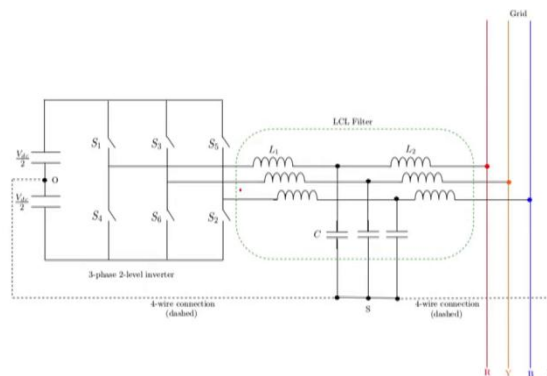



Power Electronics and Distributed Generation
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Lecture - 39
Higher Order Passive Damping Design for LCL Filters

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Voltage Source Inverter With LCL Filter



 3-phase 2-level inverter connected to the grid through a LCL filter in:
3-wire configuration (solid lines) and
4-wire configuration (dashed lines).

Welcome to class 39 of topics in power electronics and distributed generation. We have been discussing filter design, and we have been looking at an LCL filter and the constraints that we have been looking at in the LCL filter are the ripple injection into the grid. The amount of reactive power that is drawn by the filter and the DC base voltage that is required. If you have very large value of the filter inductance the amount of reactive power required would be more, you also have consideration of what is a pass band and stop band frequencies that you would like for the filter. Then you are also looking at what would lead to a very efficient filter in terms of what value of parameters for L would reduce to the power loss in the filter. And based on this one can come up with the preliminary values for L, L₁, L₂ and C. And we saw that L₁ is equal to L₂ is equal to the total L divided by 2 would be a suitable design and the C would then be related to the resonant frequency and the value of the inductance that is used.

So, what is shown over here is a 3 phase power converter with a LCL filter; in the solid line you have the configuration for a 3 phase 3 wire system. And in the if you add the

dotted lines; where you have the connection to the DC base being point. And to the output to the neutral then you have a filter for a 3 phase 4 wire system. So, just if you look at the ideal LCL filter there are possibilities of resonant that can happen such an LCL filter; one possibility might be when say you are your inverter is not switching. So, you could consider your L 1 side to be open and you could have a resonance between your L 2 and C; and if the filter is still connected to the grid.

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Resonances in the LCL Filter

- When the converter is not switching, when connected to the grid

$$\frac{1}{j\omega_s L_2 + \frac{1}{j\omega_s C}} = \infty$$

where, $\omega_s = \frac{1}{\sqrt{L_2 C}}$.

- When the converter switches as a VSI, when connected to the grid

$$\frac{1}{j\omega_p L_2 + \frac{j\omega_p L_1 \cdot \frac{1}{j\omega_p C}}{j\omega_p L_1 + \frac{1}{j\omega_p C}}} = \infty$$



where, $\omega_p = \frac{1}{\sqrt{L_p C}}$.

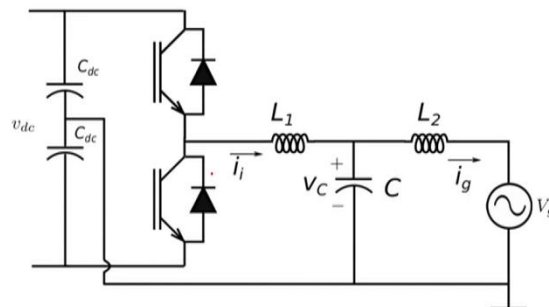
So, in that particular situation essentially the resonant frequency in this particular case would be 1 by square root of L 2 C. And at the resonance you have this particular condition for the impedance in the circuit. But this particular scenario might to some extent can lead to over voltages on the capacitor but you have the anti parallel diodes of the switch; which can actually absorb some of the energy also you have the possibility that if you have such a response if you have a upstream contactor or a circuit breaker; if that is open then the excitation to such a cycle would be prevented. So, it depends on how you are operating the power converter.

Another possibility for resonance is when you when the power converter is operating as a voltage source inverter; in which case what you would have is a your L p which is essentially the parallel combination of L 1 and L 2. And this particular impedance condition being infinite would lead would lead to operation of this filter at resonance; this resonance can be excited both from your inverter side and also from the grid side.

And you can have over currents in the filter and you could have say voltages across the capacitors or the filter components. This particular configuration cannot be prevent cannot be mitigated the opening of a upstream breaker. Because you are actually operating the power converter and you are interested in sending power exchange between your converter and the grid. So, you need to actually damp this particular resonance effectively.

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Implications of Resonances in the LCL Filter

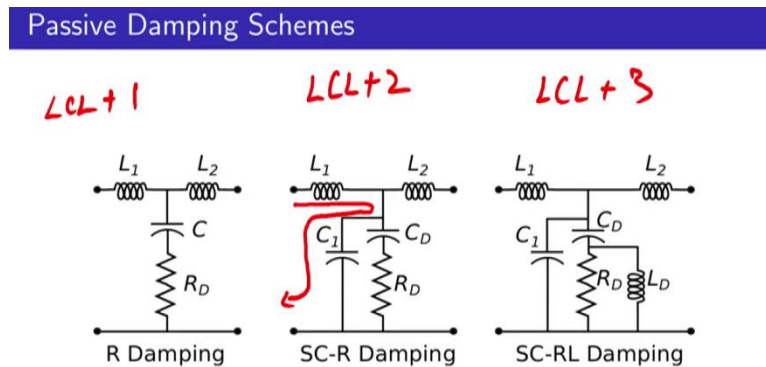


So, if you consider the excitations that can happen for the resonance; it can be both from the power converter side or it can be from the grid side. If you look at the power converter side the primary excitation is at the fundamental frequency and the switching frequency. However you have non idealities of the inverter such as a dead time, the on state voltage drops, the turn on the turn off delays etcetera; which introduce additional frequencies in the output of the inverter; you also have delay in the control timings, you have contestation of the in the control. So, many such factors can introduce noise at the output of the power converter which can actually need to a excitation.

