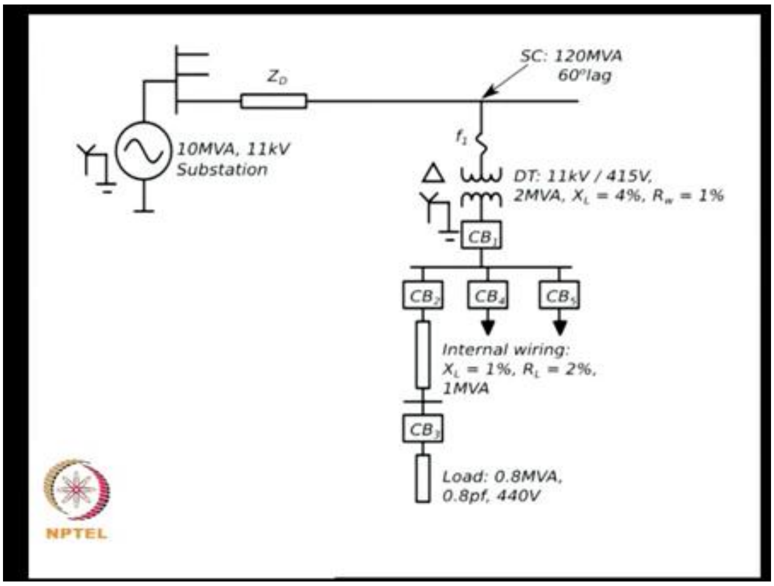
Power Electronics and Distributed Generation
Prof. Vinod John
Department of Electrical Engineering
Indian Institute of Science, Bangalore

Lecture - 6
Symmetrical Component Analysis and Sequence Excitation

Welcome to class six on power electronics topics in Power Electronics and Distributed Generation, in the last class we were looking at distribution system example.

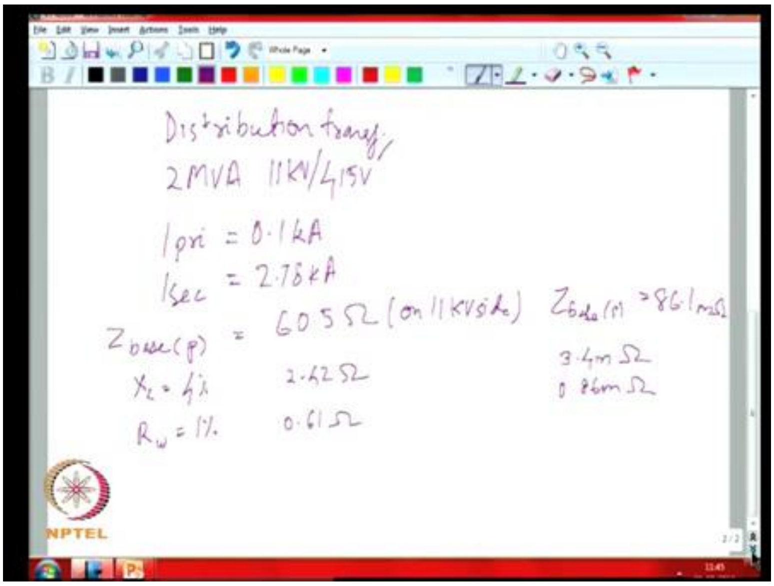
(Refer Slide Time: 00:33)



And we the objective was to look at the sizing the ratings of the protective elements, that we would need if you are connecting a facility to a feeder, in this case there is a fairly large facility being connected to the feeder. And you want to look at what should be your rating of the fuse, the circuit breakers, etcetera, coming back in from the facility. And in the last class we had looked at what is the impedance looking back from the point of common coupling taking the connection at the feeder as the point of common coupling to evaluate what would be the fault current that the fuse would see.

Example, if you have a primary winding fault in the transformer, essentially the fuse would need to carry a fairly large current, which would only be a limited by what is the upstream impedance? So, we had looked at the values of the per unit values of the impedances seeing back from the point of common coupling, and we were about to look at the distribution transformer, and we will do that in today’s class.

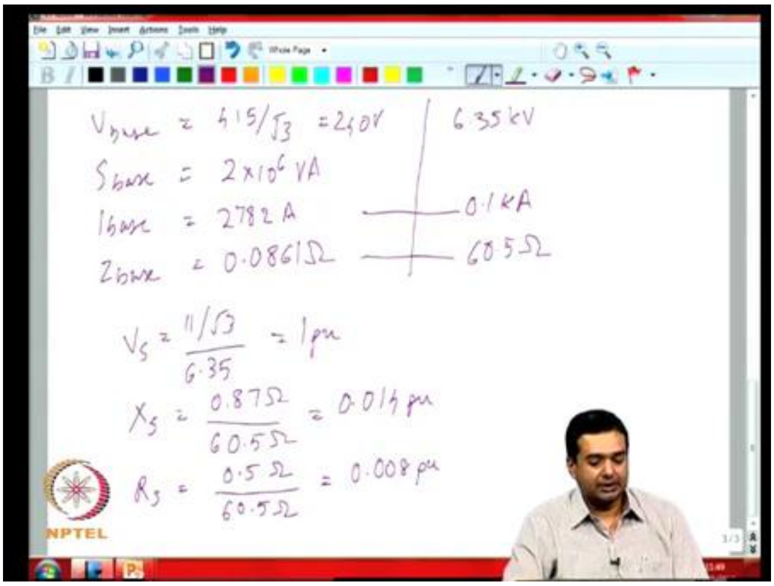
(Refer Slide Time: 02:14)



So, if you look at the distribution transformer, it is a 2 M V A 11 K v slash 415 volt transformer. So, you know your nominal current level in your transformer, your primary current level I primary is 100 amps and your secondary current is almost 2.8 kilo amps. So, if you look at say taking the overall system based to be your distribution transformer base, then your z base on your primary side would be 60.5 ohms, and if you look at the low voltage side, your z base would be 86.1 milli ohms.

So, if you are looking at a transformer your given your transformer, percentage, reactance and winding resistance you are given X L is 4 percent and your R of your winding is 1 percent. So, essentially this would then correspond to 2.42 ohms on the high voltage side for the reactance and 0.61 ohms on the low voltage side you would have 3.4 milliohms and 0.86 milliohms for the reactance and the winding resistance respectively. And the base quantities that we would consider for the analysis would be your transformer quantities.

