Pulsewidth Modulation for Power Electronic Converters
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Lecture - 12
Low switching frequency PWM - II

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters. So, after a fairly long discussion on various kinds of power electronic converter such as DC-DC convertors and DC-AC converters multi level converters etcetera, we have started working on pulse width modulation now. We are focusing on a three-phase voltage source inverter and we are trying to do some pulse width modulation assuming very few switching cycles within a fundamental cycle now.

So, let us look at a three-phase inverter. So, what we are doing is basically a Low switching frequency PWM that is we are trying to modulate the inverter such a way that the switching frequency is very very low. So, as I mentioned earlier this helps us understand the idea the concepts behind pulse width modulation better, when we deal with low switching frequency, when you deal with very few switching cycles within a fundamental cycle. Also this is relevant for high power drives so where the power levels are so high, now the energy lost per switching transition is so high, you cannot do too many switching transitions have too many switching transitions in a cycle.

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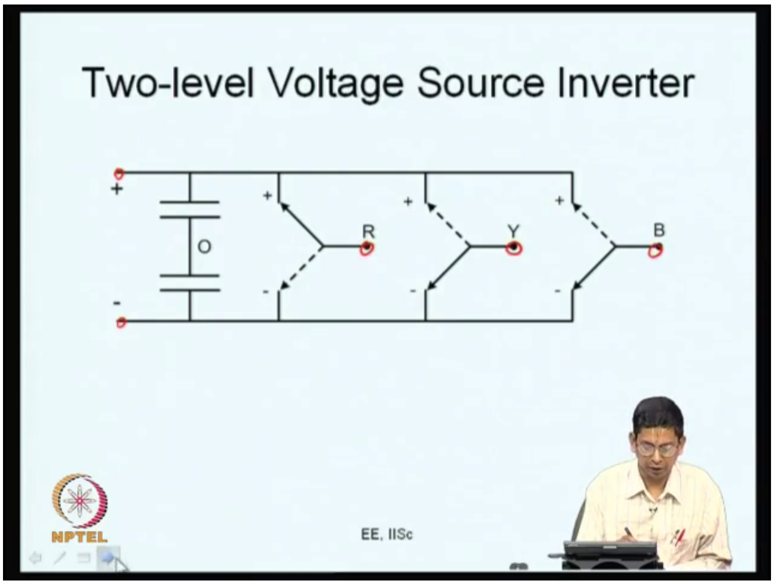

Yes. So, let us just take a relook at a Voltage Source Inverter.

So, voltage source inverter as DC on 1 side and three-phase AC on the other. So, these are the load terminals as we know and these are the DC terminals we know and these are going to be modulated, what do you mean by modulating the R face are any of the poles could be connected either to the top throw or to the bottom throw. Went to change over from top to bottom this is what we are trying to do in pulse width modulation.

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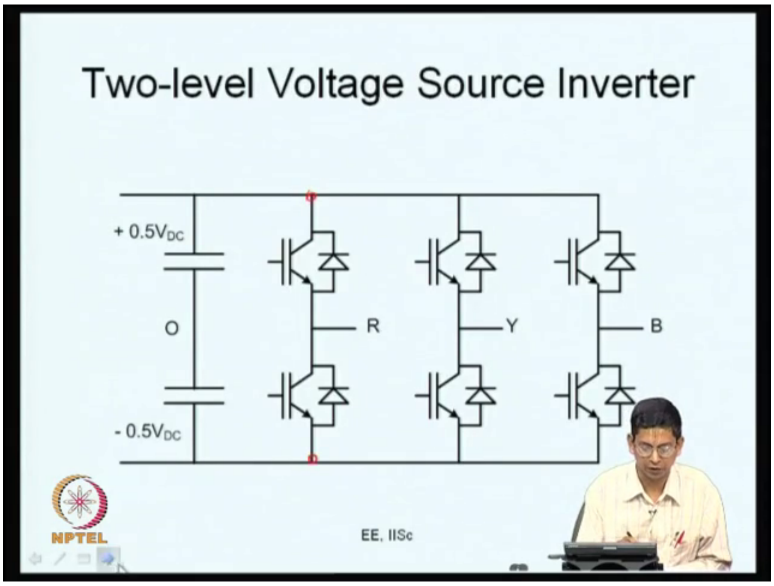

Instead of looking at it as generic switches if you look at it at actual width actual transistors and diodes this is how the circuit looks like. So, every leg has basically 2 transistors with antiparallel diodes and these conducting both the directions and they block voltage in just 1 of the directions as we have seen number of times before.

So, this is like a single pole double throw switch because I if the top is on the bottom will be off vice versa. So, R will be either connected to the positive DC bus or will be connected to the negative DC bus. So, this is a single pole double throw switch as we have seen.

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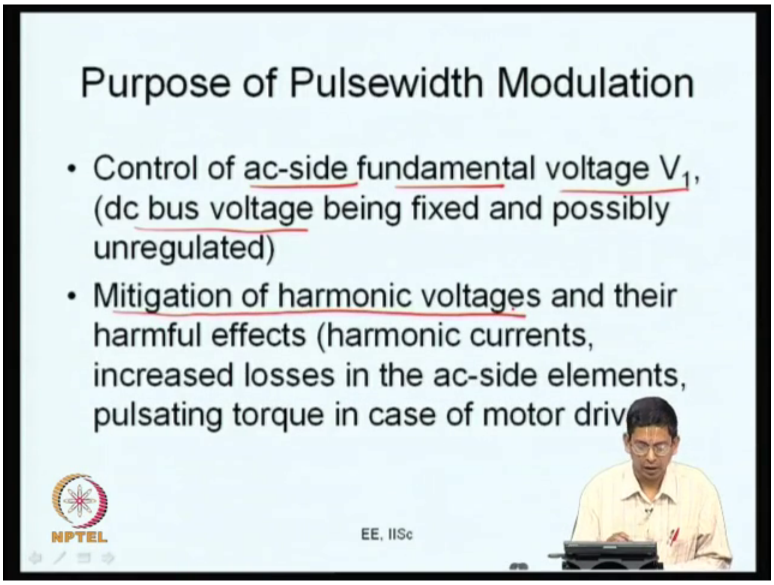

So, this purpose of pulse width modulation is what we want to do is we will have the DC voltage being fixed are you know may be unregulated regulated or unregulated whatever. So, it does not vary by itself. So, I mean it cannot be controlled we want to control the AC set fundamental voltage the question is how to do that. So, the pulse width modulation should help us achieve that desired amount of fundamental voltage now that is something that pulse width modulation is should help us now.

Apart from controlling the fundamental voltage there are several other harmonic voltages which we do not want them we might not want them, but then still they exist. So, what we try doing is we cannot totally avoid them, but we try to reduce or mitigate those reduces harmonic voltages are mitigate their harmful side effects. So, these are the 2 main purposes of pulse width modulation as we have seen earlier.

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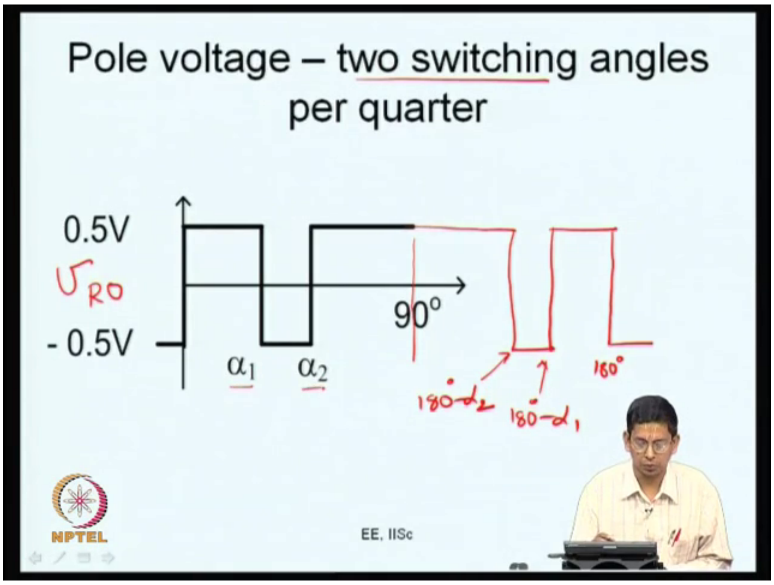

So, what we have been doing us first if you look at a square wave or you can look at just 1 switching angle per quarter cycle you can control with square where you cannot control the fundamental voltage, with 1 switching angle per quarter you can just control the fundamental voltage and you cannot influence the harmonics.

So, now we have been looking at 2 switching angles. So, what we are looking at here is the pole voltage V RO R is the load terminal; R is this is R and O is this. So, here what we are looking at is the voltage at the midpoint of a leg, measured with respect to the DC bus midpoint. So, it is 0.5 vs V R minus 0.5 V where V stands for the DC bus voltage now. So, you have 2 switching's per quarter that is alpha 1 and alpha 2. So, the waveform is all the symmetries and therefore, we are representing only 1 quarter, in case you wish to know how it will look in the other quarters what you can do is about this 90 degree you just reflect this waveform you reflect this wave from about 90 degrees.

So, you will have something like this. So, this is at 100 and 80 degrees and this is 100 and 80 minus alpha 1 and this is 100 and 80 minus alpha 2. So, you will have something it is a you reflect this you can get the other quarter and using half wave symmetry you can get the second half now. So, that is how the actual full wave form is we look at just 1 quarter because that is sufficient for us the switching angles in the different quarters are related to 1 another. So, these are the only 2 independent control variables alpha 1 and alpha 2.

