Pulsewidth Modulation for Power Electronic Converters

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Lecture - 10
Purpose of PWM - II

Welcome back to this lecture series on pulse width modulation for power electronic converters. So, the first seven or eight lectures, we focused on various power electronic converters dc to dc converters and dc to ac converters voltage source and current source and also multilevel converters. Now, we have started to focus on pulse width modulation.

Firstly, we have been looking at the purpose of pulse width modulation our focuses will be on you know among all the power electronic converters we will focus on a three phase voltage source inverter that would where you know our focus will be on; and we are trying to look at certain modulation methods for that. So, to start with we are looking at the purpose of pulse width modulation. So, in the last lecture, we saw few things in terms of why we need such kind of pulse width modulation and this is going to be a continuation on that.

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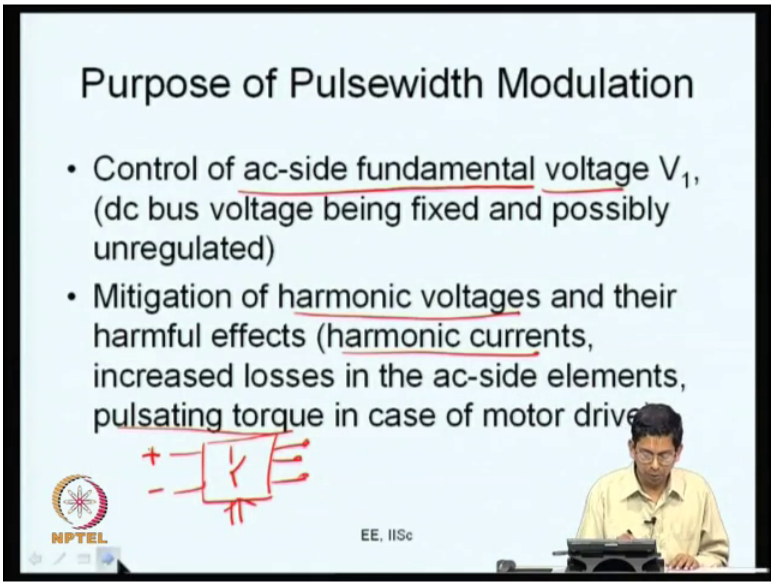

So, in pulse width modulation what we do is basically you know you have a dc bus voltage you have an inverter let say let me put an inverter like this you have certain inverter and this inverter has certain dc voltage, it has certain dc voltage excuse me it has certain dc voltage. And what you have is you need certain ac voltage here. Now, how do you get the required amount of ac that is through pulse width modulation that is by gating all these signals. So, what needs to be given by the pulse width modulator is basically the gating signals for the various devices in the inverter. So, the gating signals should be such that your ac output voltages as this side with the fixed dc bias voltage.

So, the first purpose of pulse width modulation is to control the ac-side fundamental voltage, and the dc bias voltage being fixed or it could sometimes be unregulated when it has been derived from let us say a diode bridge rectifier you know or some rectifier and a open loop. Now, the second purpose is what you are going to get here on the ac-side is going to be a pulsed waveform is not going to be a sinusoidal waveform it is a waveform is going to have certain non sinusoidal components. So, those are harmonics other than the fundamental you are going to have certain harmonic voltages. These harmonic voltages are going to cause certain harmonic currents.

These harmonic currents again could increase the copper losses in the various elements through which they flow. And you know the harmonic fluxes and the harmonic currents interacting with the steady current or steady flux can produce pulsating torque if you are talking about a motor drive. So, they have lot of effects you know which are undesirable effects of harmonic voltages. The second purpose of pulse width modulation is to reduce or mitigate these harmonic voltages are at least one or more of their harmful effects.

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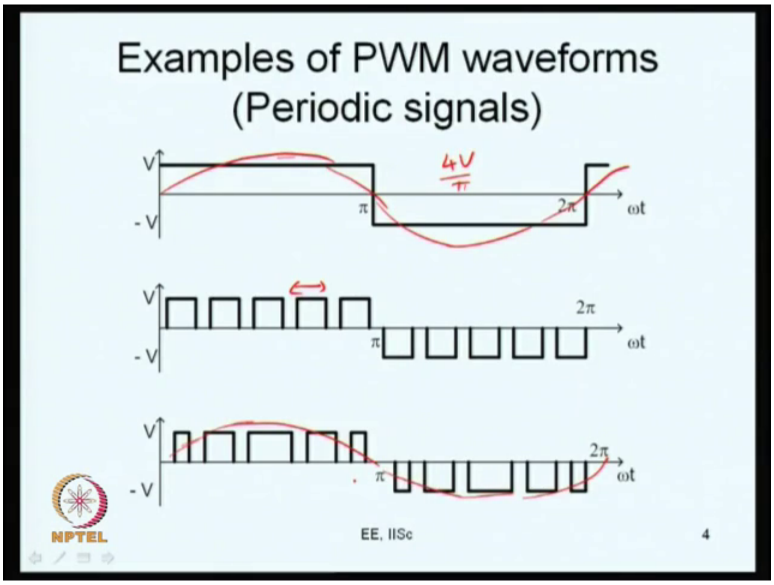

So, what we are going to look at these are the two broad purposes now and we are going to quickly review whatever we saw in the previous weeks now. So, how are you going to analyze the PWM waveform? That is what here we are seeing some very, very simple PWM waveforms the actual waveforms could be much more complicated. But these, such simple PWM waveforms are of great instructional value. And you know and also in very, very high power converters, you know you still have PWM waveforms which are reasonably simple because the switching frequencies are low in hypo converters.

Well, now this is an example is what I have given is a square wave and as I have said before the square wave has certain amplitude like this. So, if V, it is plus V and minus V this amplitude is going to be something like 4 V by pi that is the amplitude that did you are going to have here. And what you can do is this is another example what is being done is it is not a complete square wave, but you have pulses, and these pulses have certain duration, all of them have equal durations.

ou have positive pulses in the so called positive half cycle; you have negative pulses in the so called negative half cycle of the voltage of the fundamental voltage. So, you can vary these bits, if you increase the widths the fundamental voltage you get is going to increase; otherwise it is going to decrease. So, it is an example of a periodic signal and it is an example of a PWM waveform and you know and this is how you control the fundamental voltage here.

And this is a another example where you can intuitively see that the fundamental voltage is controlled and also the harmonics are probably better controlled, because your fundamental component is going to be something like this. When the fundamental component is high, those pulses durations are higher; and then when the fundamental is going through zero, the pulses duration are lower. So, you do certain things like you vary the pulse widths width of pulses in certain fashion so that the you get the desired fundamental voltage, but the harmonics are suppressed now.

So, one way I can say is you would look at these widths there are pulses and they have certain widths and those widths are modulated in certain fashion and that is why we call it as PWM. Modulating the widths of these pulses in certain fashion such that you get the desired fundamental voltage and you get you know the harmonics are reduced now. So, all these signals that we see are essentially periodic signals all the signals we see are essentially periodic signals. So, we want to analyze we want to understand how much is the fundamental voltage quantitatively and how much are the harmonics, what are the various harmonics, how much are there.

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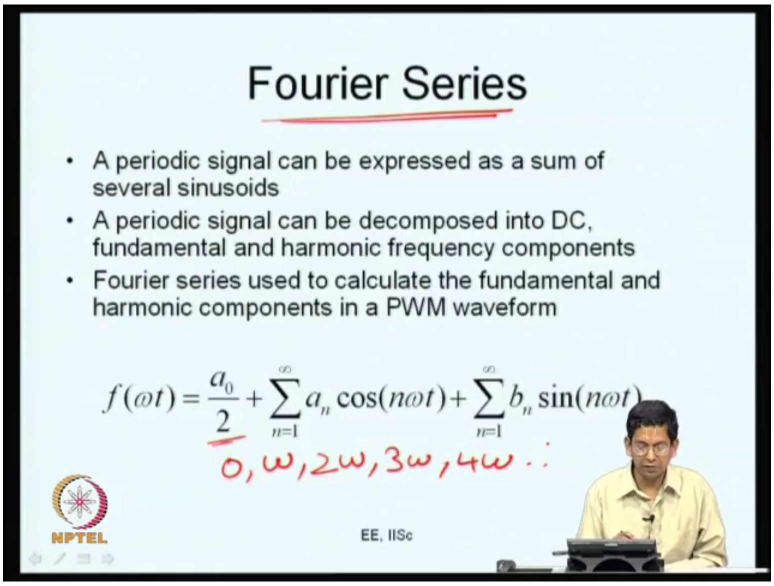

How do we do that being periodic signals, they can be expanded into a Fourier series they might have a dc component in general a periodic waveform might have a dc component and might have a fundamental component 0 omega might have 2 omega, 3 omega, 4 omega etcetera. Where omega is the fundamental angular frequency 2 omega, 3 omega, 4 omega correspond to the harmonics it goes on like this. Now, see in general it can have all these harmonic components, but the waveforms we saw previously waveforms they have zero dc value, they have no average value.