(Refer Slide Time: 04:52)

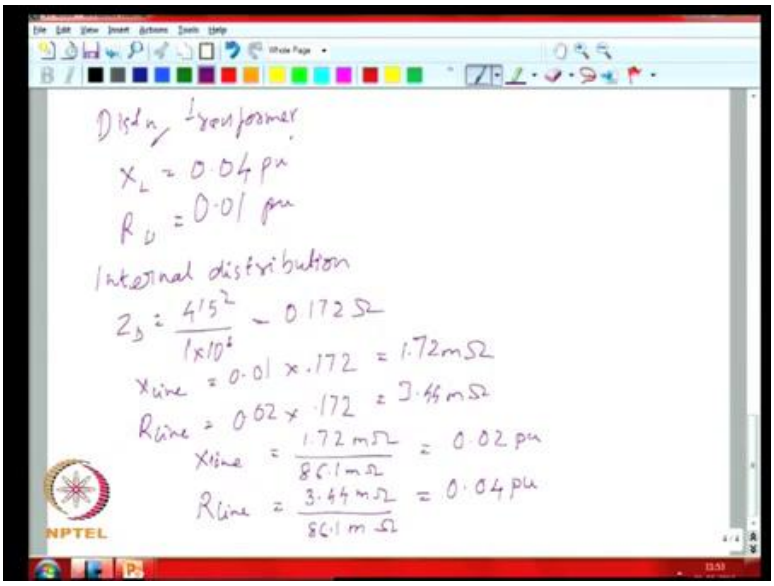


So, your v base for your low voltage side is will take it as 415 by root 3 this is 240 volts and your s base is 2 MVA and your corresponding I base is 2782 amps and you get your Z base. So, 86 milliohms is your z base, so if you look at it on the high voltage side your base voltage is 11 K v is line to line your phase voltage is 11 by root 3 that would turn out to be 6.35 K v your s base stays the same your I base would be 100 amps and your z base would be 60.5 ohms.

So, depending on from what side your normalizing you would have to use a appropriate base quantities. So, here if you want to normalize the source side, we calculated the source parameters in the last class your voltage on a normalized bases it is a 11 K v voltage divided by the 6.35 on the high voltage side is 1 per unit. So, that is what you would have as the voltage base and if you look at your source side impedance your X s is we calculated this to be 0.87 ohms in the last class and your base quantity is 60.5 ohms.

So, this turns out to be 0.14 per unit or 1.4 percent, if you look at your R s your R s turns out to be 0.5 ohms divided by 0.008 per unit. So, if you compare with the numbers that we had in the last class, your X s and R s are seen by now your reduced power level distribution transformers compared to a substation power level, the per unit count it is actually lower now. Because we have now changed to a lower power base, so this is what we had mentioned when you change over to a lower power base, you would see this effect which is what you notice in terms of those numbers.

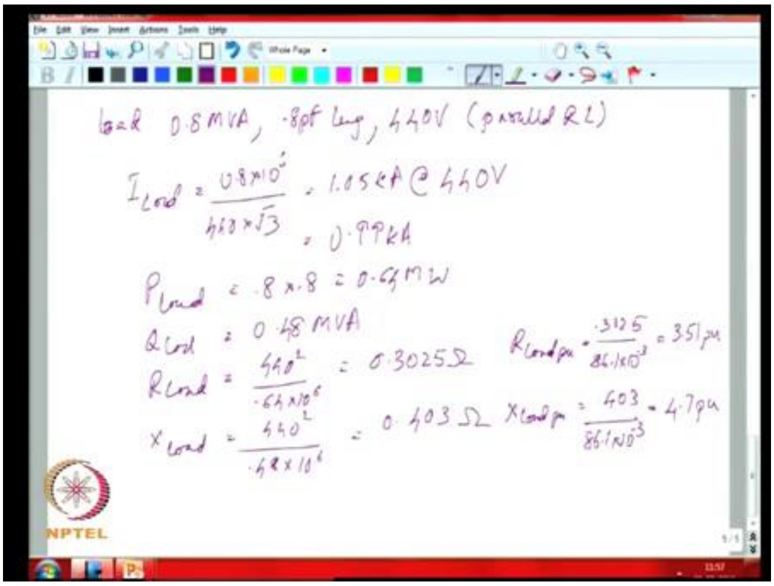
(Refer Slide Time: 08:30)



If you look at the distribution transformer your X L and R L it is the same because your making use of the same the transformer values as the base quantities. So, your R w, so it is stays the same because your base quantities are related to the transformer qualities itself. So, the next thing is your internal distribution, so to look at your internal distribution you have to look at what it is quantities are related to you are told it is was 1 percent for your reactants and 2 percent for your resistance.

But, now that is related to it is power rating, which is 1 M V A per level, so if you look at the base quantities for corresponding base quantities that would be 415 square divided by 1 M V A. So, this would be 0.172 ohms, so your actual x line is, so to bring it now to your common base, you have to now normalize to your base value of a transformer, so your x line is now. So, you can see that because now you went from a 1 M V A base level, where it was specified to a higher base your 1 percent reactants of the line, now got scaled up to 2 percent because you are going up to your larger base quantity. So, this is again consistent with, so you could either go to the physical quantity and derive it by the actual base quantities or you could use the change of base equation that was mentioned in the previous class.

(Refer Slide Time: 11:59)

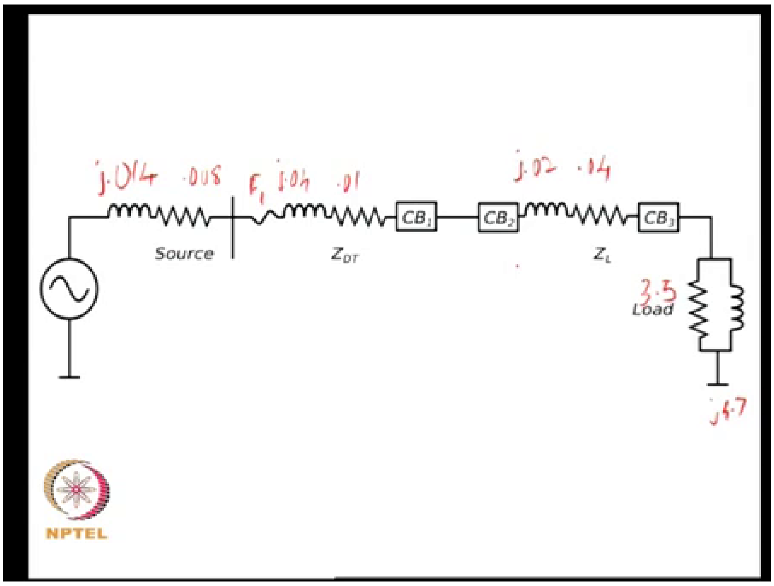


So, the next part in this particular circuit is the load, we will assume that the load is a parallel R L load and the load is given to the 0.8 MVA or 0.08 power factor lag, 440 forty volts and we will assume that it is a. So, if it is running at 440 volts your load current would be 0.08 at 440 volts, if you are assuming it is just a load now you apply a reduced voltage of 415 volt. So, then may be your current level will come down 2.99 kilo amps, say suppose it was a induction machine, then instead of the current level dropping it might would have constant power, so the current might actually go up.