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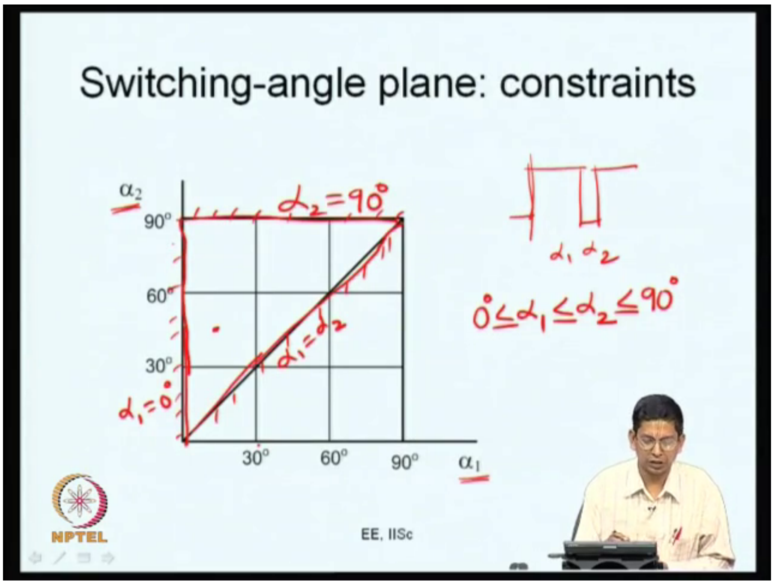

So, we saw that it is convenient to look at all these possible wave forms on a plane, where it is alpha 2 is represented against alpha 1. Every point on this plane you take any arbitrary point on this plane, it represents a pair of alpha 1 and alpha 2 it basically represents some wave form like this. It represents a pole voltage like this with alpha 1 and alpha 2. In this case the alpha 1 I have chosen is roughly 15 degrees and alpha 2 is roughly 45 degrees in what I have indicated here. So, every point represents actually waveform like this now.

So, there are limits on these angles alpha 1 and alpha 2 as we saw last time. So, the constraints are 0 degree should be less than or equal to alpha 1, alpha 1 certainly cannot go below 0 and again alpha 1 is less than or equal to alpha 2. So, alpha 1 cannot exceed alpha 2 or alpha 2 cannot go below alpha 1 and alpha 2 again cannot exceed 90 degrees. So, you have this condition 0 less than or equal to alpha 1, less than or equal to alpha 2, less than or equal to 90. Now this is the line alpha 1 is equal to 0. Now this is the line alpha 1 is equal to alpha 2 and this is the line alpha 2 is equal to 90, this is alpha 1 is equal to 0, this is alpha 1 is equal to alpha 2, and this is alpha 2 is equal to 90 degrees as we have seen before.

So, all the points that we consider are you know they should be within this region. So, outside this region is not acceptable. So, these are our constraints we need to operate within this triangle that is our constraint now. So, we look at all possible waveforms within this and try to get a waveform which gives us our desired fundamental voltage and then we can see whether it can give a better harmonic voltage also, with whether with the harmonic voltage can be reduced also, say if we desire 80 percent fundamental voltage can we get 80 percent fundamental voltage with much reduced fifth harmonic or 7th harmonic voltages for example now.

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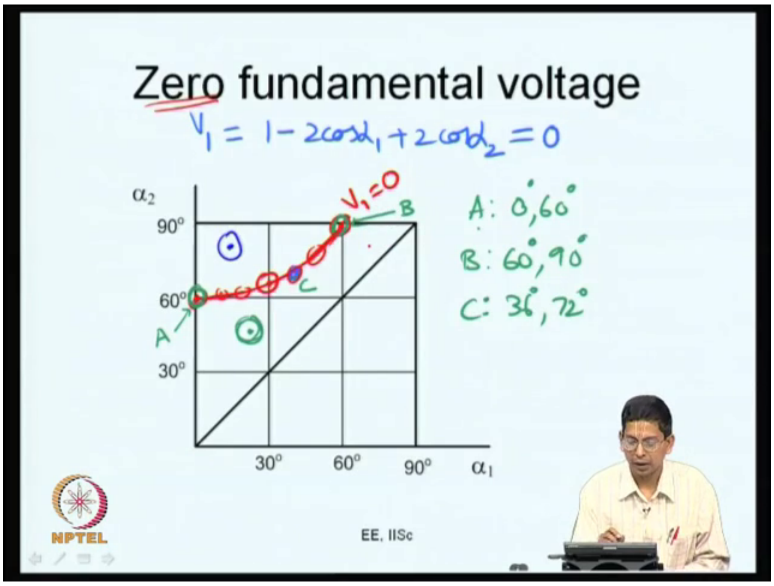

So, this is the bounds within a switching plane. Now for example, if your desired fundamental voltage is 0, the fundamental voltage has this relationship V 1 is equal to minus 2 cos alpha 1 plus 2 cos alpha 2 equals 0, I mean this is for the 0 fundamental voltage.

So, this is the equation for V 1 for we know based on that is how we calculate the Euler Coefficient in the Fourier series. So, you have this equation now if you want V 1 is equal to 0 we got several points, you can plug in several values of alpha 1 starting from 0 to 60 you get corresponding values of alpha 2, go in somewhere between 60 and 90 then we got this curve indicated as V 1 is equal to 0 here this curve we got this is the curve we got. Along this line you have waveforms all of which give you a fundamental voltage of 0 amplitude, the waveforms instantaneous value is not 0 the pole voltage waveforms instantaneous value is either plus V dc by 2 or minus V dc by 2, but it is fundamental amplitude is 0 that is that is what you mean here now.

So, we have taken some examples also 0 comma 60 degree this by you know indicated as point A and 60 comma ninety is the other extreme point indicated it is point B and some points say which is 36 comma 72 degrees.

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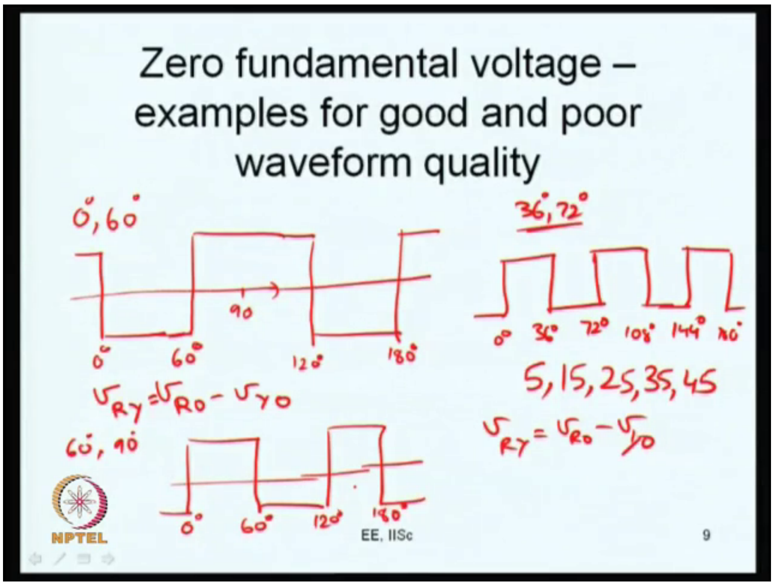

So, we saw that there were some good and bad examples of 0 fundamental voltage now you should take 0 comma 60 degrees, if you take 0 comma 60 degree what happens is there is actually a switching at 0 and at alpha 1. Now alpha 1 becomes 0. So, between alpha 1 and alpha 2 the waveform is going to be negative. So, what you get here is the waveform is going to look like this is 0 and this is alpha 1 which is equal to 60 this is alpha 2, which is equal to 60 degrees and it is like this 190. If you draw this beyond this at 120 degrees it goes low at 180 degrees it goes high, this is the nature of the waveform you have.

So, this is actually a third harmonic square wave as we saw the other day, this is a good example of a waveform with 0 fundamental voltage, why it is having a 0 fundamental voltage and it does not have fifth harmonic component for example, it does not have 7th harmonic component for example, it only has third harmonic components and the third harmonic components vanish when you subtract V RO minus V YO to get your line voltage V RY. So, third harmonic common when V RO is phase shifted by 120 degrees all the triple n harmonics the third harmonic ninth harmonic etcetera, are all shifted by certain integral number of cycles the third harmonic is shifted by 300 and 60 degrees at it is own frequency. So, when V RO and VIOR subtracted the resultant wave form has no third harmonic the resultant wave form will have no 9th harmonic and 15th harmonic etcetera.

So, V RY will be just 0 and it will not have any 5th or 7th or anything like that, all right the same way if you take 630 comma 90 that would be again a third harmonic square, where it switches from low to high at 0 at 60 degree it is from high to low, then at 120 degree it goes back and at 180 degree it comes low. So, this is also a third harmonic square, these 2 are in phase supposition, but that that does not matter both are equally good because when you subtract V RO I mean when you V YO from V RO at 1 pole voltage, from another pole voltage you are not going to get any third harmonics are any harmonics going to be there now right.

Now, we also considered the other example of 36 comma 72, here it switches at 0 degree and it down at 30 degree goes up at 72 if it goes up at 72 72 is 18 degrees before 90. So, 18 degrees after 90 is 100 and 8. So, it comes down at 100 and 8, if it comes down at 100 and 8 it will go back up at 144. So, basically 144 is 180 minus 36 and then it comes down at 180. So, this is 1 half of the waveform and you can see that 1 half of this waveform is actually 2 and half cycles of a square wave at the fifth harmonic frequency.

So, this waveform is also an example of a waveform whose fundamental amplitude is 0 all 3 are examples of the fundamental amplitude being 0 the first 2 have no harmonics, but if you look at the last 1 that I just drew it has a lot of harmonics. It has fifth it has 15th it has 25th, it has 35th and all these harmonics. So, V RO contains all these harmonics, when you subtract V RO minus V YO to obtain your V RY; V RY will also have 5th harmonic, V RY will also have 15th harmonic, 15th harmonic may know will go away because it is a Tripler frequency, but V RY will have 25th harmonic.