However, whatever comes from the power converter is to a large extent controlled by the designer. If you have excitations from the grid side it could be because of harmonics from connected loads, it could be from operation of neighboring converters which are injecting voltages at the input of the filter; you could have non idealities of the equipment. For example, you have a transformer over here and the non idealities of the

non-linear b h loops or power resonance in the transformer can actually introduce harmonics from the grid side. You could also have upstream contactors or breakers which are cycling and introduce frequencies which can excite the filter; these external excitations from the grid side is not under our control it depends on what is connected to the external side. And is important to address the resonance and ensure that you do not lead have overloading of the filter. Because of over currents or higher voltages appearing across the components; which can actually lead to damage of inverter component or tripping of the inverter. So, if you look at the resonance in a in such a LCL filter one way of addressing the resonance is by introducing a resistor into the network.

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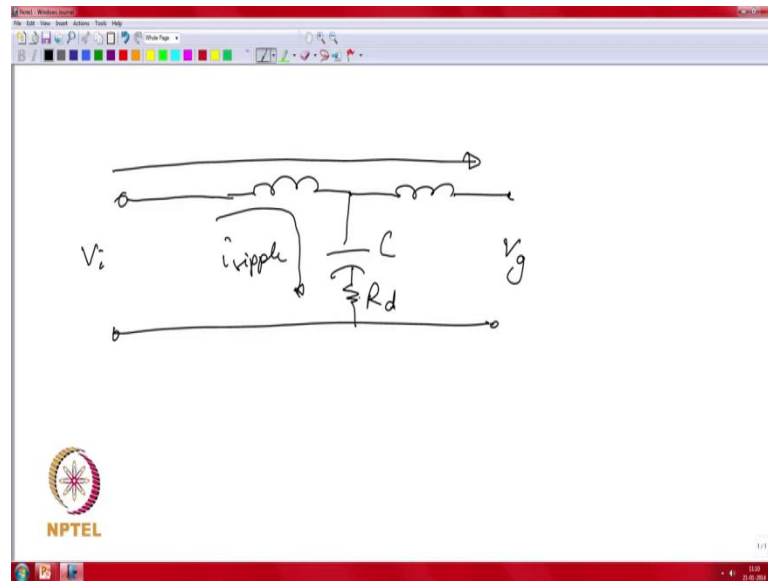


Evolution of split capacitor-RL damping scheme.

Objective of passive damping to reduce the quality factor (QF) with minimum power loss

So, as soon as you introduce a resistor in a network your power dissipation goes up. And objective of a passive damping is to reduce the quality factor or Q F with minimum power loss. So, in this particular where you have the resistive damping we are considering adding a single resistor to the LCL filter.

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And, a single resistor can be introduced in multiple ways. So, if you have an LCL filter you could think about say adding resistor in series with the inductor. So, this could be one possibility; so in this particular case whatever power flow is happening between a power converter and the grid will introduce power loss in this resistor. So, you would have a increase in losses in this particular resistor. So, another possibility of introducing a resistor damping element could be say in series with the capacitor; in which case the fundamental power flow between the inverter and the grid will not affect the power loss in the resistor to a large extent; you would of course have your ripple current going through your capacitive branch. And the ripple current would now introduce losses in the damping resistor also the capacitor would draw some fundamental reactive current which would also cost losses in the resistor.

So, if you look at the configuration between putting a resistor in series with the inductor or a resistor in series with a capacitor; the one with the resistor in series with capacitor would lead to lower losses for a given level of damping. And you can see that in this particular case you have your LCL filter plus one component which is essentially the damping resistor; if you now think about one way in which you could reduce the losses in the damping resistor is to provide a path for your ripple current to flow which is in parallel with damping resistor. So, by connecting say your resistor C_1 parallel with the damping branch or essentially what you have done is you have split your capacitor C into 2 branches C_1 and C_D . And you connect your resistor in series with one branch and the

capacitor C_1 is directly connected across you are as the LCL filter. So, your ripple current essentially would primarily go through L_1 which reduces the power loss in the damping resistor.

Here, your complexity has gone up so you have your LCL filter plus 2 components. So, you would expect the power loss to come down but the number of components have actually gone up. If you look at then case of the S C R L damping you have a LCL filter plus 3 components; you have L_1 , L_2 and C_1 . Then, you have 3 components C_D , R_D and L_D . So, you can see that the complexity is increasing as you go from a simple resistive damping to the split capacitor resistive damping to the split capacitor R L damping.

So, if you look at the as you increase the number of components your complexity of the circuit goes up also often the cost of circuit would go up; if you have more components. So, your it does not mean that for all situations you have to go in for the most complex network for lower power levels; where say for example you might be operating at power levels of less than a kilo watt. Then, you might go in for essentially the direct resistive damping. And as the power level increases you would go in for say for example you are looking at the order of 10 kilo watts maybe you would go for split capacitor damp resistive damping. And if you are talking about 100 of kilo watts to mega watt power levels then the added complexity would not matter that much. But the reduction in power loss would be significant; so you would go for the more complex forms of damping. So, in each of the situations you will have to look how to select your R_D , L_D , etcetera. So, we will start with the simple resistive damping in which case if you do a transfer function analysis; you would see that the transfer function is relatively straight forward.