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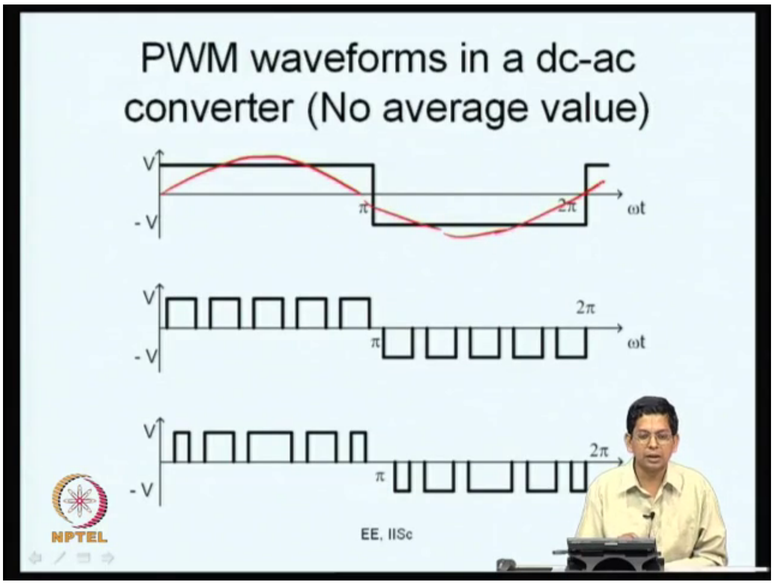

So, the dc is 0 here. So, they only have a fundamental component and then they have the other harmonics they are fundamental and they are the other harmonics now.

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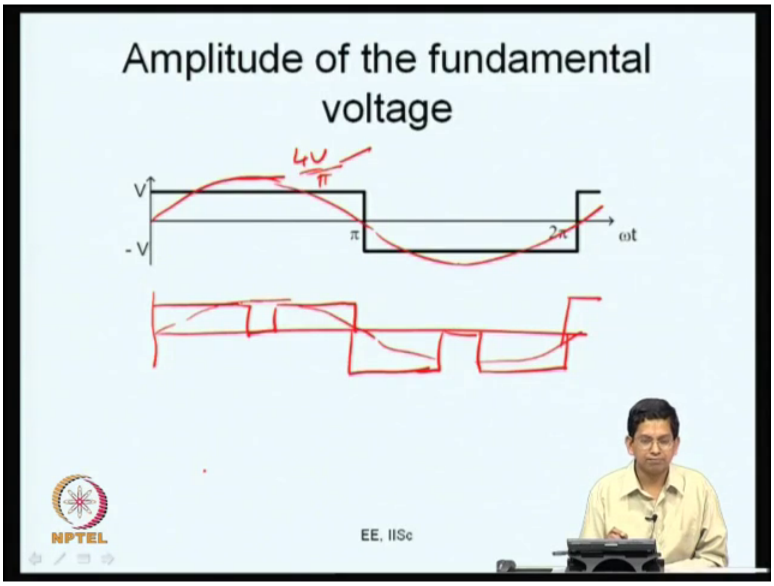

So, what is the amplitude of the fundamental I once again as I told you this amplitude of the fundamental is for such a waveform it is going to be the peak value is going to be four V by pi. So, this is the amplitude now, you cannot vary this amplitude if you want to vary this amplitude we should vary V; that means, the dc bus voltage is being varied, it is not the case that we are considering. We want you know V to be fixed. So, what we have to do here is as we will see we have to introduce certain notches, this is the maximum fundamental voltage that can be obtained for certain 4 V by pi.

So, what we can do is we have to introduce make certain values zero or certain negative values. Now, for example, if a change this waveform such that it is 0 here, and 0 here; this waveform has certain amplitude - fundamental amplitude, this fundamental amplitude is lower than 4 V by pi, this fundamental amplitude is lower than what we had in the previous case that is 4 V by pi. So, how much lower it depends on the notch, how big is the notch now. So, if the notch is 0, it will be 4 V by pi. As the notch goes on increasing, it will come down now. So, you control your amplitude of the fundamental doing like this now.

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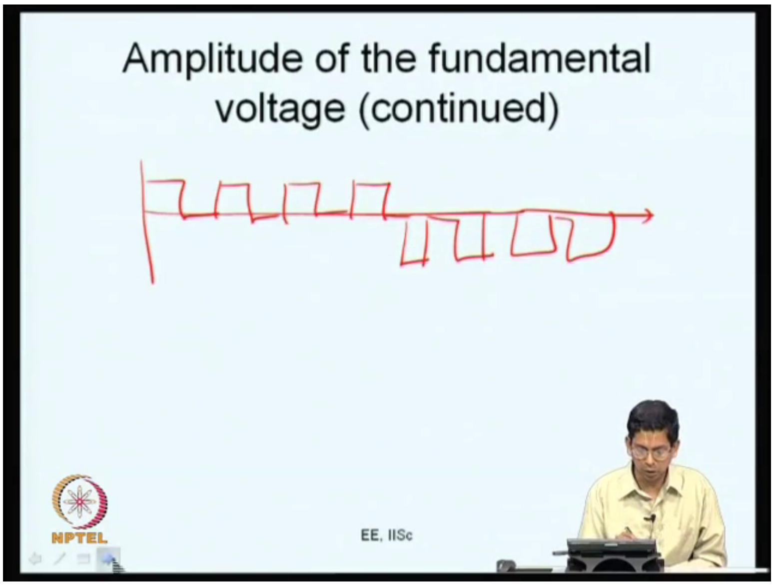

So, this is another part you know how do you want to control the amplitude of fundamental voltage now. So, there are various ways that in which you can do that that there I had introduced a single notch now whereas, you can also introduce several notches like what we said we can introduce several notches like this. So, this is an example which is very similar to the previous examples that we saw here. So, what you can do is you know you introduce some notches few notches more number of notches and more number of notches means the device is inside the inverters which is more number of times, so that places an upper limit.

So, in the olden days, the device is switched at very low frequencies. So, in a 50 hertz cycle one could have very few notches or. So, nowadays in modern power electronic devices, they can switch at much higher frequency particularly the power levels are not too high I mean the power levels are not like 10 of megawatt or something like that you can certainly do that now. So, what we do is basically we you know this widths of the pulse is basically determine what you want to do. So, you control the amplitude of the fundamental voltage by doing this now.

So, this is about the dc value we are looking at waveforms which have no dc value these are the ac-side waveforms in a PWM inverter. So, you know they do not have dc value and the fundamental voltage can be controlled as given here.

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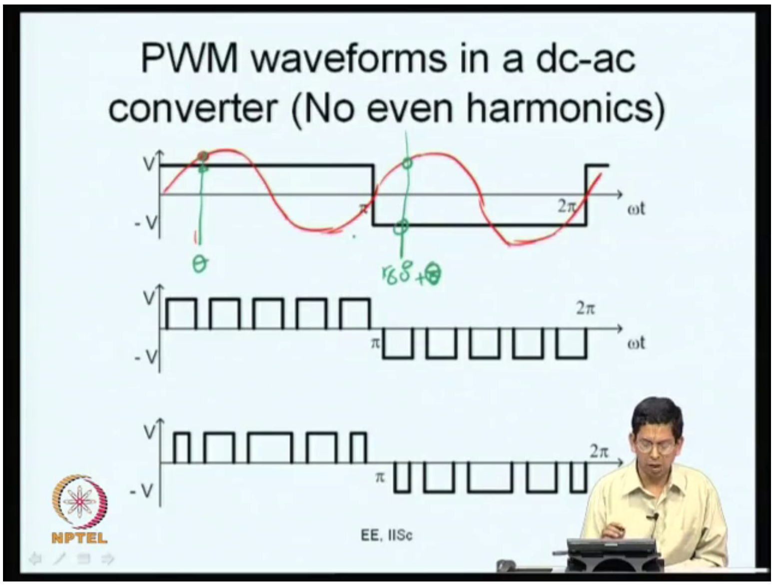

Now, and further they have no even harmonics or they have half wave symmetry this is what we saw in the last class now. Assuming now let us say you want to check whether it has a second harmonic. What you do you multiply it by a second harmonic as shown here now. Now, take some arbitrary instant say here and the same instant 180 degree is later on the same instant let me just change the color of ink. This is certain instant theta let me take some other instant 180 plus theta at 180 plus theta. The second harmonic has the same value, but the waveform in question has plus V here and it has minus V here.

Therefore, if you take the product at theta and if you take the product of the two at 180 plus theta the sum is 0. So, the same is valid. So, when you integrate this over the over entire like you know this is valid for any theta start from zero go all the way up to 180 degree, it is 0. So, it does not have any even harmonic.

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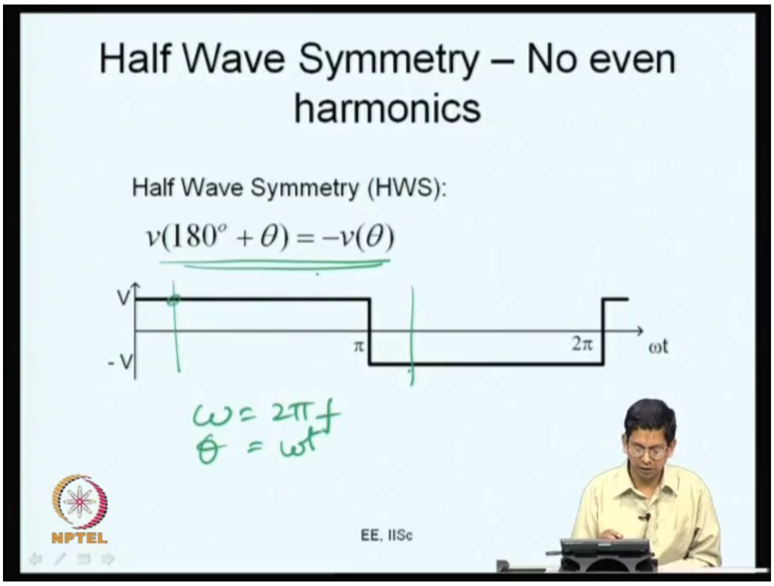

Now, this is what we say half wave symmetry and this is the definition for waveform with half wave symmetry. If V is the voltage waveform and theta is a measure of time it is in terms of the fundamental angle, it is omega t, where omega is equal to 2 pi f where if f is your fundamental frequency 2 pi f is your fundamental angular frequency. And theta what we mean here is omega t. So, you take it at any theta, if you take it as theta if you take it as 180 plus theta, you see that at 180 plus theta its value is the negative of whatever it was at theta. So, such waveforms satisfy half wave symmetry, and they do not have even harmonics.

