Power Electronics and Distributed Generation Prof. Vinod John Department of Electrical Engineering Indian Institute of Science, Bangalore

Lecture - 27 Common Mode and Differential Mode Model of Inverters

Welcome to class twenty seven on topics in power electronics and distributed generation, we have been looking at the switching and average model of the power converter.

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So, if you look at the switching model, it captures detailed information about the operation of the power converter, it has all the details about the wave forms of interest. Especially, if you are looking at quantities at the switching frequency or even high frequencies then the switching model is very useful. Also if you are looking at things like ripple switching ripple P W M spectrum EM I related issues, then the switching model can actually provide you a lot of information.

We saw that the inputs to the switching model are the DC voltage of the inverter voltage source inverter and the AC current out, AC output current. These are continuous quantities your duty cycle is the output of a controller, which is also typically p i d controllers etcetera. This is also a continuous quantity your P W M action provides the switching function.

The switching function is what then turns the actual switches in a on off manner, the outputs of the converter is actually your output voltage or your DC bus currents. These quantities are discontinuous quantities and that is one of the issues of the power of such a model because you have discontinuous quantities, if you are actually trying to model, it you need to model a lot of points close to the discontinuity, which means that it takes a long time for computation simulation etcetera.

Often one is interested in longer time frames of computation, especially when you want to decide on a attribute such as is a control behaving correctly. You may want to look at what is the operational loading over a longer time frame etcetera, in which case an average model can actually lend itself to the analysis. You can look at the low frequency wave forms look at all the lower frequency issues, control settings, whether the gains are and the average model is useful in this particular situation.

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Again the inputs to the model, you have that the DC out positive and negative bus on the DC side. You have the AC side output, which is the output of the of the center point of the leg the of the power converter. This is shown for one leg it can be extended for multiple legs the inputs are again the duty cycle. The DC bus voltage the output current, which typically are measured quantities, you have DC bus senses in a voltage source converter you often have current measurement.

The action of the power converter is modeled with a controlled current source and a controlled voltage source, the outputs in this particular case the V o is a continuous quantity. Similarly, if you look at the currents that are coming into the terminals of your DC bus this is d times I out what is coming here would be 1 minus d times I out, so these are also continuous quantities because they are continuous quantities. You can actually not have to sample very at a very large number of points closed to discontinuities etcetera, in which case now you can have speed ups of 100 to 1,000 times on your simulations or the computations that you do.

Also in these average models, which are used when you are looking at control issues or longer term issues? One has to also consider the delay effects of the power converter, you have delays because of the intrinsic P W M action. You have delays because of the computation is especially when you are implementing the computation in a digital platform. So, you have to incorporate both these factors when you are doing your analysis, especially when your switching frequency is close to your band width that you are trying to target. So, if you are there is large separation between your switching frequency and the band width, then these delays might be ignored in a preliminary analysis later on you might be able to find tune things.

Here, the switching frequency gets close to your band width, and then you definitely will have to include this in your model or your controller action or your controller models, may not give you accurate prediction of how the system would behave. So, with this we will look at in D G application what we have seen is essentially what we are trying to do is transfer power between two sources, between input and output. We are looking at different ports to which you are trying to transfer power and you have inputs outputs and you want to identify the voltages at these different terminals.

We have referred to terms such as common mode and differential mode; this terminology can be applied to both the voltage and current signals for a power converter. It can and looking at it from the common mode and differential mode perspective can give better insight into the capabilities and limitations of a power converter, so will look at an example.

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If you look at an example, where you have a power converter now shown over here with two input terminals V i 1, V i 2 3; output terminals V o 1; V o 2 and V o 3, all these voltages are given with respect to some reference. The reference is commonly the ground and typically the ground and in a power converter an inverter the ground is often the cabinet or heat sink. We in our discussion on grounding earlier we have seen that grounding is a important consideration from safety perspective, you need to ensure that the tech potentials stay low. There is a much higher chance of a human being coming and touching say the cabinet of the power converter of or if there are large heat sinks, which can be touch.