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Resistive damping

$$\frac{V_c(s)}{V_i(s)} = \frac{(sL_2)(1 + R_d C s)}{s(L_1 + L_2)(1 + R_d C s + L_p C s^2)} \quad L_p = L_1 \parallel L_2$$

$$\omega_r = \frac{1}{\sqrt{L_p C}}$$

$$Q_f = \frac{\left. \frac{V_c(s)}{V_i(s)} \right|_{s=j\omega_r}}{\left. \frac{V_c(s)}{V_i(s)} \right|_{s \rightarrow 0}} = \sqrt{1 + \left(\frac{\sqrt{L_p C}}{R_d} \right)^2}$$

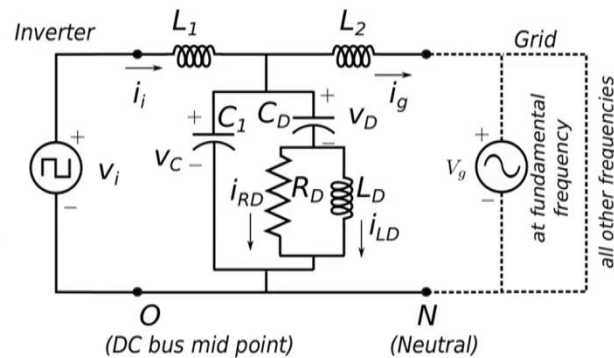
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If you look at your capacitor voltage to a inverter voltage transfer function essentially you would have sL_2 divided by $1 + R_d C s$ divided by $sL_1 + sL_2 + 1 + R_d C s + L_p C s^2$; in this particular case L_p is the parallel combination of L_1 and L_2 . The natural resonant frequency of this particular transfer function can be obtained in a straight forward manner; where the numerator is the second order 10 at the origin another based on the $R_d C$ time constant; the denominator again we have pull at the origin. And then you have a second order transfer function from which one can easily find out the location of the poles.

The undamped natural resonant frequency of this filter is ω_r is 1 by square root of $L_p C$; and you could obtain an approximate expression for the quality factor at this particular frequency ω_r . So, you could define your quality factor Q_f for this particular circuit as $V_c(s)$ divided by a $V_i(s)$ at s is equal to $j\omega_r$ divided by $V_c(s)$ divided by $V_i(s)$ at s going to the origin. So, you could evaluate this in a straight forward manner and you get an expression where you have 1 plus square root of L_p by C divided by R_d the whole square. So, you can see that when R_d is extremely small you have a very high quality factor; which means that you are filter can oscillate very easily when or your added value takes on a larger number; then your quality factor comes down essentially damping out your resonance.

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Modelling and analysis of SC-RL damped LCL filter



Model of the LCL filter with SC-RL passive damping circuit.

Current through the damping resistor is considered the output for evaluating power loss.

So, if you have a more complex passive damping circuit; such as this split capacitor R L damping or a split capacitor R damping one can actually do an analysis by looking at the model of the filter. And one particular model might be a state space model of such a filter. And for the state space model we will consider one possible set of state variables might be the inverter side current, the grid side current, the voltage across the capacitor C_1 , the voltage across the capacitor C_D and the current through the inductor L_D ; the inputs to this particular model could be the excitation from the inverter side and the excitation from the grid side. So, if you take inverter side excitation it is essentially your PWM voltage; if you are considering the line to DC base voltage you are looking at excitation of either plus $V_{DC}/2$ or minus $V_{DC}/2$; from the grid side if you are assuming the grid voltage is a pure sin wave at the fundamental frequency.

So, you have that particular frequency component at say 50 hertz; for all other frequencies essentially the voltage source can be considered as a short circuit for your analysis. And looking back at the inverter again we assume the inverter to be a voltage source; again assuming that this is a voltage source you are ignoring the control related impedances that can be seen in an inverter. But often the control bandwidths of the inverter are lower than the resonant frequency and approximating it as a voltage source is a good approximation.

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State Space Model of the SC-RL Damped LCL Filter

$$\begin{aligned} \dot{x}(t) &= \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}x(t) + \mathbf{D}u(t) \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & 0 & 0 \\ \frac{1}{C_1} & -\frac{1}{C_1} & -\frac{1}{C_1 R_d} & \frac{1}{C_1 R_d} & -\frac{1}{C_1} \\ 0 & 0 & \frac{1}{C_d R_d} & -\frac{1}{C_d R_d} & \frac{1}{C_d} \\ 0 & 0 & \frac{1}{L_d} & -\frac{1}{L_d} & 0 \\ 0 & \frac{1}{R_d} & -\frac{1}{R_d} & 0 & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} i_i \\ i_g \\ v_C \\ v_d \\ i_{L_d} \end{bmatrix}; \quad \mathbf{u} = \begin{bmatrix} v_i \\ v_g \end{bmatrix}$$

5x5 *i_{RD} R_D = P_{loss in R_D}*

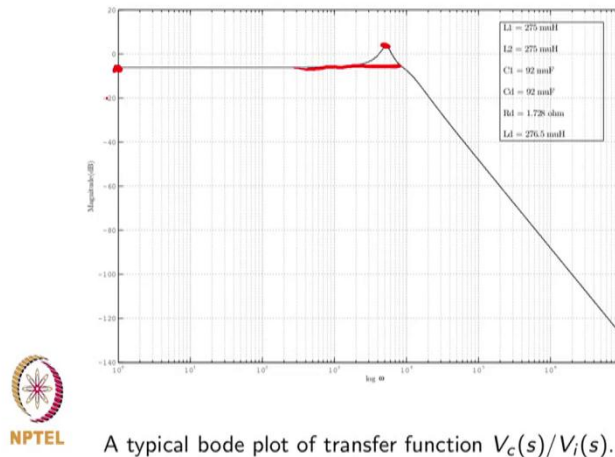
$\mathbf{D} = [0 \ 0]; \quad y = i_{R_d}$



So, if you look at the state space model of this S split capacitor R L damped LCL filter; you have a equation state equations of the form \dot{x} is equal to $A x$ plus $B u$; y is equal to $C x$ plus $D u$ you are a matrix is 5 into 5 matrix; the 5 states in this particular model you have 2 inputs. So, your b matrix is of this particular form you are inputs are your inverter voltage and grid voltage. And your state variables are given by is selected to be these 5 variables one particular output of interest is essentially a your current in the damping, resistor; the current in the damping resistor is good information. Because essentially the power loss in the damping resistor is given by your $i R_d$ square R_D gives you your power loss then R^2 . So, again keep in mind the objective of a passive damping network is to reduce your power minimize your power loss and achieve the lowest possible, lowest quality factor in a circuit. Again, the definition of the quality factor we will take it as the peak value of your transfer function.

