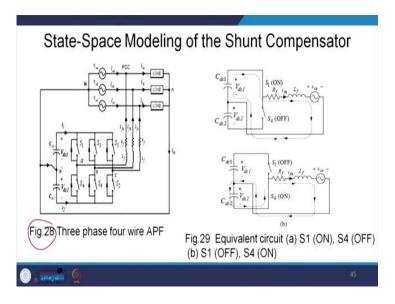
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Lecture - 31 Shunt Active Power filter- III

Welcome to our NPTEL courses on the Power Quality Improvement Technique. Today this will be the third class on the Shunt Active Power Filter. We have already discussed reference generation tracking. Now that part is mostly focused into the power system people and now, we shall discuss something that may be a great interest of the control people. Once the reference has been generated, how to track the reference?

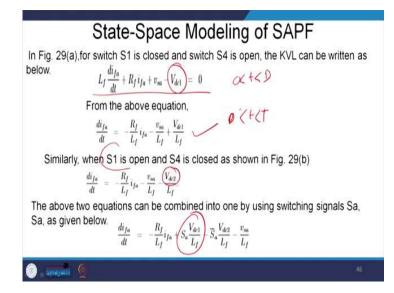
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So, we shall discuss about the tracking, current tracking. Let us take this figure. This figure number continues from the previous class for this reason it is 28. So, this is a three phase four wire system and you can have a four leg and the three-leg operation. We have considered the three-leg operation and you just split it like this and let us analyze it.

So, you have a C_{d1} and C_{d2} and once this upper switch S_1 is closed then current will flow like this. So, ultimately you have the resistance that associated with the shunt active power filter. Thereafter the inductor. Then the v_{sa} and the current will flow through this path.

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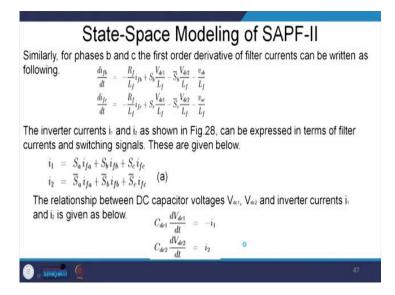


Similarly, when the lower switch is on, then current definitely will flow through this lower capacitor. So, thus we can write the state space model of this. Since this is the differential equation that will be $L_f \frac{di_{fa}}{dt} + R_f i_{fa} + v_{sa} - V_{dc1} = 0$. Similarly, you just take in a state space form. So, it is $\frac{di_{fa}}{dt} = -\frac{R_f i_{fa}}{L_f} - \frac{v_{sa}}{L_f} + \frac{V_{dc1}}{L_f}$.

Similarly, when S_1 open and S_4 is closes, so you will have V_{dc2} and also this is the only change you will get and there will be a sign change. Because it was a plus V_{dc} and it will be a minus V_{dc} . Otherwise everything will change and thus what we can say is, above two equation can be combined and this is called the average modeling.

Average modeling you have a sum for the duration D for 0 < t < D, upper switch will be closed and for D < t < T, so the lower switch will be closed. In that way if we combine that will be average modeling and ultimately it will be S_a into V_{dc1} and since this switch is a S prime, so it is a complementary. So, it will be $-\frac{V_{dc1}}{L_f} - \frac{v_{sa}}{L_f}$.

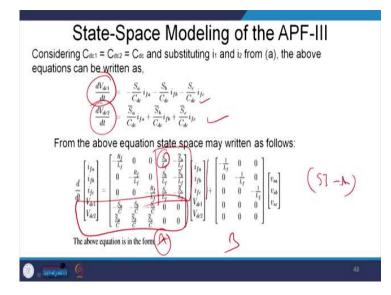
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Similarly, you can write this equation for the other phases. This will be the phase b and this will be for the phase c. So, it is $\frac{di_{fb}}{dt} = -\frac{R_f i_{fb}}{L_f} - \frac{v_{sb}}{L_f} + s_b \frac{V_{dc}}{L_f} + \overline{s_b} \frac{V_{dc2}}{L_f}$. Similarly, for V_{dc} and we know that there will be another logic you can go back, here i₁ it is flowing. So, i₁ is the essentially the DC current.

