Pulsewidth Modulation for Power Electronic Converters
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Lecture - 11
Low switching frequency PWM - I

Welcome back to this lecture series on Pulsewidth Modulation for Power Electronic Converters. So, after initial about seven or eight lectures on power electronic converters, we have got started off with pulsewidth modulation. Our specific focus is actually been on voltage source converters. So, we have been looking at the pulsewidth modulation techniques for three phase voltage source converters. And today we will be discussing on low switching frequency PWM. The last couple of lectures we looked at the purpose of PWM and certain essential PWM such as the Fourier analysis and the harmonic you know the tools for calculation of harmonic currents etcetera.

Now, we are trying to I mean achieve the purpose of PWM that is to control fundamental voltage and to mitigate the harmonics and their side effects. And we want to do this at low switching frequency. What is the low switching frequency we are going to look at an inverter and the inverter has devices; in these devices switch at certain frequencies now, and these frequencies are going to be very low.

Why do we consider at lower switching frequencies one reason is that at very high power levels if you operate the inverter at very you design an inverter for you know very high power level, then you know you cannot switch these such high power devices at very high frequencies. Whenever you turn them on and turn them off, there is certain amount of energy lost. So, the devices get heated up a lot, you cannot switch them beyond certain frequency.

So, high power devices can generally be switched at lower frequencies. So, when you make high power inverters you can only switch them at low frequencies that are one reason. The second reason and which is more important to us is these low switching frequency methods help us learn certain essentials of pulsewidth modulation very well. You can develop certain amount of understanding certain amount of insight and intuition if may we may venture to call so by studying low frequency PWM methods. So, these are the reasons why we are venturing into that.

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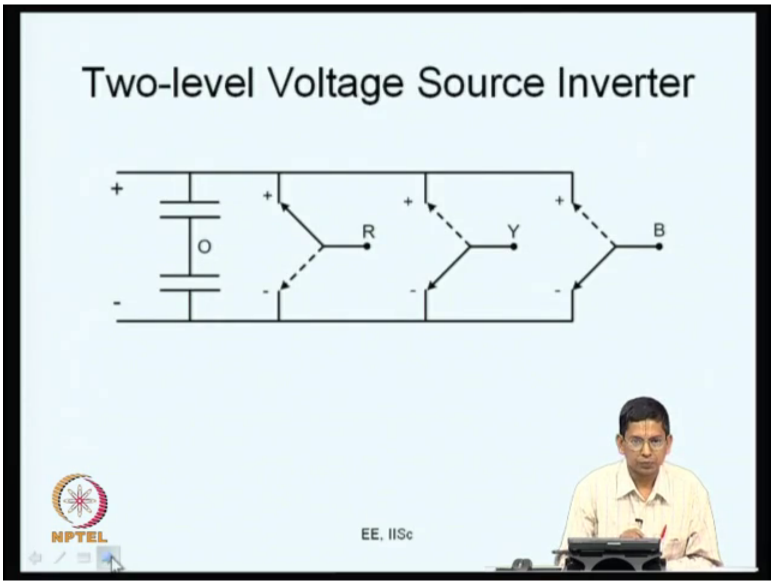

And now let us see what we are trying to do, this is the voltage source inverter three phase voltage source inverter. The three phase loads are connected to R, Y and B and the dc supply is here, this is what has been our focus. In the next few lectures, we will certainly view you know dealing with pulsewidth modulation methods only for such three phase inverters now. And in this inverter every leg is a single pole double throw switch has indicated here.

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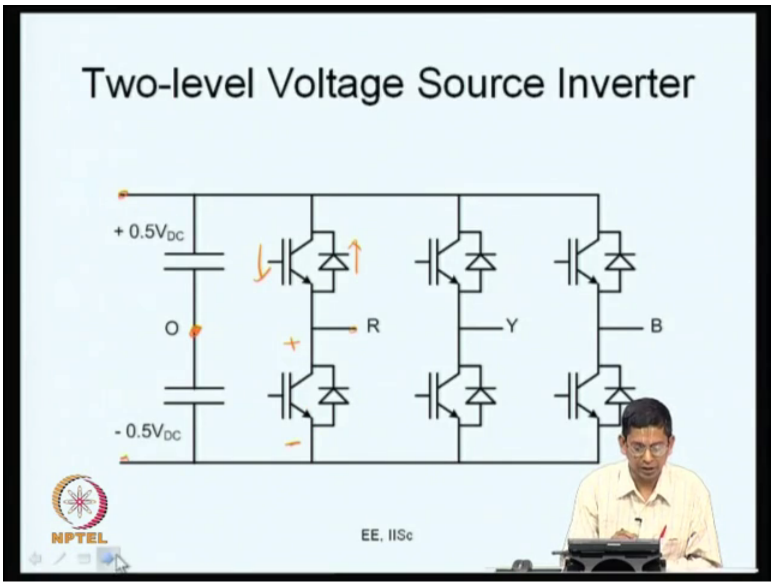

And the single pole double throw switch is electronically realized as shown here. There are two IGBTs and these IGBTs have anti parallel diodes. So, they are capable of conducting in both the directions as we have already said and we have seen in several times. And when the top IGBT is conducting, the bottom IGBT will be blocking a voltage of this polarity and the same thing vice versa. So, these devices have bidirectional current carrying capability and a unidirectional voltage capability and at the pole you can apply either this plus 0.5 V DC or minus 0.5 V DC measured with respect to the dc bus midpoint O.

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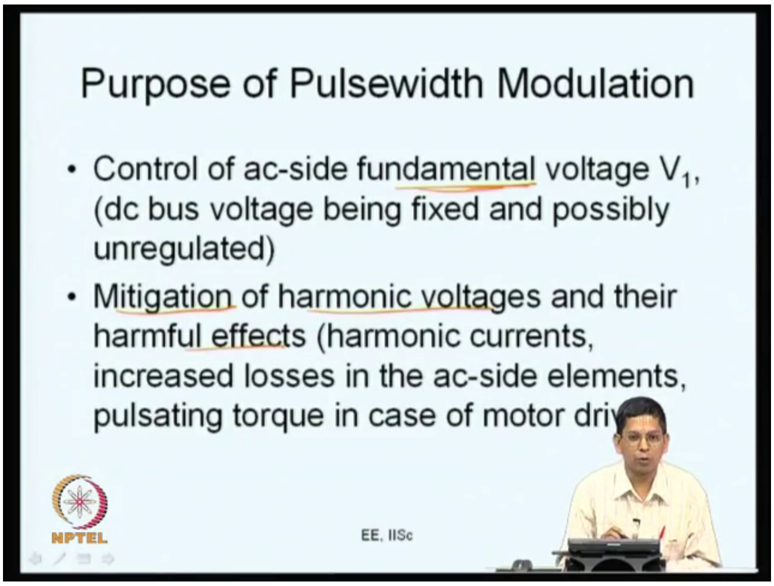

So, we defined pole voltages, we defined the purpose of pulsewidth modulation as we have already said it is to control the fundamental voltage. What you want to do is we want to switch these converters these devices will become in inverter in such a fashion that for the given dc bus voltage we want the desired fundamental voltage, we want the desired fundamental voltage on the ac-side. And not just that if possible and to the extent possible we want to reduce the harmonic voltages and their harmful effects, this is what we want to do.

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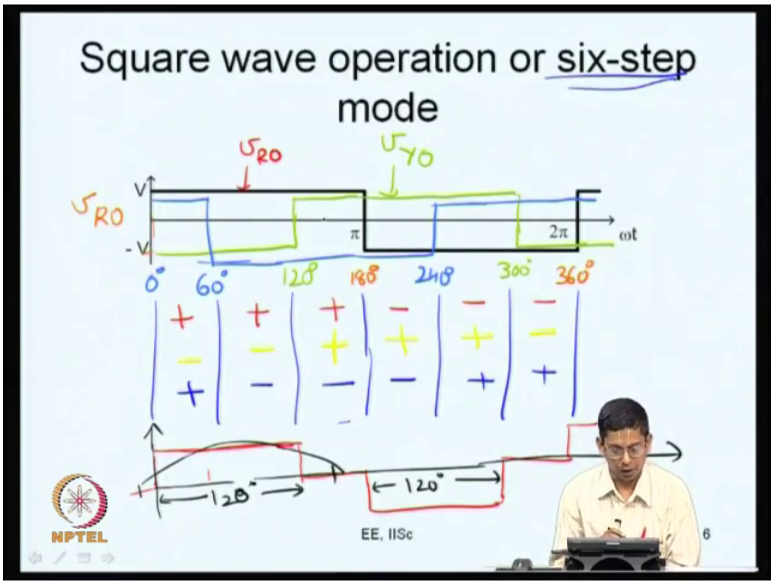

So, the simplest way you can operate an inverter is a square wave mode where let us call this as V RO that is the R phase fundamental voltage. The R phase fundamental voltage varies like this it switches from plus V to minus V at 0 and 180 degrees it comes it switches from positive to negative and 360 degrees it goes back now. So, if the this is supposed to be a square wave operation, in an inverter if the R phase like the switching like this that is the top device is conducting between 0 to 180 and the bottom is conducting between 180 to 360.