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Bode Plot of Transfer Function $V_c(s)/V_i(s)$



A typical bode plot of transfer function $V_c(s)/V_i(s)$.

So, here the transfer function that is plotted is the capacitor voltage by the into grid inverter voltage. So, you are looking at essentially this particular ratio of the peak value at the resonant frequency to the peak value at D C to be your quality factor. And if your quality factor is close to 1; it means that you're not having any peaks over there exciting resonance or essentially you are not amplifying any input when you are at the particular resonant frequency.

So, there are actually a couple of ways in which you could look at the quality factor of such a S C R L damped a passive damping network. And one is to again use this particular definition of the quality factor in this particular ratio; again the assumption that your resonant frequencies stays at ω_r ; where ω_r is equal to $1/\sqrt{LpC}$ is actually not exact. Because you know that adding passive damping components to the circuit will actually shift your resonant frequency to some extent. So, this is an approximation but still you could make use of that to evaluate what your quality factor is.

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Quality Factor

- $QF = \frac{\left| \frac{V_C(j\omega)}{V_i(j\omega)} \right|_{\omega=\omega_r}}{\left| \frac{V_C(j\omega)}{V_i(j\omega)} \right|_{\omega \rightarrow 0}}$

- $\frac{V_C(s)}{V_i(s)} = \frac{R_d L_2 s + L_2 L_d s^2 + R_d C_d L_2 L_d s^3}{R_d(L_1+L_2)s + (L_1+L_2)L_d s^2 + R_d[L_1 L_2(C_1+C_d) + L_d C_d(L_1+L_2)]s^3 + L_1 L_2(C_1+C_d)L_d s^4 + R_d L_1 L_2 C_1 C_d L_d s^5}$

- $QF = 2 \times \sqrt{\left(1 - \frac{K \cdot \omega_{fu}}{\omega_r}\right)^2 + 1}$

- where, $K = \frac{R_d}{\omega_{fu} \cdot L_d}$

- Assumptions: $L_1 = L_2 = L/2$, $C_1 = C_d = C/2$, $R_d = \sqrt{\frac{L}{C}}$



- Resonance frequency stays the same as the ideal LCL circuit,

$$\omega_r = \frac{1}{\sqrt{L_p C}}$$

And keeping that in mind one can define the quality factor in this particular manner; the transfer function $V_C(s)$ by V_i is actually now a ratio of a polynomial; it is third order in the numerator, it is fifth order in the denominator. So, it is not easy to directly simplify it to get the roots of such of a polynomial. So, these approximations help in giving you a comparatively insights into what the quality factor is.

So, assuming we will make a couple of assumptions; one is we will define the term K to be the ratio of R_D by L_D ; R_D by L_D is actually if you look at the passive damping network is the corner frequency of essentially this R_D , L_D branch. And if you look at the ratio of that particular corner frequency to the fundamental frequency; you have the term K which gives you a feel for what the how the R_D , L_D filter time constant should be placed with respect to your fundamental frequency; we also saw in filter design that taking L_1 , L_2 is equal to $L/2$ to be equal to L by 2; it would be a suitable design guideline. Also we will see that taking C_1 to be equal to C_D to be which would be C by 2 would actually also lead to a fairly good design. And we will also see that selecting R_D to be square root of L by C would also be a good design choice.

So, with these assumptions in this particular transfer function we could plug in and then take this particular ratio and simplify the quality factor. And it comes out to be a fairly simple expression twice the square root of this term this quadratic term plus 1. And again

we are assuming the resonant frequency has not shifted they are still at square root of L/C .

So, from this particular expression we can see that a couple of things are possible; one is by a appropriate selection of the R/D by L/D term; it might be possible to make this particular term equal to 1 which means that your quality factor would have a value close to 2. So, by suitably selecting this particular L/D you might be able to make your quality factor get close to 2. Another thing that it shows is there is a selection of K ; which can be related to your ω and or fundamental frequency which is your main frequency at which you are trying to reduce the excitation the flows through the R/D branch to actually reduce your quality factor.

And, this normalization term K is given as R/D by L/D into one by your fundamental frequency. Note that this is an approximate way of a looking at the quality factor; you can have a alternate method of looking at the quality factor; which would be by directly going to a numerical approach you plot the boarding plot gain plot of this particular transfer function. See where your peak of your resonant peak of your transfer function is and evaluate this particular peak value to your D/C value ratio that would give you a exact value for your quality factor. So, that would be numerical as this gives you an approximate field for what the quality factor is. Now, that we have a framework for evaluating the quality factor.

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Damping Power Loss

- Damping power loss: fundamental (P_{fu}) component and switching ripple (P_{ri}) component.
- V_c is taken to be 1 p.u. at fundamental frequency. Then, P_{fu} can be found using simple circuit principles.
- $P_{fu}(p.u.) = \frac{C_d^2 R_d}{(K - C_d R_d)^2 + 1} = R_d [V I^*]$
- P_{ri} is computed using the state space model.



The next item of importance is to evaluate the power loss in this damping filter network. And the power loss in the damping filter again primarily involves the excitation; that the main excitation of this filter is the 2 frequencies which would be the fundamental frequency and the resonant frequency. So, your fundamental frequency excitation can then be evaluated by assuming that in your LCL filter; we can assume that at fundamental frequency the inverter voltage is close to 1 per unit also your grid voltage is close to 1 per unit. And we can actually find out what would be the current that flows through this particular resistor; in this R_D at the fundamental frequency from a direct phase of analysis.

And, using a phase of analysis you could actually then get an expression for your fundamental current flowing through the branch. And we have a power which is equal to the re of V_i conjugate. And V is 1 per unit and evaluating your i conjugate in terms of the expressions of that particular branch; you can get a simplified expression as a power of the fundamental frequency on a per unit basis is $C_D R_D$; $C_D^2 R_D$ divided by $K^2 - C_D R_D$ the whole square plus 1; where these terms are again in per unit as form.