So, from there we can say that is i_1 equal to this when upper switch is on, then $i_1 = S_a i_{fa} + S_b i_{fb} + S_c i_{fc}$. Similarly, $i_2 = \overline{S_a} i_{fa} + \overline{S_b} i_{fb} + \overline{S_c} i_{fc}$. So, again you know that C dv/dt equal to i. So, $C_{dc} \frac{dV_{dc1}}{dt} = -i_1$ and similarly you got a minus sign for this is in $C_{dc} \frac{dV_{dc2}}{dt} = i_2$.

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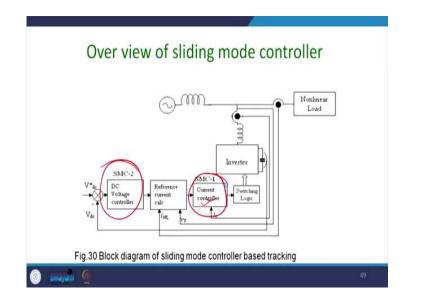
So, you can replace here this equation here and thus $\frac{dV_{dc1}}{dt}$ equal to minus S_a C_{dc} i_{fa} minus S_b C into i_{fb} this and similarly for the lower capacitor C_{dc2} it will be d v/d t minus S_a by this equation. Thus, from these equations of the state space, we can have this equation and please understand it. It is a not a linear equation. Because you know this is an average modeling essentially. You can do the average modeling and you can find out that transfer function.

So, but it will give an approximate result. You can write down these equations and you can find out the eigenvalues of it and from there you can see that what kind of perturbation you can have. This is your A matrix and this is your B matrix, you go back to the transfer function. Once you want a transfer function it is the average modeling and you go for the Nyquist, Bode any the root locus, any kind of treatment that it is available to you.

But please understand you know. These are the switching logic incorporated in this state space model. So, you can say that I can write a Cayley Hamilton theorem, like (SI - A) and then polynomial you got this. The polynomial of λ . So, λ is the eigenvalue. From there you calculate the eigenvalue and from there you can arrive at the different values of the stability.

But strictly speaking this is not so, because of the switching logic and this switching logic can be best justified with the non-linear control and it is better to use this sliding mode control. That is one of the prominent members of this non-linear hands control.

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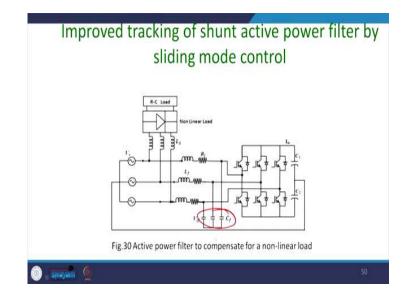
So, for this reason let us see the overview of the sliding mode control. So, you have same thing. Here you will have this current tracking which was doing in queue with a PI controller that will be replaced by a sliding mode controller. So, SMC stands for the same way. This DC bus voltage as we have seen this transfer function. This part is a part of that DC bus transfer function and this part of the system is your switching logic of the inverter.

So, you have to combine both. For sake of simplicity we will consider a three phase four wire system. Of course, you have a choice you can transform this 'abc' frame to the $\alpha\beta$ frame, to get some benefit over it. But ultimately since it is a three phase four wire system, you have a α , it does not have much benefit, because you require to have 'abc' instead of the dq0.

So, '0' will definitely come and ultimately you have to deal with a zero-sequence component. Thus, not much merit is there having the coordinate transformation. So, for this reason I am keeping in the 'abc' phase only. But if it is a three phase SISO system and if you convert into the $\alpha\beta$ or the dq frame, then zero axis component is not present. So, you have a definite advantage. The size of the matrix will be reduced, so there is a computational benefit.