So, 5th, 25th, 35th such harmonics will be presenting V RY. So, though you are applying 0 fundamental voltage you are applying certain harmonic voltages are, let us say you are applying not exactly 0 a small amount of fundamental voltage, but what you will end up applying is a large amount of fifth harmonic component and the operation of a motor could be very very bad in that in terms of you know there could be lot of pulsating park and so on.

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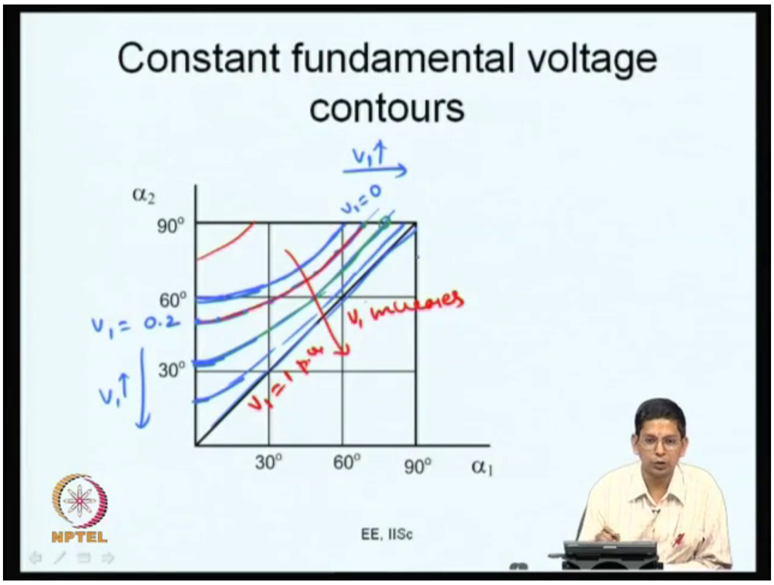

So, this is look at the other constant fundamental curves now. So, the first line that we looked at which is this the blue 1 this is the line V 1 is equal to 0 which we just looked at. And as we go on you know at 0.2 for example, or some such number you know you for a higher fundamental voltage you look at a constant curve some V 1 is equal to constant you got a curve like this you go for still higher value of V 1 and for V 1 is equal to constant you get a value like this and it go on like this now.

So, there are several V 1 is equal to constant curves that we plotted last class and V 1 increases in this direction as you more along and V 1 is equal to 0 along this line and V 1 is equal to 1 along this line. So, between those 2 lines you have any V 1 is equal to constant contour will be between those 2 curves, now let us say we had a look at this slope now. So, what is the slope of this curve it is changing all this V 1 is equal to constant contours of constant fundamental voltage contours their slopes keep changing.

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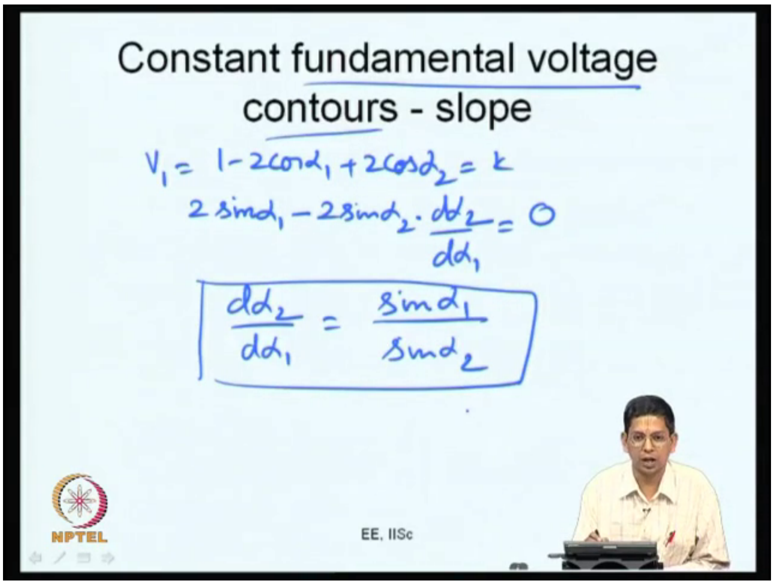

So, let us take some V 1 is equal to constant. So, what is that V 1 is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 this is some constant let us call it k.

So, if you differentiate both sides you end up getting something like 2 sin alpha 1 minus 2 sin alpha 2 d alpha 2 by d alpha 1 is equal to 0. Therefore, you get d alpha 2 upon d alpha 1 is equal to sin of alpha 1 by sin of alpha 2, this is the slope of constant fundamental voltage contour at any given alpha 1 and alpha 2 if it passes through some alpha 1 and alpha 2 sin of alpha 1 by sin of alpha 2 is the slope of the contour. So, whenever it cuts the vertical axis alpha 1 is 0. So, the slope should be 0. So, if you see the slope is actually 0 the last 2 curves shown here it seems like they do not have a 0 slope, but that is basically a drawing inaccuracy and you need to be careful about it and I am sure you can do better drawing than what I have done.

So, it is basically it should be something flat and going like this it should be basically something flat and going like this. Now whereas, on the other end if you look at alpha 2 is equal to ninety it is sin alpha 1 by sin alpha 2. So, sin alpha 2 is sin 90 is 1. So, the slope is simply equal to sin alpha 1 which is 1 of the last observations that was made in the previous lecture now and it is more or less it is like a line with slope is equal to sin of alpha 1 at those points now. So, that is what we may have to observe about the slope of all the constant fundamental voltage contours now.

So, shall we say what happens when we go beyond that though we are not really interested we are not interested beyond this region, but let us say if we go beyond what will happen this curve will go on increasing and what will happen to the slope, at some angle at some angle sin alpha 2 can become 0 what is that angle at which sin alpha 2 can become 0 after 0 it is 180 degrees.

So, at 180 degrees at 180 degree these curves they will go on and there they might become vertical these curves will tend to become vertical and again when you come back when you come back it is going to be 0 when it cuts the x axis. So, let us look at that when we actually look at the constant harmonic voltage contours what I am trying to say is these curves are. In fact, close curves they not open curves you are only seeing a portion of those closed curves and this will be clear when you look at the constant fundamental voltage contours which we are going to see in a little while from now.

So, the slopes are really changing like this and the slopes will help you plot them very very easily.

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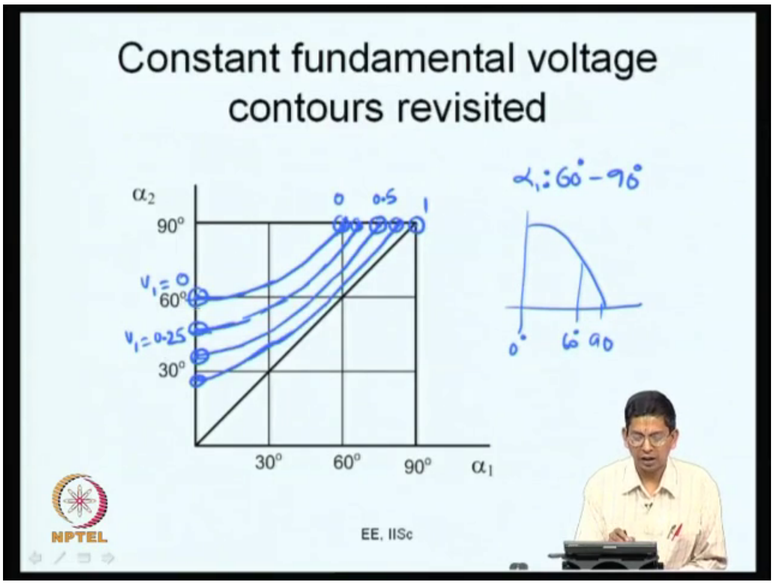

So, now, what we need to do let us say I want to plot all of them now, I have a good knowledge of the contours the nature of the contour some knowledge of them at least.

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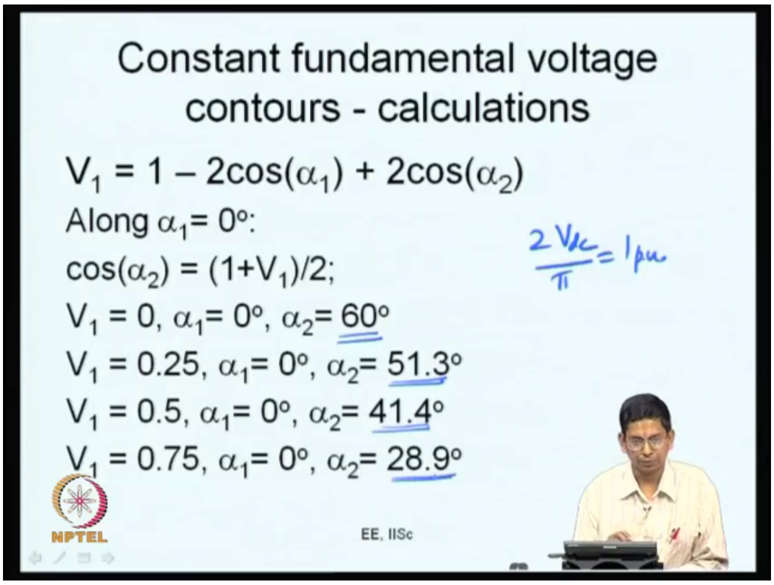

So, what I need to do is let us say, I look at the contours some do some calculations what is that V 1 is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 this is in per unit term here 1 per unit stands for the amplitude of the square wave during I mean amplitude of the fundamental during square wave operation. So, it is basically 2 V dc upon pi. So, that is 1 per unit that is the base quantity here all right.