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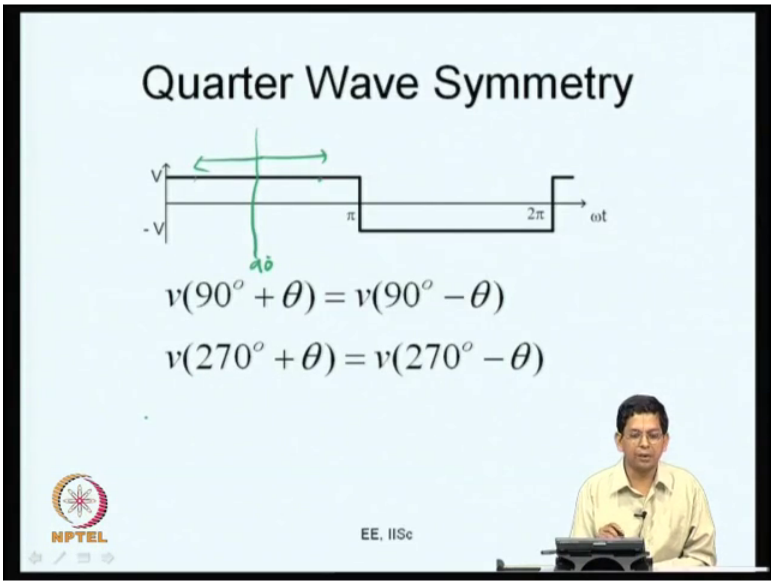

So, of the various you know harmonics or frequency components we saw they have no dc, they are fundamental, they have no even harmonics. So, the waveforms in question may have only odd harmonics. The frequency components are omega, 3 omega, 5 omega, 7 omega, 9 omega etcetera these are all the frequency components that you will find here now. So, in addition to half wave symmetry you see that these two halves are symmetric that is you take this half and you invert it by 180 degree you will get this half now.

Further to this if you look at about these points, it is pi by 2 or 90 degrees you move certain distance here in the same distance this side to the left or to the right, you find that the values are equal. So, this is what you call as quarter wave symmetry, this is again what we discussed last class now.

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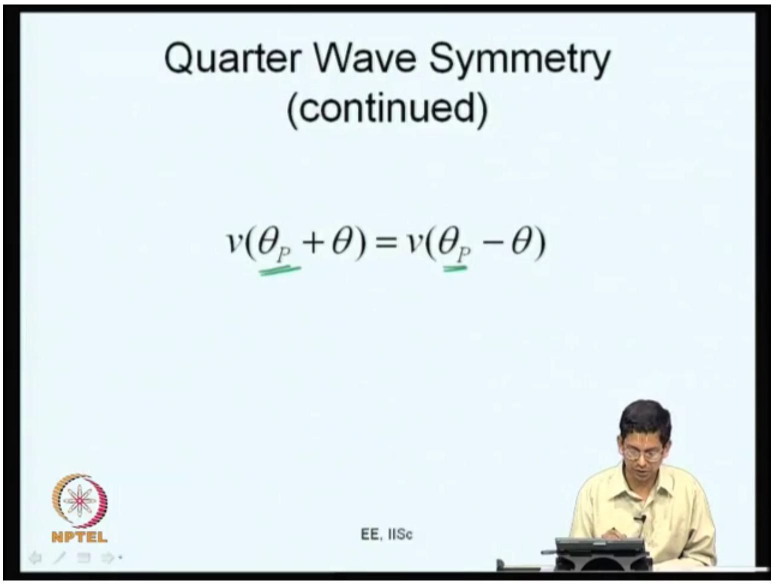

So, it is not exactly at 90 degrees. So, you can consider any theta p theta p is an instant where the fundamental component of the waveform V has a positive peak or a negative peak. So, it could be theta is equal to 90 degrees or 270 degrees going by our convention now. So, about this so you can say that V of theta p plus theta is equal to V of theta p minus theta this is what we have for quarter wave symmetry.

So, now on top of half wave symmetry if you have this quarter wave symmetry, your calculations become simpler, you can just use one quarter of the waveform to do your calculations. It also means that whenever your fundamental has a zero crossing, the fundamental component of the waveform has a zero crossing the harmonic components also have their zero crossings.

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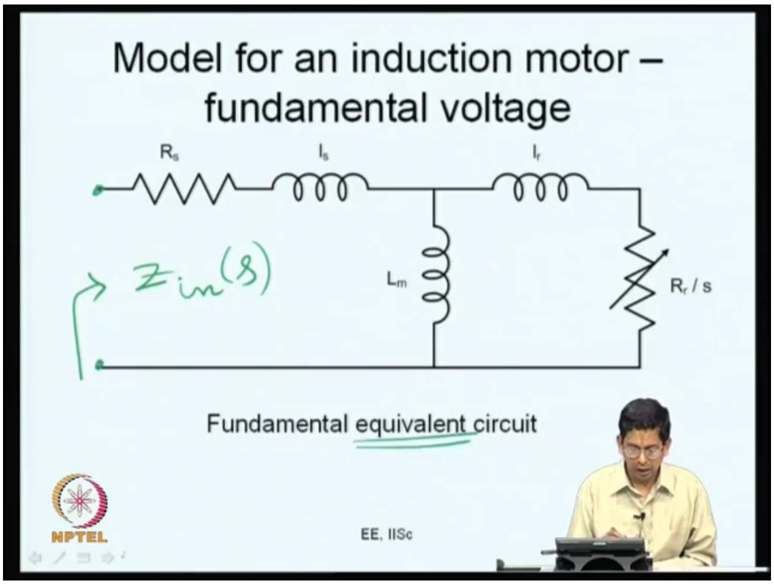

So, now up to this we have seen you know it is about Fourier series and waveform symmetries and how you calculate the various harmonic components. You know we have we now have an ability to calculate given a PWM waveform which is actually a periodic waveform we are able to calculate the fundamental and the various harmonic voltages now. Now, when harmonic voltages are fed to certain loads which we normally come across you know how would the harmonic currents would be is what we are looking at now is the next thing.

Let us say you apply a harmonic voltage you know non sinusoidal voltage to in a induction motor this is what happens in an inverter feeding an induction motor you do not get to apply pure sinusoidal waveforms. You apply waveforms which have sinusoids plus a lot of harmonics. The fundamental component of that of the inverter output voltage sees this induction motor as its fundamental equivalent circuit this is the well known equivalent circuit that you would have studied in your undergrad courses on basic electric machines or the basic electrical technology kind of courses you would have studied this.

So, the machine you know the fundamental component is seen as something like this its basically it is seen as certain impedance. It is seen as certain impedance and this impedance is a function of the slip. As I said at no load the slip is 0; as the motor gets loaded and loaded the slip increases, and the rated slip could be something small like about 5 percent or so it is a typical value. So, what you have is you know you we have the Z n as a function of s.

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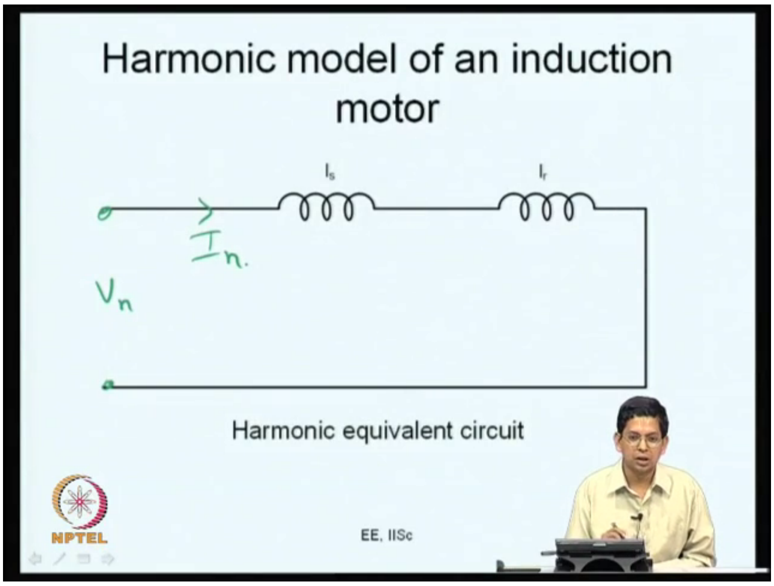

On the other hand instead of fundamental, if you considered the harmonics what happens is the harmonics see a much simpler equivalent circuit, which is only the leakage inductances, because for the harmonics if you look at this, the slip is very high. And therefore, R r is very small therefore, this is basically a shot. Now, when this leakage inductance of the rotor comes in parallel with the magnetizing inductance this is small and therefore leakage inductance is small, therefore, this dominates. And the reactance is dominate over resistances at high frequencies considering these factors the whole thing simplifies to this now.