So, one can see from this expression that as K takes on a large values your power loss through the at fundamental frequency can actually come down. Because you have a quadratic term the denominator. So, as this particular term increases your power loss can actually come down; the second component that is to be evaluated is essentially the power loss because of your ripple frequency. So, what P_{ri} indicating the power loss in the damping network due to the ripple component, due to the switching action of the power converter. And this can be evaluated using the state space model of the power converter.

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$$\dot{x} = Ax + Bu$$

$$y = iR_d = \frac{V_c - V_d}{R_d}$$

include winding resistance in L_1 & L_2 , A matrix is invertible -

$$u = \begin{bmatrix} V_i \\ V_g \end{bmatrix}$$

$$V_i = \begin{cases} f \frac{V_{dc}}{2} & t \in [0, T_{on}] \\ -\frac{V_{dc}}{2} & t \in [T_{on}, T_{sw}] \end{cases}$$

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$$T_{on} = d T_{sw}$$

$$V_g = V_{dc} (d - 0.5)$$

So, if you look at this state space model; the model of the power converter of the filter is essentially \dot{x} is equal to $Ax + Bu$ and y is equal to iR_d . And this is essentially equal to $V_c - V_d$ by R_d . So, in this particular case if we include the winding resistance; you could have generate a matrix this invertible. And your inputs are u is essentially V_i and V_g and we have V_i is equal to plus V_{dc} by 2 or t belonging to the duration 0 comma T_{on} ; where t_{on} comma T_{sw} . So, this particular duration is your T_{off} and if you look at your grid side voltage; we will take our grid voltage to be equal to V_{dc} into d minus 0.5; where d belongs to the range 0 to 1.

So, this means that your assuming that at your inverter voltages exactly balanced by your grid side voltage. So, you are assuming a cautious steady state at each duty cycle of operation of the power converter with the LCL filter. So, you could then obtain a solution for the state equations.

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$$X(T_{sw}) = e^{AT_{sw}} X_0 + \int_0^{T_{sw}} e^{A(T_{sw}-\tau)} B u_1(\tau) d\tau \quad \left[\begin{array}{c} + \frac{V_{dc}}{2} \\ V_g \end{array} \right]$$

$$+ \int_{T_{sw}}^{T_{sw}} e^{A(T_{sw}-\tau)} B u_2(\tau) d\tau \quad \left[\begin{array}{c} - \frac{V_{dc}}{2} \\ V_g \end{array} \right]$$

$$= X_0 \text{ under steady state.}$$

Numerically evaluate $i_{rd\ rms}$ using n sub intervals in T_{sw}

$$i_{rd\ rms} (T_{sw}) = \sqrt{\frac{1}{n} \sum_{m=0}^{n-1} i_{rd}^2(m)}$$
 for a given duty cycle d



So, you would have you could write your X at your switching interval is e to the power a T s w times your X naught plus integral 0 to d T s w; which is your t on into the power of A T s w minus tau b u 1 of tau d tau plus d T s w to T s w into the power of A T s w minus tau b u 2 of tau d tau again you want is essentially your plus V d c by 2 comma V g. And V 2 is essentially minus V d c by 2 comma V g. And if the system is in steady state it means that at the end of the switching interval at T s w; you get back to the same point where you started so this would be equal to your X naught. And given that your e matrix is invertible; we can actually solve this particular equation to obtain what your X naught is.

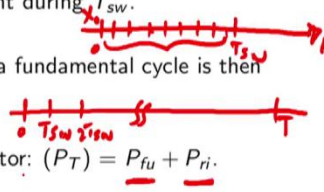
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Damping Power Loss

- The state space model with i_{Rd} as output is used to evaluate P_{ri} for a given duty cycle of the inverter leg.
- Initial conditions are calculated using equilibrium conditions, $x(T_{sw}) = x(0)$.
- i_{Rd} is then numerically evaluated for a given duty cycle, d . This is used to evaluate the rms ripple current during T_{sw} .
- The rms ripple current, $i_{Rd,rms}$, over a fundamental cycle is then evaluated. $P_{ri}(p.u.) = (i_{Rd,rms})^2 R_d$.



Total power loss in the damping resistor: $(P_T) = P_{fu} + P_{ri}$.



So, once you know your X naught your initial condition then you could use that to evaluate your current in your damping branch; numerically at points at n points between 0 and T_{sw} . And this is this is then used to evaluate your current R M S current ripple during the duration T_{sw} . So, essentially what you are doing is if your time is proceeding like this 0 T_{sw} to T_{sw} will be further down; essentially your dividing it into n points. And using the state evolution equation you know your value of X naught at. So, you know your X naught at this particular point; you can use that to evaluate your state variables at all the subsequent points. Because you know your excitation you know your initial conditions you can solve your dynamic equation of the filter and essentially the output it is evaluated is your i_{Rd} .

So, once you have your i_{Rd} evaluated at these points; you could then generate your R M S currents over a switching interval over T_{sw} using n sub intervals in T_{sw} . And you have i_{Rd} R M S over T_{sw} is summation over the n points of the square and you average it; and this is for a given duty cycle. So as your duty cycle is waiving between zero and one this particular i_{Rd} R M S would vary over the different switching periods different T_{sw} 's; as one goes over the fundamental cycle.

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Over a fundamental cycle

$$i_{rd-rms} (f_a) = \sqrt{\frac{1}{T} \sum_{j=0}^P i_{rd-rms}^2(j) T_{sw}} \quad P = \frac{T}{T_{sw}} \leftarrow \begin{matrix} 20 \\ \text{ms} \end{matrix}$$

$$P_{Rd} = i_{rd-rms}^2 R_d$$

So, if you look at a fundamental cycle you have i_{rd-rms} over a fundamental cycle to be equal to the square root of $\frac{1}{T} \sum_{j=0}^P i_{rd-rms}^2(j) T_{sw}$; where T is 20 milli seconds of j is equal to 0 to P ; i_{rd-rms} square over the duration switching the duration T_{sw} over all the P points times T_{sw} . So, essentially your P is equal to T divided by T_{sw} ; where T is 20 milli seconds for 50 hertz and T_{sw} depends on your switching frequency that you are using for your particular filter. And essentially once you have your i_{rd-rms} your power loss at your ripple frequency is $i_{rd-rms}^2 R_d$.