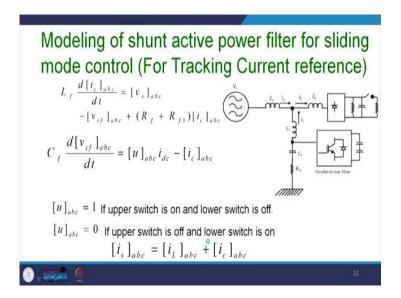
Since the computation has been done nowadays by the computer, you may not be actually bother about those computational benefit, and you can write this state space equation in the 'abc' frame only. So, that is a reference generation technique. You got the reference generation then you can track it by SMC, because essentially what you have this A matrix which is a non-linear matrix. Ok. Now let us see how to derive it.

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So, this is the same thing and you know we have connected a small capacitor across it and this capacitor is to suppress the switching losses. You can actively damp it or there is a small resistance with that, that will help you to mitigate the switching losses.

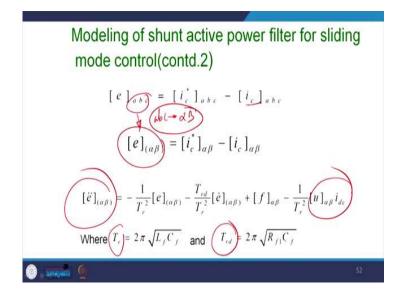
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Now, the same equation we are rewriting it and this ensure the load change operation and $L_f \frac{d[i_c]_{abc}}{dt}$. Here first we are there for normal standard model. I was maintaining the symbol of i_f. Since it is a sliding mode control, I wanted to just change the symbol, here compensating current I considered to be the i_c. So, $L_f \frac{d[i_c]_{abc}}{dt} = [v_s]_{abc} - [v_{cf}]_{abc} + (R_f + R_{f1})[i_c]_{abc}$. Similarly, $C_f \frac{d[v_{cf}]_{abc}}{dt} = [u]_{abc}i_{dc} - [i_c]_{abc}$.

So, from this condition you can see that if we have used the function 's' and here since this is an input, we consider it is an 'u'. So, when u = 1, the upper switch is on and lower switch is off and when just reverse it is 0. Then upper switch is off and lower switch is on and we have a logic here you can apply the KCL here. You can see that $i_s = i_L + i_c$.

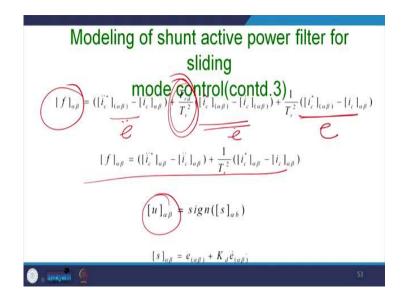
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So, let us consider the instantaneous tracking error as $[e]_{abc} = [i_c^*]_{abc} - [i_c]_{abc}$ and that error if you convert to the $\alpha\beta$ frame, then of course, you have advantage of having a less number of equations as I told you. So, I have written here. They are purposefully in abc frame and here I have changed this abc to $\alpha\beta$ frame for calculation sake. We shall see that the benefit after it. So, the error in the $\alpha\beta$ frame is $[i_c^*]_{\alpha\beta} - [i_c]_{\alpha\beta}$.

Similarly, you can differentiate it, this equations and we can have $[\ddot{e}]_{\alpha\beta} = -\frac{1}{T_r^2} [e]_{\alpha\beta} - \frac{T_{rd}}{T_r^2} [\dot{e}]_{\alpha\beta} + [f]_{\alpha\beta} - \frac{1}{T_r^2} [u]_{\alpha\beta} i_{dc}$, where u is the switching logic. So, where I have abbreviated it. So, it is $T_r = 2\pi \sqrt{L_f C_f}$ and T_{rd} is essentially the R_f and C_f.

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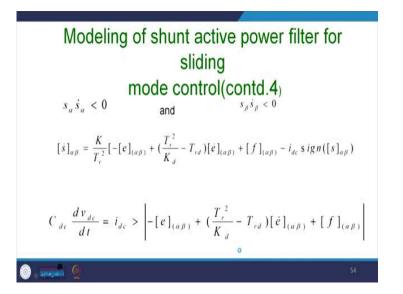


So, if you expand it you know, you take this function then you can have this value and here you know, since this ratio is quite small. Ultimately what you essentially have? This damping coefficient will be off. Please note that here you have a double differentiation. It is not showing properly. Sorry. This is a single differentiation and this is the error.

