower ranges, the number of solutions should go on increasing at lower ranges, the number of solutions is certainly higher; that is what we can really say from here now.

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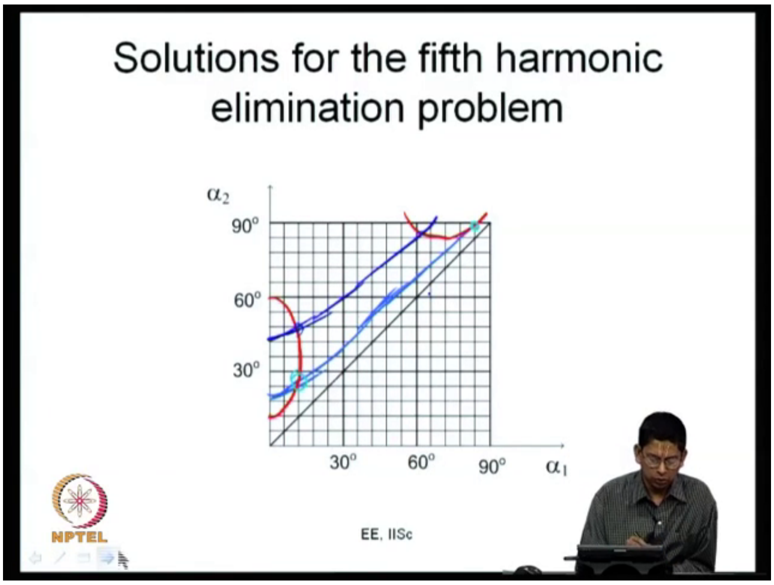

So, what you generally do is, to solve fifth harmonic problem. This is a graphical approach, and this is mentioned may meant only for to switching angles.

For anything you know you can always solve those equations, it is two equations in our case, when there are two switching angles, when there are three switching angles that are going to be three equations, and you have to solve that iteratively. Now it depends on what you want, let us say you have something like V 1 is equal to 0.8; something like V 1 is equal to 0.8 or so. So let us say this is the curve that you have. This is the fundamental voltage curve you may have. So, now, you have the other v 5, is equal to 0 is like this, and here it is like this. So, it depends on where you are starting from, and how the convergence happens with your solution. Let us say now you might have started, it depends on your initial guess on the numerical procedure.

So, depending on whichever starting point, sometimes you may end up with this point as one solution, and at the rest you may end up at that point as the solution. So, it depends on your initial guess also. So, it is possible that when the problem is very complex, still certain solutions have not been found, you know you really, you do not know you can go on doing many things, it depends on your initial guess here. So, you depending on that, you are going to get solutions now. So, sometimes you know if yours initial guess is very bad, it will take more time to come up if. So, your initial guess is better, you may be able to solve this problem much more quickly. So, when you have an idea of the solutions, then your initial guess could be better. So, that is one of the reasons why we are trying to do all this. So, with this kind of an approach you will see that you know.

You will start slowly learning how to make a good initial guess. So, this, that is one of the intentions here now. So, it depends on your initial guess, and also if there are some other V 1. Let us say you take another value of V 1, this could be another value of V 1. So, you may want to solve for V 1 is equal to 0.3, and V 5 is equal to 0. So, it will depending on your initial guess, your iterative procedure will bring you to either one solution, which is here or the other solution that is there. So, if it is more than 0.95, you may not get any solution at all. So, this is how you get it for the fifth harmonic elimination problem now.

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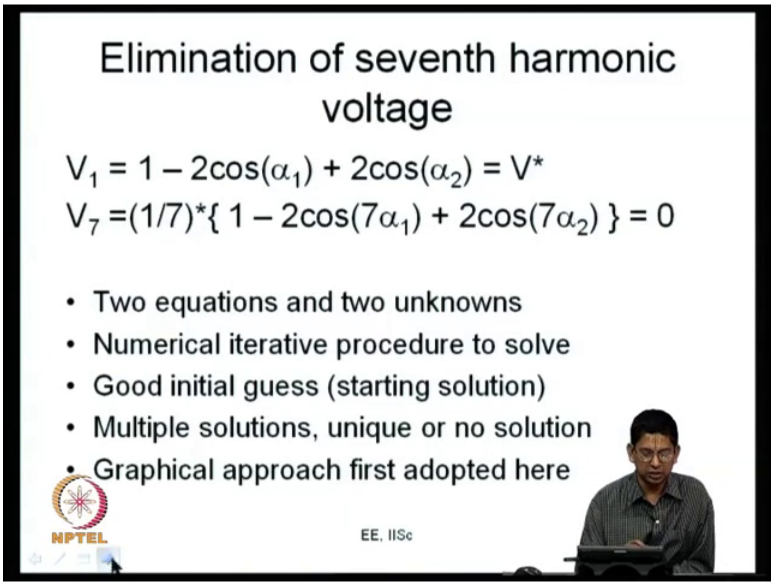

So, let us look at how do you do, a is eliminate seventh harmonic instead of fifth harmonic, for some reasons you might want to choose a seventh harmonic eliminate.