If you apply certain harmonic voltage then the harmonic voltage sees the motor as such an equivalent circuit which is simply its total leakage inductance. So, if you apply as this n th harmonic, you can calculate the n th harmonic current using this now. These are the models for the fundamental and the harmonic components for a induction motor.

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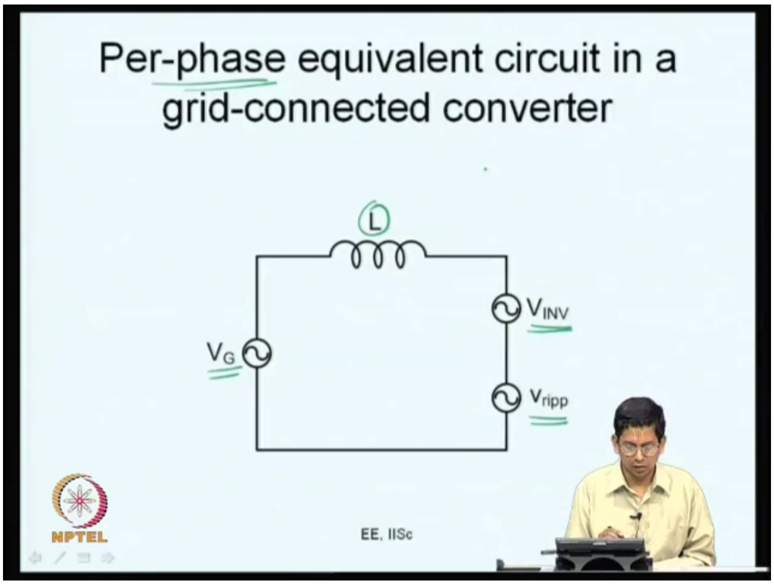

Now, in inverter you can consider a grid connected converter that is an inverter is connected to a grid through line inductors and this is the line inductor L. Now, the invert this is the grid voltage, we can assume it to be sinusoidal though it is not always so grid voltages are distorted and they could be unbalanced, but here let say for the time being we assumes balanced and sinusoidal grid voltages now because V G is the grid voltage which is sinusoidal. Since they are balanced we can consider a per-phase equivalent circuit.

So, now, what you do is this is the sinusoidal voltage. Now, how about the inverter terminal voltage the inverter terminal voltage has sinusoidal as a sinusoidal component, it has a fundamental component plus also lot of harmonics. So, let me say the sinusoidal component or the fundamental component there let me give this name V inverter and all the harmonics added up constitute this V ripple. So, this is sinusoidal and this is sum of several harmonics, and it is non sinusoidal now. This is the per-phase equivalent circuit for a grid connected converter now.

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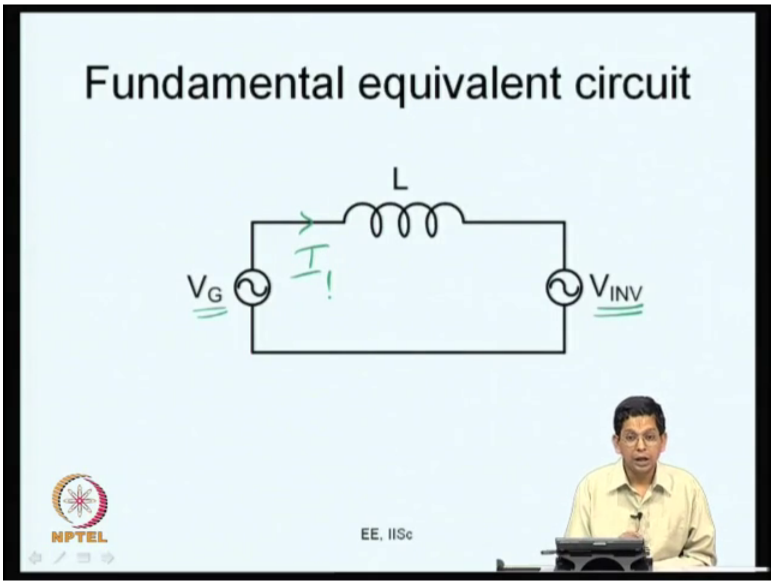

So, if you are interested in the fundamental current you want to see how much fundamental current is drawn the ripple voltage produced by the inverter makes no sense here they are at various frequencies. So, what you have is the grid voltage and this side what you have is the inverter output fundamental voltage, the fundamental voltage in the inverter line side and then the leakage inductance comes in between. This circuit helps you calculate the fundamental component.

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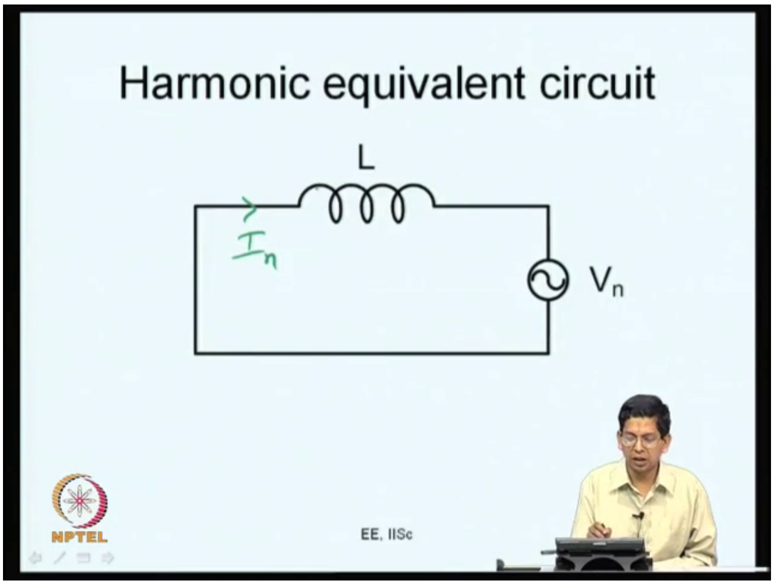

If you are interested to calculate the harmonic components, now the grid voltage is a sinusoidal voltage at 50 hertz or 60 hertz and whatever the line frequency is in that country. Now, that is not going to affect any nth harmonic component. So, if you want to calculate some nth harmonic component how much it flows, the grid voltage is zero there, it has no effect. And the fundamental component of the inverter output voltage has no effect. So, it is just a specific harmonic voltage produced by the inverter and the line inductance. So, this gives you basically V n upon omega L is going to be the amplitude of the current that was flow here right.

So, this is how we can calculate harmonic currents that is first we are able to calculate harmonic voltages or we are considering a voltage source inverter which applies certain voltage waveforms, these voltage waveforms are periodic waveforms to the extent we have considered, but they are not sinusoidal. So, we know how to calculate the fundamental and harmonic components you know that is a Fourier series helped us. And once the harmonic voltages are known the harmonic currents can be obtained through the models we just discussed now.

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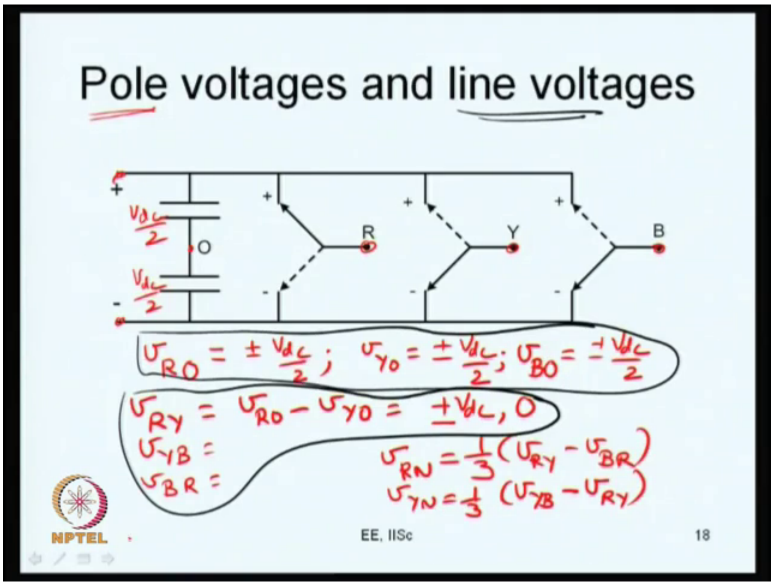

So, to repeat we have done two things, one is given the fundamental voltage I mean given a PWM waveform. We know how to calculate the fundamental voltage and harmonic voltage. And given the fundamental voltage and harmonic voltage, we know how to calculate the fundamental current and harmonic current. We have considered two situations here; one is the inverter feeding an induction motor, the other is the induct the you know it is connected through grid the line side is connected through the grid through inductors.

So, both cases have been one is a motor drive case and the other one is a grid connected converter, both the cases we have considered and we know how to calculate the fundamental current and harmonic current based on the fundamental voltage and harmonic voltages. So, with these two capabilities we now move onto the more serious business of controlling the fundamental voltage first, and to see if we can reduce the harmonics to certain extent. Now, this is the three-phase inverter that we have been looking at or we will be looking at for a few more lectures to come now.

So, as you can see that you know there is dc applied here, and these are the load terminals R, Y and B, the three phase loads are connected here. And as you can see that every load terminal is basically the pole of the leg you have a every leg is a single pole double throw switch and the load is connected to the pole. So, we will commonly use the term pole henceforth. So, we will define certain terminology certain voltages here, we will or you know we will identify certain nodes. If you look at what are the various nodes that you have in this circuit, the nodes are R, Y and B which are basically the load terminals or the midpoints of the three legs or the poles.