It is essentially e, it is essentially \dot{e} and it is essentially \ddot{e} . But this ratio you can see that. What is T_{rd} ? Please go back, $T_{rd} = 2\pi\sqrt{R_{f1}C_f}$, and $T_r = 2\pi\sqrt{L_fC_f}$. So, this value of the resistance is very minimal. You have not physically connected the resistor. It is an intrinsic resistance of this circuit, of this inductor and for this reason, you can get rid of this quantity \dot{e} . So, damping part is over. \dot{e} is basically the damping part.

So, you are left with this and let us defined your switching surface as S equal $[u]_{\alpha\beta}$ equal to sin of S the switch $\alpha\beta$. Where we require two damping out, because of this damping effect is quite less and thus, we define the sliding surfaces $[S]_{\alpha\beta} = e_{(\alpha\beta)} + K_d \dot{e}_{(\alpha\beta)}$.

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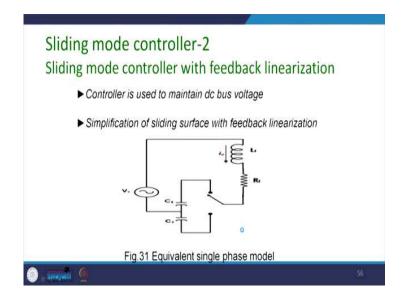
This is a condition of the convergence of the surface. You want that S, \dot{S} should be less than 0 and for α plane as well as the β plane. So, you rewrite this equation again and thus it is a $[\dot{S}]_{\alpha\beta} = \frac{\kappa}{T_r^2} [-[e]]_{(\alpha\beta)}$. Here again you get the sliding surface hence you have taken the K_d. So, it is T_r square this value, then $\alpha\beta$ and thereafter the dc sign. why do you get it there you know? You have seen that the sign.

So, i_1 and i where essentially i_{dc} stands for the i_1 and i_2 in our previous state space modeling. Essentially, we required to ensure that $C_{dc} \frac{dv_{dc}}{dt} = i_{dc}$ should have this condition to satisfy, that you get your surface convergence or the this is a definition of the stability as far the sliding mode control. (Refer Slide Time: 18:00)

Modeling of shunt active power filter for sliding mode control(contd.5) $1 \ge \int_{0}^{\infty} \frac{T_{c}^{2}}{K_{d}} \approx T_{cd}$ $C_{dc} \frac{dv_{dc}}{dt} = i_{dc} \ge \left|-[e]_{(a\beta)} + f_{(a\beta)}\right| \qquad \frac{1}{T_{c}^{2}} \ge 0$ $f_{a\beta} = \frac{1}{T_{c}^{2}}([i_{c}^{*}]_{a\beta} - [i_{c}]_{a\beta})$ $C_{dc} \frac{dv_{dc}}{dt} = i_{dc} \ge \left|[f]_{(a\beta)}\right|$

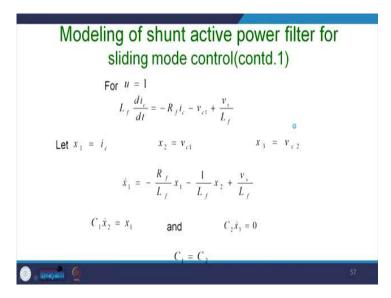
So, since as you can consider that it is much less than 1 and we can say that this value is T_{rd} and from there we shall say that $C_{dc} \frac{dv_{dc}}{dt} = i_{dc}$, greater than this condition and this is our condition $\frac{1}{T_r}$ is a much greater than 0, because you can see that it is under root of L_f by C_f .