So, you have to make the assumptions on about what you are actual load is to determine what your current would be. So, your p load your actual real power of the load is 0.8 MVA into 0.8 power factor 0.64 mega watt and your reactive power of the load is your 8 MVA square minus 0.64 kilo watt square under square root. So, that turns out to be 0.48 M V A, so if you look at your R and L of your parallel R L load your R load is 440 square divided by...

So, you can determine what your R and L is and then if you look at it on your common base on your low voltage side your R load and per unit would be 0.3025 divided by 86.1 into 10 to the power of minus 3. So, this is 3.51 per unit and your X load in per unit is 0.403 divided by a base quantity on your low voltage side, which is 86 milli ohms, so this turns out to be 4.7 per unit. So, now, that you have all the parameters of your circuit, now normalized to a common bases you could actually then draw a final single line equivalence circuit of your system.

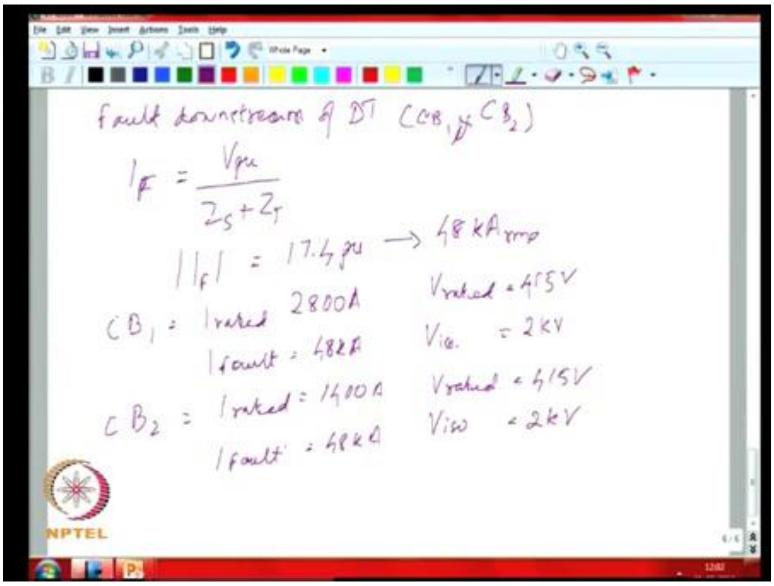
(Refer Slide Time: 16:03)



So, if you look at your final single line circuit, essentially you have the source, which is now it is reactants is j.014, the resistance is 0.008 your fuse F 1 is over here. And then you have the distribution transformer impedance, which was j.04 reactants and resistance of 0.01, then you have circuit breaker 1 followed by your distribution panel, which goes to a bank of multiple breakers. So, circuit breaker 2 is also essentially at the same point, then you have your internal wiring your Z L, which is j.02 reactants and resistance of 0.04 and your load is a 3.5 per unit and your reactants is j 4.7.

So, now you have the parameters of your circuit on a single line on a consistent base, so now, you could you make use of this to look at what happens, if you have a fault at say the distribution transformer or just somewhere along the internal wiring sections or somewhere closer to the load point. So, the next thing that we will look at a, so we made use of if you have a fault over here, you could evaluate what the rating of the fuse F 1 it should be, so you next need to evaluate what should be the rating for your circuit breaker 1 and circuit breaker 2.

(Refer Slide Time: 18:29)

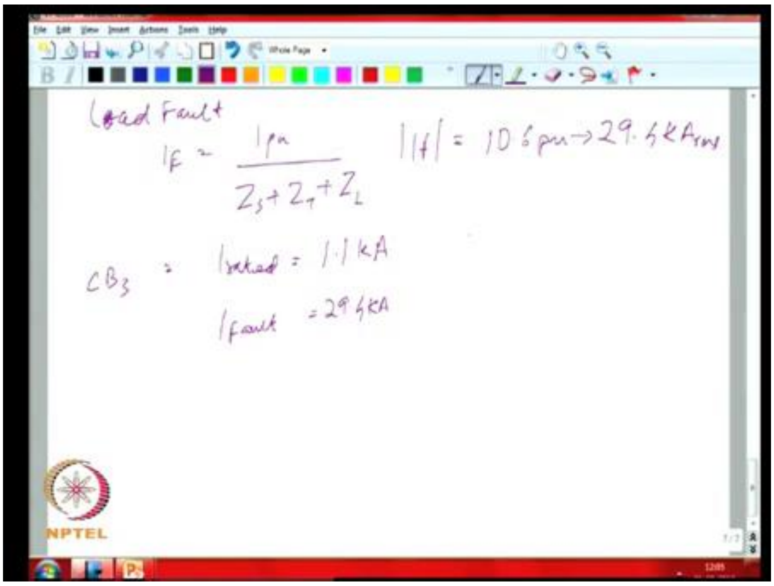


So, if you have a fault downstream of your transformer, if your distribution transformer, essentially this is for yours C B 1 comma and C B 2 your I F is by z of the source plus z of the transformer. So, if you look at it is magnitude this turns out to be 17.4 per unit which corresponds to 48 kilo amps r m s, so you get your peak current that you would expect your circuit breakers to interrupting when you have a fault, just downstream of your distribution transformer to be 48 kilo amps. And you know what your ratings of the circuit breakers 1 is it has to carry a 2 M V A of load current.

So, for your C B 1 your rated current is 2800 amps what is rated to carry, if your fault current is 48 kilo amps r m s, your V rated is 415 volts and your isolation should be at least twice your rated voltage with some additional margin, say your could take it as say 2 K v might be or 1800 volts might be the standard values that you might get at for this particular voltage. So, if you look at the circuit breaker 2, the circuit breaker 2 is now rated for feeding a internal distribution line, which is rated to carry 1 M V A of power.