The other terminals that we have are the dc terminals. So, these are the various nodes and you also have the dc bus midpoint O. So, this dc bus midpoint O need not have to be a physical point I mean available for making a connection it could just be a point you know which is for a theoretical purposes, we will use that as a reference point with respect to which we can express the other voltages. Now, let us say we want to say what is the voltage at this terminal R phase terminal or this pole R, so we call that as pole voltage the voltage at pole are measured with respect to the dc bus neutral O, this is what we would call as pole voltage V RO.

Now, this pole voltage if the pole is connected to the top throw as indicated by the solid line here then the potential will be plus V dc by 2 the entire dc bus voltage is V dc. So, you have V dc by 2 here and V dc by 2 here. So, when the pole is connected to the top throw V RO is plus V dc by 2; and when the pole is connected to the bottom throw shown through the dashed lines here, you get minus V dc by 2. So, V RO can be either plus V dc by 2 or minus V dc by 2. Similarly, you can define V YO this is another pole midpoint of the Y phase leg and V YO can also take the values of either plus V dc by 2 or minus V dc by 2. Similarly, you have V BO voltage at the pole B measured with respect to the dc bus neutral O, this is also equal to plus or minus V dc by 2. So, these are pole voltages.

Now, what are line voltages is basically V RY, if you want to get V RY what you have is V RY is essentially V RO minus V YO. Similarly, V YB is V YO minus V BO and V BR is V BO minus V RO you can you like that things similarly. Now, if R is connected to the top throw and Y is connected to the bottom throw then you have V RY is equal to plus V dc. So, V RI can take the value of plus V dc and V RO is plus V dc by 2, and V YO is minus V dc by 2 or this is connected to the bottom throw. The other way, if R is connected to the bottom throw as indicated by the dashed lines here, and Y is connected to the top throw as indicated by the dashed lines there, then what you have is minus V dc V RY will be equal to minus V dc.

On the other hand, you know these are only two combinations that is R can be either connected to the top or bottom, Y can be either connected to the top or bottom, you should take the two phases together you have two multiplied by two - four possibilities are there these are only two possibilities. What are the two other possibilities, yes, R can be connected to the top throw and Y can also be connected to the top throw. What would be R Y then both are connected to the same point, so V RY is going to be is zero. Not just that when R and Y both can also be connected to the bottom throw in that case also V RY is going to be zero. Thus the line voltage V RY takes three different values plus V dc, minus V dc or 0. So, these are the possible value line voltages now.

You have a pole voltage if the pole is connected to the top throw it is plus V dc by 2; if it is connected to the bottom throw it is minus V dc by 2 there is nothing you do not have any redundancy there. Whereas, if you start seeing the line voltage you start finding one redundancy, plus V dc there is no redundancy R top has to be connected and Y bottom has to be connected similarly for minus V dc. But when you want V RY is equal to zero you are able to achieve it in two different ways; one is by connecting both the poles to the top throws, or both the poles to the bottom throws. So, this redundancy will be later on exploited as we will see later.

The similar definitions are possible for V YB and V BR as I mentioned before now. So, these are the pole voltages and the line to line voltages to be specific now. So, all of them will take values, the line voltages will take either plus V dc, minus V dc or 0 as the possible values now. Then you can have further voltages defined here that is let us say you assume there is a three phase load, let us say what we have is a three phase load. And the three phase load is balanced and star connected load, let us assume a balanced star connected load. In that case its possible for us to express V RN as one by three times of V R Y minus V BR.

The same way V YN and V BN can also be expressed now. So, we have different things here. So, one of them is let me just we have the voltages V RO, V YO and V BO. What are these, these are the pole voltages these are the voltages at the poles measured with respect to the dc bus neutral, and we have these voltages V RY, V YB and V BR. What are these voltages, these are the line-to-line voltages, these are the line-to-line voltages. So, if you consider a balanced star connected load, we will further have V RN, V YN and V BN. And how will those be V RN is as given by this equation one-third of V RY minus V BR.

Similarly, if you want V YN that is going to be one-third of V YB minus V RY, similarly you will have V BN also. So, these are the line to neutral voltages applied on the load in case of balanced star connected load now. So, these are the various voltages we just needed to define them and we have been able to define the pole voltages, line-to-line voltages, and line to neutral voltages applied on the load.

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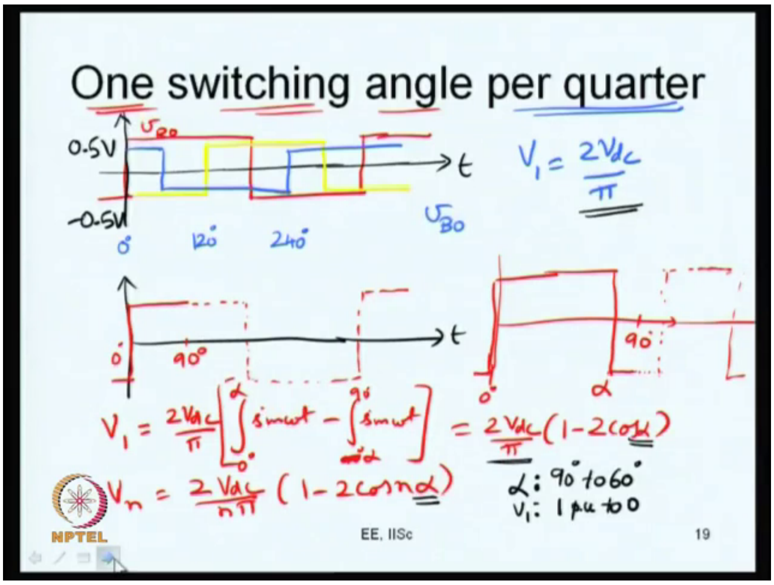

Now, with this let us move on to see how we can control the fundamental voltage now. Firstly, if you just take a square wave, let us take a square wave. If your inverter is switched like a square wave, let me take the horizontal axis. Let me draw it with black ink, this is the time axis, now this is the voltage axis. So, this is equal to 0.5 V or 0.5 V dc and the lower level is equal to minus 0.5 V. If I switch the inverter in a square wave fashion, what I get is you know I get of the pole voltage to be 0.5 V dc for half the time and minus 0.5 V dc for the remaining half the time and this goes on like that.

So, I have shown this in red color for the red phase. If this is my V RO, how will my V YO be this is V RO, red phase V RO; how will my V YO be that is the Y phase let me try and choose the yellow color here. So, V YO has to be exactly phase shifted by 120 degrees now. So, this is 180 degree and 120 degree is here V YO has to be phase shifted the yellow color might not be very clearly visible, but I mean I do hope that you are able to see this waveform that I am drawing now right. So, V YO is going to be phase shifted by 120 degrees it goes on like this now how will V BO be let me choose blue color here that will be phase shifted by further 120 degrees now.

So, if R had a zero crossing at zero degree Y had a zero crossing at 120 degree Y B will have its zero crossing I mean or rather what I mean by zero crossing is switching from low to high R switches from low to high at 0, Y switches from low to high at 120. So, B will be switching from low to high at 240 degrees. So, this is how your V BO will be. So, if it is switching low to high at 240 degrees it will be switching high to low at sixty degrees now. So, this is how the three phase voltages will be. What I have drawn now with blue ink is V BO, blue ink is V BO and the yellow ink is V YO. So, these are the three phase pole voltages when you operate with a square wave mode.

And what is the fundamental component it is the same for all the three and how much is that the fundamental component is simply equal to 2 V dc upon pi as we saw earlier. We cannot control this fundamental voltage. So, to start controlling the fundamental voltage what we do is we are going to introduce one switching angle in a quarter cycle. See we want to have waveforms with you know further to three-phase symmetry, we want to have a half wave symmetry and we also want to have quarter wave symmetry. Now, if you want to have a quarter wave symmetry, there should be equal number of switching angles in all the quarters. So, we will go for first is one switching angle in a quarter, we will consider a quarter cycle be quarter cycle, if you look at one switching angle per quarter.

So, what is that one switching angle per quarter? Now, this entire waveform I am going to represent in a simpler fashion now. Let me just see here that is ok. So, here what I am going to do is this is one switching angle per quarter. First let us look at the square wave this V RO. What I can do is let me change back to the black color and indicate the voltage in the time axis let me indicate the voltage and the time axis along this now. So, instead of drawing the entire waveform, I can only draw one quarter of this, it is negative just before 0; and at 0, it switches from low to high and then it is high up to 90 degrees it is high up to 90 degrees. If I draw this much, it is enough because the waveform has quarter wave symmetry.

So, what does it mean it means I can just simply extend this? If this part of the waveform is given to me I can just reflect it about the 90 degrees and I can get the next quarter, and now I have one-half. If I have one half then through using half wave symmetry I can construct the other half. So, what is given, what I have drawn in solid red line is all that you need to given; and once that waveform is given for one quarter cycle we know what it is for the another quarter and the rest of the half cycle. So, what we will henceforth do is only to use one particular quarter now. So, let us focus only on 90 degrees, so the first 90 degrees now.