Thus, it is a capacitor which is in microfarad and inductor is in millihenry, so T_r^2 gives you a quite big value. So, if $\alpha\beta$ by T_r square should be a converging case, because this value is 0 and thus $C_{dc} \frac{dv_{dc}}{dt} = i_{dc} > |[f]_{(\alpha\beta)}|.$ (Refer Slide Time: 19:10)



Then there is another controller that is tracking current. Another controller was to track maintain the dc bus voltage and we wanted to have a linearized sliding surface with the feedback linearization.

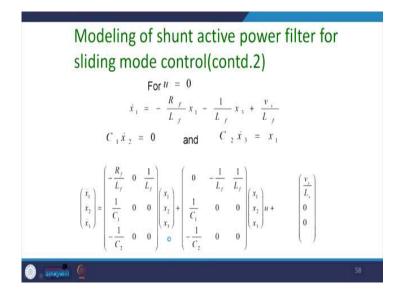
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So, we again rewrite this state space model or the differentiation $L_f \frac{di_c}{dt} = -R_f i_c - v_{c1} + \frac{v_s}{L_f}$. Let us consider state as i_c and v_{c1} and v_{c2} . So, $\dot{x}_1 = -\frac{R_f}{L_f} x_1 - \frac{1}{L_f} x_2 + \frac{v_s}{L_f}$. Similarly,

 $C_1 \dot{x}_2 = x_1$. That is C d v/d t called i and $C_2 \dot{x}_3 = 0$. And we assume that $C_1 = C_2$ definitely the value of the capacitor should be same.

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From there you got a same transformation matrix with the u sign. What you get it there? Ok? From there we required to do this sliding surface analysis. So, it is nothing new in this part here.

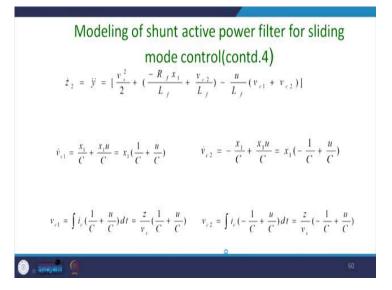
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Modeling of shunt active power filter for
sliding mode control(contd.3)
$$\frac{d(\frac{1}{2}C[v_c^2]_{1,2} + \frac{1}{2}L_f i_c^2)}{dt} = v_s i_c - i_c^2 R_f$$
$$y(t) = \int v_s i_c dt \qquad \dot{y}(t) = v_s i_c$$
$$\dot{y}(t) = \dot{v}_s i_c + v_s \dot{i}_c = [(-\frac{R_f x_1}{L_f} + \frac{v_{c2}}{L_f} - \frac{uv_{c1}}{L_f} - \frac{uv_{c2}}{L_f}) + \frac{v_s}{L_f}] + \dot{v}_s i_c$$
The output function Y obtain the degree r=2
$$z = (z_1 z_2)^T = (y, \dot{y})$$

Again, you just write in terms of the energy. Then it becomes power $\frac{d(\frac{1}{2}C[v_c^2]_{1,2} + \frac{1}{2}L_f i_c^2)}{dt}$. So, that is the power essentially supplied or exchanged that should be equal to the real power supplied by the source that is v_s into i_c minus the loss.

Again, let us consider that this power integration is energy, because you are considering the energy as y. Thus, double differentiation of the power, rate of change of power is essentially $\dot{v}_s i_c + \dot{i}_c v_s$ and thus you differentiate it. We require the double differentiation. There you had \ddot{e} , here for this reason you got an energy double differentiation. The output Y has even a degree 2. So, we can say that $z = (z_1 z_2)^T = (y, \dot{y})$.

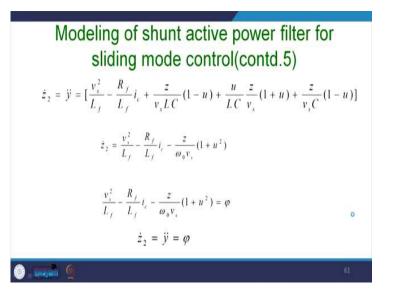
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So, from there we can write. This is for the sake of transformation. We just say that $z_2 = \dot{y}$. So, that is $\left[\frac{v_s^2}{2}\left(\frac{-R_f x_1}{L_f} + \frac{v_{c2}}{L_f}\right) - \frac{u}{L_f}(v_{c1} + v_{c2})\right]$. Similarly, you have v_{c1} equal to x_1/C plus this is input signal x_1/C equal to you can combine.