So, you could actually look at this particular evaluation. So, essentially what you are doing is now that you know your rms current over a switching interval what you are doing is your now looking at $0 T_{sw}$ to T_{sw} over the overall fundamental cycle T . And looking at the rms the power loss and the ripple frequency from the rms loss through this particular damping branch. And the total loss power loss in this damping resistor is because of your fundamental frequency power loss and the ripple frequency power loss. So, essentially what you are trying to do is to minimize your quality factor while keeping your power loss in this particular total power loss to be a small value. So, that is essentially the overall frame framework. And now we have a method for evaluating both the quality factor and the power loss could actually go about looking at how to actually select the components of the LCL filter.

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Variation of Switching Ripple Current in R_d

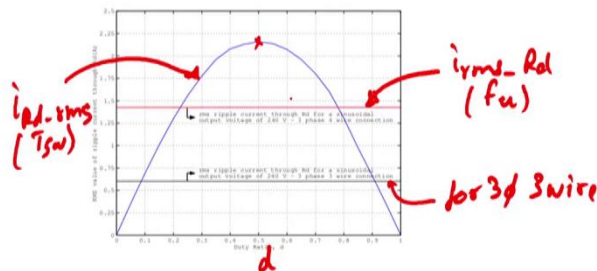


Figure: Rms ripple in i_{R_d} with duty ratio for $L_d = 500\mu H$.

- Worst ripple loss occurs at duty cycle of 0.5
- Total rms ripple current over a fundamental cycle calculated using rms ripple at each switching interval
- Total rms ripple current for a sine output voltage for the 3-phase 3-wire is less than the 3-phase 4-wire case

So, what is shown over here; if you look at the power loss over the different switching durations, over different switching T_s 's. And as a function of your duty cycle d what is plotted over here is i_{R_d} rms. So, you can see that the losses in the damping branch is maximum for a duty ratio d . But because you are as you are going over your fundamental sine wave your duty cycle is sweeping a value depending on your modulation depth; it may be between say 0.1 to 0.9 or 0.2 to 0.8 depending on your modulation depth or the inverter. And then you evaluate the rms components of over the different switching situations and that particular value is what is plotted over here i_{R_d} rms over the fundamental cycle.

So, this is over a switching duration T_s which is now a function of d . So, if you then take the overall rms we will then get a value of the current which is lower than the peak the worst case being at a duty cycle point five for a 3 phase 4 wire system; if you do again a rms evaluation for a 3 phase 3 wire system the power loss in the damping branch is further reduced. So, if you are doing a 3 phase 3 wire design adopting this particular approach to give a consecutive value for the design; in the sense that the actual loss is would be lower than what you would have if you are thinking analyzing as a 3 phase 4 wire power converter.

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Comparison of Damping Schemes

Quantity	Notation	Base Value
Power	P_{base}	40 kVA
Voltage	V_{base}	240 V
Current	I_{base}	55.6 A
Impedance	Z_{base}	4.32 Ω
Inductance	$L_{base} = Z_{base} / (2 \cdot \pi \cdot 50)$	14 mH
Capacitance	$C_{base} = 1 / (Z_{base} \cdot 2 \cdot \pi \cdot 50)$	737 μF
Frequency	ω_{base}	$2 \cdot \pi \cdot 50 \text{ rad/s}$

Base values used in the filter analysis in design example.



So, to actually then proceed with the design we will look at an example design; the example that we would choose is a 40 kilo watt power converter with a line to neutral of voltage 240 volts. So, given the power level and the voltage you can then calculate your base current; from which now you had base voltage and base current you know what your impedance base is. And then you can calculate your this inductance and base capacitance and your base frequency is essentially 50 hertz or in radian to 50 radian's per second.

Now, the use of the base per unitized notation for a power filter is actually useful because you want to compare different filtering up range. And this looking at in milli Henry and micro Farads you would not make the comparison directly easy to compare whereas doing a comparison on a permanent basis can lead to a simpler and easier comparison.

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Comparison of Passive Damping Schemes

Parameter	R Damping	SC-R Damping	SC-RL Damping
f_{sw}	9.75kHz	9.75kHz	9.75kHz ~10kHz
f_r	1kHz	1kHz	1kHz
$L_1(p.u.)$	0.02	0.02	0.02
$L_2(p.u.)$	0.02	0.02	0.02
$C(p.u.)$	0.25	-	-
$C_1(p.u.)$	-	0.125	0.125
$C_d(p.u.)$	-	0.125	0.125
$R_d(p.u.)$	0.0718	0.484	0.4
$L_d(p.u.)$	-	-	0.0201
$ i_g/v_i _{\omega=f_{sw}}$	-59 dB	-65 dB	-65 dB
QF	3.0	3.0	3.0
$P_{Ru}(\%)$	0.45	0.75	0.0016
$P_{r1}(\%)$	1.09	0.05	0.065
$P_r(\%)$	1.54	0.80	0.0666



For a given QF and switching ripple attenuation, power loss can be reduced by SC-RL damping.

So, for the design that we are considering looking at a switching frequency of close to 10 kilo hertz. And a resonant frequency of 1 kilo hertz you are taking considering 3 possible damping operations of just the R damping; then the split capacitor resistive damping. And the split capacitor R L damping and we will take the value of C to be 0.25 per unit. And in case it is split we will consider C 1 is equal to C d is equal to C by 2. So, it is just 1.25 the value of R d is selected to get quality factor of close to 3. And if we could see that under this particular condition both the S C R damping and the S C R L damping it is possible to meet the given attenuation requirement into of this case minus 65 d b. Whereas in the R damping because of the damping resistor position you are not able to get that level of attenuation.