You need to exchange air for cooling, then you need to ensure that the individual, who comes into contact with these points do not get a electric shock. So, you need to actually ground these points in power converter in a appropriate manner. So, safety is actually the primary concern in all electrical equipment including inverters distributed generation equipment etcetera. So, if you look at the output voltages in this particular case you have three differential mode voltages. When you are looking at differential mode you are looking at differences between the terminals.

So, if you look at the output differential mode voltages, you could look at V o 1 with respect to 2 or V o 2 with respect to 3 or V o 3 will be with respect to one thing that you can see is these three differential mode signals are not independent.

You sum them together you get 0, so there is a constraint there actually only two independent quantities over here. So, you look at the summation of your V o over i j the terminals of your particular power converter for all your i j variables, then they would sum up to 0. So, essentially you lose one piece of information and the piece of information, which you are essentially losing, is essentially what the voltage with respect to the ground is. You are looking at it from a line to line perspective, you lose the information about the ground and a quantity, which can actually preserve such information, is the common mode signal.

When you are typically looking at the wave forms of a power converter in a text book or in many illustrations, what you are looking at typically is the differential mode wave forms you are looking at nice DC voltages or smooth AC voltages. These are typically observed on line to line bases or on a differential mode bases. Often, the common mode signals, which are a little bit more difficult to observe are ignored till the last minute. Often, these are the signals that can actually result in E M I problems or circulating currents to grounds, etcetera.

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- Diffesientrel mode signal can be viradingedon a fire lo line basis. - Common mode signals - circulating currents (LF) - High Frequency common mode -> EME The common mode voltage for a group of terminals can be defined as

So, if you look at your differential mode signals, so on a line to line bases the differential mode signals can be easily visualized. Those are the signals that you would typically see if you look at your common mode signals.

You can have problems, because of circulating currents, especially low frequency circulating currents are consider the problem with can cause such ration in magnetic etcetera, your high frequency common mode signals essentially lead to problems of EMI. So, one way of defining what a common mode variable would be for a group of say n terminals that one is interested. For example, in our case we had three output terminals, so if you want to define a common mode voltage for those three terminals, we could define it as summation over all the terminals V i by n, which is essentially the average that you do.

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Let us say in our particular in this particular case, in our example your input common mode voltage V c m of your input would be V i 1 plus V i 2 by 2 and V common mode for your output would be V o 1 plus V o 2 plus V o 3 by 3. So, one thing that you one could do are similar to the common mode voltage. We could also define common mode and differential mode currents.

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So, if you are looking at again a blocks, which might be power converter you could your terminals coming out of it, it might be DC terminals AC terminals some group might be the input some might be the output. One is interested in determining how to define the signals on a differential mode in a common mode bases.

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The common mode current through a group of noterminals can be defined $icm = \sum_{i=1}^{n} ii$ The differential mode current can be defined as $idm_i = i_i - \frac{icm}{n}$

So, for the currents, so your common mode current can be defined, so for a group of n terminals your common mode current is essentially the summation of those currents to that group.

Let us say if you are looking at the currents through some group with n terminals you could add those currents. So, the summation of that is what is coming in common through that particular group. Then, one could define the differential mode current can be defined as i differential mode going in to terminal i considering a particular group is essentially, the current in that particular terminal minus i common mode. For that group divided by the number of terminals, that are been considered in that group so far example, in the example that we had...

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For the example, we have i c m at your input is essentially i i 1 plus i i 2 and i. If you then look at what is i d m one is i 1 i 1 minus i c m input by 2, which actually is i i 1 minus i i 2 by 2 substituting what i c m is if you look at i d m 2. So, this is equal to i i 2 minus i c m of your input by 2, which is now i i 2 minus i i 1 by 2, so you can see that if you now again sum your differential mode currents they would add up to 0.

So, essentially the differential mode currents are what circulate through the group where as common mode currents actually flow through. So, the question is if you have a current that flows through, you cannot just flow currents from some particular point to end the particular point, you need a close path for current and where is the closed path for current.