Similarly, for the other capacitor. Differentiation of the capacitor essentially if you multiply with the C. This is basically the current sinking or giving out from the capacitor that is $\dot{v}_{c2} = -\frac{x_1}{c} + \frac{x_1u}{c} = x_1(-\frac{1}{c} + \frac{u}{c})$. Thus, $v_{c1} = \int i_c(\frac{1}{c} + \frac{u}{c})dt = \frac{z}{v_s}(\frac{1}{c} + \frac{u}{c})$. So, $v_{c2} = \int i_c(-\frac{1}{c} + \frac{u}{c})dt = \frac{z}{v_s}(-\frac{1}{c} + \frac{u}{c})$.

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So, expand it further to $\dot{z} = \ddot{y}$ and thus you have v_s^2 by L_f minus R_f by L_f ic plus z by v_s LC and if you combine you know from there this terminology, you essentially get some switching function that is non-linear. So, how to get rid of it? Then what we do generally? When you got a non-linear system, you linearize in a particular neighborhood and let us assume that this equation is equal to some constant value ϕ . So, $\dot{z}_2 = \ddot{y} = \phi$.

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Modeling of shunt active power filter for
sliding mode control(contd.6)

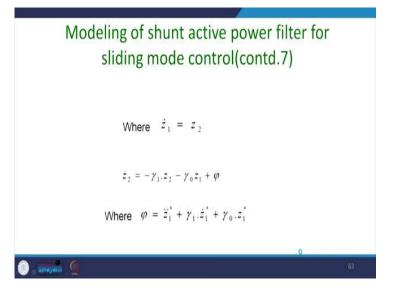
$$(\ddot{z}_1 + \gamma_1.\dot{z}_1 + \gamma_0.z_1) - (\ddot{z}_1^* + \gamma_1.\dot{z}_1^* + \gamma_0.z_1^*) = 0$$

 $\dot{s} = (\ddot{z}_1 + \gamma_1.\dot{z}_1 + \gamma_0.z_1) - (\ddot{z}_1^* + \gamma_1.\dot{z}_1^* + \gamma_0.z_1^*) = 0$
 $\dot{s} s < 0$
 $s = (\dot{z}_1 - \dot{z}_1^*) + \gamma_1.(\dot{z}_1 - \dot{z}_1^*) + \gamma_0 \int (z_1 - z_1^*) d\tau$
 $u_{eq} = \frac{\omega_0}{z_1} [(\gamma_1 + \frac{\dot{v}_s}{v}).z_2 + \gamma_0.z_1 + \frac{v_s^2}{L} - \dot{z}_1^* - \gamma_1.z_1^* - \gamma_0.z_1^*)$

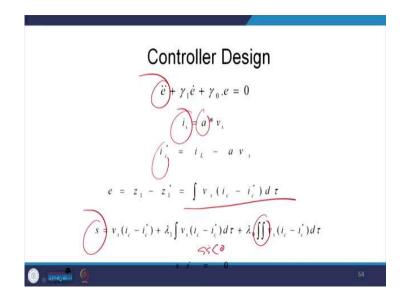
Then you rewrite this equation and you linearize it. Because of that considering it has a value it is \dot{S} dot and S_1 , γ_1 , z_1 these are the eigenvalues of it and ultimately u equivalent.

You can see these quantities with the linear entity. So, it is ω_0 , where ω_0 is the natural frequency of the system. Hence, $u_{eq} = \frac{\omega_0}{z_1} \left[\left(\gamma_1 + \frac{v_s}{\dot{v}_s} \right) \cdot z_2 + \gamma_0 z_1 + \frac{v_s^2}{L} - \dot{z}_1^* - \gamma_1 \cdot \dot{z}_1^* - \gamma_0 \cdot \dot{z}_1^* \right]$.