See sometimes it is possible that you can use passive filters tuned passive filters which can selectively bypass or filter out certain harmonics. So, maybe for fifth harmonic you might have some filtering. So, you might want to filter out the seventh harmonic. So, let us say, you want to eliminate seventh harmonic. So, what do you want to do? You have the equation slightly different. So, V 1 is 1 minus 2 cos alpha, it is the same thing and instead of v 5 is equal to 0, you.

Now I have v 7 is equal to 0, the same old story two equations, two unknowns you will have to use a numerical procedure to solve this. And you need a good initial guess, and once again you might have multiple solutions or a single solution or no solution at all. And once again it is possible to solve it graphically, because we are still with two switching angles only. So, let us just try to doing that.

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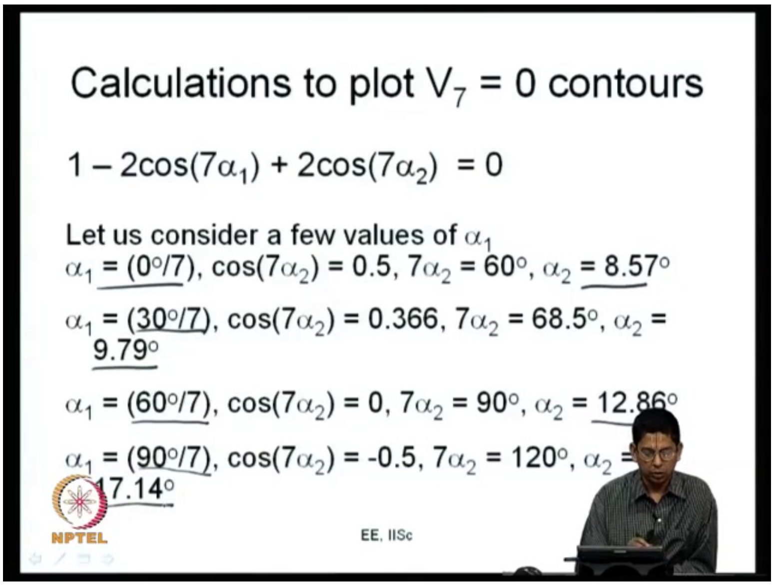

So, if you follow a very similar kind of an approach that we followed earlier; that is now we can follow, we can calculate and plot V 7 is equal to 0 contour. So, how do we do that. The same way, you take 1 minus 2 cos 7 alpha 1 plus 2 cos 7 alpha to 0, and fill in feed values of alpha 1, and try to find out what those values of corresponding values of alpha 2 are. So, if you, see I have taken alpha 1 is 0, and the next value I am trying to take is alpha 1 is 30 by 7, it is easier that way, because 7 alpha 1 will, then be 30 degrees, and I am taking the next value 60 by 7.

So, 7 alpha 1 will be 60 degrees. So, its kind of easy to do my calculations and all that. So, we have just taking some numbers like that. So, 30 by 30 by 7 is essentially something like 4.3 and 60 by 7 is something like 8.6 are 8.7. Again 90 by 7, 90 by 7 is something close to 13, little less than 13 degrees. So, the corresponding solutions are all given here now. So, let us look at what those solutions are. So, at alpha 1 is 0. The corresponding solution is alpha 2 is 8.57, it is the same procedure. Now alpha 2 is 9.79. So, alpha 1 is 60 by 7, as I said it is a little more than 8 degrees, something like 8.6 or so.

So, the corresponding alpha 2 is 12.86. Now again when you have alpha 1 is 90 by 7 which is little less than 13 degrees.

Your alpha 2 is 17.14.