So, 2 V dc upon pi is taken as 1 per unit. So, that is what is taken there and the 2 V dc by pi term essence not shown explicitly. So, if you take along alpha is equal to 1 your cos alpha 2 is simply when alpha 1 is equal to 0 this 1 minus 2 cos alpha 1 becomes totally minus 1. So, cos alpha 2 is 1 plus V 1 the whole divided by 2 1 plus V 1 the whole divided by 2. So, here you can plug in several values of V 1 to get the corresponding values of alpha 2.

So, you want V 1 is equal to 0 at alpha 1 is equal to 0. So, when V 1 is equal to 0. So, you have cos alpha 2 is equal to 1 by 2 that is 0.5 therefore, alpha 2 is equal to 60 degrees it is fairly easy. Then if you are looking at V 1 is equal to 0.25 then your cos alpha 2 equals 1.25 by 2 that is something like 0.625 and the corresponding alpha 2 is 51.3 degrees and if you are looking for V 1 is equal to 0.5 along alpha 1 is equal to 0 then the corresponding value of alpha 2 is 41.4.

Similarly, if you are putting 0.75 if you are interested in V 1 is equal to 0.75 you going to get alpha 2 is equal to 28.9. So, let us go back now if you want V 1 is equal to 0 along alpha 1 equal 0 this is your V 1 is equal to 0 let me call this is 0 here right 0 here this is V 1 is equal to 0. Then if I want 0.25 along this line what I got was alpha 2 is equal to 51.3 degrees. So, this is something like 51 degrees let me say. So, this is my V 1 equals 0.25 and if I want V 1 is equal to 0.5 the angle is something like 41.4 degrees. So, the angle is 41.4 or here something like this again there are certain drawing inaccuracies anyway this is 41.4.

So, now you go to V 1 is equal to 0.7 5 it is a little lower than 30 degrees there is 28.9 somewhere here is there. So, we know that these are the points for V 1 is equal to 0.2 5 etcetera. So, if I want to finish that on the other side V 1 is equal to 0 what I need to do is let us say I can say how it passes through alpha 2 is equal to 90.

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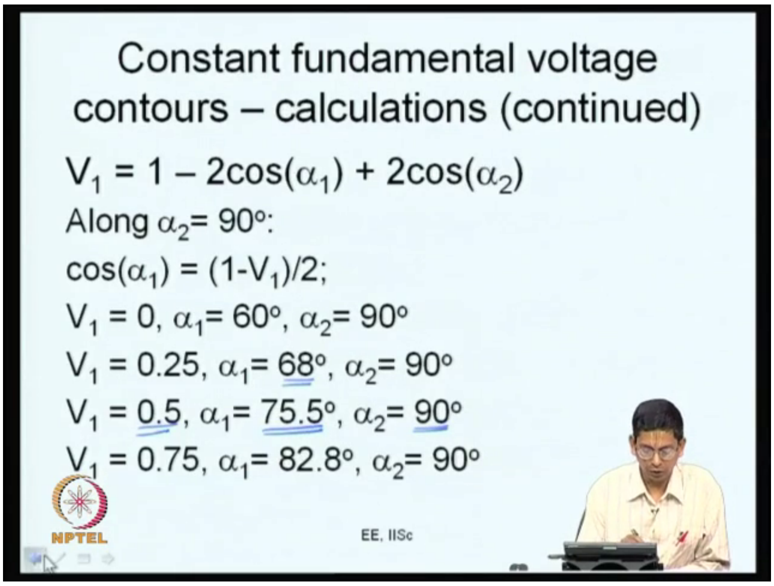

It is easy for me to consider alpha 2 is equal to 90. So, V 1 is the same expression 1 minus 2 cos alpha 1 plus 2 cos alpha 2 if I consider along alpha 2 is equal to 90 degree I have cos alpha 2 is 0. So, that makes the calculation simpler. So, cos alpha 1 is equal to 1 minus V 1 the whole divided by 2.

So, if I want V 1 is equal to 0 along alpha 2 is equal to 90 alpha 1 is 60 if I want V 1 is equal to 0.25 along alpha 2 is equal to 90 alpha 1 is 68. Again for V 1 is equal to 0.5 along alpha 2 is equal to 90 it is alpha 1 is 75 degrees. So, if I look at it here on the on 2 here this is my 0 this is my 0 and this is my 1 just square wave operation there .If I want 0.5 I can roughly take the middle of this which is around 75 degrees alpha 1 is equal to

75 why can I take it as how can I take it as roughly in the middle along this line what varies is just alpha 1 alpha 2 is 90 and what varies alpha 1 what is the range over alpha 1 varies alpha 1 varies between 60 to 90 degrees. So, if you consider cos of alpha 1 and it is variation between 60 to 90 it is a fairly it is fairly linear. So, if you look at cos alpha 1 this is at 0 this is at 90.

So, between 60 and 90 it is fairly linear it is a sinusoidal curve any sinusoidal curve closer to it 0 crossing it is fairly linear. So, that is what is happening now. So, if you 0.5 do not be exactly at 75 degrees just a little further as we got here it is you know it has been 75.5, again if you want V 1 is equal to 0.2 5 it will be quarter the distance from here to there. So, which I would say 67.5 the actual number could be 68 degrees. So, which is not too far similarly for 0.7 5 I will have it here which is 3 fourth the distance from 0 to 1 along this line.

So, I have all these points I can actually go about joining them the other thing that I know is the slope what is the slope here.

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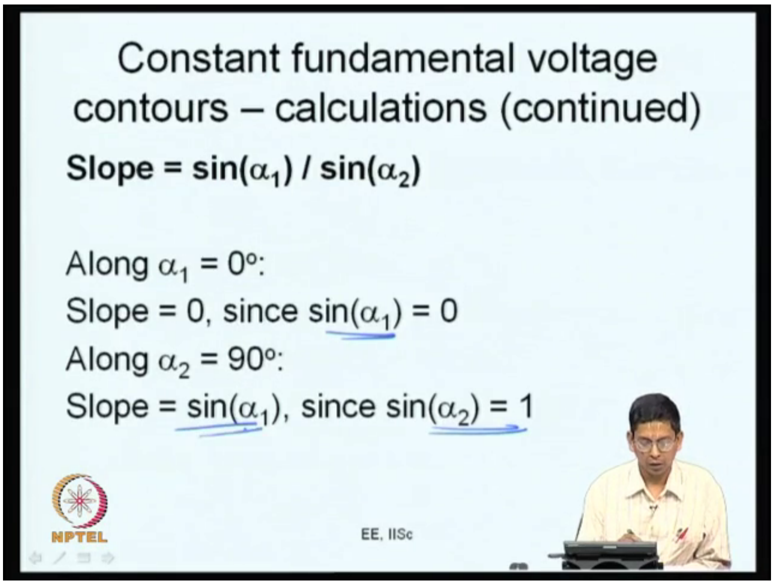

The slope is sin alpha 1 by sin alpha 2 just as we derived a little back while back. So, along alpha 1 is equal to 0 your slope is 0 because sin alpha 1 is 0. Again along alpha 2 is 90 you have sin alpha 2 is equal to 1 therefore, slope is just equal to sin alpha 1. So, if you go back here I know that the slopes here are 0 I know that the slopes here are 0 the curve is flagged them. And how about the slopes there here the slope is equal to sin 60 that is root 3 by 2 here the slope is equal to 1. So, in all these cases the slope is somewhere between root 3 by 2 and 1 that is basically equal to sin alpha 1 now.

So, with these 2 idea you know I can roughly sketch all these curves that I want to I am just going to sketch these curves. So, this is just for us to clear up our understanding of the constant fundamental voltage contours now. So, these curves are you know there are of 0 and then that there is some slope here as we go on and these are the varied kinds of curves now. So, once you have these what we can do is let us see what we want mean by a PWM method you have several of these contours.

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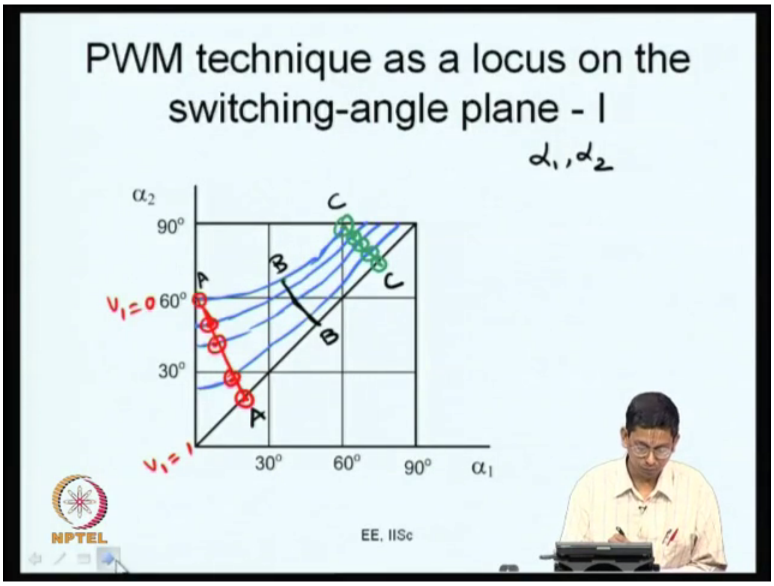

So, let me not let me just draw 1 or 2 of them let me just draw a few of them something like this. So, what do we mean by PWM technique? You should be able to control the fundamental voltage, how can you control the fundamental voltage, you want a design fundamental voltage. Let us say I want this fundamental let me take a different colored ink let me say I want 0 or something very close to 0. So, let us say I want operate here.