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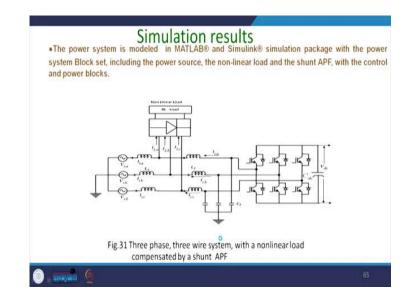


So, you just try to derive this equation. It can be followed and this paper has been published in 2016. So, you can refer to this paper. It is published by one of my B Tech student Anirban Sinha and myself. We have done this job. So, $\dot{z}_1 = z_2$ detail are given in the paper. So, $\dot{z}_2 = -\gamma_1 z_2 - \gamma_0 z_1 + \phi$ and thus, you got a quadratic equation of the z_1 . So, where it has to be linearized at this entity. So, it is $\ddot{z}_1^* + \gamma_1 \cdot \dot{z}_1^* + \gamma_0 \cdot \dot{z}_1^*$. (Refer Slide Time: 26:15)



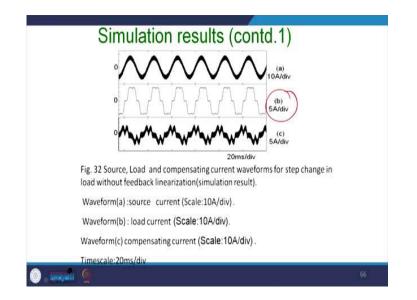
So, you say that this is an error and that should be equal to 0 and in this case, you know that 'is' should be some constant into vs, because it has to be in a same phase. So, ic should be $i_L - av_s$. So, e equal to z_1 minus z_1 prime. So, you have this equation and thus your sliding surface become your $v_s(i_c - i_c^*) + \lambda_1 \int v_s(i_c - i_c^*) d\tau + \lambda_0 \iint v_s(i_c - i_c^*) d\tau$ and you should have a condition $ss \doteq 0$.

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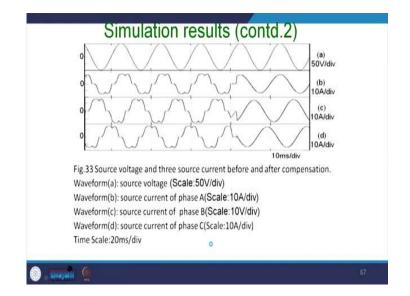
So, based on that we required to design it. So, let us see the simulation result, then only we can appreciate it. How does it work?

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So, see that due to the chattering, you know there is a lot of chattering, this is your load current and this is a source current. Even though it is sinusoidal, these are a huge chattering. This means we have not designed the surface properly. Even though it is tracking, but there is a lot of switching losses.

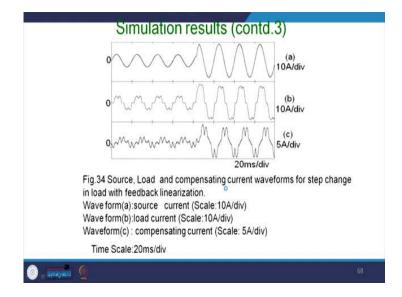
So, we required to also minimize that switching losses by this. If you do not properly design this value here, then this chattering will come. So, you have to linearize this equation in a close neighborhood.



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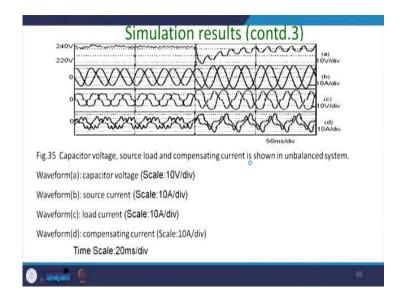
Thereafter you can see that, here you have started. This is upper one is a voltage and lower is the source current. Here the source current after compensation and it does not have much chattering.