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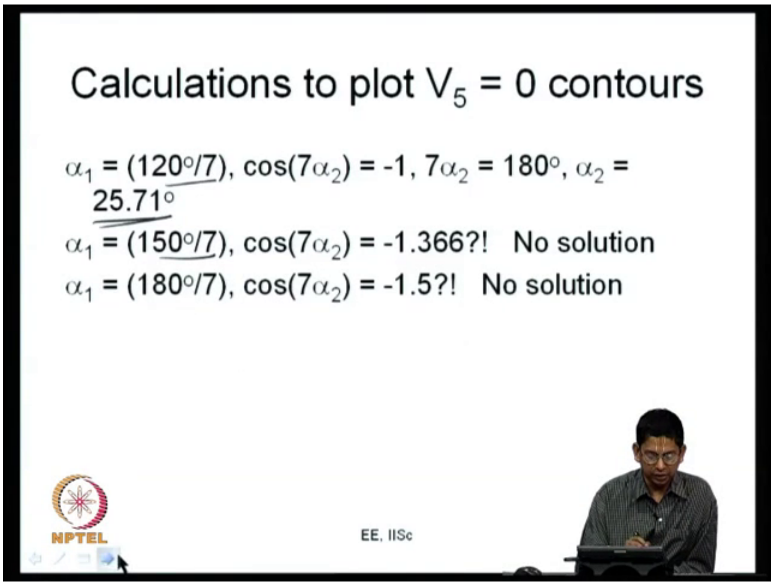

So, you go for more values, you take 120 by 7. So, for this, they have corresponding value of alpha 2 is 25.71, then you have 150 by 7 for which you go, and you know it says its beyond solutions. So, for certain values of alpha 1, you may not have, no solutions at all, it is really like this. Now and. So, let us look at just plotting this. So, first is 0 and alpha 2 is 8.57 degrees.

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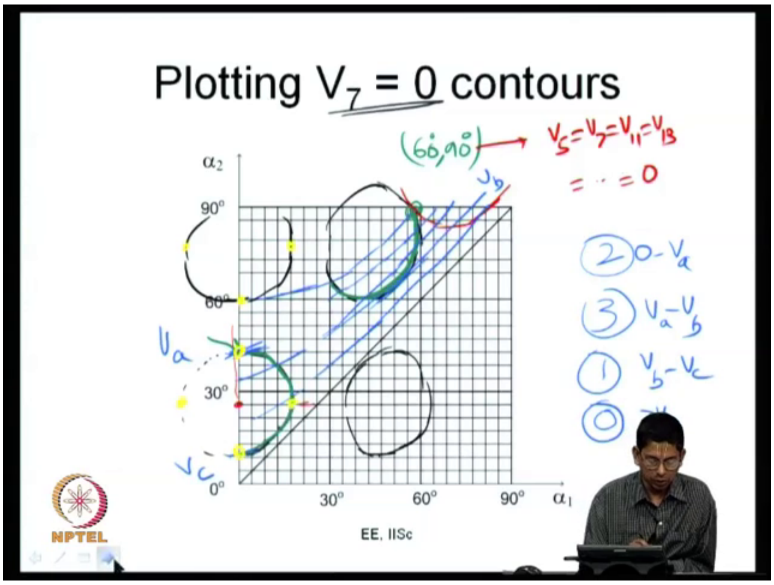

So, alpha 1 is 0 and alpha 2 is 8.57 degrees, basically more than that what I can say is 7 alpha 2 is 60 alpha 260 by 7. So, what I have done here is.

I have divided every 30 degree into 7 grids. I have just made a suitable kind of a grid here. So, everything is you know 30 by 7 and you get. So, one grid here is 30 by 7. So, 60 by 7 is here, my point is somewhere here. So, if I go next, what is the next point alpha 1 is 30 by 7, my corresponding alpha 2 is something like 9.79 degrees. It is alpha 2 is some little more than that 9.79, you see the slope is very small, the slope is very small now, right then I go to the third point 60 by 7. So, here alpha 2 is 12.86 degrees, are basically 7 alpha 2 is 90. So, the point is here 7 alpha. So, the point is here now. So, the curve is gradually going up as you can see.

So, if I go to 90 by 7, 90 by 7, then, which is roughly 13 degrees, alpha 2 is 17.4 degrees, alpha 2 is 17.4. So, it is somewhere like that. So, then you come to 120 by 7, you have alpha 2 is 25.71 degrees. So, 25.71 is something like here. So, this is the curve that you have. This is the curve that you have, and this is just one of the solutions. So, if you go the other side, you will have 2 3 4 1 2 3 4. So, you have this coming around, and going back to this. This is a V 7 is equal to 0 contour. And of course, you have this curve going to the left also, this going to the left also.