So, what we want to do is we are looking at a waveform with just one switching angle per quarter that is when we want to control the fundamental voltage. Let us say V RO is negative and switches from negative to positive at zero degrees like what it happened did here now. Then before 90 degrees, what it does is it switches down from high to low and stays low. So, there is one switching angle here and let us call this angle as alpha. So, what do you have I am representing only one quarter I hope that you can see how it is going to appear in the next quarter for example, if I want to do that all that I need to do is I need to use you know mirror symmetry across about 90 degrees. So, I can just do it like this. So, this is how the waveform will look like about 90, I can go on till 180 and I can use half wave symmetry to constrain the other half now.

So, in one switching we have only you know in one quarter we have only one switching angle. So, the waveform will switch from zero to you know at zero degrees and it will also switch at 180 degrees, it will also switch back at 180 degree from high to low other than these two there will be one switching in a quarter. So, this waveform is an example of one switching angle per quarter now. So, this alpha depending on what the value of alpha is you can control the fundamental voltage now.

So, what will be the fundamental voltage, if you use Fourier series, you can say this is V 1. And you all that you need to do is you need to integrate this waveform as we were doing we are considering only up to pi by 2 and the waveform value is V dc by pi. So, what we have is 2 V dc by pi, and you need to integrate this sin omega t starting from zero to alpha and minus sin omega t from this is sin omega t you integrate this from zero to alpha and again minus sin omega t from 90 to sorry from alpha to 90. This is what essentially you are going to have.

So, what is this going to be its going to be 2 V dc by pi times 1 minus 2 cos alpha if I integrate sin omega t the integral of sin omega t is cos omega t and I am trying to put its limits over zero to alpha. So, what I will get here is 1 minus cos alpha if I consider the second term its again you know it is cos omega t. So, what I will get here is cos alpha; so cos 90 is anyway 0. So, what will happen is this whole thing becomes 1 minus 2 cos alpha. So, this is what we have. And if you consider the nth harmonic instead of the fundamental, this is going to give you something like 2 V dc upon n pi times 1 minus 2 cos n alpha, this is just applying Fourier series. There is a waveform has quarter wave symmetry, we need to consider only one quarter of the waveform and we are able to calculate V 1 and V n or any n th harmonic as given like this now.

So, what do you get here you need it you need certain fundamental voltage let us say I want V 1 is equal to 2 V dc pi that is the maximum which is possible with square wave. Let me change the ink color to black. So, I want 2 V dc by pi. So, if I want 2 V dc by pi what do I have to do now this is alpha and this alpha I have to make it as 90 degrees. If I make this alpha as 90 degrees then this essentially becomes a square wave what I want you can see that 1 minus 2 cos alpha is simply equal to 1. So, at alpha is equal to 90 degrees, it gives me a square waveform. And as alpha is reduced from 90 degrees let us say is made 89 degrees you get some amount of V 1.

So, as alpha is reduced further and further from 90 degrees, your V 1 goes on increasing till what when your alpha is 60 degrees 2 cos alpha is 1, and 1 minus 2 cos alpha is 0. So, alpha can be varied over the range 90 degrees to 60 degree. And your fundamental voltage correspondingly will go from 1 per unit to 0. What do you mean by 1 per unit, 1 per unit is the fundamental voltage that you get with square wave operation this 2 V dc upon pi thus is what I take as the base and this is one per unit now.

So, here you have a capability to vary the fundamental voltage. What we have is a capability to vary the fundamental voltage. We choose this there is only one control variable namely alpha. We choose this alpha to give the desired value of fundamental, the fundamental may be a 100 percent 1 per unit or it may be 0.8 per unit, it may be 0.5 per unit whatever it may be we choose a particular value of alpha which will give us the desired fundamental voltage. Once we have chosen this alpha, we need to plug in the same alpha here and that will give us the harmonics that are being generated. So, we have no control over them. We got to accept whatever harmonic voltages are for this value of alpha.

And you see with alpha this variation is co sinusoidal I mean it is a transcendental function, but here alpha varies only from 90 to 60 degree. If you look at for example, fifth harmonic again for this variation of alpha between 90 to 60 degree, five alpha will vary between 450 degrees to 300 degrees. So, that is a larger amount of variation. So, for a small variation in alpha n alpha will have a substantially larger variation here. So, what you will see here in the fundamental will be a small cycle you know, but whereas here you will see you know larger cycles you will be able to see certain cycles of a sinusoidal co sinusoidal functions here coming up here. So, V n etcetera will change much more drastically than V 1 would change that is what I would like to tell you.

If alpha is changed by 1 degree, V 1 will change to certain extent, but if you take the fifth harmonic it is cos 5 alpha 5 alpha changes to by 5 degrees if alpha changes by 1 degree, 5 alpha changes by 5 degrees, therefore the change in V 5 will be much more now. So, all this it is possible for you to plot these are very fairly simple functions for you to plot and you can try plotting them. Now, what I am going to see is I am going to look at the next two thing of looking at an alternative to get this one switching angle.

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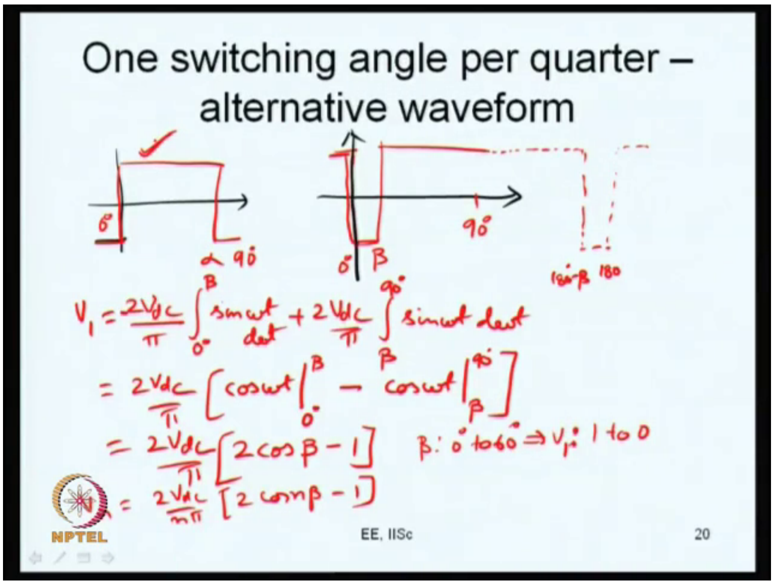

So, what we do in the earlier case was a situation like this. Let me just change the color to red here. So, the waveform was switching at zero degree and it is switching back at an angle alpha and this is 90 degrees. On the other hand, one could also consider a different kind of a waveform namely something that switches from high to low and low to high and still about 90 degrees. First let me indicate the horizontal and the time axis now. So, this is the voltage axis, this is the time axis now.

What I can do is here what I have is V RO is positive and it is switching from high to low, and it is remaining low and then it is going back to high, let me call this as beta and this is 90 degrees right. So, in this kind of a waveform, if I were to projected how will it look in the second quarter I can simply project it, this is just mirror reflection. So, at 180 minus beta it will switch back; and at 180 degrees it will go higher; and beyond this is and beyond this is you can go through half wave symmetry.

So, this is another kind of waveform. You can see that here if then switches from zero and stays high till alpha; and from alpha to 90 it is negative. Whereas, in the other case the waveform switches from high to low at zero degrees, and it remains low for a certain angle that we call as beta here and then it is up to 90 degrees now. So, for this waveform if you have to write what is the fundamental component, you can see that it is V 1 is 2 V dc upon pi. Now, what you need to do is you have to integrate this is minus the first term is negative of sin omega t and this runs from 0 to beta. And the next thing term is you will have plus 2 V dc upon pi you will have to integrate it sin omega t I mean there is a d omega t here please note that it even if I have missed out, you can take it to that beta to 90 degrees now.

So, what you have here is basically 2 V dc upon pi and this is integral of sin omega t is minus cos omega t you have a again a minus sign here. So, this is cos omega t limits from 0 to beta, and this is another you have minus cos omega t evaluated from the limits beta to 90 degrees now is it right. So, what I am going to get is 2 V dc upon pi. So, this is cos beta minus 1 and what you have is another cos beta coming up here this is 2 cos beta minus 1, this is what I am going to get for such a function.

So, let us just check if beta is equal to 0. So, this waveform becomes a square wave. Beta is smaller and smaller becomes close to 0, so there is no switching at all this becomes square wave. And when beta is equal to 1 here this is 2 cos beta is 2. So, 2 cos beta minus 1 is 1 you get square wave. When beta goes on increasing, let us say it goes to 60 degrees. So, 2 cos beta is 1, 1 minus 1 is 0. So, beta now can vary from 0 to 60 degrees if beta is varied from 0 to 60 degrees you can get a corresponding variation in V 1. The fundamental voltage from 1 per unit to 0, again one per unit has the meaning of 2 V dc by pi whatever it is equal to the square wave now. The same thing if you want the nth harmonic this becomes you know instead of 2 V dc by pi there is an n pi term and you have n omega t you are going to integrate it with respect to n omega t.

So, you will have an additional n coming up here for any V n, you may have 2 V dc upon n pi and you may have another cos n beta coming up here minus 1 this is what would be your corresponding voltages now. So, these are two different ways of controlling. And what we will do is by enlarge you will use the first option here. So, we will look at these things a little later when you are doing selective harmonic elimination, we will use both types for certain number of switching angles now. So, for the time being this is what we are going to consider now.