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So, if you look at the model that we had over here we know that current cannot just originate from one point, just go into some particular point it has to actually flow somewhere. Often the place from which your currents would flow in or flow out is essentially your ground. When you have a power converter you have different equipment connected to the one group of terminals. You might have other equipment connected to other group of terminals; you would have parasitic impedances between your inputs your power converter itself and your outputs.

So, essentially your common mode currents can actually one thing it can actually flow in to the ground through your inverter depending on the parasitic of your inverter etcetera. You could also have ah these common mode currents flowing through ground not just to within your equipment. It can also spread out to a adjacent equipment, which is essentially what causes interference when you start one in one device, you would have interference with a neighboring device. So, you have to be conscious about the powers that are available for these particular common mode currents.

Again, look at what the voltages are which actually excite these particular currents. So, if you look at the example of simple inverter that we have a single phase capacitor center tap inverter. We can actually define common mode differential mode signals for the inverter, so you have the inverter was essentially you have a capacitor bank, we have two switches will consider them s 2 switches as this.

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Then we will consider the AC terminal coming out, so this is V out you have some inductive filter you have your grid connected to ground or your reference, so this is your neutral wire, which is coming back in which goes back to the source. At the input, you have say your p V p I p V n negative bus your current going into your negative bus, this is your I out and lets call this I on, which is the current coming out of your neutral. So, you could then know that you defined your quantities you can actually look at what your common mode and differential mode terms would be, so if you look at it your voltage your common mode voltage.

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input side Vdm = Vp-Vn loni = ig tip iono = ight f lon Neglicity stray capacitoneds to ground on the inverter iem = (ip + in) = (but + ion) iem = ip - ion = ip - in ipdm = ip - ion = ip - in

If you look at your input side, your V differential mode is V o minus V n, which is essentially your DC bus voltage. So, when we look at the voltage being input voltage being DC at some particular value 800 volts, which we discussed. That is essentially your differential mode voltage, which you are looking at from the top of your DC bus to your bottom of your DC bus. If you look at your V s common mode it is V p plus V n by 2 again V p and V n etcetera are with respect to reference, which is ground in this particular case.

If you look at your common mode input i c m at your input it is i p plus i n i c m at your output is i out plus i on, which is your essentially a current going out to your neutral. Let us say, if you neglect the parasitic between your inverter and the ground assuming that the negligible stray capacitances etcetera. You have i c m is equal to i p plus i n on your input side, which is i o i out plus i on, which is going in to your output side. So, essentially it is going through the inverter from your input side into your output side. So, anything that is flowing and a common mode bases would actually flow through around a loop going into the ground and come back to through your source, which might be connected to your input.

So, your excitation on your input excitation on your differential mode bases if you look at i p on a d m differential mode basis. It is now i p minus i c m, which is i p minus i n by 2, which is essentially the current, which now circulates through your DC bus, which is now responsible for power transfer. So, essentially when we are considering power transfer or ripple across the bus we are looking at the differential mode currents. (Refer Slide Time: 25:59)

Vdr_cm = Vpr fVnr Vdc-dm = Vpr-Vnr A VAC-CM 0 r = - Vnr - Vds Nole-dm Ve

If you look at then the voltage we had V DC c m was V p with respect to your reference V n with respect to your reference by 2. So, if you if V p r is equal to minus V n r is equal to V DC by 2, which is the objective of how would you control your center tapped single phase inverter. Then you can look at your common mode voltage would add up to 0, so essentially in this particular topology V DC on your common mode bases would essentially be 0.

So, on your input side you would not have a common mode excitation the differential mode excitation V DC d m would be a flat quantity, which would be your V d c. So, this gives you a feel that yes, it is something relatively smooth for this particular topology.