This curve is in fact, symmetric, about this point it is symmetric about that, its not a circle. It does not have a radial symmetry, but every quadrant here is symmetric. So, if you draw ones quadrant, you can get the other quadrants by reflecting them. So, every quadrant is symmetric about that. So, you can get this now. So, again what we have understood earlier, if this is the point, let us say a, let me go back to the same colour that I chose earlier. So, if I shift alpha 1 now by 360 by 7, 360 by 7 is something like 52 degrees, it is a little less than 50 to 51 point. So, 4 degrees or. So, if I shifted by 51.4 degrees what happens is, correspondingly 7 alpha 1 increases by 360 degrees, and cos 7 alpha 1 is unchanged. So, all these curves can really be shifted.

So, it is somewhere around 51 degrees, you will get that which is, should I say here. So, you may get a curve which is going around. So, this is 25.7. So, this will further get shifted here. So, what will have is, you may have some curve which is like this, some kind of curve you will get like this, which is exactly you can shift it to the right by (Refer Time: 40:35) degree. This curve is not within our region of interest, because you know it comes in a range, where alpha 1 is greater than alpha 2, which is not possible for us. Now let us if you shift this curve up, you will get a corresponding curve there right. So, what will that curve be, you will get in fact, 4 5 6 you will actually get another curve, which comes from the other side.

So, you will have multiple curves coming up now. So, this curve, if you really see it will get shifted up, let me just blue line. So, one of this curve if I shift up, it will actually be like here. This curve will actually go through this point, one thing I want to tell you, is all these curves will go through one particular point; that is the green point, I am indicating here y, what is that point that point is 60 comma 90; all this V. See V 5 is equal to 0 contour, also went through that point, the curve was different, if I redraw the curve here, that curve went through like this, something like that, whereas, this curve is going around like this now; so why all these curves will go through that; because 60 comma 90 corresponds to the third harmonic square wave.

So, third harmonic square wave contains no harmonics; neither fifth harmonic nor seventh harmonic, neither eleventh harmonic not thirteenth harmonic. So, any Vn will be equal to 0 anywhere, n is any non turbulent harmonic order. So, at this point you have V 5 is equal to 0, V 7 is equal to 0, V 11 is equal to 0, V 13 is equal to 0, and all of them will be equal to 0. So, many a times that is a good starting point, when you want us to solve for numerical solutions, because the harmonic equations are straight away. So, you know satisfied inside that particular point now. So, you can also shift this curve a little this side, and you will come up with yet another curve, which passes through 60 degrees now.

So, what you will get is, I will go back to the same black colour. So, I will have another curve like this. We have another curve going around like this now. So, this is yet another curve now. So, I now find that, there are a few different curves, let me choose a different colour to highlight, which of these curves fall within my region of interest, this curve falls within my region of interest. This is basically, one half of a complete curve and this again part of this curve falls, within my region of interest part of this curve, falls within my region of interest now. So, at this point, you have some value of V 1 at this point. You will have some value V 1, what is that point actually; you can just find it out, it is for alpha 1 is equal to 0 and the corresponding value of alpha 2, that we got that is alpha 1 is equal to 0 7 alpha 2 is equal to 60 or 7 alpha 2 can be 300 degrees. So, what you have, is alpha 2 is 300 by 7.

So, that point is actually alpha 2 is equal to 300 by 7; so at this point. So, if you take this point, the fundamental voltage is low and let me use an arrow here. So, at this point, the fundamental voltage is low, as you go around, moving this curve, as you go on moving around this curve, your fundamental voltage will increase and will come to some other value; excuse me they will come to some other value. So, there is a range over which the fundamental voltage varies along this curve and that range starts from somewhere higher than V 1 is equal to 0. Now, you take this curve; now, this is V 1 is equal to 0, the blue line is V 1 is equal to 0, excuse me, the blue line is V 1 is equal to 0. So, as we move from both the sides. So, for something like V 1 is equal to 0.1, this curve itself offers two different solutions is that clear.

Let me just use blue ink and draw another line. Now, let me say I have some other V 1 is equal to 0.1 or. So, that V 1 is equal to 0.1 or. So, is going to intersect this particular green curve itself, in at two different points. So, this curve itself offers two different solutions and there can be a set of values of V 1, for which you may have like this, it will intersect twice on this curve and once here. So, sometimes you may have two solutions and sometimes you may have three solutions and when you go beyond this particular point for some higher value of V 1, which does not cut this intersect, this green curve at all, but it intersects only here, you may have one solution and finally, when your V 1 goes very close to the 6 step voltage, there can be 0 solutions.