How will the Y phase be the Y phase is in such a fashion that three-phase symmetry is satisfied? So, at 120 degrees what happens is the Y phase goes high, it switches high from low? So, if this is 120, 120 plus 180 is 300 degrees; so at 300 degrees, it comes back, so this is 300 degrees. So, what I have drawn here is the y-phase pole voltage that is the voltage at the midpoint of the leg Y measured with respect to the dc bus neutral O. So, this is the Y phase voltage which I have drawn with some greenish yellow kind of color.

Now let us move on to see how the blue phase should look like. For convenience let me choose blue color ink here. So, R phase switches from low to high at 0, Y phase switches from low to high at 120. So, B phase should be switching from low to high at 240 degrees. So, this is 240 degrees. So, at 240 degrees, the blue phase switches from low to high; and it remains high for another 180 degrees which is all the way plus you know

240 plus 180 is 420; up to 420 it continues to be high. And it is low for 180 degrees before 240 that is up to 60 degrees that is up to 60 degrees this is low, the Y phase is low. So, at 60 degrees, it basically switches from high to low. So, this is how you have the three pole voltages.

So, what you have here is V RO that is let me indicate it in red ink itself. This is the R phase fundamental voltage when you are considering square wave operation and Y phase is shifted by 120 degrees and B phase is shifted by yet another 120 degrees. So, these three phases are really like this now. So, it is also called a six step mode. Why do we call it a six step mode? You can see that there are six different switching’s' here. Firstly, if you considered this first 60 degrees, you see that R phase is positive, R phase is positive. The next 60 degree is also R phase is positive and again between 120 to 180, R phase is positive; between 180 to 360 degrees, the R phase is negative, this is what you have.

Now, let me look at the yellow phase let me choose this yellow or one of this greenish colors let me choose yellow. So, how about Y phase, the Y phase goes low to high at 120 degrees. So, between 120 to 180 degrees, it is high; the same way between 180 to 240 degrees, it is high, here also it is high. At 300 degrees, it switches from high to low and therefore, between 300 and 360 it is low. Similarly, between 0 to 60 and 60 to 120 it is low.

Now, let us look at the blue phase, let us look at the blue phase the blue phase is high for the first 60 degree duration and then the blue phase is low from 60 degree all the way up to 240 degrees it is low then what happens subsequently it goes high. So, what I am trying to say is between 0 to 60 degree, this is between 60 to 120 degree, this is between 120 to 180 degree. So, every sixth of the fundamental cycle, the inverter is in the same switching state. If you consider 0 to 60 degree, it is in one particular switching state in which the top device of R phase is on, the bottom device of Y phase is on, and once again the top device of B phase is on.

On the next switching 60 degrees what happens R and Y continue to be whatever they have been they were. So, R top is high and Y bottom is gated high. Now, B just switches, so this inverter state from what is plus minus plus it now changes to plus minus minus. And once again it changes state because who switches here at 120 degrees at 120 degrees Y phase switches. So, from plus minus minus, it moves onto plus plus minus again at

180 degrees R phase switches. So, plus which was you know R phases which was positive all along now becomes negative. So, plus plus minus here becomes minus plus minus here. And similarly, you have the state minus plus plus between 240 to 30, and 300 to 360. So, it does not switch you know it is switches only one since 60 degrees and only one of the phases switches and you see that there are six different intervals like this during each interval there is one switching state one reason why you may call this as a six step mode.

Further, you can even draw the R phase you know this is these are the pole voltages you can subtract V RO and V YO and get your line to line voltage V RY how will the line to line voltage V RY look like? If you look at the first 120 degrees if you look at the first 120 degrees on this waveform V RO is positive and V YO is negative. So, the difference V RO minus V YO is V DC by 2 minus of minus V DC by 2, therefore, it is equal to plus V DC. So, if you look at V RY it is between 0 to 120 it will be plus V DC and what happens between 120 to 180 V RO and V YO are equal, therefore V RY is 0.

Similarly, between 180 and 300, V YO is positive and V RO is negative, therefore, V RY will be equal to minus V DC. And once again for the last 60 degree duration V RY will be 0. So, what do you find, if you look at V RY, it will be 120 degrees. In the positive of cycle, it will you have you will find a positive pulse for 120 degrees and similarly you will find a negative pulse let me just try sketching that here itself. So, this is what I am trying to do is V RY I am sorry let me.

So, if you look at V RY let me choose a color, should I choose red. So, V RY is going to be a positive pulse like this for 120 degrees and then it is going to be 0 for the next 60 degrees then it is going to be negative for the next 120 degree and it is 0 again now, then it goes high this is what you have. So, this is what you have. So, this is how V RY looks, this is for 120 degrees it is positive; and this is the 0 axis let me choose a different color to draw the horizontal axis. So, this is the horizontal axis and this is the vertical axis at times 0. So, this is high for 120 degrees.

So, if you look at this its fundamental is something like this, its fundamental voltage will be something like this. So, for the first 30 degrees in the positive of cycle V RY is 0 and the last 30 degrees in the positive of cycle once again it is 0 and in the middle 120 degrees of half cycle, this is 120 degrees V RY is going to be a positive pulse. The same way it is you will find this negative pulse for a hundred and 30 degree duration this is how the V RY waveform looks like and V YB will look similar just shifted by 120 degree. And the V B R is also going to look like the same thing further shifted by 120 degree, so that is how you are going to look at.

You can subtract V RY and V BR and scale them to come up with V RN. If you consider that the inverter is connected to is feeding a balanced three phase star connected load then you have the star to you know phase to neutral voltage V RN, V YN, V BN, if you see them they will be a six stepped waveform once again. So, during the first 60 degree duration it will have a value of plus V DC by 3. And then it is 2 V DC by 3, then V DC by 3, minus V DC by 3, minus 2 V DC by 3, minus V DC by 3 and so on that is the kind nature of waveform that you will have for V RN.

So, once again you see that every 60 degree the level changes, so that is another reason now this is why you call it as a six step mode. So it is either, you call it square wave operation or you call an six step mode, this is the most fundamental method of controlling this. You cannot control the fundamental voltage here you accept what are fundamental voltages; so the fundamental component of the pole voltages 2 V DC upon pi. So, the same is the fundamental phase to neutral voltage applied, it is of 2 V DC upon pi.

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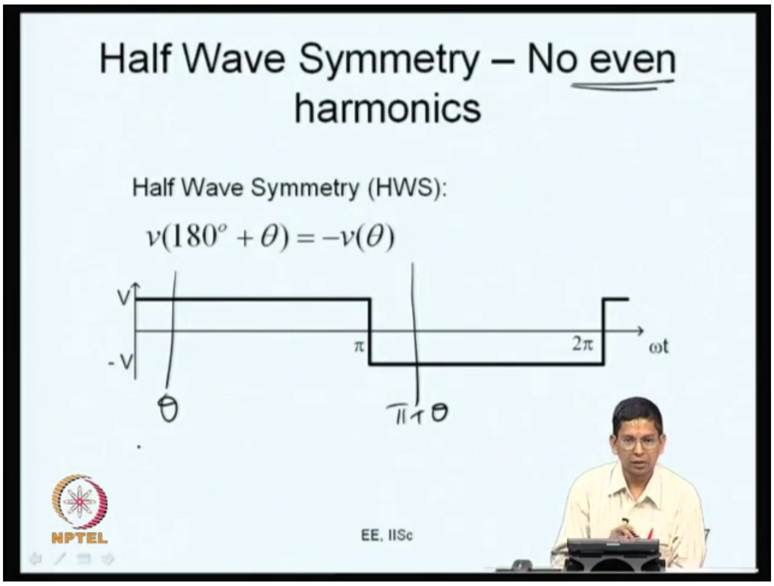

So, now for controlling is where you need notches and all that, but the waveforms all the waveforms we consider here have half wave symmetry; and when you say half wave symmetry whatever it value it has a theta, the same value it has a 180 plus theta with just a negative sign. So, this is half wave symmetry and it ensures that there are no even harmonics. So, the PWM waveform has three phase symmetry.