So, if you look at your I rated for circuit breaker 2, it is actually now 1400 amps, but your I fault is still 48 kilo amps. So, you can see that even though the continuous rating of your circuit breaker 2 is lesser it is fault current that it has to interrupt is a same as the circuit breaker C B 1, your V rated and the isolation voltage stays the same. So, in the circuit that we were looking at , so in fact, all the devices, all the breakers C B 2, C B 4, C B 5 there is any other breakers that might have they might appear in parallel. They might carry potentially lower current levels on a continuous bases, but the sizing to interrupt fault current is quite large, depends on essentially the short circuit current level that can happen immediately after the transformer.

(Refer Slide Time: 22:31)



So, the next point at which you would need to calculate a fault, if a fault occurs right at the load, then you make use of the fault current level at the load to now decide on your circuit breaker 3. So, your fault current level is now your 1 per unit of your voltage divided by Z s plus Z of a distribution transformer plus Z of your internal wiring and your magnitude of I f turns out to be 10.6 per unit, so this corresponds to 29.4 kilo amps r m s.

So, if you look at your circuit breaker 3 it is I rated is now corresponding to your 0.8 M V A load. So, you get current level of roughly 1.1 kilo amp, your fault current level is 29.4 kilo amps and your voltage rated voltage and isolation voltage stays the same, in fact, ((Refer Time: 24:12)) if you look at this particular circuit, you see that if you look at a breaker often the ratings of the breaker related to how much fault current it needs to interrupt, because the amount of energy that gets dissipated when the contacts open, corresponds to the current that it is trying to interrupt.

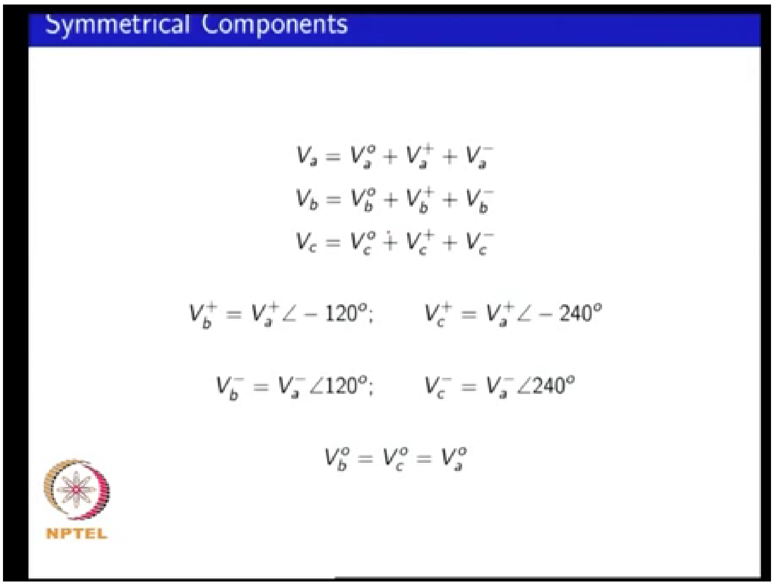
So, even though C B 4 and C B 5 may be carrying much lower current, the cost can be substantially if your count level it needs to interrupt is quite high. So, sometimes people what do is they could add say series inductance, worth just a stream of the breaker, so the purpose of the inductance might be to reduce your fault current level. So, the cost of your inductance might be lesser than the cost savings that you incur in your circuit breakers because potentially a incoming feeder which might go to a large number of branches, so there are potentially large number of breakers.

So, you might see suddenly in the wiring inductance is being added in some sections, so this is intended to handle the fault current level and manage it to an appropriate level. ((Refer Time: 25:32)) So, at this particular point we did a per unit analysis of the system and we calculated the fault current levels, we did it for a balanced case and some of the assumptions that we had in when do a per unit analysis, this is the radial distribution system, so we do not have loops. So, if you we assume in general that there are no loops in with net transformer ratios gains, when you were doing a per unit analysis.

And also of your this especially the case when you might have auto transformer, tap changes, etcetera, you want to ensure that you are doing it at the nominal level. So, that you do not have circulation within loops, also if you have phase shifters, you have star delta transformers, etcetera, again you are assuming that the net phase shift in a loop is adds up to the same irrespective of which direction you are going, so these are some of the assumptions that you are making.

And the even though fault current level is steady state fault current level, you are not looking at unbalanced effects of this particular point and to address the unbalanced effects we will have to look at sequence components. Also you are not looking at the dynamical effects for example, depending on where the fault is occurring and depending on your x by r ratio of the line you can have peak fault current, which are much higher than your steady state fault current levels. So, the dynamic effects are also ignored and what we are calculating is the steady state fault current level.

(Refer Slide Time: 27:34)



So, what we have looked up, so far is the a three phase fault and if you want to look at unbalanced faults and a common fault is single line to ground fault, you will have to look at the sequence components. Again the sequence components along with on which the symmetrical components analysis is based, one needs to keep in mind that it addresses the unbalanced issue, but it is still a steady state concept it is not a. So, the quantities you are looking at a still faces r m s quantities, etcetera.

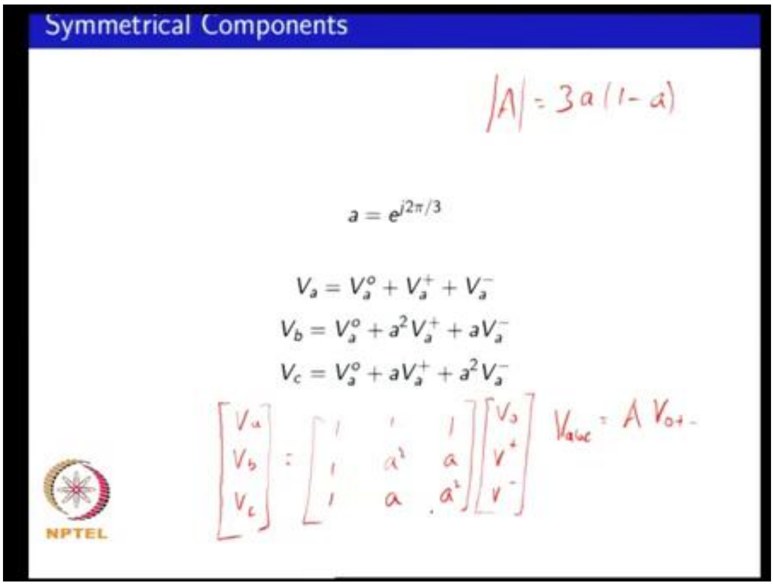
Rather than, peak instantaneous values etcetera, which you would need for dynamic analysis to look at what might be the worst case peak current that may be your breaker needs to interrupt. So, if you look at the symmetrical components, essentially what you are saying is if you have unbalanced your phase quantities might be not equal and not phase shifted by 120 degrees, you might have 3 arbitrary quantities, say if you take the voltage your voltage b a, V b and V c might be arbitrary.