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If you look at the output side AC differential mode is V out minus V on, this might be your voltage of leg a of a multi phase power converter etcetera. We know that V on in this particular example the neutral is actually connected to the ground. So, that voltage is actually 0, so if you look at your differential mode voltage it is essentially going between plus V DC by 2 and minus V DC by 2. So, depending on the switching action of your P W M converter, it is essentially a step we form like this. If you look at your V AC common mode, you are AC side common mode voltage. It is essentially V out plus V on by 2, which if you plot is going between plus V DC by 4 and minus V DC by 4.

So, you can see that in this simple case ah it gives you some information that. Say, for example if you have parasitic to ground on your DC side it may not affect you as much where as if you have say. For example, your output filters inductor in this particular topology if it had capacitance between you is winding to ground. Then these adjusts could actually cause currents to be injected to ground through those switching adjust.

So, plotting this on a common mode and differential mode bases gives you information on where, what side of the inverter can actually be more (()) or can actually generate more concerns, especially related to high frequency injections into ground. So, the single phase inverter that we have looked at so far is the simple capacitor midpoint topology. We could consider a topology, which is slightly more complicated more complex by adding one more leg. So, instead of having just one leg we could now consider a topology, where we have two legs and instead of having a capacitor bank, which has midpoint, now will have essentially a simpler capacitor bank.

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So, essentially we have a two leg inverter topology; now this can be considered as so two single pole double throw switches as for modeling. This particular inverter it still a single phase inverter, where your try transferring power between your DC side and your single phase AC side. So, you have in this particular case less complicated capacitor bank arrangement, but you have a more number of semiconductor devices.

So, in this particular topology, one thing that we might start off with this, how would one do your modulate the switches of this particular power converter. Essentially, the question is what would be the basic duty cycles, that are required to be provided for controlling leg a and leg b, so that we get the decide sinusoidal output, which would be required in typical D G applications.

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To modulate liga and lyb $d_a = \frac{1+MCoont}{2}$ $d_b = \frac{1-MCoont}{2}$ $M \text{ is } + \text{ and } \leq 1$ $da, db \in [0, 1]$

So, in the example we had our leg a is connected to the AC source than leg b is connected to ground or essentially, it is the neutral point of the particular converter in connected to our single phase system. So, for leg a and leg b there because, now we have two legs there are many possibilities for how you could modulate will look at. One possible way of modulating is to have your duty cycle d a to be equal to 1 plus, some m cosine omega t by 2 taking your output signal to be some sinusoidal cosine sinusoidal signal d b to be 1 minus m cosine omega t.

The modulation method that we would use is still the sine triangle modulation in this particular case, we will assume that m is positive and is less than or equal to 1. So, your d a and DC d b signals are still lying between 0 and 1, so your triangle is again going between 0 and 1, so your d a and d b belongs to the range from 0 to 1, so we could do again the sine triangle modulation for the signals.

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Essentially what you are doing now is because you have two signals, d a is this red curve over here and d b is the blue curve over here. Now, you have switching functions for two legs S a for leg a S a plus for the top switch of leg a and S b plus for the top switch of leg b. We are considering the in this particular illustration we have d a to be greater than d b. Essentially when you are trying to apply a positive voltage at the output terminals of this particular two leg inverter. So, when you have the output voltage in this particular case is essentially the voltage between terminals a and b. You can see that because your d a is larger the width widths of the switching function S a plus is wide because d b is lower the widths of the switching function S b plus is narrower.

So, if you look at the difference between S a plus and S b plus that essentially is what drives the output voltage. Now, you end up with pulses between your terminals a and b in this particular case the pulses are positive, because your d a is greater than d b, which means your S a pluses wider than S b pulses in case your d b was greater than d a. Then, essentially S b pluses would be wider. Then, S a minus S b, which would be your voltage between b a and b would then be flipped over to the other side and would be negative, so one thing that you could observe in this particular operation of this particular power converter.