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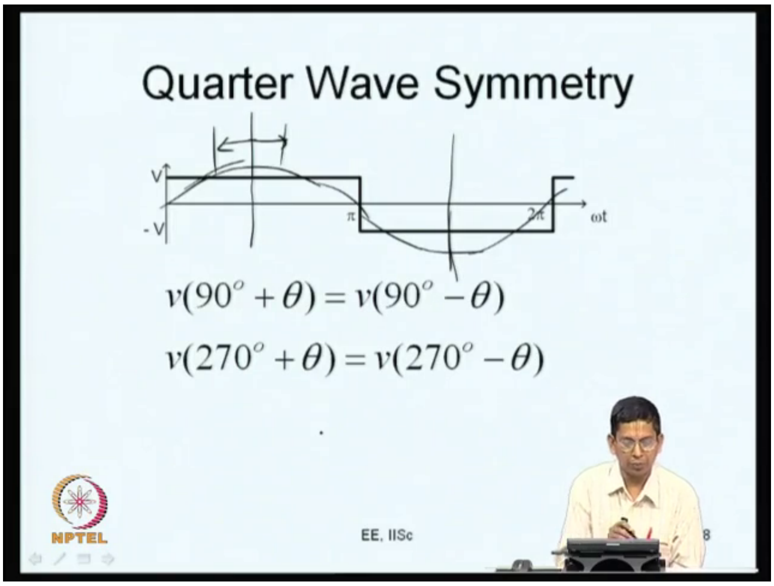

And it has half wave symmetry and it also has quarter wave symmetry which we have already discussed now. If it is quarter wave symmetry, you consider this 90 and 270 degrees that is when the fundamental has its peak that is when the fundamental has its positive peak and negative peak. So, above these if you look at that it has symmetry. So, if you move in this direction and if you move in that direction to same distance, you find that the waveform is identical. This is quarter wave symmetry it makes your calculations easier you need to consider only one quarter of the waveform may be starting from 0 to 90 degrees now, so that is what we are going to do now.

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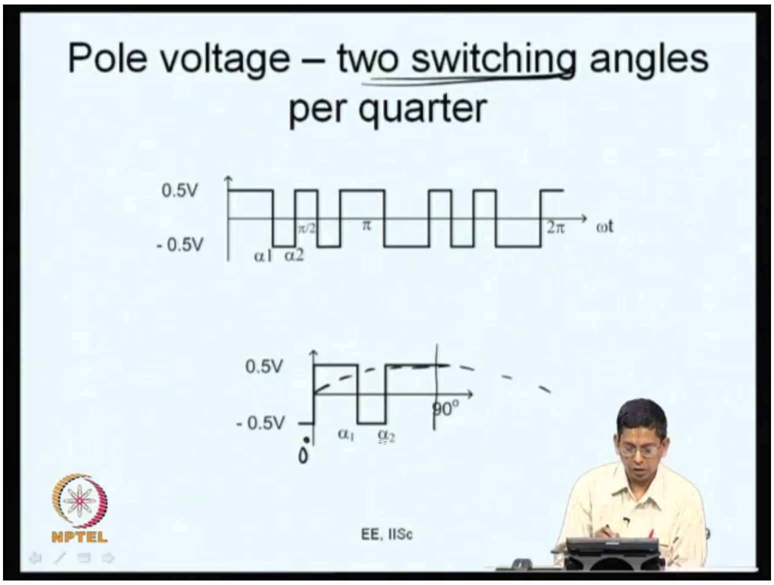

What we are trying to do is we are trying to control the fundamental voltage. So, we need at least one switching angle. So, we saw it in the last lecture that with one switching angle you can control the fundamental voltage, but you cannot influence the harmonic voltages you have to accept whatever they are. So, now, we want to be able to influence the harmonics to certain extent at least and hence we go for two switching angles in a quarter. For two switching angles in a quarter, the complete pole voltage waveform looks like this, this is something we saw in last class. So, there are two switching between 0 to 90, there is a notch.

So, you do not have because of half wave and quarter wave symmetries it is unnecessary for you to draw the entire waveform, we can consider only the waveform starting from 0 degree to 90 degree; and its fundamental is something like this, its fundamental is something like this. This is at 90. So, you are considering a quarter cycle of this waveform, this has two switching angles alpha 1 and alpha 2. So, this waveform is completely described by alpha 1 and alpha 2. What we mean here is we are considering a pole voltage and the pole voltage is switching from negative to positive at 0 degree and it has two switching one at alpha 1 switches high to low; and at alpha 2 it switches back from low to high. And till 90 degrees, it has no switching; and it is symmetric about 90 and so on.

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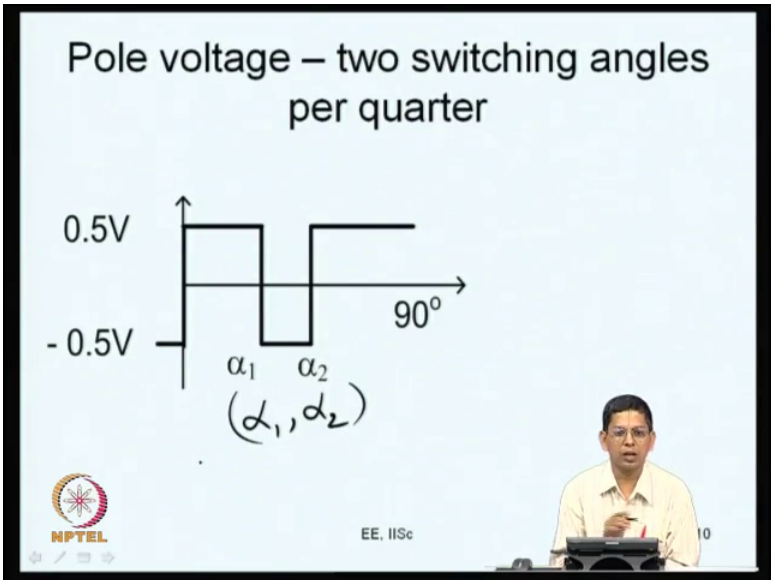

So, since there are two variables here this is the same thing drawn here. So, what we can do is the waveform is almost completely defined by alpha 1 and alpha 2, the family of waveforms. We are considering a family of waveforms, and for various pairs of values of alpha 1 and alpha 2.

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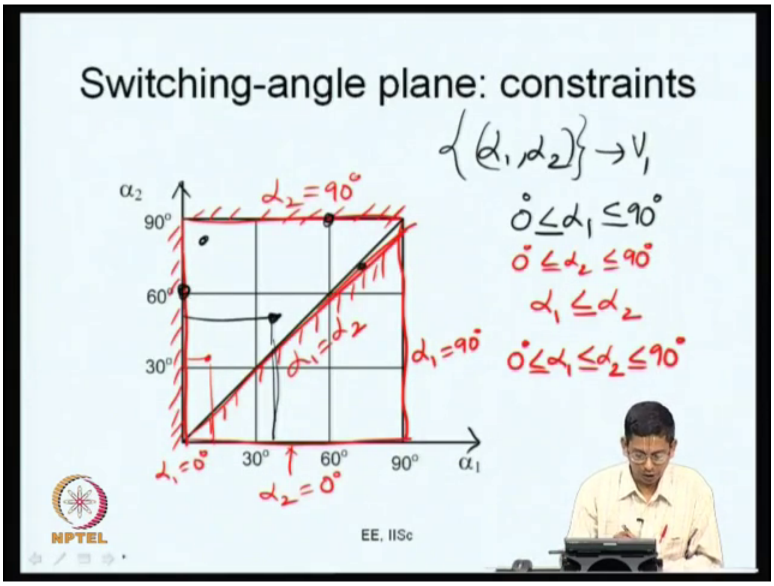

So, now to study this waveform, we can look at the switching plane now. Why do you want to look at the switching plane, different pairs of alpha 1 and alpha 2 can lead to the same fundamental voltage. Let us say we want a particular fundamental voltage that particular fundamental voltage is obtainable through several values of alpha 1 and alpha 2 several pairs of alpha 1 and alpha 2. One thing that we can first understand is; what are all the values the set of all pairs of alpha 1 comma alpha 2 that lead to a particular fundamental voltage this is of interest to us. Alpha 1 comma alpha 2 is a set of all alpha 1 and alpha 2 that can give a particular value of fundamental voltage that we want say 0.8 per unit or you know 0.5 per unit or whatever now.