And what we are doing is we will take this 3 phases V a, V b, V c and represent it in terms of 9 quantities V a 0, V a plus, V a minus, V b 0, V b plus, V b minus, V c 0, V c plus, V c minus. So, if you look at it you will think that this is actually a bad way to go you taking three quantities and you representing as nine quantities, which is making it more complex.

But, the main advantage is that this V a 0, V a plus, V a minus and the b and c corresponding quantities are all related the V a plus V b plus V c plus form a positive sequence growth in the sense that V b plus is V a plus with a phase lag of 120 degrees V c plus is V a plus with a phase lag of 240. If you look at the negative sequence component your V b plus is V a with a phase lead of 120 degrees and V c minus is V a with a phase lead of 240 degrees and if you look at V b 0 and V c 0 they are actually equal to V a 0.

So, if you look at the number of independent quantities over here, you really have just three independent quantities. So, you are representing three arbitrary quantities in the these phases in terms of three quantities on a sequence bases.

(Refer Slide Time: 30:32)

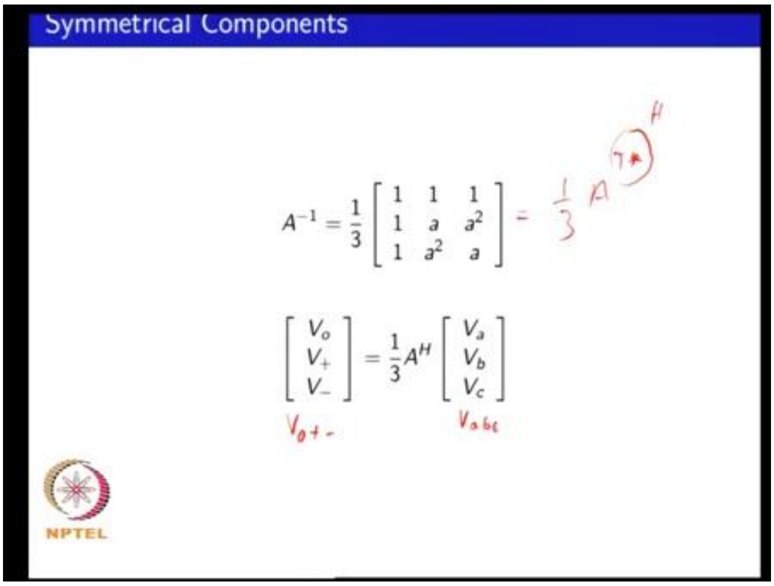


So, based on what we just mentioned now if you represent 120 degree phase shift by the number a small a your V a is V a zero plus V a plus plus V a minus. If you look at V b V b 0 is the same as V a 0 V b plus is 120 degree lagging V a plus, which means that it is now a square times V a plus and as similarly you get a times V a minus for the negative sequence value of the b phase quantity. And similarly, you can write it now in a matrix form for your transformation now from a b c to 0 plus minus.

And because you are now looking at a quantity is for all the three phases, you could also drop the such script a and just look at call it V 0 v plus V minus, the implicit assumption is that you are referring to phase a, when you are looking at 0 plus minus. So, you could write now this in a matrix form as and essentially in a compact form, you could called is as V a b c is the matrix A times V 0 plus minus. So, essentially what you are looking at is a transformation going from a b c to 0 plus minus.

If you look at the determinant of the matrix A, you could you calculate that in a fairly straight forward manner, you would get 3 into a into 1 minus a, which means it is non-zero and matrix is invertible. In fact, what you have is a linear transformation from a standard bases to a new set of bases, which leads to your symmetrical components. So, if you look at the matrix A if you this matrix is actually is symmetrical, so you take a transpose would be the same as A. In fact, you can calculate what you are A inverses A inverse turns out to be this particular matrix over here.

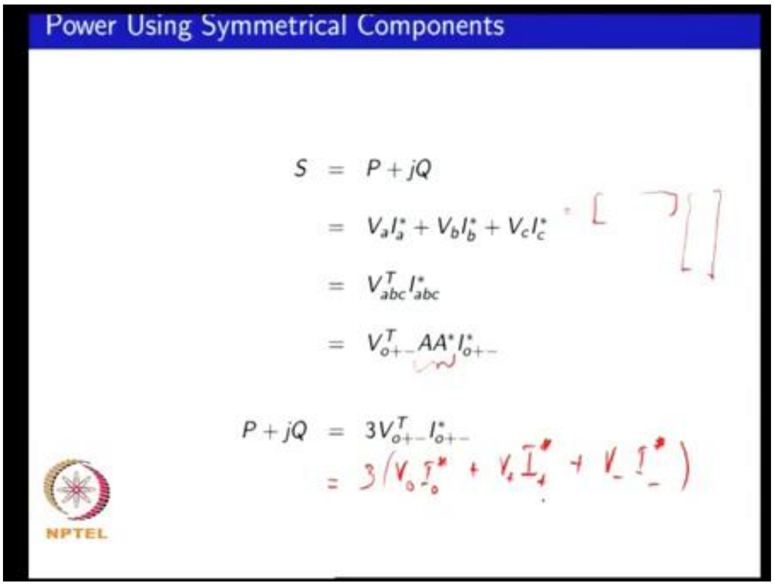
(Refer Slide Time: 34:04)



If you look at what it is, it is actually 1 by 3 A transpose conjugate, sometimes people referred to the transpose conjugate as the Hermitian of a matrix. So, A H is essentially it is transpose conjugate, if you have a matrix which where A inverse is equal to A H then it is called a Hermitian matrix, here it is not exactly a Hermitian matrix. Because you have the factor 1 by 3, but it is nearly Hermitian, because you are A inverse is related to the inverse transpose of A and if you take a times A inverse transpose you would get three times that identity matrix.