So, wa here, what we find is at low voltages, starting from V 1 is equal to 0 to some value, you can, you will find two solutions, that value is basically the whatever is the fundamental voltage at this point this curve. So, from that fundamental voltage, let me just write down some values. So, that, it is clear as to what we are trying to say. So, let me call this fundamental voltage is Va and let me call this value, which is tangential as Vb, that is the line Vb fundamental voltage equal to Vb is tangential to this green curve now, and then you may have some voltage Vc. So, between 0 and Va between 0 and Va you may have two solutions and for the range Va to Vb. You may have three solutions, because this will intersect twice, in this curve and it will intersect once in this left top, left bottom curve and if we go.

If we have gone beyond Vb and all the way up to Vc, your V 1 is equal to constant curve will only intersect, this particular green curve that is the left bottom curve and it will not intersect the right top curve. So, you may get only 1 and if you are fundamental voltage is even greater than Vc. You will have no solution, this is greater than Vc, you may have no solution. So, what this values of Va Vb in Vcr, I would leave them to you as exercises to find out, they are reasonably simple exercises and the interesting point is, you know at Vb you will find that the curve, the constant voltage curve V 1 is equal to Vb and V 7 is equal to 0 will be tangential to one another. So, that is the condition, you must use to come up with what the exact value of Vb is;s o the seventh harmonic solution problem.

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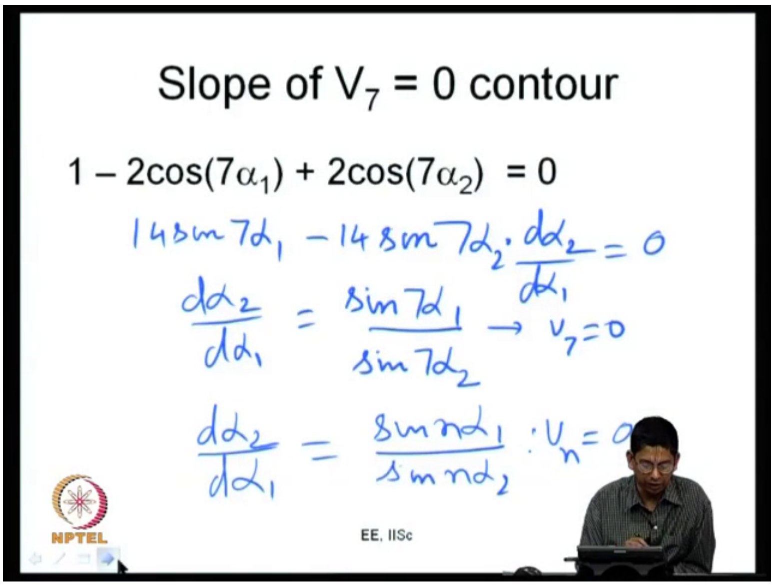

If you look at it, it offers a lot of issues like you, know there are multiple solutions and there are no solutions, etcetera and it is also good, if you look at the, you know what is the V 7 is equal to 0 contour slope is going to be. So, it is the same thing, 1 minus 2 cos 7 alpha 1 plus 2 cos 7 alpha 2 equal to instead of 5, it is 7 in this equation. Now, if you want to find the slope, what you need to do; this differentiates both sides of this equation, with respect to alpha 1. So, you are going to get something like 10 times or I am sorry. It is 14 times. So, 14 times sin of 7 alpha 1 minus 14 times sin of 7 alpha 2 times d alpha 2 by d alpha 1 is 0. So, once again you will have, d alpha 2 by d alpha 1 is equal to sin of 7 alpha 1 by sin of 7 alpha 2.

This is true for any V 7 is equal to 0 contours. So, you can see that it is very similar, what was it in case of V 5 is equal to 0 contour, the slope of the V 5 is equal to 0 contour, we found was sin 5 alpha 1 by sin 5 alpha 2. Now, it is sin 7 alpha 1 by sin 7 alpha 2. So, you can very easily generalize, if you replace this as wa, you know n alpha 1 n alpha 2 for any nth harmonic, you will find that the curve is d alpha 2 upon d alpha 1 is equal to sin of n alpha 1 divided by sin of n alpha 2. This is for any particular Vn is equal to 0, it can be V 11 is equal to 0 or V 13 is equal to 0 and whatever. So, this is how the slopes are for any V 7 is equal to 0 contour are V, any Vn is equal to 0.

And from this slope, it is very easy for you to find out where these curves are going through zeros. I mean 0 slope and where it is infinite slope and you can very easily come up with certain sketches, that you really want to have here. So, this kind of sketches that we really saw, it is easier to come now.