So, let us look at the whole thing in a switching plane with where this is alpha 2 is taken on the vertical axis, and alpha 1 is taken on the horizontal axis now. So, in this switching plane, we are not going to consider the entire switching plane because alpha 1 is restricted you know it is the range, it is not everything. So, alpha 1 if you take individually alpha 1 can never go less than 0; and alpha 1 can never go greater than 90 degrees. So, alpha 1 is equal to 0, let me choose red ink here, so that it will be clear on this. So, this is the line alpha 1 is equal to 0, this is the line alpha 1 equals 0 degrees. And this is the line alpha 1 is equal to 90 degrees. So, no switching angle is will go beyond these two, we want these two switching angles within this region now. So, these are two limits now.

Similarly if you take alpha 2 you have 0 less than or equal to alpha 2 less than or equal to 90. Take alpha 2 alone alpha 2 can never go less than, so this line is alpha 2 is equal to 0 degree. Similarly, alpha 2 cannot go beyond 90 degrees also. So, this is alpha 2 is equal to 90 degrees. So, we need to look at only one quadrant and only one portion of this quadrant 90 etcetera and that is not all and alpha 1 and alpha 2 have some constraints again relating one another. What is that constraint, alpha 1 has to be less than or equal to alpha 2 that is the constraint. If I put all the constraints together, it is actually 0 less than or equal to alpha 1, less than or equal to alpha 2 less than or equal to 90 degree, this is the constraint on the switching angles.

So, this is the line alpha 1 is equal to alpha 2. So, you cannot operate at any value of alpha 1 lower than I mean any value of alpha 1 that is greater than alpha 2. So, if you look at it, you cannot operate in this side. Why, alpha 1 cannot go less than 0. You cannot operate on this side of this line, why because this line is alpha 1 is equal to alpha 2 you can operate only when alpha 1 is lower than alpha 2 or not otherwise, so this is one. And the other thing is you cannot know your alpha 2 is maximum values is 90 degrees. So, if it is beyond 90, you are not going to consider that. So, this right triangle is the region of our interest this is what we saw and it is basically it is a kind of recollection on this now all over the forms are going to be within this now.

So, what do we mean, any point here is a PWM waveform. If we consider some point, this point has particular value of alpha 1 and particular value of alpha 2. Let me consider another point; let me take yet another point. So, this point has a particular value of alpha 1 there is a another value of alpha 2. So, this point is another PWM waveform of the type we are looking at let us take this here also it has some value of alpha 1 and some value of alpha 2. Let us take a point here, let us take a point there. So, all these points have some values of alpha 1 and alpha 2. All these are PWM waveforms of the variety we are looking at ok.

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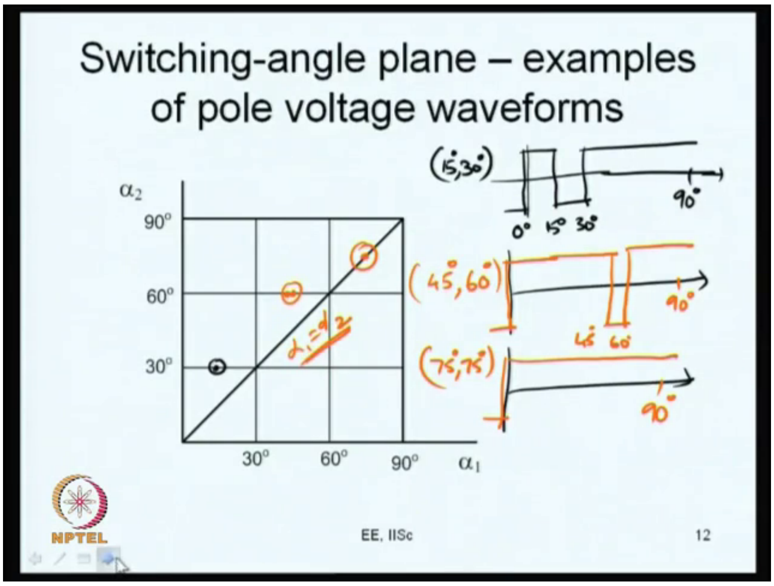

So, now let us look at some specific examples as to how the pole voltage waveforms look like how they look like. So, let me take this take a point here this is a point now at this point it is 15 comma 30 degrees, I will considered the point 15 comma 30 degree that is alpha 1 is 15 and alpha 2 is equal to 30. So, what is the PWM waveform, what is the pole voltage I am looking at, the pole voltage is like this at 0 it switches from high to low; at 15 degrees, it comes back; at 30 degrees, it goes high; from 30 to 90, it stays high it is 90 degrees.

So, let us just simply work out a few more such examples now. Let us just work out a few such examples now. So, let me say consider another point. This point let me say is here, this is at 45, this is 45 comma 60 degrees now; alpha 1 is 45, alpha 2 is 60. So, what we have is it is switches from at 0 from low to high; and it is high at alpha 1 is equal to 45 degrees it comes down and alpha 2 is equal to 60 it goes high, this is 45, this is 60 and this is 90, this is how this waveform is going to look like.

Let me take yet another example. Let me say an example is here itself. This point is here, let me call this point as c or this is 75 comma 75. So, what is it that I am saying 75 comma 75; alpha 1 is at 75 degree, alpha 2 is also at 75 degree that basically means there is no notch at all. So, this waveform is a square wave form. In fact, this is the line alpha 1 is equal to alpha 2, you may consider any point on this line alpha 1 is equal to alpha 2, if you consider any point on this line alpha 1 is equal to alpha 2, you are going to have a square wave. So, this line corresponds to basically the square wave operation. So, you consider various points, you can just consider the few may several arbitrary points here in different regions, and you can plot them. And you know you can just get a feel for how the waveforms really look like now.

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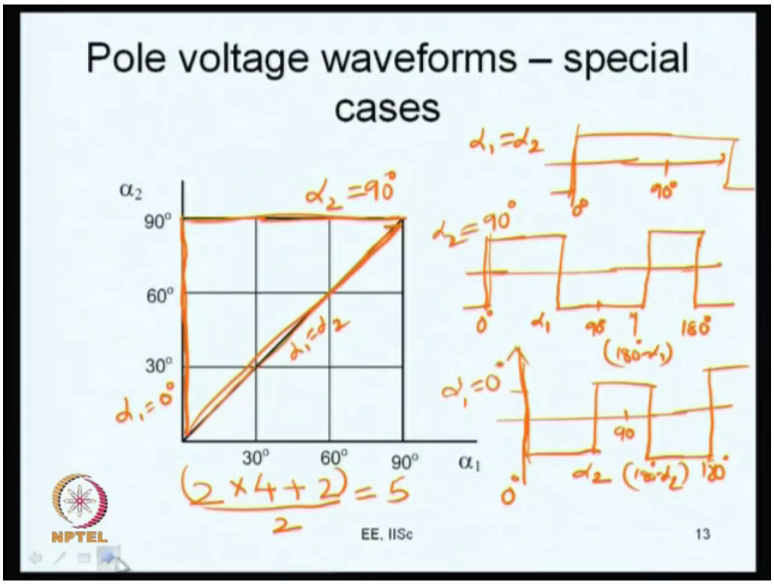

So, this is some examples of pole voltage waveforms, let us move to some special cases. The special case one special case is something that I have already indicated that is this line alpha 1 is equal to alpha 2. When alpha 1 is equal to alpha 2, you have it as a square wave which we just saw, this is a square wave; at 0 degree it switches, and till 90 degrees it is high, this is a square waveform. The other special cases that we have are let say let us look at the line alpha 2 is equal to 90 degrees, this is alpha 2 is equal to 90 degrees. What do we mean by this? You have a PWM waveform it switches like this it switches from low to high at 0 degree. So, it switches down at alpha 1, but its alpha 2 is at 90 degree, its alpha 2 is at 90 degrees. So, due to quarter wave symmetry, you know if there is a switching at alpha 2, there is another switching at 180 minus alpha 2. So, when alpha 2 is 90, there are two switching at 90 degrees that basically means there is no switching at all is that right.

So, the waveform continues like this, the waveform continues like this, I am drawing it beyond this. So, this is 180 degrees and this is 180 minus alpha 1. So, what happened to alpha 2 alpha 2 became equal to 180 minus alpha 2, when they are 90 degree; and alpha 2 is 90 degree alpha 2 same as 180 minus alpha 2 and one notch has been removed from your waveform from what is really this thing here from the original waveform with two switching angles now. So, this is one special case this is how the pole voltage looks like when you are operating on the line alpha 2 is equal to 90 degrees.