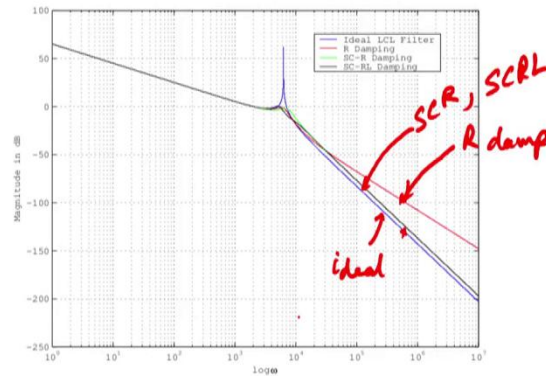
And, we will see that from the transfer function why that is the case you can also see that in case of the R damping the power loss is about 1.5 per unit. So, if you are talking about a 1 kilo watt power converter, you are talking about a power dissipation of about 15 watts which might be reasonable to handle. Whereas, if you are not now talking about a mega watt converter you would this loss might be much higher; you are talking about is of 15 watts, 15 kilo watts.

So, going to higher more complex damping network would lead to a lower much lower power loss of 0.07 percent. So, you can see that as the complexity of the damping

network is increased, your power dissipation in your damping circuit is actually coming down.

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$I_g(s)/V_i(s)$ Comparison for Different Damping Schemes



Reduction in attenuation for R damping when compared to SC-R, SC-RL.

So, to look at the recent why I your one when you have your one S C R damping what is shown over here is a plot of ideal LCL filter. So, this is the ideal LCL filter and what is shown over here is your resistive damping. So, at low frequencies the transfer function between your grid current to your inverter X voltage is falling off at 20 D B per decade; for the resistive damping at the higher frequencies the role off is at minus 40 d b per decade whereas, for the ideal LCL filter it is actually minus 60 d b per decade. So, if you are switching frequency is 10 kilo hertz; you are talking about a lower attenuation of this particular S C R damping network compared to the ideal LCL filter.

If you look at the case of your S C R L, S C R and S C R L type of damping network they both give a similar level slightly lesser damping than your ideal LCL filter. But the attenuation at high frequency is at minus 60 d b per decade. So, that is the reason why this particular case you are not able to achieve the decide damp level of attenuation with the R damping. But you could get a higher attenuation with the S C R and the S C R L damping.

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Component Selection Procedure for SC-RL Damping

- A multi-parameter optimization is a direct approach
- Damping components are added in steps to the ideal LCL filter for better design insight
- At each step, the additional component is chosen such that the quality factor and the total power loss in the damping resistor are low
- The ideal LCL filter is first converted into an SC-R damped circuit. Then, the SC-R circuit is modified to get an SC-RL circuit parameters



So, one approach to look at the selection of the parameters of the damping S C R L damping say LCL filter is to look at multi parameter optimization. So, the objective of the optimization would be the same to minimize your quality factor and to minimize your power loss. So, you can then look at a multi parameter optimize optimization approach; what gives you minimum values gives you the range of L D, R D, C D etcetera that gives you puts you at the minimum. But often the output of such an optimization engine may not be intuitive the optimization engine will give you particular design. But it might not give you the intuition behind why the particular design gives you lower value of losses.

So, the approach that we would take is actually to think of the more complex S C R L damping as adding one complexity at a time to the ideal LCL filter. And you use that to get a insight to what exactly one is trying to achieve by the passive damping network. So, essentially you are adding one component to and simultaneously looking at you factor performance factor such as your quality factor and your power loss and the implication of adding one component at a time. So, you take your LCL filter converted to into a S C R damp circuit and then convert that into a S C R L damp LC L filter.

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Ideal LCL Filter Components

- The ideal LCL filter components are chosen to meet filtering objectives
- $L_1 = L_2 = 0.02$ p.u. can be chosen from minimum power loss perspective and other constraints
- $C = 0.25$ p.u. based on designed resonance frequency

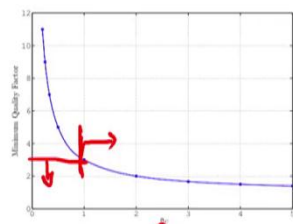


So, again the filter values that we are starting off with is with L_1 is equal to L_2 0.02 per unit in our particular design example. And C to be 0.25 per unit again based on the resonant frequency constraints that we have chosen in our design example.

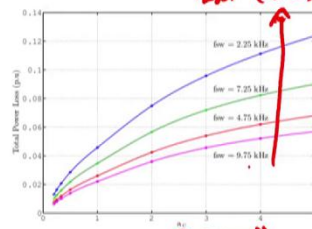
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Selection of SC-R Damping Circuit Parameters

- Let C be split into C_1 and C_d such that $C_d = a_c \cdot C_1$ $C_1 + C_d = C$



(a) Variation of QF with a_c .



(b) Variation of total power loss in R_d with a_c .



$a_c \geq 1$ for $QF \leq 3$

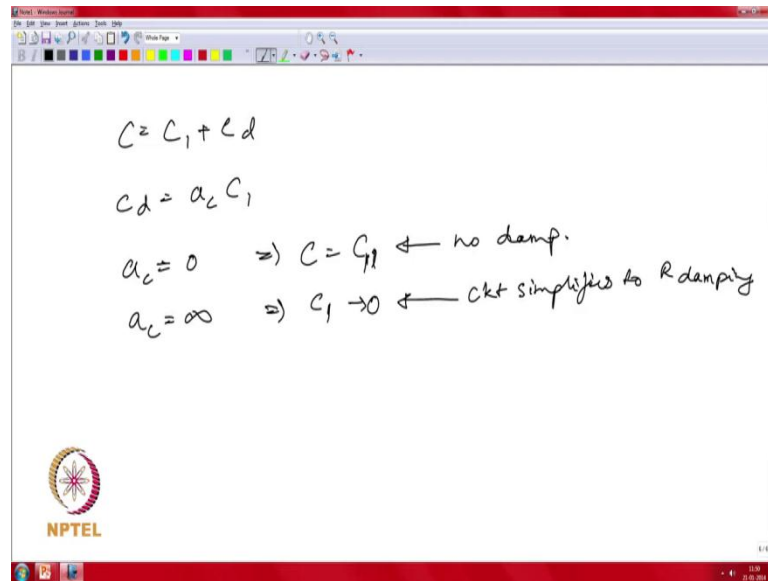
$a_c = 1 \Rightarrow C_1 = C_d = C/2$

Damping power loss increases with a_c

- QF versus a_c is independent of F_{sw} if R_d selected to minimize QF

So, a starting point for the design would be to consider how to split C into C_1 and C_d . And one could consider C_1 and C_d to be in a ratio of C_d equal to a_c times C_1 and also to C_1 plus C_d to be equal to C . So, the total value of the capacitance is kept at C and they split between C_1 and C_d in the ratio of C_d is equal to C_d by C_1 is equal to a_c .