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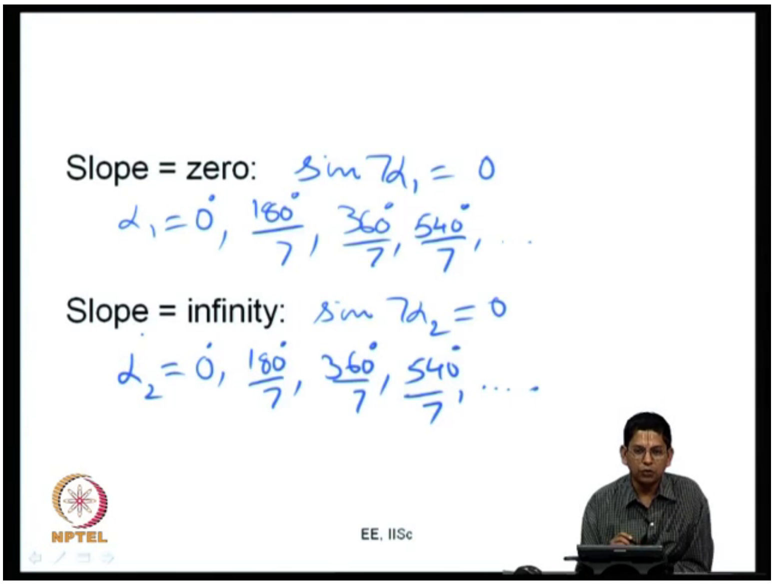

So, for example: if slope is 0, what we mean is, in this case we say sin of 7 alpha 1 is 0. Here, we mean sin of 7 alpha 2 is 0. So, there are values of alpha 1 for which you get 0 slope that is alpha 1 is 0, then the next value is 7 alpha 1 is 180 or you have alpha 1 is 180 80 by 7, then 360 by 7, then 720. I am sorry 540 by 7, so 540 by 7 and so on.

So, for these various values of alpha 1, you find out the corresponding values of alpha 2 and I thought all those points, you will find that, this V 7 is equal to 0 contour, has 0 slope. Similarly, you look at wherever the slope is infinity, the curve is the vertical. So, sin 7 alpha 2 is 0. So, what you will have is. So, sin 7 alpha 2 is 0. So, same way, if this is satisfied, whenever you have alpha 2 is whatever alpha 2 can be 0 and alpha 2 is 180 by 7 alpha 2 is 360 by 7 5 40 by 7 and so on. So, at these values of alpha 2, you plug in these values of alpha 2 in your V 6 is equal to 0 equation and find out the corresponding values of alpha 1. So, at these is points alpha 1 comma alpha 2, you will find that you know it is V 7, has infinite slope now and wa, if you are able to spot them, you get those curves basically, you know the contours are like this.

All that we are trying to do here is let me use a slightly different colour to indicate that. So, you can very easily come up with these yellow points, you can very easily come up with these points, which are really the corners. So, once you have the four corners available, you know what the nature of curve is fairly quickly. So, this is about your infinity and things like this now.

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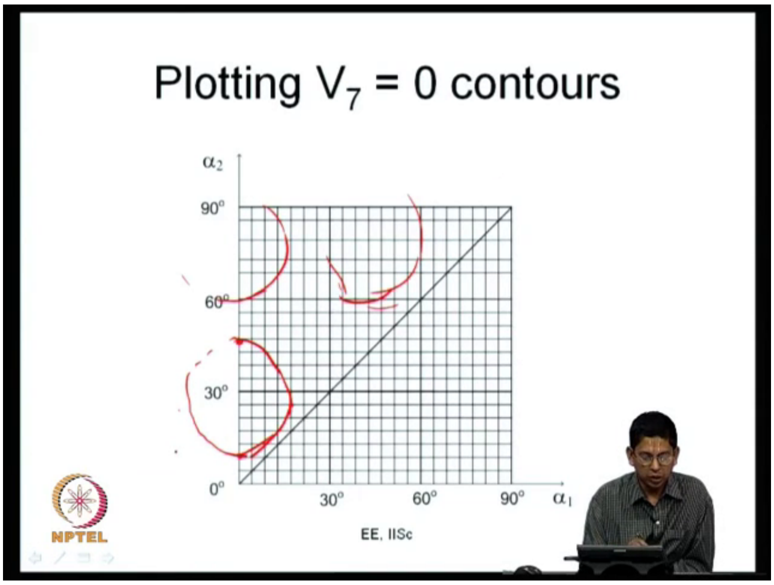

So, you as I said that you can use that idea to plot it very quickly; so you know that it is a 0 slope here, and you know that it is infinite slope here, and you know that it is some 0 slope. So, once you know that, you know that the nature of the curve is going to be something like this and once you know that, this is the other side of the curve.