Now I want some other fundamental voltage I can operate on any point on this curve I can operate on let me just draw a few more curves for clarity. So, let us say here. So, let us say I want a higher fundamental voltage now. So, higher fundamental voltage corresponding to the second curve that I have drawn here; I can operate basically on any point along this to get that fundamental voltage. So, let me say I operate on this point again I want fundamental voltage equal at to the what the third curve gives again I can choose any point along this third curve let us say I choose this red point.

Similarly, for the fourth blue curve it has some constant fundamental voltage let us say it is something like 0.8 or so I want V 1 is equal to 0.8 I can operator along at any point along this curve on this curve let us say I choose this point now. So, basically then I want to go to say full voltage per 100 percent voltage or 1 per unit 2 V dc upon pi it is basically square wave. So, I can do this now. So, what I can do is it is basically what I am doing is I am moving from here all the way to here.

So, this red line now represents a PWM technique, we are considering a three-phase voltage source inverter, we are considering that the it is switched with all the waveform symmetries, and we are assuming that there are only 2 switching angles per quarter. Under these conditions if we want to define a PWM technique, I can just define a PWM technique, as a locus of a point on this plane and this locus begins at V 1 is equal to 0 it can also begin from some V 1 is equal to V minimum if we are not interested in 0 and ends in V 1 is equal to 1. It could be some V 1 is equal to V max this is different less than 1.

So, this is you can define a PWM technique as a locus of a point moving on this plane, it starts from V 1 is equal to 0 contour and finally, ends in V 1 is equal to 1 contour or V 1 is equal to V max contour whatever your V max could be alternatively this is not the only possibility there are several other possibilities let me choose a different color ink here we go.

Now I say I operate here when I want 0 fundamental voltage after all the red point is a good starting point for V 1 is equal to 0 this green point is also good starting point for fundamental voltage is equal to 0 as we had seen a little earlier . So, now, I want to operate at V 1 is equal to 0 I can choose to operate at this green point here. Now I want a slightly higher fundamental voltage something like 0.2 or 0.25 as indicated by the second curve now, I can choose to operate at any point along this curve, but let us say I choose to operate here. Again on the third curve let us say I choose operate here on the fourth curve let us say I choose to operate here and I choose to operate here that is it becomes square wave now.

So, this is again the locus of a point, this is again locus, this locus again starts at V 1 is equal to 0 and ends at V 1 is equal to 1, this is another PWM technique this is another PWM technique . So, you can define a PWM technique as a locus of a point moving on the switching angle planes starting from 1 point on the curve V 1 is equal to 0 and ending at 1 point and the curve V 1 is equal to 1. So, that is another way. So, let me take another point here like here let us say shall I use a different colored ink yes.

So, let us say I use this top this can also be the 1 that I have drawn towards the end can also be a PWM technique. So, this is also because it is a locus of points. So, what all these curves represent certain relationship between alpha 1 and alpha 2, how many independent variables do we have we have 2 independent variables alpha 1 and alpha 2 and every 1 of this curve or every 1 of this locus use certain relationship between alpha 1 and alpha 2. So, they are actually reducing our problem to a single variable problem. So, V 1 can is now related just to alpha 1 or just 2 alpha 2 once you have chosen A particular curve. So, it simplifies the problem for you.

Now along this curve which is good which is bad we need to look at the harmonic contents already we have seen along this like for example, let us just call this as A B and C these 3 curves is A B and C. We have already seen A is good, A is good as far as 0 voltage is concerned, C is also good as well as 0 voltage is concerned, but B is not. So, good we have already seen this now, but this is at 0 voltage which can be extended this knowledge can be extended to small amount of fundamental voltages if this is going to produce A very high amount of harmonic at V 1 is equal to 0 what it means is basically for small values of V 1 like 2 percent or 5 percent or 10 percent you can still expect the harmonics to be very high that is with B.

If A and C are expected to produce low amount of harmonics at 0 they can be expected to produce low amount of harmonics at small amount of fundamental voltages also we will be comparing A and C sometime later, but these are just some examples of PWM techniques which I wanted to give now. There could also be other examples now here I have somehow tended to draw I have had a tendency to draw all of them straight lines because straight lines the equation between alpha 1 and alpha 2 simplest and the calculations are a lot simpler right.

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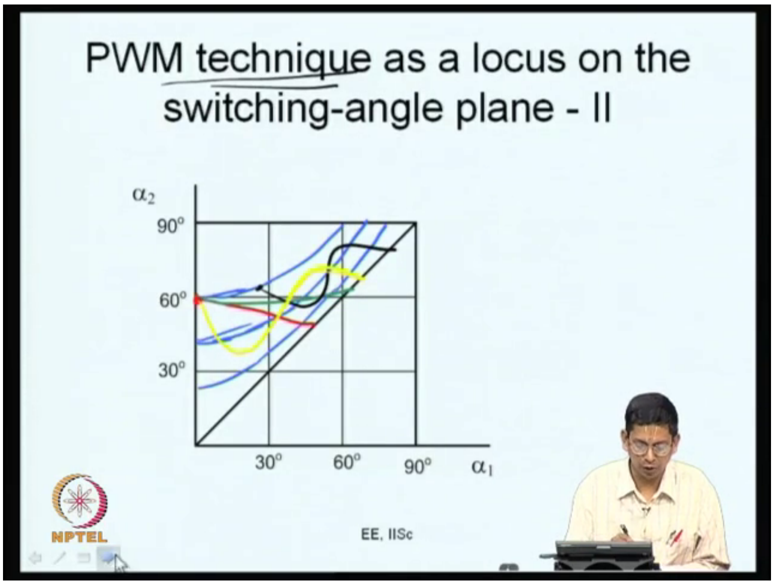

Now, I do not have to do that now let me say I go in for many more like the same kind of V 1 is equal to constant contours let me take we can now plot many of them fairly quickly right. So, let us say we have this. So, let us say we have these contours going like this just a few of them have drawn here it does not have to be a straight line, it can be anything else also. For example, I can say I can give 1 example maybe I can start here and end up here, I can start here maybe are another example I will indicate with a different color, I can start here and I can end somewhere there else also all these are PWM techniques.

Let me take another example I can start here and I can go up come down now what is the problem with this black 1 well the starting point is not very good there is another issue. So, let me say let me say the starting point is all right let me follow this yellow curve with this yellow curve the starting point is, but then as you move along this locus V 1 is not monotonically increasing. So, you have multiple values sometimes V 1 goes on increasing here and V 1 reaches some local maximum and after that V 1 V 1 starts decreasing and as you move along here. And finally, V 1 which you receive some minimum on once again it starts increasing there.

So, it is not monotonically increasing what you prefer to be is all these curves are all examples of low say along which V 1 increases monotonically, if you move from point a and let us call this point as A dash, let us call this other point as A dash, this is B let us call this B dash, let us call this C C dash you move from A to A dash the fundamental voltage is monotonically increasing B to B dash, or C to C dash it is monotonically increasing. So, at a you know for every fundamental voltage you have only 1 point along the locus that gives you that whereas, if you take this example of the yellow curve here there are several points, you know there are multiple points, that give you the same fundamental voltage and this can cos confusions and create troubles when you are implementing them.

So, by and large we would want the PWM technique as a locus on the switching plane starting from V 1 is equal to 0 and ending with V 1 is equal to. So, some point and V 1 is equal to 0 and ending at some point on V 1 is equal to constant not just that we also want we want to be increasing monotonically as the point moves from V 1 is equal to 0 to V 1 is equal to 1. So, that is the other 1 that we want to say.

So, you can look at several examples any curve that you are draw here is going to be a PWM method whether it is good or bad, it depends on the harmonic analysis we can do some harmonic analysis and find out if it is good or bad what is sense there.

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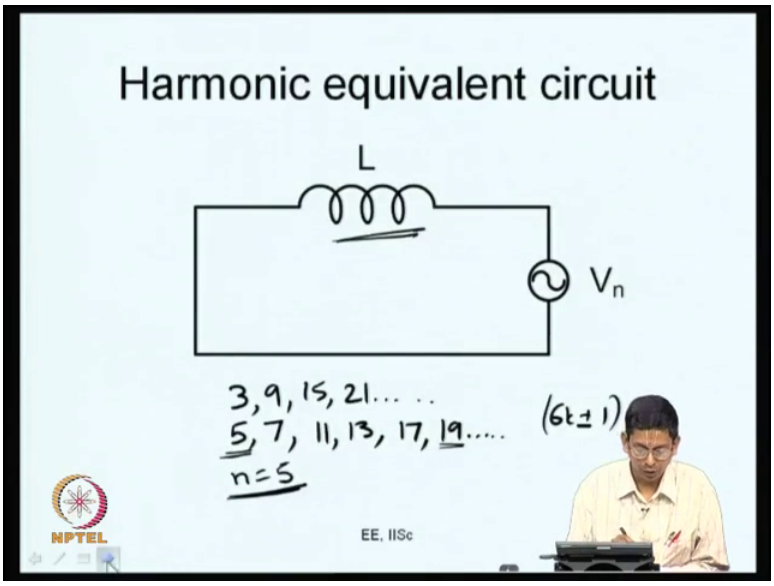

So, when you say a harmonic analysis we have to look at V harmonic which harmonics there are. So, many of them this 5 there is 7 there is 11th, I mean the thirteenth and so on. We are not worried about 3 9 etcetera we are not worried about 3, 9, 15, 21 etcetera why because if the 1 pole voltage contains 3, 9, 15 etcetera even if the pole voltage contains them the line to line voltage would not contain means. So, that will not get applied on to the motor, on the other hand we will be worried about voltages harmonic orders 5, 7, 11 13, 17, 19 etcetera. These are all basically 6 k plus r minus 1 you can all these are you can call them a 6 k plus r minus 1 where k is any integer or as a positive integer and you say plus r minus 1. So, 5 7 11 13 etcetera these are the harmonics which will cause you trouble now.