So, if you look at it in compact form, you can determine your 0 plus minus quantities as 1/3 A H times V a b c. So, the next thing that you could ask is what happens when you look at power on a sequence bases rather than on a phase bases.

(Refer Slide Time: 35:47)



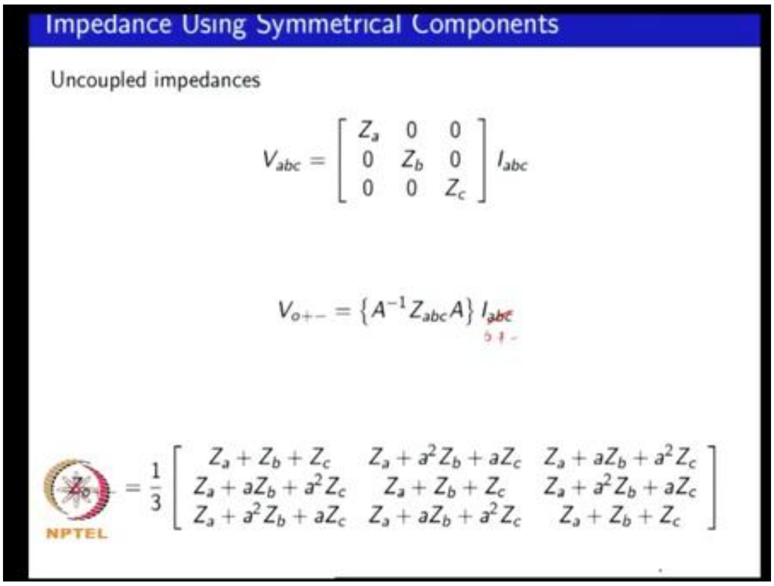
So, your power is essentially s S P plus j Q and on a per phase bases this is V a times I a conjugate plus V b I b conjugate plus V c I c conjugate. So, if you write it in a vector form this can be though off as multiplication of a row vector times a column vector and the row vector being V a b c transpose and your column vector being I a b c conjugate. So, the dot product of V a b c and I a b c conjugate gives your power, now you could substitute for V a b c and I a b c in terms of the change of bases transformation to go from your a b c reference to your sequence reference.

So, you can write V a b c as a times V 0 plus minus and because you have the transpose over here, the a moves over to the other side and a transposes the same as a. And if you write I a b c conjugate that is A conjugate times I 0 plus minus conjugate and we know that A conjugate is 3 times the identity matrix. So, your P plus j Q is now 3 times your V 0 plus minus vector transpose into I 0 plus minus transpose, so if you just a expand the sort this is 3 times V 0 I 0 conjugate plus...

So, if you look at a typical circuit, typical circuit you might have unbalanced loads, but under normal conditions your voltages should not be unbalanced, if you want to maintain reasonable power quality to your loads. So, your V plus is the dominant term over here, V 0 and V minus should be quite negligible, so even with unbalanced loads your dominant part of your power comes from V plus I plus conjugate.

So, your positive sequence would determine most of your power in under normal conditions of course, if you have a severe conditions such as a fault, then the value of V plus V minus etcetera can also become large.

(Refer Slide Time: 38:51)

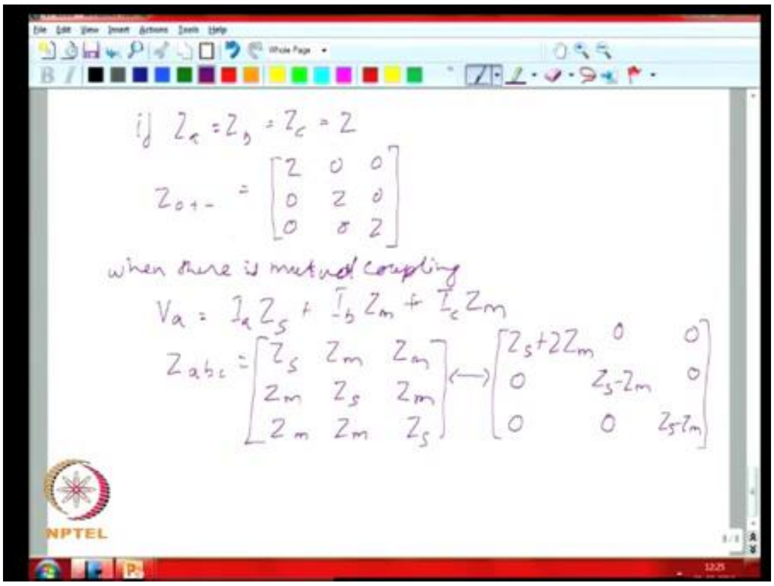


So, the next question that you could ask us is that what happens you have impedances and if you look at impedances in the normal per phase bases and then if you look at impedances on a sequence bases how would it transform. So, if you look at uncoupled impedance say V a is equal to Z a I a V b is Z b I b and V c is equal to Z c I c, essentially you can write it in a matrix form like this. And then you can write it your V a b c as A times your V 0 plus minus and you are I a b can also be written this is actually I a b c can also be substituted as A times I 0 plus minus.

So, you could then transfer over the A you get A inverse Z a b c times A and if you carry out the multiplication for the matrix, this particular matrix for that is given over here. Then essentially what you would get is a fairly full matrix of this particular from, which is shown at the bottom of this particular section. So, if you look at this you will feel that this is may not be a good idea, where you have a simple nice diagonal matrix and you do a some transformation and you end up with matrix, which is full.

You could then look at some simplifications, where what happens when say the value Z a Z b Z c are equal, say in a situation such as that, then you know that a property of the quantity small a is 1 plus a plus a square 0. So, when a b c are equal then the half diagonal terms would drop off, again you would end up with a diagonal matrix.