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The image shows a whiteboard with handwritten mathematical equations. At the top left is the NPTEL logo. The equations are:

$$C = C_1 + C_d$$
$$C_d = a_c C_1$$
$$a_c = 0 \Rightarrow C = C_1 \leftarrow \text{no damp.}$$
$$a_c = \infty \Rightarrow C_1 \rightarrow 0 \leftarrow \text{ckt simplifies to R damping}$$

So, for example if you are value of so if you are a c is equal to 0 it means that C is equal to C_1 because your C_d would be 0. So, C is equal to C_1 which again would correspond to the condition where you have no damping. Because if C_d is 0 and C is equal to C_1 your essentially your damping circuits damping branch is open circuit. So, if you look at the other end of this extreme case a c effect takes on a very large value this implies that C_d is 10 into 0; this implies that your circuit would simply to essentially your R damping. So, when a c takes on a very large value essentially your C_1 would be 0; so essentially your circuit would simplify to your R damping.

So, what it means is if you look at a the trade off in this particular case essentially a c having a small value would mean that essentially you do not have any damping, your quality factor is very large. And a c taking on a large value essentially you have R resistive damping. So, your quality factor actually comes down if you look at the knee of this curve at a value greater than 1; you do not have a significant decrement in the value of a c . So, region of a c greater than 1 might be considered a suitable design such that you would get a quality factor which is less than 3.

So, selecting a value of a c greater than or equal to 1 would give a quality factor less than 3; the other thing that you could see is as you are a c value is increased essentially the losses in the damping circuit goes up, because as a c is increased essentially the capacitor C_d is being increased. So, your losses in the damping branch goes up; also you can see

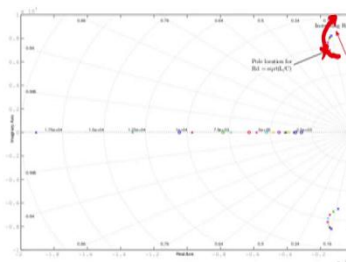
that as your switching frequency is reduced you have essentially increased losses in your damping resistor branch which is to expected. Because your switching frequency is getting closer and closer to your resonant frequency.

So, as the switching frequency is reduced your power loss is going up; power loss is going up as your switching frequency is reduced. So, this is looking at your power loss as a function of a c for different switching frequencies; what is interesting to note is if you plot your minimum quality factor versus your factor a c; the minimum quality factor actually order minimum quality factor curves by one on top of the other; which means that the quality factor is independent of your switching frequency. If R d is selected to actually in such a manner that minimizes your quality factor term.

This also implies that a design point of say if you say guideline a c is equal to 1. So, selecting a c is equal to 1 which means that as C 1 is equal to C d is equal to C by 2 would be a good design selection irrespective of your switching frequency; whether it is 2.5 kilo hertz or 9.10 kilo hertz of what would be in a range of switching frequencies that are commonly used in power converter design; this particular selection of dividing C 1 to C d would actually lead to a good design.

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Pole Loci with Varying R_d in a SC-R Damped LCL Filter



Poles of $V_C(s)/V_i(s)$ for R_d variation from $(0.6 \text{ to } 2) \times \sqrt{L/C}$:



Two Real poles with one at the origin, and a pair of oscillatory poles

NOTE $R_d = \sqrt{L/C}$ for maximum damping of oscillatory poles when $a_c = 1$

So, then the next thing one can look at is you could once you have actually plotted your taken your C 1 is equal to C d. Then, you could actually try varying your R d we initially saw that R d is equal to square root of L by C might be a reasonable design choice to

validate that we could vary R_d in a neighborhood from 0.5 or in this particular case 0.6 root L by C to twice root L by C . And you can look at the variation of the location as your R_d is varied from a small value to a large value. And you can see that the point at which you get loose damping because lines of constant damping is in this particular orientation. And as your damping is improved you will get closer and closer to your real axis.

So, you can see that the point at which your you get just damping is when R_d is equal to square root of L by C ; when you are a c takes a value of 1. So, making use of your state space model of the system you can actually look at your pole locations and see what value of R_d gives you your best damping. And this confirms that selecting R_d of L by C gives you a good value of damping of you are a $S C R$ damped network.

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Filter Parameter Values for SC-R Passive Damping

- The combination $C_1 = C_d = C/2$ and $R_d = \sqrt{L/C}$ is good in terms of quality factor and power loss in R_d for an SC-R damping scheme.

Parameter	$L_1=L_2$	$C_1=C_d$	R_d	ω_r	ω_{sw}
Per Unit Value	0.02	0.125	0.4	20	195
Physical Value	275 μ H	92 μ F	1.728 Ω	6283 rad/s	61261 rad/s

Filter parameter values for SC-R passive damping.



So, if you look at then a good starting point for your design choosing C_1 is equal to C_d is equal to C by 2 and L_1 is equal to L_2 would be a good starting point. So, here the values are given in physical per unit values and physical values; the resonant frequency is taken as 20 which 20 times your 50; which is your fundamental frequency would be 1 kilo hertz and your switching frequency in this particular case is 10 kilo hertz. So, this gives you a initial design point for the LCL damped a split capacitor resistive damped LCL filter. So, in the next class we will look at how we could then add the term L_d to such a $S C R$ damped network; again in manner which gives you a design that minimizes

your quality factor. And reduces the power loss in the damping filter branches simultaneously.

Thank you.