The other special case that you can look at this when you are on the boundary of this boundary of this plane that is alpha 1 is equal to 0. So, the waveform by definition is supposed to switch from low to high at 0 then it switches from high to low at alpha 1; if alpha 1 is 0 both of them happen at the same time, so the waveform actually looks the other way. It is only at alpha 2 it goes high and then 90 degrees it is like this then here this is 180 minus alpha 2 this is 180 minus alpha 2 it is like this at 180 degrees its switches high. This is how the waveform looks like; this is 0 degree. So, you do not find alpha 1 at all alpha 1 is merged with 0.

So, this is again you know it looks like a third harmonic wave or whatever, the number of switching is less the originally the waveform add two switching per quarter cycle in these two cases when alpha 2 is equal to 90 instead of two switching per quarter cycle it becomes only one switching per quarter cycle. Once again when you have alpha 1 is equal to 0 instead of two switching per quarter cycle, you have only one switching per quarter cycle. But these two waveforms at alpha 2 is 90 and alpha 1 is equal to 0 seem to have different phases.

So, then you have the other case of alpha 1 is equal to alpha 2. When you have alpha 1 is equal to alpha 2 the number of switching per quarter cycle is 0 because both of them are coincide and there is no notch at all there and so that becomes a square wave. So, these are the special cases of the pole voltage waveforms, and these special cases are basically what we are looking at is along the boundary of this permissible region of our region of interest on the alpha towards is alpha 1 plane. So, these are the special cases of pole voltages, where basically two switching per quarter gets reduced to one switching per quarter or 0 switching per quarter.

Now what is two switching per quarter mean in terms of switching frequency you have two switching per quarter cycle, and how many quarters are there, there are four quarters. So, this is 2 multiplied by 4 plus there are two switchings that is one switching at 0 degree, and one switching at 180 degree. So, a waveform totally switches time I mean 2 times 4 plus 2 that is 10 times in a full cycle and that divided by 2 is the number of switching cycles. So, this is 5.

When a waveform switches with two switching angles per quarter it is actually switching at five times the fundamental frequency, this is what is called as pulse number or sometimes. So, the number of the switching frequency to be fundamental frequency is 5 now. This gets reduced to 3 when alpha 2 is equal to 90 or alpha 1 is equal to 0. And this gets reduced to 1, if alpha 1 is equal to alpha 2. If alpha 1 is equal to alpha 2 it is a square wave; and if it is a square wave operation, it switches only at 0 degree and switches back at 180 degree. So, if there is nothing that happens here. So, there are the only two switching’s in a cycle and only one switching cycle in a fundamental cycle. So, the pulse number or the ratio of switching frequency to fundamental frequency is one now.

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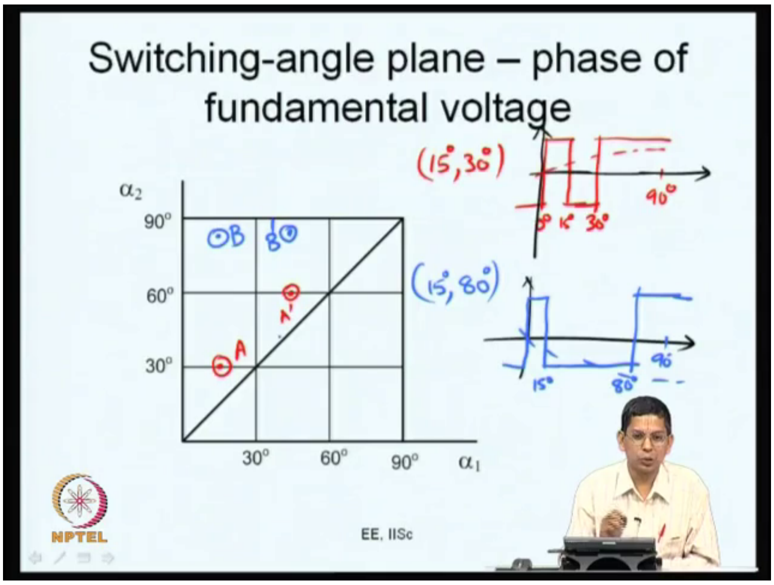

So, let us move on to see the phase of the fundamental voltage that is what we are trying to see here is we will again consider some examples now. First let me consider one example that is here, it is quite similar to what I had taken, just a moment. So, let me consider alpha 1 is equal to 15 degrees and alpha 2 is equal to 30 degrees, this is the point I have considered now. So, let me put a circle around that, so that it is clear. So, how does the pole voltage look like at 0, it switches from negative to positive; and at 15 degrees it switches back; and at 30 degrees it once again goes high, then it is going to be high throughout till 90 degrees, going to be high throughout till 90 degrees. So, this is how your waveform is going to be, this is for 15 to 30 degrees now. So, let me indicate the horizontal and the vertical axis on this clearly where is the horizontal axis, this is the horizontal axis; and where is the vertical axis, this is the time 0 it is like this now.

So, let me consider another example, where shall I use a different color here. Let us say like this. This is something like 15 degrees comma 80 degree alpha 1 is 15 and alpha 2 is 80 degree. So, how will you have this waveform it switches from low to high at 0 and high to low at 15, but it switches back only at 80 degrees here, and it is high till 90 and beyond. So, this is how this waveform looks like. What is the striking feature here if you look at the fundamental component in the red waveform if you look at the fundamental component of the red waveform the fundamental component is likely to be like this. It will have a 0 crossing here, but it will have a positive phase, it will be in phase with this now. On the other hand, if you consider this blue waveform you see in the so called positive half cycle in one quarter of the positive half cycle it is negative more often actually the fundamental voltage could be something like this the fundamental voltage could be something like this.

So, the phase of this it could really be out of phase that is what we call a zero degree in the red case is very well the positive zero crossing of these fundamental sign, but what we call a zero degrees here in the blue case is actually the negative zero crossing of this phase now. So, what was cause the difference the difference between alpha 1 and alpha 2 in the red point here let me call the red point this point as B and let me call the red point as A. In case of point A, the difference between alpha 1 and alpha 2 is not very high and it is more or less positive the waveform is positive in the first quarter for most part of the time. And in the second case the waveform is more negative in the first quarter 0 to 90 degree, so that is one difference that you find here now.

You can take several such examples now. I can give another example like let say I may have my alpha 1 is 45 degrees and alpha 2 is equal to 60 degree, this point I may call as A dash. And I may take a another point which is say like here the same 45 degrees and B dash this is let say the same 45 degrees as A dash about 80 degrees now. So, you will find that this A dash is of the same variety here. So, the fundamental has the phase as indicated here it is in phase and here what you can call as out of is that the fundamental phase is different, at zero crossing it is not the at 0 degrees it is not the positive zero crossing on the fundamental, but it is the negative zero crossing of the fundamental law. So, since you have quarter wave symmetry you know you will you will have a zero crossing of the fundamental there and of all the other harmonics there, but the just a question of whether it is a positive zero crossing or negative zero crossing. So, these are some differences that you have now.

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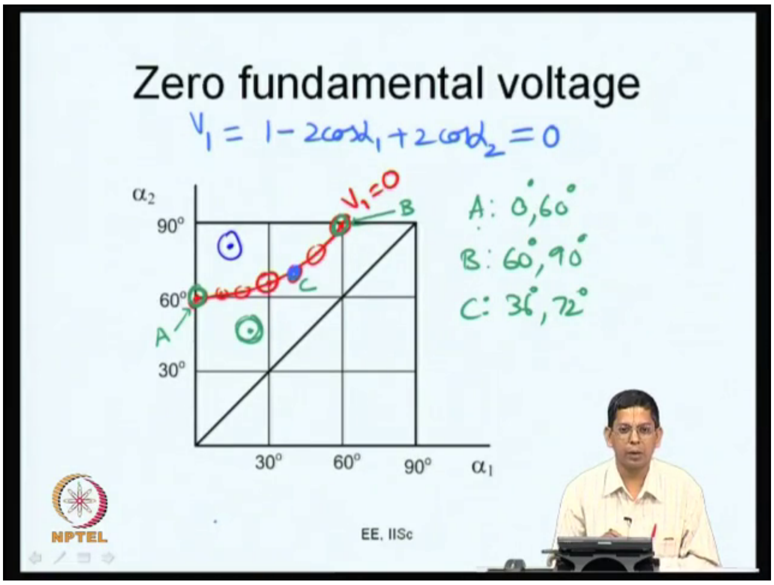

See in between these two you must have a line somewhere there along which the amplitude is 0, the amplitude of the fundamental voltage is 0. Let us just look at that because that could be the start of our the exercise on controlling the fundamental voltage. Let us say I want the fundamental voltage V 1 to be equal to 0. What is my V 1, V 1 is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 this is the fundamental voltage I want it to be 0. Let us see what we can do to get this here now. So, let us do a few calculations to do that now. So, how can we obtain, one simple way to do this is substitute some value of alpha 1 and find out what is the value of alpha 2.