So, what we have seen is the harmonic voltages see such a harmonic equivalent circuit. So, if you say fifth harmonic it sees an inductance and the reactance is 5 times the fundamental reactance. If you take 7th harmonic this reactance is 7 times the fundamental reactance. So, if you take 19th the 19th harmonic voltage sees 19 times the fundamental reactance. So, as the harmonic order increases the reactance increases therefore, what can you say it is the lowest order harmonic n is equal to 5 that is going to see the lowest value of harmonic reactance and therefore, it could end up producing quite a substantial amount of harmonic current.

So, we are looking at 5th harmonic 7th harmonic 11th 19th all these are problematic for us. So, the 1 that is arguably most problematic is the fifth harmonic voltage because for a given amplitude of voltage fifth harmonic voltage is capable of producing or is going to produce of much higher fifth harmonic current than let us say 7th or 11th or 13th.

So, we would target n is equal to 5 we will say our immediate attempt would be to look at n is equal to 5 and see how good or how bad it is and we will see 4 to achieve the same fundamental voltage can we reduce fifth harmonic voltage.

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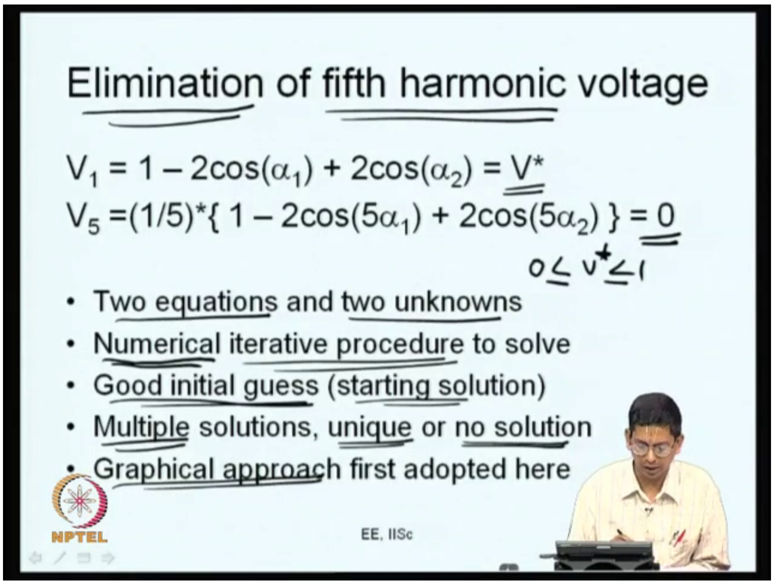

So, what goes without saying is if you want to reduce can it straight away make it 0. So, fifth harmonic voltage is the most problematic voltage in the sense that for a given amplitude fifth harmonic voltage will produce greater amount of the highest of greatest amount of harmonic current among all the harmonics is just 5 7 11 13 and etcetera. So, I want to reduce.

So, the best thing to do is to totally eliminate this at least from this point of view well there is 7th 11th are also going to cos some problems, eliminating fifth harmonic by itself might not be a very good idea you know it can cos to you know pulsating torque in motor drives as we will see a little later, but let us say I want to eliminate now fifth harmonic I am not worried about all that, I am just now my intent is to eliminate fifth harmonic. So, I am making that is equal to 0.

So, what I want is I want a fundamental voltage V 1 and that is given by 1 minus 2 cos alpha 1 plus 2 cos alpha 2, I want it to be equal to some reference voltage or desired voltage this V star is might decide voltage it could be 0.2 5 it can be 0.5 it can be 0.7 5 or 0.8 or whatever I want, but I want also the fundamental I mean the amplitude of the fifth harmonic to be 0 this is what I want can I get this yes I have 2 equations here and I have 2 unknowns.

So, I can solve for them I have 2 equations and 2 unknowns and I can solve for them, but how do I solve them these are not linear equations these are non-linear equations these are trigonometric equations these are transcendental equations. So, you can not solve them straight away. So, what we would normally do is we will adopt some numerical iterative procedure we will adopt some numerical procedure and I am sure you are all familiar with some numerical methods are the other that has Newton Raphson method and Gauss method and there are several such methods which are taught in the undergraduate course on computational methods or numerical methods in almost all universities.

So, I am sure you would have done them as part of this and you know there are other courses on NPTEL forum which certainly you know give you a few things about computational methods there are lots of computation based courses available well they can give you a very good feel for these various computational procedures. Now what I am going to say is all that you need is numerical iterative procedure though we are not going to get into the details of this year. So, the numerical iterative procedure you will formulate the equations for solving and you will start with the good initial guess and with this good initial guess and you go on doing iteration and it will finally, go on give you a solution which is very close to the solution the actual solution and you will stop some were there.

So, what you need to do this numerical solution is you need to start with a good initial guess or what is called as a starting solution you need a good starting solution from there. So, to solve these equations you can actually solve from any starting equation I can just say alpha 1 comma alpha 2 equals 0 degree that is what comes to my mind, but it may or may not be good for example, the alpha 1 is equal to alpha 2 is equal to 0 yes the square wave operation I may be interested in V star is equal to 0.5 the reference voltage is 0.5 I want eliminate fifth harmonic voltage I might be fairly far away from the point where my final solution lies. So, if I have a feel for the problem if I have a feel for these equations then I might be in a position to come up with a better initial guess now.

Now, if I solve I will come up with some answer if I change my initial guess let us say I can start with an initial guess of alpha 1 is 10 degrees and alpha 2 is 15 degrees I may end up with some solution I may start with an initial guess of alpha 1 is 70 degrees and alpha 2 is 75 degrees if I do that I may end up with another solution. So, it is possible for you to sometimes get multiple solutions maybe more than 1 solution exists now. Sometimes the solution could be unique that is no matter what your initial guesses sometimes it may not convey whenever it can finally, converge to a particular solution only. So, the solution could be unique also you know you never know then there could be no solution also. For some range for certain values in 1 if I can give you an example straight away take V star is equal to 1 per unit take V star I want square wave operation, with square wave operation V 1 is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 is equal to 1 that is alpha 1 is equal to alpha 2 is what a square wave operate it is just square wave. In a square wave a fifth harmonic cannot be eliminated see square wave is a square wave it has a fifth harmonic whose amplitude is 1 fifth of the fundamental voltage. So, it is not eliminated you are not switching at all. So, how can you eliminate the fifth harmonic voltage?

So, when V star is equal to 1 or V star is something close to 1 like 0.9 9 or 0.9 8 you should expect that there could be no solutions at all to this problem then. In fact, there are no solutions to the problem and in some regions there could be unique solution in some regions there could be multiple solutions 2 or maybe 3 I do not know at this point of time.

So, 1 question that I have on my mind is can I at least this is the second equation there is nothing then the first equation there is V star there is V star and this V star has a complete range that is 0 to 1 V star 0 less than or equal to V star less than or equal to 1. Is there a range of V star for which I have no solution can identify that range, is there a range of V star for which I might have a unique solution can we identify that range, is there a range of V star for which there are multiple solutions say 2 or maybe 3 can identify that range those are the questions that I have on my mind.

So, let us see we can answer those before solving all this you can actually solve these problems you can put V star is equal to point data whatever you want and you can go about solving them now our approach is going to be little different, we are going to take a graphical approach first now, you going to take a graphical approach first we can take a graphical approach because we are only looking at 2 switching angles at this point.

So, this will give us some more understanding some better understanding better feel of this problem now. So, let us take a look at that.

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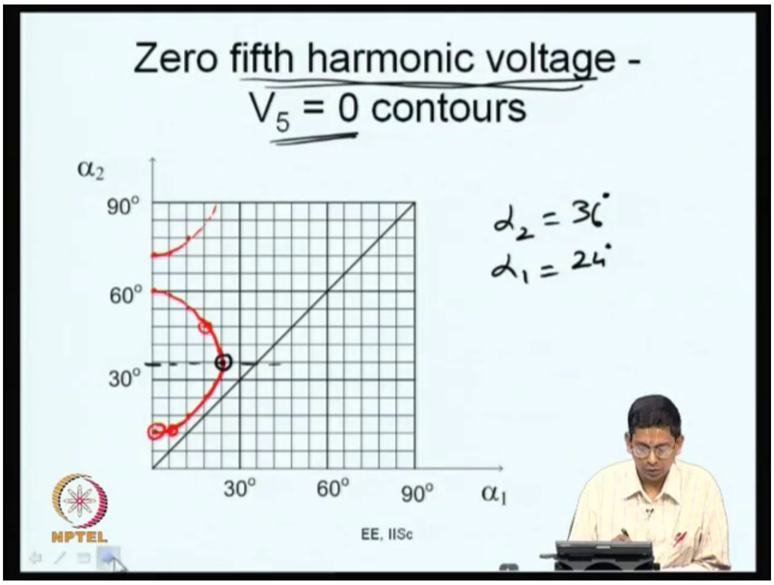

So, what do I mean by graphical approach now we had earlier drawn V 1 is equal to constant contours certain constant fundamental voltage contours.

Now, I want to see I want to focus on fifth harmonic voltage I want to focus on fifth harmonic voltage and I want the fifth harmonic voltage to be 0. So, what I want to do is I want to identify the set of all points on this alpha towards is alpha 1 plane on which V phi is equal to 0, can is identify the set of all points in this plane and within the region of interest where alpha V phi is equal to 0 that is my question now how can I do that well we can follow an approach similar to what we did for constant voltage counters.