When you have two legs is that now it is possible to have pulses, which are either in one polarity in the positive side or in the negative side in our center tapped topology. You

had pulses, which were going between plus V DC by 2 n minus V DC by 2 where as in this particular case it is going towards plus V d c. When your signals required being positive or it could go to minus V DC and between minus V DC and 0. When your signals need to be negative another thing that you could observe in from this switching signals of this particular topology is that both the triangular carrier has p d t S w. So, its frequency corresponding to a frequency of f S w, this switching functions S a and S b turned the switches turn on once and turns off once in one switching cycle.

So, the switching frequency of the leg a and leg b is f s w, but if you look at the number of output pulses in one particular duration, you would get two pulses every t s w. So, essentially the output switching frequency seen at the terminals of this inverter is essentially double, this switching frequency. So, for this inverter we could again look at your switching's functions and write down the equations corresponding to the switching functions.

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$$\frac{1}{2} \frac{1}{2} \frac{1}$$

So, you could write your voltage of terminal a with respect to reference is s a plus times V DC plus V n with respect to reference. We could write your voltage of leg b with respect to the reference is S b plus times V DC plus V n r also note that V b r the leg b is connected to ground.

So, essentially V b r 0 in this particular case, so if you look at your V a b is S plus a minus S plus b times V d c. So, this gives you ah the switching model information,

which relates your output terminal voltages to your DC bus voltage. Similarly, you can have write an expression for what your DC bus a positive DC bus current is. So, this is equal to S a plus times i a plus S b plus times i b and your i a is the i out and i b is essentially i out because, with a negative sign because this is coming back in the loop. So, essentially you have S a plus minus S b plus times i out of t. So, essentially these set of equations give you the switching model of the power converter.

So, the of the two leg single phase inverter and to look at it on an average bases, it is actually quite simple you can look at the corresponding equations for your average model. If you look at the average model of your quantities from your terminal quantities V a b, you will get the expression for your average voltage as V a b on an average bases is essentially d a minus d b times V d c. Essentially, your i DC plus of your average model would be d a minus d b times i out of t.

So, you can see that the average model and the switching model they look very close. One has to keep in mind that the quantities over here are on the switching model is discontinuous quantities the quantities that you are looking at on your average model or continuous quantities. So, they if you actually plot them they look quite different, where as if you look at the expressions, they look very close, some of the properties that we saw of this particular power converter are that.

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1. finpple = $\frac{2}{1800}$ 2. output is between 0 and Vac if da 7 ds 1. is between 0 and Vac if da 7 ds 1. is between 0 and -Vac if da < ds 3. Vac = $\int 2Vac^{R} 1.05 \times 1.05 \times 1.1$ $\rightarrow 300V$ No series capacitors.

One property was that your ripple current is essentially twice your switching frequency. So, essentially you get a higher effective switching frequency, because of this particular way, which the two leg converter and the way in which you are modulating this particular two leg converter. Another quantity that you can observe is that your output voltage is between 0 is switching between 0 and V d c. If d a is greater than d b and your output voltage is between 0 and minus V d c, if d a is less than the other thing that you could observe.

It can observe from your average model is that the relationship between your output voltage V a b the AC voltage. Your DC bus voltage is now d a minus d b, so the value of d a and minus d b can now take on values of either plus 1 or minus 1. Let us say for example, when d a is maximum and d b is minimum, then this quantity takes a value of plus one when d a is minimum close to 0 and d b is maximum close to 1, the value of V a b takes on a value of minus 1. So, in fact the AC voltage that you can generate with this particular ability to take your effective d a minus d b between a value of plus 1 to minus 1.

You have now a range of two, because you are multiplying something with a range of two for your V DC the required V DC is now essentially half of what you had for your DC bus capacitor midpoint topology. So, essentially we have V DC and we looked at a number of factors, so we saw that we had factors for your peak voltage root two times V AC plus factors for your grid voltage variation, for your dead band for your filtering etcetera. So, we might effectively require a V DC of about 400 volts compared to V DC of 800 volts for the capacitor midpoint topology.