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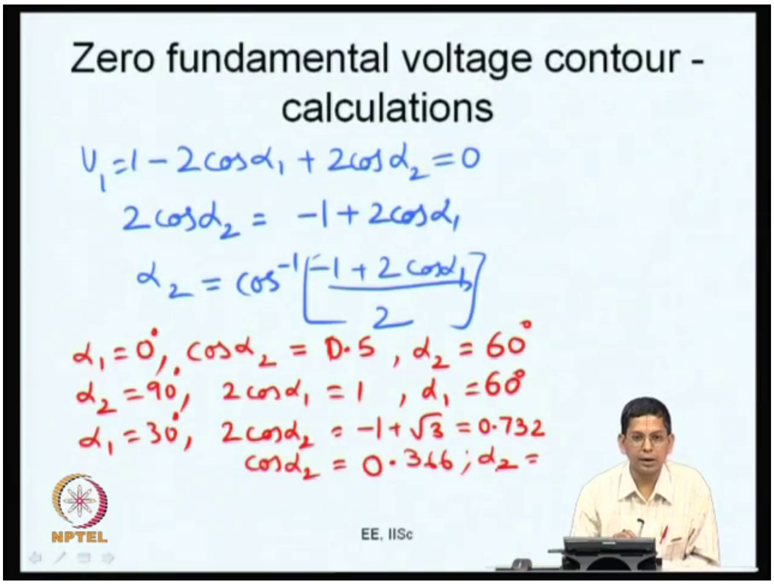

So, you have 1 minus 2 cos alpha 1 plus 2 cos alpha 2 is equal to 0 this is your V one and you want it to be 0 now. So, what is your alpha 2 basically the 2 cos alpha 2 is minus one plus 2 cos alpha 1 and what is your cos alpha 2 is half of that alpha 2 is basically minus half 2 cos alpha 1 divided by 2 and the cos inverse of this, and a cos inverse of this. So, cos alpha 2 is minus 1 plus 2 cos alpha 1 by 2, and you take the cos inverse you get this now. Now, you can plug in several values of alpha 1 and you can get the corresponding values of alpha 2 is that right.

Let us come up with some values here let me take alpha 1 is equal to 0, it is a simple value to start. If I say alpha 1 is equal to 0 then what do I get here cos alpha 1 is 1, therefore it is 2 cos alpha 1 is 2. So, you have minus 1 plus 2 is 1, you have cos inverse of half. Alpha 2 is cos inverse of half or you can say cos alpha 2 is 1, and you know a 0.5 I am sorry, so cos alpha 2 is so minus 1 plus 1 this is so it is half and alpha 2 is cos inverse of point five cos inverse of 0.5 is 60 degrees.

So, if I consider alpha 1 is equal to 0, I want V 1 is equal to 0. If I consider alpha 1 is equal to 0m I get my corresponding alpha 2 to be 60 degrees. So, where is that let me go back to the previous curve, let me say here this is alpha 1 is equal to 0 and alpha 2 is equal to 60 degrees, this is the point I get now. Now, let me say another point, let me take another point very, very quick calculations are actually possible if we consider let us say alpha 2 is equal to 90 that is also very easy place that you can come to. If I consider alpha 2 is equal to 90 degrees, and then what happens is the 2 cos alpha 2 term is 0, therefore you get 2 cos alpha 1 is equal to 1.

So, 2 cos alpha 1 is equal to 1 or alpha 1 is equal to 60 degrees this is what you get. So, let us go back and see this point here, this is alpha 1 is 60 and alpha 2 is 90. So, these are the two points mean at which V 1 is equal to 0, I should say these are the two points these are two of the several points at which V 1 is equal to 0. So, let us say there are many other points if we go on substituting values of alpha 1 in between you will get corresponding values of alpha 2 from here that is what we can do here.

One simple value we will substitute is alpha 1 is equal to 30 degrees. When you substitute alpha 1 is equal to 30 degrees, so what you have is you have your 2 cos alpha 2; and 2 cos alpha 2 is basically minus 1 plus 2 cos alpha 1 that is minus 1 plus root 3 is that right, it is minus 1 plus root 3 alpha 1 is 30. Therefore, cos alpha 1 is root 3 by 2, 2 cos alpha 1 is root 3. So, you have minus 1 plus root 3. So, this is 0.732 therefore, your cos alpha 2 is something like 0.36 cos alpha 2 is something like a 0.366 roughly. And your alpha 2 is going to be some value which is slightly higher than this 60 degrees is that right it is going to be some value like this.

You can find out what is your exact value with your calculator we can plug this in you will find that it is coming somewhere like this. So, if you take several points in between what you will find is you will find you can get many such points like this. And you can join these points together and this will be your V 1 is equal to 0. So, this basically gives you the set of all points with this property of V 1 is equal to 0 that is we consider any alpha 1 comma alpha 2, you consider any pole voltage with this value of alpha 1 and with such a value of alpha 1 and alpha 2 that pole voltage will give you 0 fundamental voltage tau. And if you go to this side of this take a point on this side that is going to have a phase shifted waveform it is the fundamental waveform is going to be out of phase.

And if you consider on this side as we saw before this is going to be of proper phase. So, at omega t is equal to 0 what we call as omega t is equal to 0, the fundamental will have a positive zero crossing, whereas in the blue case the fundamental will have a negative zero crossing and. So, this is the line along which you have zero fundamental voltage now. So, let us say you want 0 fundamental voltage you know it is a slight slightly funny I would like to clarify this in most open loop we do have tried you may not come to zero fundamental voltage at all you may start with some minimum fundamental voltage. It may be 2 percent, 3 percent, 5 percent or whatever it could be its not going to be zero fundamental voltage, but there are closed loop situations where you may have one zero fundamental voltage to be applied or certain small amount of fundamental voltage to be applied that is a different point now.

So, why we consider zero fundamental voltages there is a kind of continuity in the properties here. Now, let us say you consider this waveform that I have given here. If you say that the waveform has certain spectral properties at that point at in the neighborhood of that point the spectral properties are going to be quite close to it. So, let us say a particular harmonic is very high or let me first say that the fundamental component of this point that I am indicating here is so much.

Let us say it is 0.4 per unit. And if you want to consider somewhere in the vicinity of this very close to it, again it will be very close to 0.4 will be slightly higher or slightly lower than 0.4. Similarly, let me say that the fifth harmonic component of this point is very high at this point is very high. So, you can say that in the neighborhood of this point the fifth harmonic component will be very high. So, if you understand V 1 is equal to 0 certain several operating points corresponding to V 1 is equal to 0, you can reasonably understand V 1 is you know the points at which V 1 is some small value.

So, if you want to consider V 1 is equal to 0.03 or 0.04 etcetera we can just start at V 1 is equal to 0. So, now, we have taken V 1 is equal to 0, and all these points along this contour we have V 1 is equal to 0. So, you need to be able to discriminate as what is a good operating point and what is a not so good operating point.

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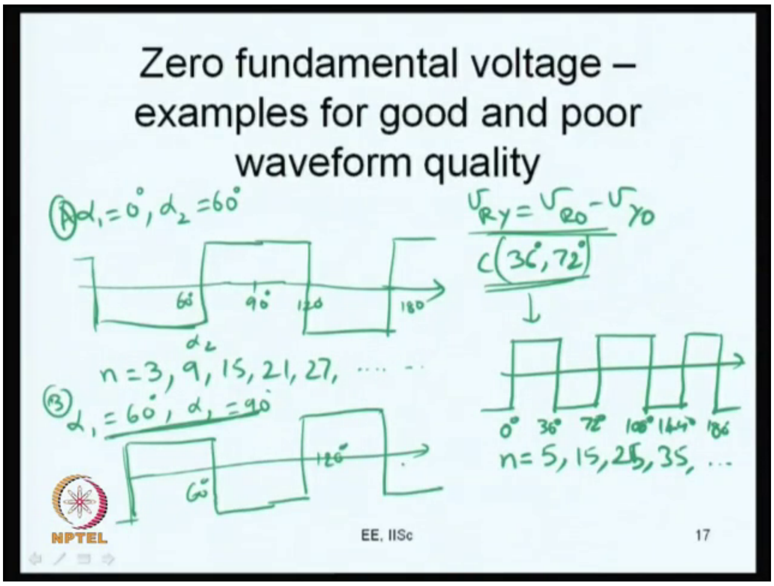

So, what is good and what is bad about it is a situation now. So, we have to find out some examples for good quality waveform and some examples for poor quality waveform. So, what are good quality waveform, what are poor quality waveform. Let us look at some example here go to points one by one. Let us say there is a particular point here and let me call this point as A, this point let me call it as A. What is this point this is alpha 1 is 0 and alpha 2 is equal to 60 now. Let me take that alpha 1 is 0 and alpha 2 is equal to 60. What is the nature of the waveform going to be here and alpha 1 is equal to 0. So, at 0 the waveform is supposed to switch from low to high and that alpha 1 is equal to 0 it is suppose to switch from high to low now these two have been merged. So, the waveform is basically the other way here. And it switches high at alpha 2, this is 60 degrees and this is 90 degrees, this is how the waveform is.