So, you can translate this curve, you can shift this by 360 by 7 degrees to the right or 360 by 7 to the left 360 degrees by 7 to the bottom or top and you go, what getting the other curves, that you want to get. Now, which is what with the burden of our argument has been write down. So, you can get the other curves, that you want is it. So, you can very easily come up with these points once you have them. Now, at east roughly, you can come up with what you want to say now.

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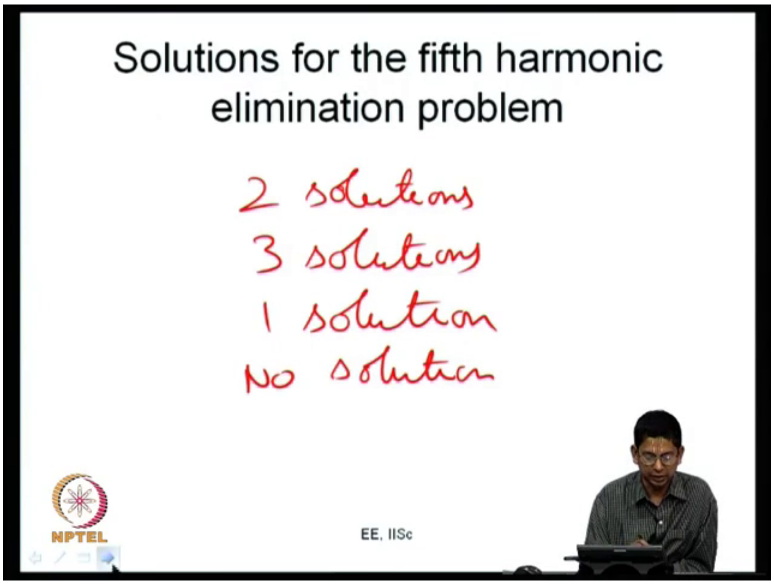

So, as I said about the fifth harmonic, please mean it is about seventh harmonic elimination problem, here again you have, you know there are two solutions and do you have, you can have three solutions, in some other range.

And you may have only one solution in some range and you may have no solution, in some other range. It is possible for you, to find out what are these ranges and things like this now.

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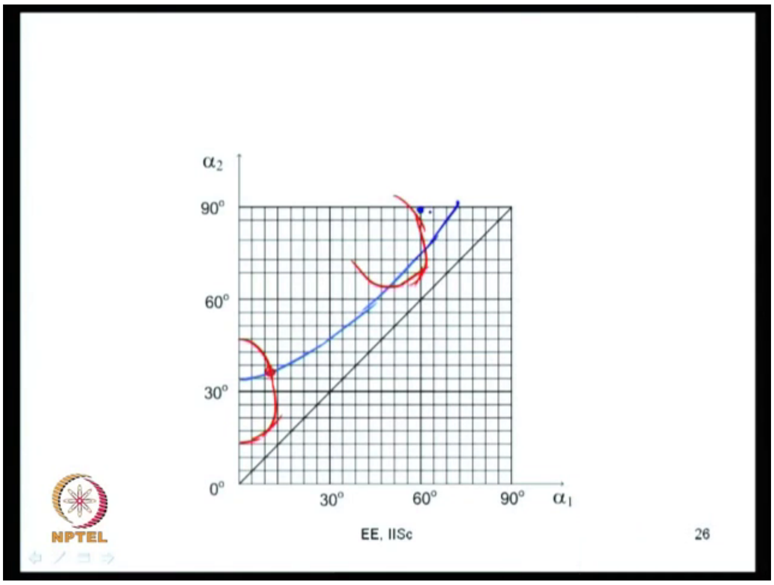

So, when you solve them what you are really trying to do is though you know you may have particular value V 1. So, that may be a curve going like this, it may be a curve going like this and you will have V 7 is equal to 0. So, that may be the curve like that. So, you are trying to find out this point of intersection, by solving those two equations iteratively. So, it may also be intersecting with the other curve that we had considered here. So, this is the other. I am sorry; the other curve that we had considered here. So, which point you get, if there are you know, if you are within the range.