So, we need just one capacitor in our capacitor bank rather than a set where you had one at the top and one at the bottom, so no series capacitors. We could also look at the common and differential mode signals of your input and output for this particular topology. Again the switching functions give us the particular information, if you look at your common mode your differential mode voltage on your output side. (Refer Slide Time: 45:35)

-----Output depurantial mode voltige $V_{AM(d)} = V_{ab} - V_{br}$ = Vab = $(S_a^f - S_b^f)V_{Ac}$ $V_{\rm cm}(o) = \frac{V_{\rm av} + V_{\rm bb}}{2}$ $= \left(S_{a}^{\dagger} + S_{b}^{\dagger}\right) \frac{V_{Ac}}{2} + V_{nY}$ using $V_{nY} = -S_{b}^{\dagger} V_{Ac}$ $= \left(S_{a}^{\dagger} - S_{b}^{\dagger}\right) V_{Ac}$

So, V d m at your output is V a with respect to reference minus V b with respect to some reference is essentially V a b, which is S a plus minus S b plus times V d c. We saw the shape of the wave form that we just plotted it in the previous slide if you look at it on a common mode bases of your output it is V a r plus V b r by 2, so this is equal to S a plus S b plus times V DC by 2 plus V n r. This is essentially from the two equations that we had written previously making use of the fact that your b phase voltage sums up to 0 you can actually substitute for V n r, so essentially you would end up there. So, essentially your common mode voltage would be S a plus minus S b plus into V DC by 2, so which is essentially your differential mode voltage of your output by 2.

So, essentially the characteristics in this particular case are similar to what we saw for the capacitor midpoint topology. Your AC side voltage is one particular value for your differential quantity and half that for your common mode quantity, if you look at your input side common mode voltage and differential mode voltage. (Refer Slide Time: 47:47)

for input side Vam-in = Vpr - Vnr = Vdc Vcm-in = Vpr + Vnr = Vdc Vcm-in = Vpr + Vnr = Vdc (1/2 - Sp)

Your V p r minus V n r is essentially your DC bus voltage, which is essentially a fact quantity that we would like in this case it is a fact quantity at a reduced value, which is essentially 400 volts for 230 volts AC system closed to that. If you look at your common mode voltage of your input this is V p r plus V n r, so this is your positive DC bus with respect to your reference, which is ground and your negative DC bus with respect to reference. So, this can be written as V DC plus 2 V n r by 2.

Again, using the expression V n r is equal to minus b plus times V d c, you can write this as V DC into half minus S b plus, so we know how our switching function S b plus looks like S b, plus essentially the blue switching function that we have over here. So, using that particular signal we can actually write down what are input side common mode signal would be.

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So, if we have again the sine triangle modulation would be, if we have our d b signal to be something like this essentially you are looking at. So, if you look at your common mode input side voltage you will end up with a signal, which is now switching between minus V DC by 2 and plus V DC by 2. So, you can see that, now on the input side you have high frequency components, which means that now. For example, if you have ah this being used in a solar inverter or in a U P S that is a large area for your panels, your batteries might take up lots of space on a battery rack.

So, you can have now capacitance to ground, so you are more susptive build to parasitic currents going into the ground on the input side. We know that your input common mode voltage has stepped wave form in this particular case, in this two leg topology with this particular type of modulation signal that we are using. So, if you are using this in an application where you now have more chances for parasitic in this particular in a particular application. Then, you have to be more conscious about E M I concerns in or currents going in spiky currents getting injected to ground in this particular case.

So, you can see that this differential mode and common mode analysis can actually provide quite a lot of information and insight into problems, which can come up at not just at the design stage, but actually at the end execution point, when you are just trying to test. Let us see whether your system is going to be functional or not your analysis upfront can actually give you a good feel for what potential problems. You can face especially at the final stage of your system design assembly testing etcetera.

So, we will look at some of these issues in more detail the how you could now modulate your single phase two leg power converters in other alternate methods to address or mitigate some of these issues. To address some of these issues, you do not just have to look at hardware related solutions; you could look at different ways of modulation, which could actually address some of these concerns.

Thank you.