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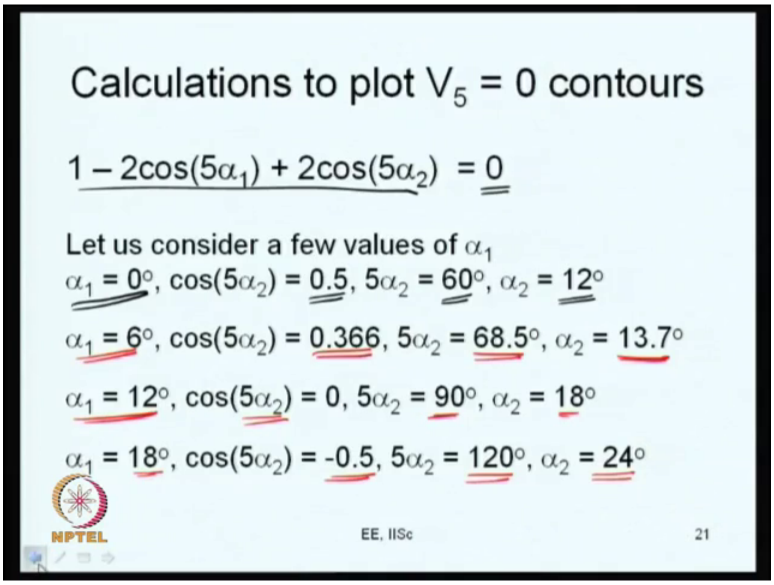

So, what did we do for constant voltage contours we start out with the equation we said 1 minus 2 cos alpha 1 plus 2 cos alpha 2 is equal to some 0.6. So, here we are focusing on fifth harmonic. So, we are saying 1 minus 2 cos phi alpha 1 plus 2 cos alpha phi alpha 2 this is your fifth harmonic right of course, it is 1 by a factor when you say V phi is equal to 0 basically 1 minus 2 cos phi alpha 1 plus 2 cos phi alpha 2 is equal to 0 this means 5th harmonic voltages 0.

Now, what you can do is you can plug in a few values of alpha 1 and find out corresponding values of alpha 2 alternatively you can plug in certain values of alpha 2 and come up with certain values of alpha 1 fine we can do that. So, we will get a feel for you will be able to find out a few points along mean at which fifth is harmonic is 0 and we can connect them and we will get some idea. So, let us start with alpha 1 is equal to 0 it is our friend 0 is our friend. So, because alpha 1 is equal to 0 cosine of phi alpha 1 or anything you know is cos 0 cos 0 is 1 we know that very quickly. So, if we can solve this now.

So, when you say alpha 1 is equal to 0 cos phi alpha 1 is 1. So, 1 minus 2 is minus 1. So, when it goes to the other side it cos phi alpha 2 is equal to 1 by 2 that is 0.5. So, when cos phi alpha 2 is 0.5 what you get is 5 alpha 2 is 60 degrees phi alpha 2 is 60 degrees, if 5 alpha 2 60 degrees alpha 2 is 12 degrees is it right. So, alpha 1 is equal to 0 and alpha 2 is equal to 12 degrees is a solution for V phi is equal to 0 at this 0.0 comma 12 you have fifth harmonic is equal to 0.

So, shall we go and plot that point on this; what is that point alpha 1 is 0 and alpha 2 is equal to 12. So, alpha 1 is equal to 0 and alpha 2 is equal to 12. So, let me use this shall I use a different color 5 let me use a red color here. So, I take this point now this is my point V phi is equal to 0. Now let me look for next point what could be next point I am taking alpha 1 is 6, why do I take alpha 1 is equal to 6 the arguments of the cosine functions are 5 alpha 1. So, 5 times 6 degree is 30 degree we are very comfortable with cos 30 cos 60 etcetera rather than cos 27 or cos 64. So, you know if I it is cos 30 it is comfortable for us we can readily come up with number we are familiar with those. So, let me take 5 alpha 1 is 30 is basically alpha 1 is 6. So, let me take alpha 1 is equal to 6 degrees.

So, when I say alpha 1 is 6 I have cos 5 alpha 1 is cos thirty and cos 30 is equal to root 3 by 2. So, 2 into root 3 by 2 is simply root 3. So, it is 1 minus root 3. So, you it goes to the other side and so you get if you plug in alpha 1 is equal to 6 into z equation you get cos 5 alpha 2 is equal to 0.3 6 6 you get cos 5 alpha 1 is equal to 0.366. So, cos 5 alpha 2 it is 0.366 mean that 5 alpha 2 is 68.5 degree this is 5th 68.5 degrees and corresponding alpha 2 is 37 13.7 all right.

So, similarly what I can do is I have 12 degrees let me just go on plot that this is 6 comma 13.7 this is 6 comma 13.7. So, this is something very close to this is 12 and 8. So, this is something like this 6 comma that is the point now maybe I should not even put a circle around that that is probably easier all right. So, when I go to alpha 1 is 12 again you know 5 times 12 is 60 degree it is easy for us to come up with. So, I am taking alpha 1 is equal to 12 and if I do that I get cos 5 alpha 2 is equal to 0. So, 1 of the solutions is 5 alpha 2 is 90 and alpha 2 is 18. So, if you go back here. So, what I am seeing here is 12 comma 18 is a solution now. So, this is 12 comma 18. So, 12 comma 18 is a solution, then I go move on let us say alpha 1 is equal to 18 this is been our approach. We are plugging in some value if alpha 1 finding the corresponding value of alpha 2 if you plug in alpha 1 is the 18 cos 5 alpha 2 is minus 0.5 if that is there 1 of the solutions for 5 alpha 2 is 1 20 degree and alpha 2 is 20 4 degrees.

So, it is 18 comma 24. So, alpha 1 is 18 alpha 2 is 24 this is the point now. Then let me go about increasing I have been increasing alpha 1 starting from 0 let me go here. So, what I am going to do I will go to the next point alpha 1 is 24 alpha 1 is 24.

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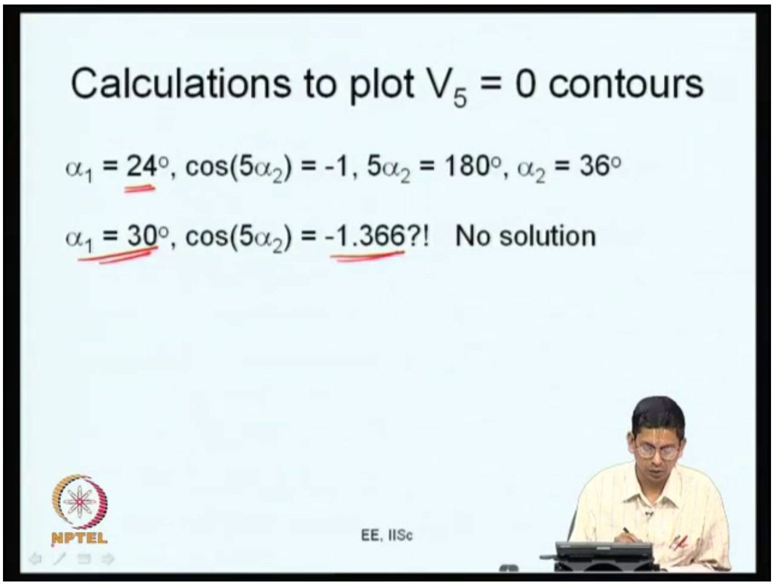

So, corresponding 5 alpha 2 is minus 1 and I am getting 5 alpha 2 is 180 or alpha 2 is 36. So, alpha 1 is 24 and alpha 2 is 36 alpha 1 is 24 alpha 2 is 36 is the point here. So, this is the nature of movement here I hope you can see these red points this is the nature of the movement as I am going on increasing alpha 1 let me go increase my alpha 1 further alpha 1 is 30. So, what do I get cos 5 alpha 1 is something more than minus 1. In fact, if I substitute any alpha 1 which is greater than 24 degrees like 24.5 or 25 I would end up getting cos by alpha 2 is beyond minus 1 more negative than minus 1 which is not a permissible value. So, there is no solution. So, what does it mean it means for alpha 1 is equal to 30 there is no value of alpha 2 which will satisfy the equation V phi is equal to 0.

So, along this line alpha 1 is equal to 30 if you go you can go at any point you will not find a point at which the fifth harmonic voltage is 0, whatever you have done is you can join these points you can join these points and be happy, you can join all these points, you at least found out a set of points along which are at which the fifth harmonic is equal to 0. Now the question is R these the whole set of points are there more set of points now, that is a very important question now, have you identified the set of all points in this plane within the region of interest or have we identified only a few of all possible points now this is something that many students do quite often.

So, when I say cos 5 alpha 2 is 0.55 alpha 2 is 60 is only the principal solution is only 1 of the solutions there are several other solutions, 5 alpha 2 is in cos 5 alpha 2 is 0.55 alpha 2 is 60 is 1 of the solutions it can also be 360 minus 60 it can also be 360 plus 60 and so on. So, if it is 360 minus 60 what happens it is 300. So, a 5 alpha 2 can also be 300 if 5 alpha to can also be 300 you can say that alpha 2 can also be 60 degrees, alpha 2 can also be 60 degrees.