Where there are three solutions, are possible, which point you, will get depends on your initial guess and your iterative procedure. So, it will take you to one or the other solutions now. So, it depends on your initial guess, you will get one or the other solution and if you are within the range, which is greater than that Vc, that I had told you, before you may not get any solution at all . So, that is the selective harmonic elimination problem, you can really extend that problem, you can generalize that problem. So, the problem that we had studied here was to solve only these two equations, instead of V 7, you can take V 11. You can take V 13, any one. Particular harmonic can be eliminated, depending on what you want to be done that depends on the application and then, if you have more number of switching angles, let us say, you have three switching angles, it is possible for you to have it as a three angle problem now.

So, now let me say.

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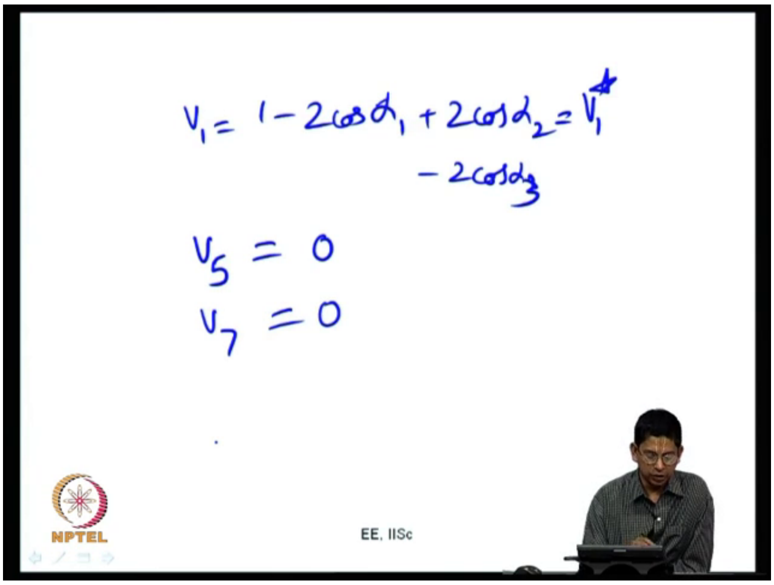

I have V 1 is 1 minus 2 cos alpha 1 plus 2 cos alpha 2 is equal to V 1 star, this is my desired voltage, there is another cos alpha 3 also here. So, I can have V 5 is equal to 0 as another equation, this is again three switching angles are going to be there, it is going to be 1 minus 2 cos 5 alpha 1 plus 2 cos 5 alpha 2 or 2 cos 7 alpha 3, again V 7 is equal to 0, can be a third equation. There are three angles; alpha 1 alpha 2 and alpha 3. So, there are three variables, it is possible for you to solve this and come up with this, now in this kind of a situation, what you will have, is it may not be in an intersection of two different curves. Now, this is alpha 1 and alpha 2, when you are considering only two switching angles, you are all on, you are on a plane.

If you are considering alpha 1 alpha 2 and alpha 3, you are you are in the 3 dimensional space now. So, your desired V 1 may not be a curve, it itself will be a surface and again, when you say V 7 is equal to 0, you may find multiple surfaces there like this might become a surface and that might become an equivalent surface. So, you are looking at and V 5 is equal to 0 will be another surface. So, you are looking at the intersections of, let us say, three different surfaces, in a three dimensional region.

So, if this idea, you can just go about extending. So, it will give you some kind of an idea, as to what you can start from. Now, in many a times you can solve from this point as I said that 60 comma 90 as I said, there is an equivalent point, at which there are, you know that the wave form is a triple end frequency wave form, it can be a third harmonic or it can be a ninth harmonic or it can be a fifteenth harmonic. You can always come up with some switching angles, for any number of switching angles, alpha 1 alpha 2 alpha 3, say it is possible to come up with that and.

So, at that particular point you will, if the wave form could be a ninth harmonic square wave and at that point, are the third harmonic square with itself. So, at that point you know whatever you, for three switching angles, you will not get a ninth harmonic square wave, but you will get a third harmonic square wave. So, that could be a good point to start your solutions many a times, because that point naturally satisfies Vn is equal to 0 equation now.

So, this is I hope, that you know this is given you some idea of what selective harmonic elimination is there, are several papers and there are books which can give you details of this and they can also tell you the way the angles alpha 1 alpha 2, etcetera, vary with this fundamental voltage, those curves are plotted. And so not doing it, here you find that information here, what I was trying to do here is to get the kind of an insight, to take it, limit it to two switching angles. So, that we can deal with it on a plane, on a graph sheet and get some understanding of this now.

So, you know one of them is to realize that sometimes there could be no solution, there could be multiple solutions and I hope you did develop, some idea of about selective harmonic elimination. So, thank you for your interest in this lecture and I am hoping to see you again in the next lecture.

Thank you very much.