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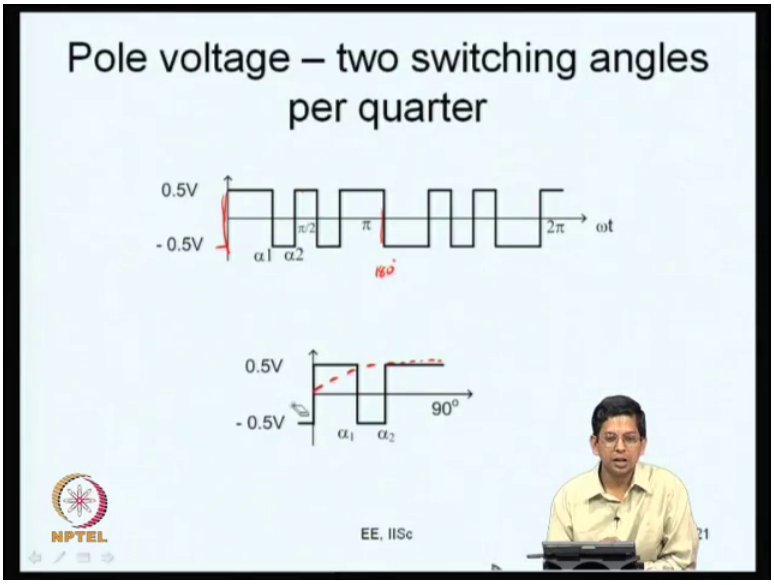

And let us move on from here to the problem of let us say two switching angles per quarter. The difficulty with one switching angle per quarter is you cannot control the harmonics, you can control your fundamental voltage, but you cannot control your any harmonic. So that is why you require a minimum of two switching angles now. So, here again what I said is this is the complete pole voltage waveform, I have just plotted it ready electronically, so that you know it is ready for us to see.

So, you have alpha 1 and alpha 2, the waveform is initially negative and it is switching here which is not very evident. So, this is switching here. Then it is switching back at alpha 1 from high to low; and low to high at alpha 2, and this is 90 degrees. So, then 90 degrees you have there is a switching this is pi minus alpha 2, this is pi minus alpha 1 and then you have pi at pi, it is switching back here this is pi, this is 180 degrees. So, you have with the half wave symmetry because the waveform has half wave symmetry it is enough to consider only one half of the waveform, because the waveform is quarter wave symmetry it is enough to consider just one quarter of the waveform.

Now, this waveform has a certain fundamental component, which is like this. This has some fundamental component like this. This fundamental component as you can see is controlled by certain value of alpha 1 and alpha 2. Now, let us just ask one question ourselves and you know look at it little qualitatively. When alpha 1 and alpha 2 are very close to one another, we will have a fundamental whose goes like this. What happens when alpha 1 and alpha 2 are much farther from one another, what happens when alpha 1 and alpha 2 are much farther from one another?

So, in that case, what may happen is the waveform will look more negative. Now, it is clear that it is we are in the positive half cycle; we are looking at one quarter of the positive half cycle of the voltage waveform. If the gap between alpha 1 and alpha 2 is very large, this will look like a quarter in the negative half cycle and indeed you may have your voltage waveform like this if you have you know the alpha 1 it depends on the relative values of alpha 1 and alpha 2. So, let me just clear this up, I just wanted you to be aware of this. In this kind of a scenario, we are going to calculate what is the fundamental voltage and what are the harmonic voltages now.

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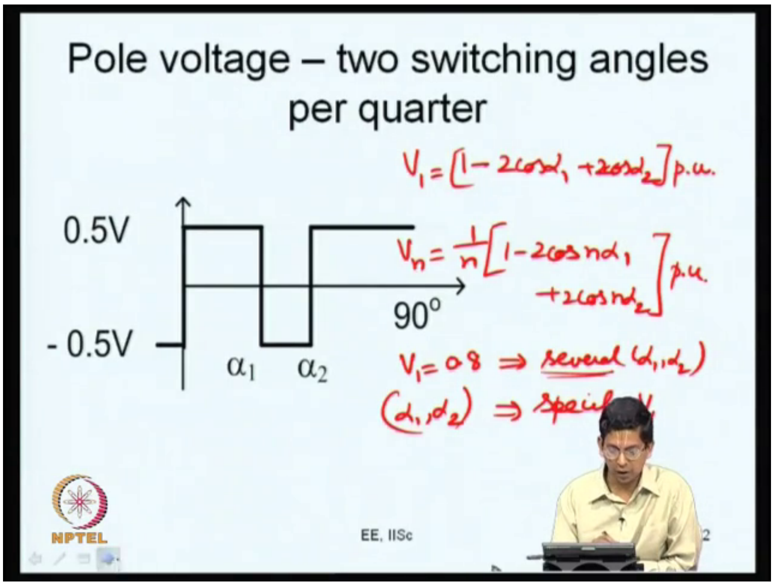

So, we have the same waveform reproduced here for just one quarter. And what is the fundamental voltage, we can say that V 1 is going to be 1 minus 2 cos alpha 1 plus 2 cos alpha 2 in per unit terms, this is what it is. And the n th harmonic component V n is going to be 1 upon n times 1 minus 2 cos n alpha 1 plus 2 cos n alpha 2 in per unit terms, again

1 per unit means 2 V dc upon pi that is the fundamental voltage obtained with square wave operation is it right.

So, now we have these are this we basically you know coming from Fourier series, we are able to evaluate here now. And for different values of alpha 1 and alpha 2, you will get different values of fundamental voltage; conversely for the same fundamental voltage let me say V 1 is equal to 0.8, if I say V 1 is equal to 0.8 there can be several values of alpha 1 comma alpha 2. If I specify a particular value of alpha 1 and alpha 2, let us say 20 degrees and 40 degrees or 60 degrees and 80 degrees, this will be giving me a specific V 1 will give me a specific V 1.

On the other hand, if I have a specific fundamental voltage such as V 1 is equal to 0.8, I can realize this not using just one pair of alpha 1 and alpha 2, but I can use it during several pairs of alpha 1 and alpha 2. So, we need to understand what are all the various possibilities, and we need to from there move on to select which possibility is better for us. So, this gives us multiple possibilities. We will choose one of them which is better in terms of harmonics that is how we start exercising some control over harmonics are their side effects now.

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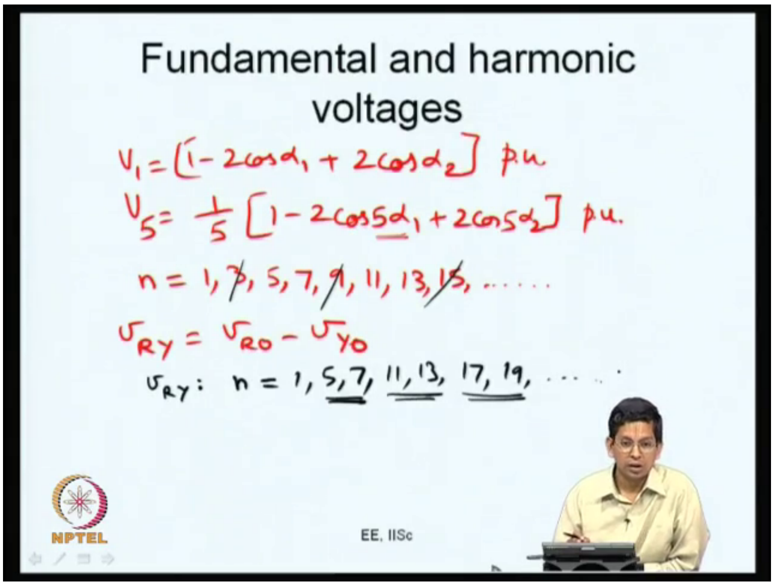

So, as I have already said the fundamental and the harmonic voltages we have them as V 1 is equal to 1 minus 2 cos alpha 1 plus 2 cos alpha 2 in per unit terms. So, you can just plot this you know by varying alpha 1 varying alpha 2, and there are many, many, many things that you can plot here. And V n is 1 by n times I am just repeating it, but I think it is all right because I am sorry this is also in per unit terms now. So, as I mentioned before for some value of alpha 1 and alpha 2, you will get V 1. If you change those either alpha 1 slightly or alpha 2 slightly, the corresponding change in the argument here in the second one is n alpha 1 is going to be substantially higher. Let us focus on a specific value that is n is equal to 5.

Let us focus on n is equal to 5. So, let me take this V n of and let us focus on n is equal to 5. So, I will say V 5 is equal to 1 by 5 times cos 5 alpha 1 cos 5 alpha 2. Now, why is it I am focusing on fifth? The waveform as such has a few components what are the components the waveform has the order of harmonics are it has fundamental it has third, it has fifth, it has seventh, it has ninth, eleventh, thirteenth, fifteenth and so on. It is the waveform on account of its half wave symmetry has only odd harmonics, it has the fundamental and all the odd harmonics. Now, what I am going to do is we are going to apply it to a three-phase load we are going to have your V RY is equal to V RO minus V YO, recall the definition of the line voltage.

Now, if V RO is what we are looking at if V RO has a third harmonic component, V YO will also have the same third harmonic component. And V RO and V YO are phase shifted by 120 degrees. And 120 degrees for the fundamental frequency is equal to 360 degrees for the third harmonic. So, the third harmonic will be the same third harmonic will be available at the same phase in both the waveforms. And when you subtract V RO minus V YO, the third harmonics will get cancelled. So, what will happen is the third harmonics will all go off. If you look at V RY this is in V RO you will have all the harmonics if you look at V RY V RY will have only n is equal to 1, 5, 7, 11, 13, then 17, 19 etcetera. So, you will see pairs of harmonics this is around 6, this is around 12, this is around 18 and so on. So, these are the kind of harmonic voltages which will be eventually seen into the line-to-line voltage.