So, what is this waveform let us just come to that. What is this waveform? If I just continue this at 120 degrees, it will switch back; and at 180 degrees, it will go back like this. So, what can you say about this waveform this waveform has 0 fundamental voltage why what is the frequency of this waveform it has actually a periodicity of 120 degrees at the fundamental frequency. So, it is a third harmonic square wave, its periodicity is one-third of the fundamental cycle. So, it is a third harmonic square wave.

So, all those you know the frequency components it contains are three and odd multiples of three why odd multiples of three it is a third harmonic square wave and this wave form has a half wave symmetry. And therefore, it does not have its own even harmonics it has only its odd harmonics. So, the wave form contains n is equal to 3, 9, 15, 21, 27 etcetera; n is equal to 3, 9, 15, 21, 27 etcetera, it does it does not contain the other ones. So, the fundamental component is 0 here.

Now, once again let me take another waveform here take let us say this particular point let me call this point is B. What is B, B is you have alpha 1 is 60, and alpha 2 is 90, this is B, this is point b. So, at this point, if you look at the waveform, the waveform switches like this at alpha 1 is equal to 60, it comes back; and at 120 degrees, it goes high. Alpha 2 is 90, therefore you know 180 minus alpha 2 is also 90, so that notch is not there. So, it goes like this now this is the horizontal line.

So, this again is a third harmonic square wave except that these two square waves are out of phase, this is also a third harmonic square wave this also has n is equal to 3, 9, 15, 21, 27 etcetera. So, if you look at the line to line voltage, what is line to line voltage V RY is going to be V RO minus V YO. V YO is phase shift at by 120 degrees, but these are all triple n frequencies, any triple n frequency you phase shift by 120 degree at the fundamental frequency, it is going to be just identical. So, when V RO you know V YO is subtracted from V RO all the triple n frequency components are subtracted. So, essentially you will find that V RY is 0 throughout. So, there are no harmonics of plane it is it is 0 fundamental voltage and the harmonics are also 0.

Now, what can be bad? Let us take something like this now. You take a point here if you let me take about this is I will call a point as C. and A is 0 comma 60, and B is 60 comma 90m and C is let us say 36 degrees. If you plug in 36 degrees, you will get 72 degrees as the output. You would get 72 degrees as the alpha 2 here. So, if you do that if you consider 36 comma 72, this is the point C. If you consider this what will you get at 0 the waveform switches; at 36, it switches back; at 72, it goes high. So, if 72, it goes high, this is 72 is 90 minus 18. So, at 90 plus 18, it will come down; 90 plus 18 is due to quarter wave symmetry is 108. And once again at 144, it will go high; and 180 it will come low. So, this is the nature of the waveform.

And what is this, this is a fifth harmonic square wave. The fundamental the periodicity of this waveform is one-fifth of the fundamental cycle. So, it is a fifth harmonic square wave, it is a square wave, it is a fifth harmonic square wave. So, what are all the values of n it contains now, it does not contain a fundamental component at all because its own period thing is 5. So, n is equal to 5. And further it has all you know it has no even harmonics because it has half wave symmetry. So, it has its own odd harmonics that is n is equal to 5, n is equal to 15, 25, 35 and so on.

So, if you take the first two cases of a and B and if you consider V RY, V RY will be 0 the fundamental voltage will be 0 and also the harmonic voltages will be 0 because the these V RO contains only the third harmonics and third harmonics get subtracted. Whereas, if you take this point C, if you take the it is a fifth harmonic square wave, so V RO has a fifth harmonic and V YO has a fifth harmonic, they have phase shifted by 120 degrees at the fundamental frequency. So, what you will get you will get a resultant fifth harmonic in V RY; so the waveform that the voltage that you end up applying to the motor will be big fifth harmonic waveform. So, it is instead of applying some 0 fundamental or a very low value of fundamental and you know what you end up applying is a very large amount of fifth harmonic wave.

So, if you look at this V one is equal to 0 curves, what turns out is these are the two points A and B which are good points good operating points. Why they are good operating points, the entire curve is capable of giving you 0 fundamental voltage, but at these points A and B, the harmonics are all 0. In all the other points the harmonics are quite high. A particularly bad example is you know yes let me just check this something like C which is 36 comma 72 degrees is a particularly because it is a fifth harmonics we instead of you know what you want is 0 fundamental voltage and do not want any harmonics you end up applying a large amount fifth harmonic. In several points along this line, except if you are close to A or close to B you are going to have fairly high harmonic content, thus we can distinguish between what could be a good operating point and what could be a not so good operating point here now.

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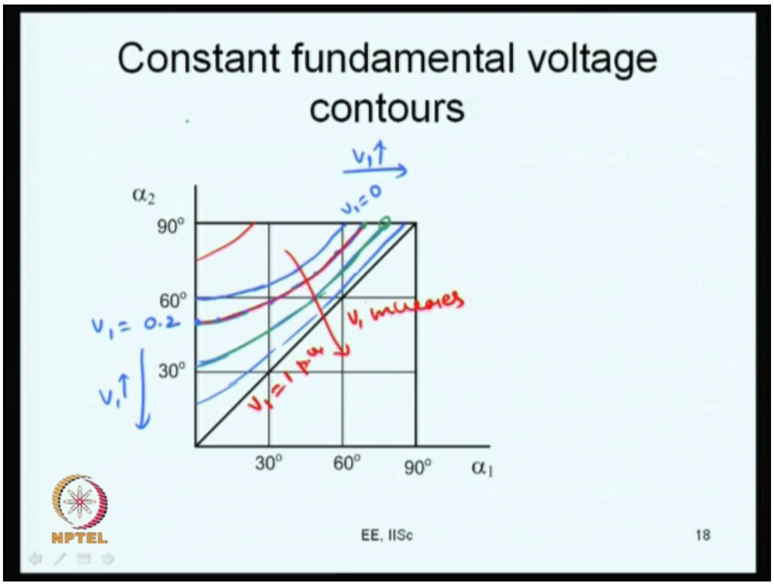

So, we have been focusing on V 1 is equal to 0. If you just draw this again, V 1 is equal to 0, what we got was this was the curve V 1 is equal to 0 we got roughly this is the kind of V 1 is equal to 0 we got. So, instead you want to get several other values of V 1.

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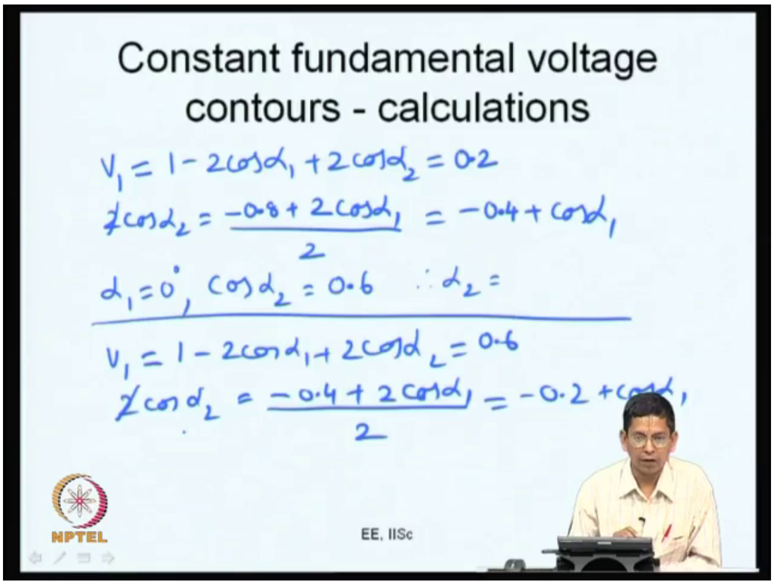

Now, let us say what you want to V one is equal to 0.2. If you want V one is equal to 0.2 all that you need to do is V one is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 equals 0.2 you substitute different values of alpha 1 and you get different values of alpha 2. So, your 2 cos alpha 2 is now 0.2 minus 1, 0.2 minus 1 is minus 0.8 plus 2 cos alpha 1. And your cos alpha 2 is the whole thing divided by 2 that is minus 0.4 plus cos alpha 1 I hope I am not making any mistakes here. So, it is 0.2, 0.2 minus 1 is minus 0.8 minus 0.8 plus 2 cos alpha 1, the whole thing divided by 2. So, you know cos alpha 2 is minus 4 plus cos alpha 1 and you have you take the cos inverse of that to get what you want.