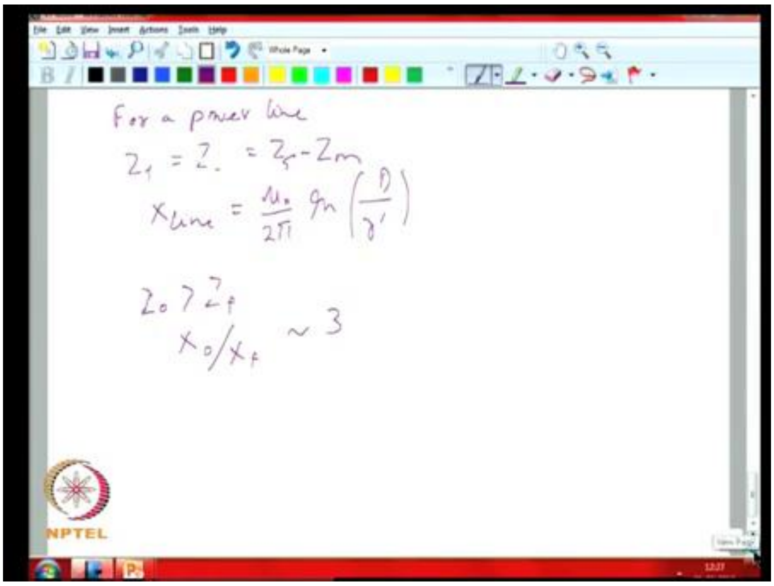
(Refer Slide Time: 41:07)



But, again this looks fairly simplistic the more interesting situation is when you have coupled system, where you have mutual coupling between the phases. So, suppose you have V a is equal to I a times Z s plus I b Z m plus I c Z m, so now, you have mutual coupling from phase b and c into your voltage in phase a and similarly for your b and c phase. So, if you look at your Z a b c matrix you would get Z s Z m Z m and if you transform it to your sequence components matrix, you end up with something which is actually now substantially simpler see you get Z s plus 2 Z m 0 0 0.

So, you can see that you end up with fair amount of simplification, if you look at systems where you have mutual coupling. And for many practical power application, where as you looking at machines, transformers, etcetera, you end up with coupling between the phases. And you seen a symmetrical components based analysis, you can expect a considerable simplification in terms of your circuit analysis, where you have a diagonalised circuit in your sequence domain, where as in your a phase domain you end up with a full matrix for the impedances.

(Refer Slide Time: 44:04)

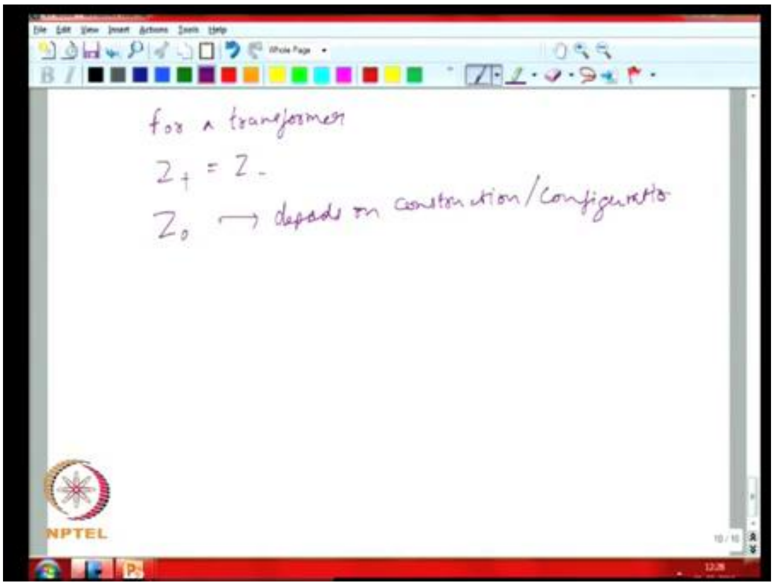


So, if you look at specifically you have power line, so if you looking at the sequence models of a power line your Z plus is equal to Z minus and is Z s minus Z m and from your courses on power system analysis, you would have looked at expressions for how to calculate the reactance of a line. So, you have expression such as mu naught by 2 pi lon D by r prime, D represents the distances between conductors and R prime is a effective a radius. So, if you look at a situation, where you are looking at the distance between conductors in a over head line.

The distance between the phase conductors might be a small, but then if you look at the height of the line above the ground, what you might need to consider for a phase two single phase line to ground fault your Z naught you will end up with a much larger value of distance compared to what you have for your positive and negative sequence components, which flow within the lines themselves. So, you end up with Z 0, which is typically greater than Z plus.

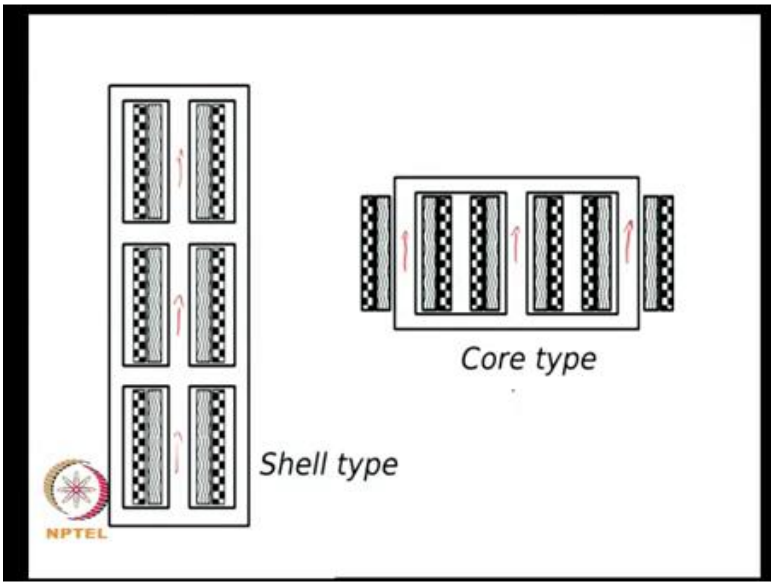
So, if you look at a typical number for line you would see things like such as X 0 divided by x plus is around 3. So, gives you higher 0 sequence the impedance compared to your positive sequence impedance for your line.

(Refer Slide Time: 46:17)



If you look at a transformer your Z plus is equal to Z minus your Z 0 depends on the type of your construction and the transformer configuration.

(Refer Slide Time: 47:07)



So, for example, if you look at a transformer you are familiar with core type or a shell type transformer. So, if you have a shell type transformer and if you have a 0 sequence flow that can flow in one of the windings, the windings of the primary or secondary winding of the transformer, essentially you will end up with 0 sequence flux. And the return path of the 0 sequence flux is essentially the same core, the same path as for the positive or negative sequence.