So, these are the ones that get applied to the motor or the load, the triple n harmonics in V RO do not matter they will get cancelled out. So, this is why I focus on V five residual on this is equal to 1 by 5 times 1 minus 2 cos 5 alpha 1 plus 2 cos 5 alpha 2. So, you can start saying that you have there are two equations and we have two unknowns and this is what has led to the famous problem of selective harmonic elimination we will deal with selective harmonic elimination in greater detail in the coming few lectures now.

This is just an introduction. So, if you want to say that you can choose your alpha 1 and alpha 2 such that is equal to let us say some 0.8 per unit. Let us say 0.8 per unit is what we want its possible for us to choose not just one value several values are possible of alpha 1 and alpha 2 which will give 0.8 per unit at the same time you can also make sure that V 5 is equal to 0. So, what you have is the you have two equations and two unknowns alpha 1 and alpha 2 you can solve these equations of course, these are transcendental equations, you may not be able to solve them straight away it might be better to use you know numerical iterative procedures to solve these, but you can solve them.

It is possible for you to come up with some value of alpha 1 and alpha 2, which will give you a fundamental voltage of 0.8 per unit and a fifth harmonic voltage is 0. Why, it may also give you not just one value of alpha 1 and alpha 2 it may give you maybe two or even you know possibly three I do not know, but maybe even two values of alpha 1 and alpha 2 might be possible or let us say for some value of fundamental voltage it might be possible. So, these are the things which we will explore later when we go on to the specific problem of selective harmonic elimination. This is a harmonic elimination because we are making fifth harmonic to be 0. So, it is harmonic elimination. We are making a specific harmonic to go to 0. So, we would call it as selective harmonic elimination, we will deal it with a little later.

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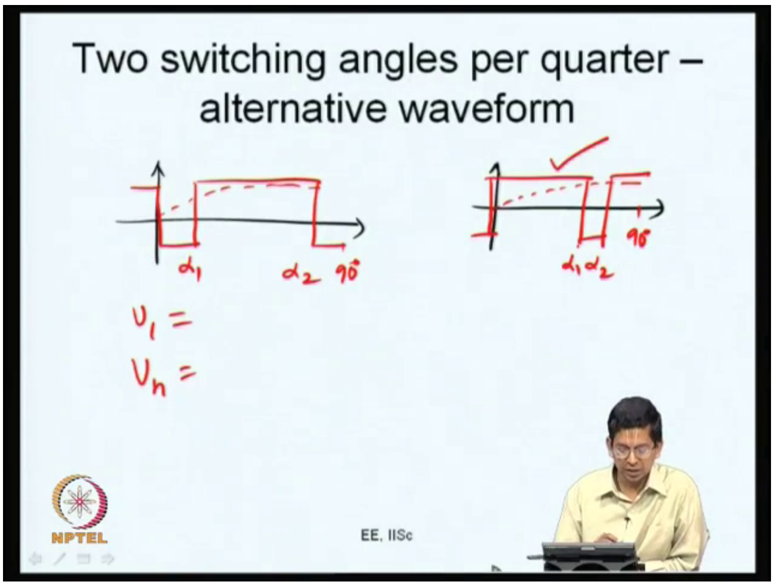

Before that let us try to understand the various possibilities, the same fundamental voltage can be obtained through various pairs of alpha 1 and alpha 2 that is what we have now. So, here you know it is better to see there is an alternative you know waveform also with two switching angles per quarter. The alternative waveform as I was saying could be like this. In the waveform that we considered last time at 0, it was switching from low to high; here in the alternative waveform, you can also it you can have it switch from high to low. You can switch like this and this can be your alpha 1 and you can have your alpha 2 here and this can be your 90 degrees. This is an alternative waveform from what we have earlier considered.

What we earlier considered was like this where the switching was from high to low and you have a not shear alpha 1 and alpha 2 is up to 90 degrees now. So, for the time being we will consider just this waveform. Here for this also you can write down V 1 and V n just as what we did before. This is once again be a function of alpha 1 and alpha 2, but will be a slightly different function. In this case, if you want a fundamental voltage with such a phase then the spacing between alpha 1 and alpha 2 has to be fairly high only then you will get a positive voltage; otherwise the fundamental may have an opposite phase here it may be going negative here.

Then if on the other hand, if you look at this waveform alpha 1 and alpha 2 have to be close to one another for you to have a positive phase otherwise your fundamental voltage might have a negative phase, it might be going in the negative direction here. So, we will be considering this for some more time to come and the equations are as we have given now.

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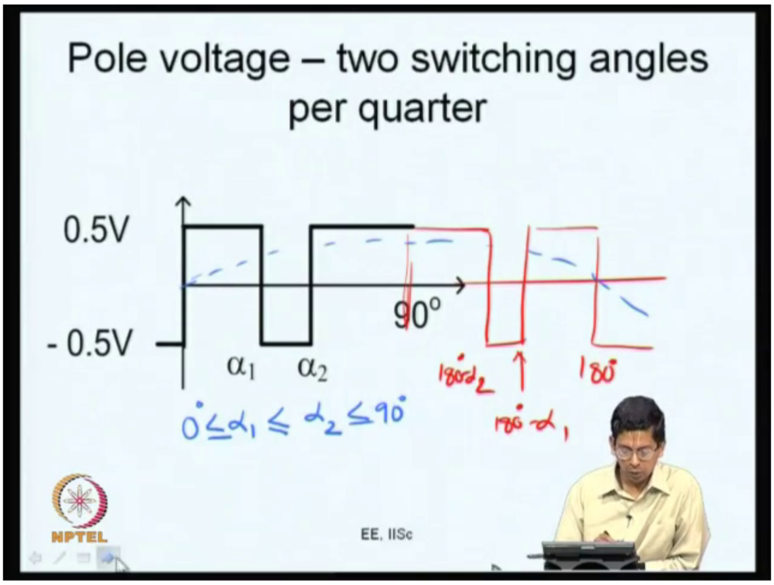

So, what we will look at is is this problem of two switching angles per quarter. And once again let me reiterate this is only one quarter is being represented; if I want to extend this I can extend it like this I can extend just like this. So, this will be 180 degrees, and what is this angle this angle will be 180 minus alpha 2, and this angle will be 180 minus alpha 1 this is one half. And this will have a fundamental component like here this will have some fundamental component like here. So, at zero the fundamental has its positive zero crossing at 180 the fundamental has its negative zero crossing; at 90 degree the fundamental is as its peak this is how it will be now all right.

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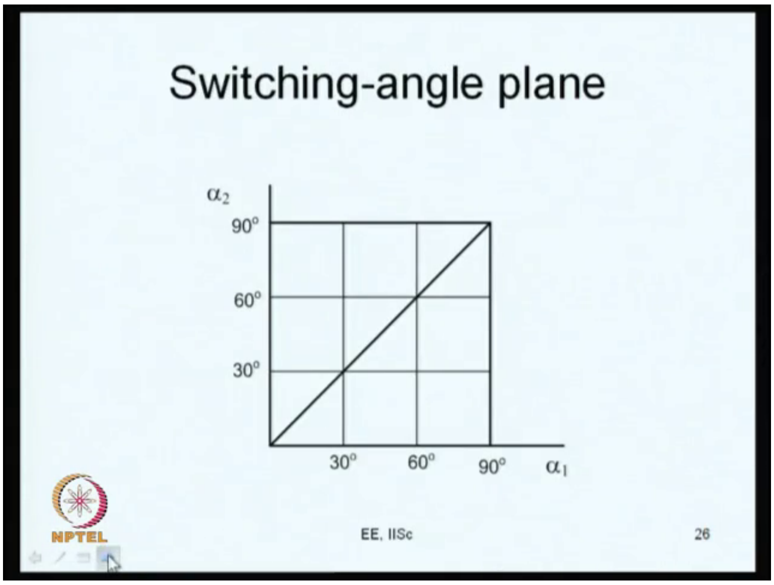

So, we are looking at different values of alpha 1 and alpha 2. First let us see; what are the relationships that we have between alpha 1 and alpha 2. Here we have certain things, what is this you have alpha 1 and alpha 2, certainly alpha 1 is less than alpha 2, as a special case alpha 1 can also be equal to alpha 2. So, this is alpha 1 less than or equal to alpha 2. And this once again has got to be zero degrees. Alpha 1 is greater than 0, similarly alpha 2 is less than 90 degree. Now, you can consider all this in a switching angle plane. If you look at a switching angle plane, what we have is we can never have any value of alpha 2 greater than 90, you can have no value of alpha 1 which is greater than 90. So, you have alpha 1 which is greater than 0. So, here and you also have alpha 2 you know this is the line alpha 1 is equal to alpha 2. Your alpha 2 has to be greater than alpha 1. So, you have this here.

So, alpha 1 cannot be less than 0. So, this is the line alpha 1 equals 0. This is the line alpha 2 is equal to 90. This is the line alpha 1 is equal to alpha 2. All that you have is these are your PWM waveforms. What we are trying to see here is a point. Any point here such as these are PWM waveforms of the kind we just saw with two switching angles per quarter.

So, we would like to look this waveform and this kind of a plane can make this very clear. So, we will deal with more of this in the coming lectures. We will use this switching angle plane to discuss fundamental angle and how they vary in harmonic voltages, how they vary etcetera move on to selective harmonic elimination.

Thank you very much.