So, you plug in different values of alpha 1, you can get the corresponding values of cos alpha 2 and alpha 2. So, you can go about doing that. If you do that exercise you can start with for example, alpha 1 is equal to 0 degree, if you say alpha 1 is equal to 0 degree, it is easy to see cos alpha 2 is equal to minus 0.4. So, what we have is 1 minus 2 cos alpha 1 plus 2 cos alpha 2 is equal to 0.2. Therefore, 2 cos alpha 2 is 0.2 minus 1 here is minus 0.8 plus 2 cos alpha 1 and the whole thing divided by 2. So, what we have is minus 0.8 by 2 is minus 0.4 plus cos alpha 1.

So, now, let me substitute alpha 1 is equal to 0 when I say alpha 1 is equal to 0 what I have is 1. So, minus 0.4 plus 1 is basically 0.6. So, I made a mistake here let me take this off. So, what I really have is, 0.6 now. Therefore, I will get some value of alpha 2 and where will that alpha 2 lie here on this curve it will be somewhere here. This is cos alpha

2 is 0.5, and now I am talking of cos alpha 2 is equal to 0.6, say the sum value lower than that here this is the value that I will get now.

So, if I go about getting several points, I will get points like this. And this would be my V 1 is equal to 0.2, this curve could be my V 1 is equal to 0.2. Similarly, I can get for any value of V one that I really want let us say I want V one is equal to 0.75. So, if I want V 1 is equal to 0.6 maybe 1 minus 2 cos alpha 1 plus 2 cos alpha 2 equals 0.6 what I what then what I should be doing is. So, here I will have 2 cos alpha 2 is equal to 0.6 minus 1 is minus 0.4 plus 2 cos alpha 1 and you divide this by 2, this is minus 0.2 plus cos alpha 1.

So, you can once again plug in the values of different values of alpha 1 and you can get various values of alpha 2 now. So, you can do it for 06, 0.8 and anything that you want. So, if you look at something like 0.6 or so, you may get another curve like this, you may get a third curve make it more curves like this. So, these are higher and higher values of V 1. So, V 1 goes on increasing, along this line V 1 is increasing. Similarly, when you come down this line V 1 is increasing.

So, I think I should better draw it this fashion. Let me take a red ink here. So, if I move in this direction, V 1 increases. And what is my V 1 along alpha 1 is equal to alpha 2 line, V 1 is equal to 1 per unit, V 1 is equal to 1; alpha 1 is equal to alpha 2 is the square wave and therefore, that is equal to 1 per unit. So, you have several such constant fundamental voltage contours you can plot. You can plot not just for V one is equal to 0; V 1 is equal to 0 maybe of very less you know practical significance because you may not be applying V 1 is equal to 0, but V 1 is equal to 0.6 or anything 0.5 or 0.8 etcetera are more frequently applied. Let us say in motor drive operating entire low frequency. So, you take those curves and many of these you can drive this is how it.

So, from V 1 is equal to 0 to V 1 is equal to 1, for any value of V one you can get curves here. If you go on this side if you go on this side what you will get is you will get negative values of V one basically that basically means the phase difference. So, whatever phase you get for fundamental and what you get there here will be out of phase right. So, now we have been able to come up with these several things to about the various I mean contours. So, V 1 is equal to 0.2, 0.5, 0.6 etcetera we can draw. So, what it basically means is if you want V one is equal to 0.2, you have all these points available for you, you have all these points available for you. And if you want some other value of V 1, and you have several points available for you. Let me take a different color and over drawn this. So, you have several points, you can pick up any of those points. How to pick up those points one thing you can do is to look at the harmonic content. So, pick up one or two of those points, which have better harmonic content than the others and use that fine.

So, we at least the choice in terms of V 1 is becoming clear to us. So, there are several values of alpha 1 and alpha 2 that could give the same V 1. We have first at least understanding what are the various values of V alpha 1 and alpha 2 that give us a particular value V 1, and then we can go on to select which value of alpha 1 and alpha 2 based on the harmonic content now which is what we will do subsequently.

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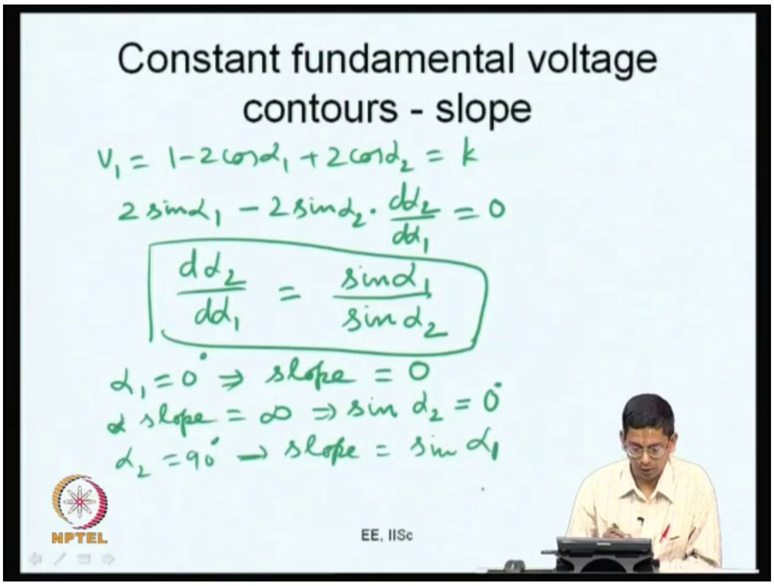

So, the constant fundamental voltage contours what we had was you know the it is a kind of a curve it will be interesting to understand the slope of this curve. Now, what we are saying is we are looking at these curves V 1 is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 this is equal to some constant let me call this k. Now, I am interested in understanding the slope of these curves. So, what do I do, I can differentiate both sides with respect to alpha 1. So, what I will get here is 2 sin alpha 1 minus 2 sin alpha 2 times d alpha 2 by d alpha 1 is 0 this is what I get by differentiating both the sides now.

So, what I essentially have is d alpha 2 by d alpha 1 is basically sin alpha 1 by sin alpha 2, this is what we get. So, the slope is something like this the slope is something like that. So, the numerator is sin alpha 1 the denominator is sin alpha 2. So, whenever you have alpha 1 is equal to 0, you can say that the slope is 0, you can say the slope is 0. Go back to the curves, you see that the slope is 0 the curves are practically flat here. There are some errors in drawing, if you actually plot them you will find them, these are 0 here, you plot them you can find out that they are 0 the slope is 0 there.

Now, you go to the other side you can go to the other side. What is that you can say that you know when is the slope going to be infinity when alpha 2 for certain values of alpha 2 0 the slope will be infinity. Let me start from that, slope equals infinity implies sin alpha 2 is 0 so that means, basically whenever alpha 2 is 0 or a 180 or such values it is going to be infinity is that correct. So, that is where you really have that there. Well, we should be careful in coming up to alpha 1 and alpha 2 both becoming 0 anyway you can see what happens here now.

So, what I am trying to say is before we know going to infinity it is best that you go further on this curve. You find that let us say this curve is 0 slope and the slope goes on increasing the slope goes on increasing. What happens is somewhere in the middle of the curve the slope is it looks flat I mean it looks more or less like a straight line now. So, if you consider this alpha 2 is equal to 90, then you say that the slope is equal to sin alpha 1 this slope is simply equal to sin alpha 1.

So, let us look at these curves. So, what is the slope of this curve at this point, it is simply equal to sin alpha 1. Again if I take this point, what is the slope of that of the curve at that point, it is equal to sin of alpha 1. So, these are constant fundamental voltage contours. In the next lecture, we will be focusing on constant harmonic voltage contours which will take us to selective harmonic elimination.

Thank you and see you again in the next lecture.