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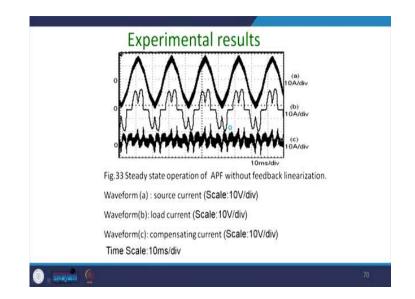


This is the load change. This is the source current and I have not shown the other phases, but it is a balance to unbalance changeover. But not only the load change, but also it changes with the unbalance phase.

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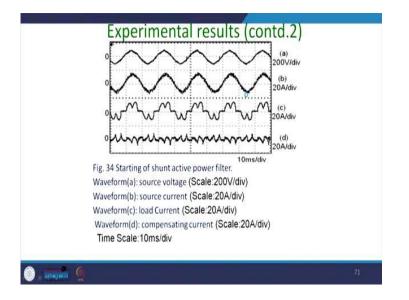
This is the condition you can see that. The dc bus voltage is maintained there and all of a sudden you made this which was balanced. So, ultimately you had a ripple. That ripple was essentially 6th harmonic, so 6-pulse converter and since you have unbalanced kind of load and thus what happened? Your dc bus voltage will have this kind of double frequency ripple essentially and this is your compensating current and you see that system is quite stable.



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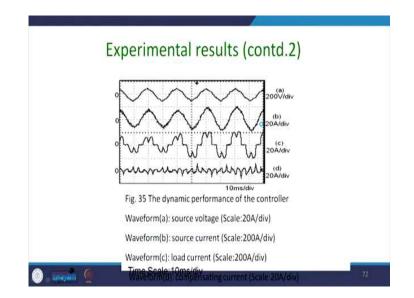
I show you the same results without the feedback linearization, where intentionally the value has not been linearized and chattering and the sliding surface was not meticulously designed. Even though it tracks, but this is an experimental result and you can see that there is a huge chattering. So, that is quite disadvantageous for the operation and ultimately you have to do it either you have to put a capacitor to suppress this. But if you properly design it, this chattering will go out.

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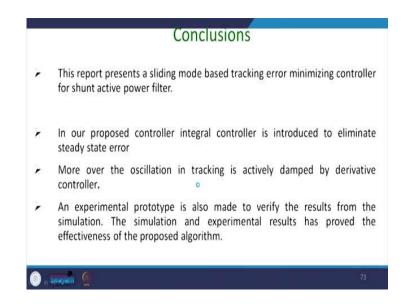
This is the source voltage and this is a source current, you can see that chattering is quite less. This is a steady state operation.

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Thereafter this is a load change, same thing. There is a load change and this is a voltage. This is a source current. That you can see that, if you zoom it out this part, you can see that there is a consistent stretch chattering. This is a compensating current. So, compensating current I have taken across this inductor, so chattering is little less visible there. But this chattering is also there, because the source was the compensating current.

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So, this report presents a sliding mode-based tracking for the error minimization of the controller for the shunt active power filter. In our proposed controller, integral controller is introduced to eliminate the steady state error. That essentially does the chop of the PI controller. Moreover, the oscillations in the tracking is actively dammed by the derivative controller.

If you can go back here in the first part of this, you have designed the sliding surface choosing. There is a two-controller mind it. One controller was, one controller for tracking current and here we have designed this value as your sliding surface as this K_d. So that will damp out the oscillation, because you have neglected the damping part of it.

For the designing the another sliding mode controller. That is essentially maintaining the dc bus voltage, that part we required to have a feedback linearization. We did the feedback linearization's and we can achieve the desired results. So, as an experimental prototype also we have made and verified the simulation and the experimental results proves the validity of this sliding mode concepts and its application to the shunt active power filter.

Thank you for attention. This is one of the examples of the higher non-linear control application of the sliding mode control. You can apply any other control technique. So, ultimately this shunt active power filter can be approached from the different background and some person is from the control background, still there are many problems arises. How, genuinely you design the sliding surface is a very big challenge.

For this reason, there is a also scope of improvement while the tracking performance improvement as well as stability. So, if you have made the system stable, of course, you can reduce the cost of the component and the size of the component.

Thank you for your attention. I shall continue your discussions with the topological aspect of the shunt active power filter in our next class.