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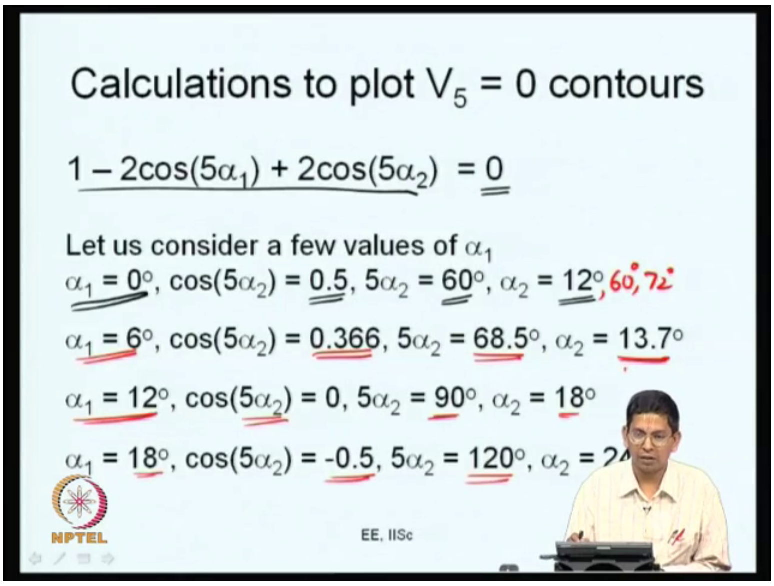

So, where is that 60 degrees here let me come here alpha 1 is 0 and alpha 2 is 60 is another point here.

Similarly, you go to the next 1 you say alpha 1 is 60 cos 5 alpha 2 is 0.366. So, 5 alpha 260 8.5 is only 1 of the solutions, 5 alpha 2 is also equal to 360 minus 68.5 360 plus 68.5 and so on. So, here also you have 12 16 is you know 360 minus 16 the earlier case it is also 360 plus 60 which is 420. So, 420 by 5 is 72 degrees. So, not just that I also had 72 degrees here so 72 degrees is like this 72 degrees is like this now.

So, let us just consider 1 other set of solution if it is 360 minus 68.5 your alpha 2 is going to be something different 360 minus 68.5 there is something like 2 91.5 to 91.5 divided by 5 is something like 58 or so what you are going to get is like this what you are going to get is like this now.

Similarly if you consider the second solution right 360 minus whatever then you will find that these are the points you will get these are the points you will get. So, if I come here alpha 1 is equal to 18 for example, cos 5 alpha 25 alpha is 120 or it can also be 240. So, 240 by 8 is 42 degrees. So, 18 comma 48 is another solution this is 18 this is 36, 42, 48 is another solution now.

So, what I will have is I can also do this like that this curve is. In fact, something like this gives a fuller picture what we want, but not full yet I mean it is not complete yet, why we have considered only 1 other set of solutions that is cos 5 alpha 2 we have come up with 5 alpha 2 is it is principal solution or 360 minus the principal solution, it can also be 360 plus that. So, if you do that you will start getting points here it will start getting a few points like this you will you will get a curve going up like that. So, what do these mean various solutions are there are several solutions for a particular value of alpha 1 you get several solutions of alpha 2. So, you go about getting many of the solutions now.

So, let us say you have drawn this now. So, let us now get a little wiser the set of all points let us now get a little wiser it is all right to calculate points like this and we have come up with some contours now. So, what we can do is.

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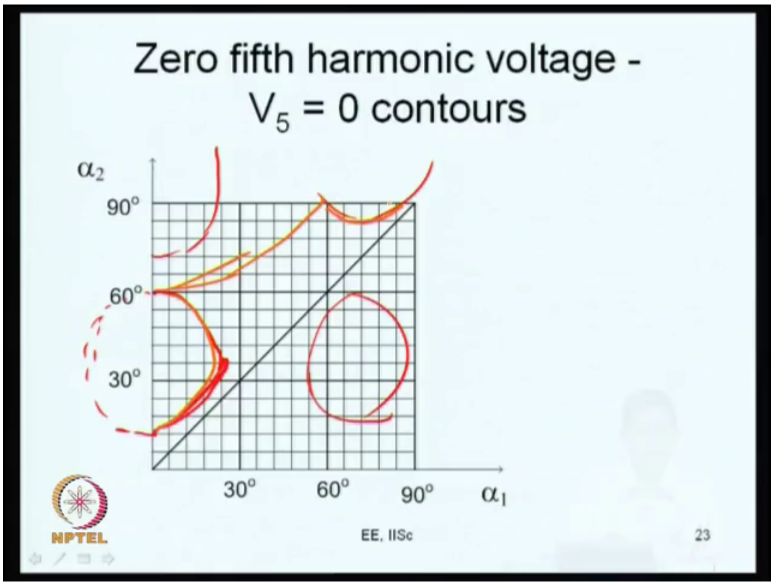

You can come up with a complete thing of all the 0 fifth harmonic voltage now. So, before that we can also see how the slope of V phi is equal to 0 raised how V 5 is equal to 0 raised.

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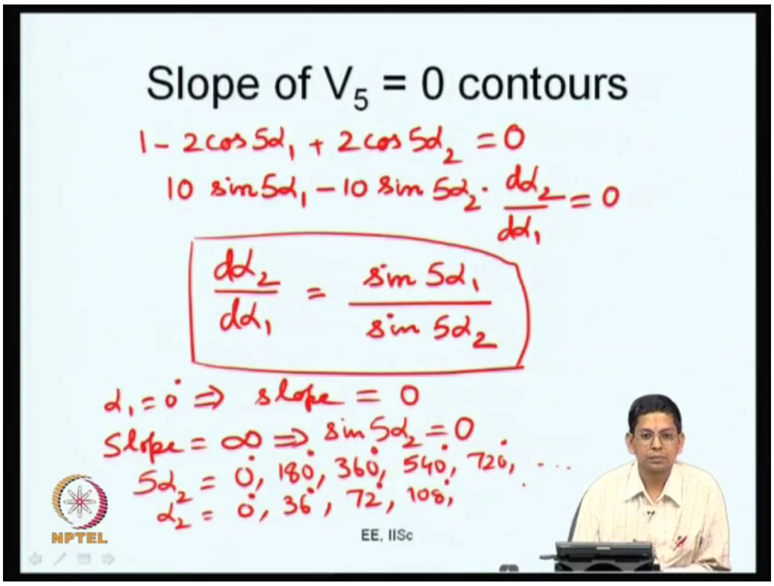

So, V phi is equal to 0 has this equation 1 minus 2 cos 5 alpha 1 plus 2 cos 5 alpha 2 is equal to 0 this is the equation V phi is equal to 0. If you want to find out the slope differentiate both sides with respect to alpha 1. So, what we are going to get is something like 10 times sin of 5 alpha 1 minus 10 times of sin 5 alpha 2 multiplied by d alpha 2 upon d alpha 1 is equal to 0 .

So, in effect what you get is d alpha 2 upon d alpha 1 is basically sin of 5 alpha 1 by sin of 5 alpha 2, earlier for V 1 is equal to constant curves we had the slope to be sin alpha 1 by sin alpha 2 here V V 5 is equal to 0 we are interested not in V 5 is equal to constant we are interesting V 5 is equal to 0 why because we want to eliminate the fifth harmonic voltage.

So, along this again we find sin 5 1 by sin sin 5 alpha 1 by sin 5 of alpha 2 this is what we are finding now. So, if you consider the line alpha 1 is equal to 0 degree you will have slope is equal to 0 because sin 5 alpha 1 is sin 0. So, slope will be 0 and when is the slope infinity slope can sometimes be infinity slope can be infinity that basically implies sin of 5 alpha 2 is 0. Whenever sin of 5 alpha 20 when can 5 alpha to be 0 sin 5 alpha 2 can be 0, when 5 alpha 2 is equal to let us say 0 or equal to 180 degrees are 360 degrees are 540 degrees and so on.

So, correspondingly the values of alpha could be 0 1 80 by 5 it is 36 360 by 5 is 72 and so on this is 100 and 8 and so on. So, for all these values of alpha 2 you have infinite slope. So, you come back here if you go to the old curve for at alpha 2 is equal to 36 degrees at alpha 2 is equal to 36 degrees let me do it with a different color here, this is alpha 2 is equal to 36 degrees it is along this line we had a curve there.

So, now what we can do is we are a little wiser we know what the points are and we know where the slopes are infinity and where the slopes are 0 now. So, if you if you plug in alpha 2 is equal to 36 degrees into your V phi is equal to 0 you will get alpha 1 is equal to 24 degrees, alpha 2 is 36 gives alpha 1 is equal to 24. So, if you put alpha 1 thing you will get alpha 1 is equal to 24 degrees that is this point. So, let us now do it earlier we were just trying to explore to find these points. Now, if you do it comfortably what you can do is you can find out all the extreme points that is 6 comma 12. So, these are the extreme points which can be easily found out 6 are sorry and it has to be 12 at 60.

So, let me erase this off. So, you can go about drawing them from here to here you will have a curve and this is at 36 and 24. So, the nature of the curve is going to be something like this is the nature of the curve, you can have yet another curve this act curve actually goes on both sides this actually goes on both sides we are not interested in the left hand side. So, now, what you can do is you can shift this curve by 72 degrees why 72 degrees if you shift it by 72 degrees it is 360 degrees for the fifth harmonic. So, you can shift it to the right are to the left or top or anything and you will get these curves again.

So, you can get lots of V phi is equal to 0 and you can look at all those curves within your region of interest. So, that is how you can get lot of your fifth harmonic voltages that not be 1 curve. So, this if you shift it up you are going to get another curve like this you are going to get another curve like this if you shifted this side there could be at another curve, but that may not be useful for you, but if this curve is shifted up you will get an at another curve which is going something like this. So, there are several such curves which will be of interest to you and this is your V 1 is equal to 0. So, V 1 is equal to 0. So, within this region you have number of solutions we will discuss. So, this part can give you some fifth harmonic solution and this curve can give another fifth harmonic some other fifth harmonic solution.

So, we will discuss these various solutions in detail in our next lecture. So, thank you very much for being with me and I hope you enjoyed this lecture. And I hope you will have continued interest to follow the next lecture.

Thank you very much.