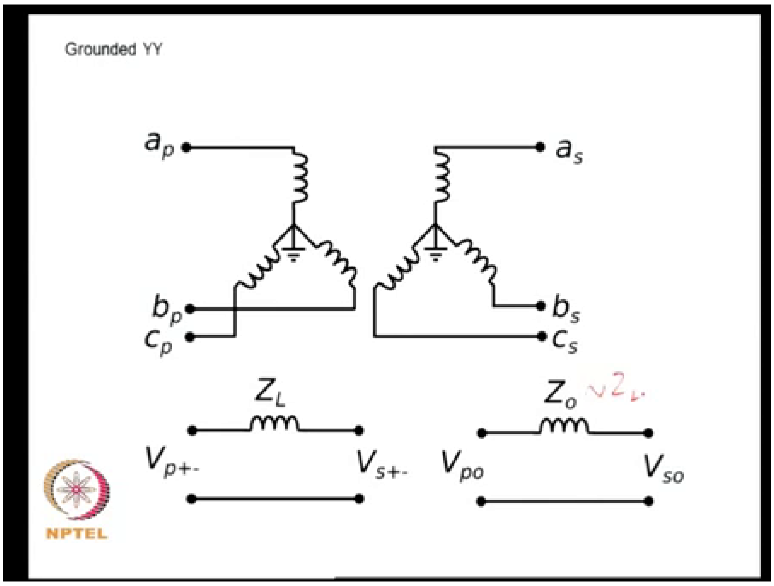
So, you end up with similar impedances for your 0 sequence as for positive or negative sequence. Whereas if you look at a transformer, which is of core type what is shown over here is what is in the white is a core and the hashed and wavy lines corresponds to the primary and secondary windings of the transformer. So, if you have again currents that can flow through the winding, which can lead to a 0 sequence flux, now the 0 sequence flux are going up on all three elements of the transformer, essentially the path for red now would be through the air, rather than through a magnetic path.

So, in one case you would see a large air gap, so you would end up with essentially much different value of your 0 sequence impedance. Also it depends on say for example, you take this core tie up and you put it in a in some sort of a cabinet or enclosure, so you might have walls of the enclosure, which is sitting nearby under flux that comes out of the transformer might now interact with the cabinet or the enclosure and cause heating or hot spots on the surface.

So, depending on your application if you want to have 0 sequence flow in your windings it might be preferable to look at shell type or if you want you could look at a single phase transformers connected in the appropriate configuration. So, the value of this 0 sequence impedance depends on the type of transformer that you are using and whether you should be using such a transformer in the first place at all, what is shown over here is the core and shell type for three phase.

You can also have core and shell type for single phase also depending on whether a windings or enclosed by a core on all three sides or whether the windings are sitting on the two limbs of a single phase transformer. So, depending on the transformer configuration and also depending on the winding type you can have different impedances for the transformer.

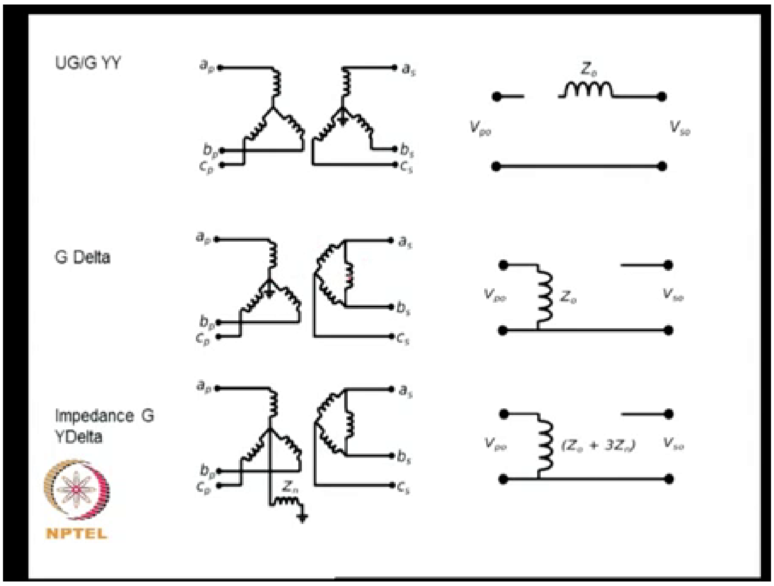
(Refer Slide Time: 50:07)



So, what is shown over here is see a Y Y transformer, where both the Y points are grounded. So, if you look at the positive and this negative sequence model going from your primary to the secondary, if you now do a per unitize analysis essentially the transformer becomes transparent. So, essentially you are modeling the leakage inductance and the winding resistance, the leakage inductance term is a dominant term.

So, you might call the impedance as Z L and you use a same Z L in a model for both positive and negative sequence. When it comes to 0 sequence you will have a Z 0 and your Z 0 would roughly be equal to your Z L, so for a grounded Y Y transformer your 0 sequence impedance turns out to be the same as what you had for your positive or negative sequence.

(Refer Slide Time: 51:22)



If you look at a see other configurations say if you have a ungrounded Y you can have a grounded Y on the secondary or you can be even ungrounded. Essentially what would have is the positive and negative sequence circuit stay the same, as what we discussed previously, but a 0 sequence circuit is now open. So, essentially this would not allow 0 sequence current to flowing through the circuit, essentially in this model we are neglecting the magnetizing branch and the core loss branch, assuming that its value is much larger than one per unit, so it can be neglected.

So, for 0 sequence practically you have now a open circuit, even if you take one grounding or from one of the Y points of the transformer. If you have now Y delta type of transformer now if you ground the Y point now from your primary side, essentially you now have a path for your 0 sequence current to flow. So, you have now Z naught connected back to your return on the primary side on a Y side, but on the delta side there is no path for the 0 sequence to flow, so it is open on the delta side.

If you have a grounded Y delta and if you are grounding through a impedance Z n, essentially now your 0 sequence currents coming through the individual phases would some of will have 3 I 0, now flowing through this particular impedance. So, essentially the model would be Z 0 plus 3 times the impedance that you are connecting to the neutral point and your secondary side stays open, so depending in this case say when you look at your positive and negative sequence model.

The impedance wise it is stays the stay the same it is still the dominated by the leakage inductance, you have to keep in mind what the phase angle is going to be between your primary and secondary voltage. In the next class will look at the phase angle, depending on your winding configuration your can have different phase relationship between the angle between your primary and secondary, will look at it what are the possibilities with a standard Y Y delta delta or delta y type of configuration in terms of phase relationship between your primary and secondary winding.

Thank you.