Microelectronics: Devices to Circuits
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Lecture 08
BJT: Small Signal Circuit Model-I

Welcome to the NPTEL online certification course on Microelectronics: Devices and Circuits. This module we will be taking care of a Small Signal Circuit Model for BJT. The obvious question asked is, why should we require this? Well, BJT as a stand-alone device has got no usage or application, so this has to be integrated onto a circuit for example BJT as an amplifier, BJT as a switch, BJT as a current source and so on and so forth.

Therefore, if you have understood how BJT works which we have already done in our previous interactions, we will understand or we will together learn how this BJT the device can be broken down into sets of active and passive elements, so that it is very easily integrated with the circuit analysis. And it is easier with circuits and it is easy to do the analysis in circuits in a much detail manner.

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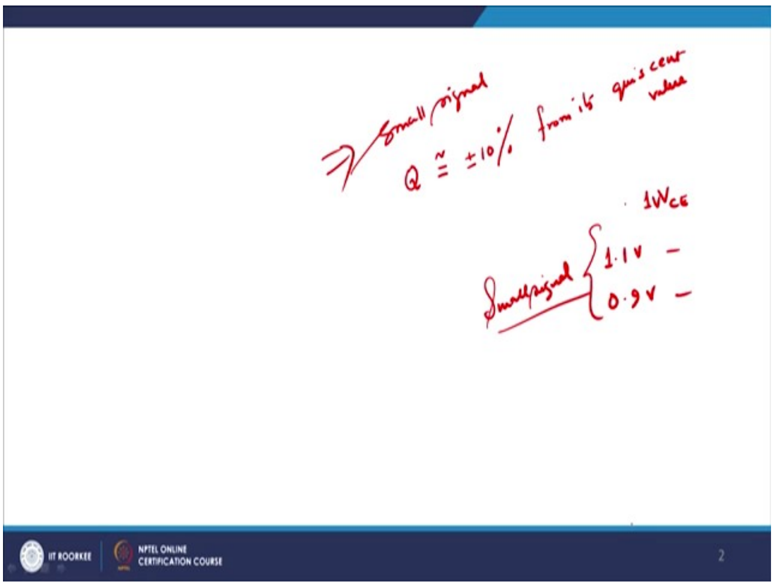

Why small signal? We will discuss why small signal now. What is the meaning of small signal? Let me first make it clear to you. A small signal primarily means that the Q-point does excursion of approximately plus minus 10 percent from its quiescent value, right? So if you have biased your device then we define this to be as a small signal, right? So if you have biased your device say at 1 volt, V_{CE}, right? 1 volt then if the input signal changes it from

1.1 volt to 0.9 volt peak-to-peak, I would expect to see that, we can define this to be as a small signal.

This is a rather not a very decent definition but nonetheless works fine for all circuit applications assuming that this much amount of variation in the voltage, input voltage will not force the device to go into either the saturation or cut-off region. So we have biased Q somewhere in the middle of active region, a 10 percent high rise or fall will result in approximately a linear change in the value of your amplification, right?

So you are still in the linear region of amplification as far as Q point is concerned. So two things one is a small signal circuit model primarily means that I am assuming that my input signal is basically small signal which means that is just a plus minus 10 percent of quiescent value and the second point which I am assuming is that there is no nonlinearity into the picture by virtue of a large signal, right?

So and the third point is that obviously we require to have a corresponding circuit elements being inserted, so that whenever we plug those BJT and use it as a switch or any device we can easily do that by doing small amount of manipulation in terms of circuit application. So let's see what the issues are.

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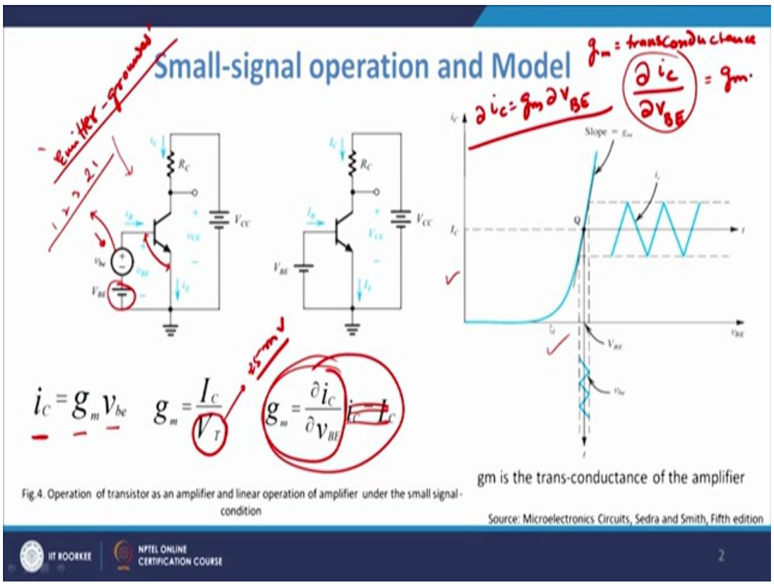

So let me come to that point in a detailed manner. I will take one by one as we move along. If you see clearly again this is, as you can see this circuit is basically a BJT in an emitter grounded, so this is basically your emitter grounded configuration or even a common emitter configuration, emitter grounded configuration or common configuration. And what you see from here is that V_{BE} is the applied AC bias which you are giving and this V_{BE} is responsible for biasing it in the appropriate Q-point.

We define therefore I_C, the output current to be equals to g_m times V_{BE}. So if you understand V_{BE} is the voltage which you have applied here, right? And this V_{BE} is varying voltage because that is what the input voltage is all about. So one time it is 1 volt, 2 volt, 3 volt then again comes to 2, 1, so on and so forth. So it is a rising and falling maybe a sine wave maybe a pulse waveform or maybe any other wave forms available to you.

And therefore this base to emitter voltage which you give that converts into a corresponding current on the collector side, how? By mathematically multiplying with a value which is g_m. g_m is referred to as a transconductance, right? And it is defined as, in this case as $\partial_{i_c}, \partial_{V_{BE}}$, right? So it is defined as rate of change of collector current with the base of voltage. So once you get this to be as g_m, right?

I simply get ∂_{i_c} to be equal to g_m times ∂V_{BE}, so higher the value of g_m of a transistor, I would expect to see a larger amplification appearing to me, right? Because higher the value of g_m will primarily mean that the transistor is more sensitive to variations in the input voltage, right? So g_m is basically a notation or a parameter which gives me how sensitive your BJT is all about.

So if your BJT is quite sensitive, I would expect to see a large value of transconductance available to me. So I define $g_m = \frac{i_c}{V_{BE}}$. So if you see it is i_c by V_{BE} and this is replaced by V_T for the lowest case because this is thermal energy which you get which is 25 mVs at 300 K, right? You can also write down, therefore $g_m = \frac{\partial i_c}{\partial V_{BE}}$ for the fixed value of I_C, right?

So at a fixed value of I_C what is the change in this thing? But you need not forget about all these things, this you just forget. g_m is basically ∂i_c, ∂V_{BE} which you see, right? Now if you plot I_C vs V_{BE}, this versus this, let's see why and how we get I_C vs V_{BE} in this graph.

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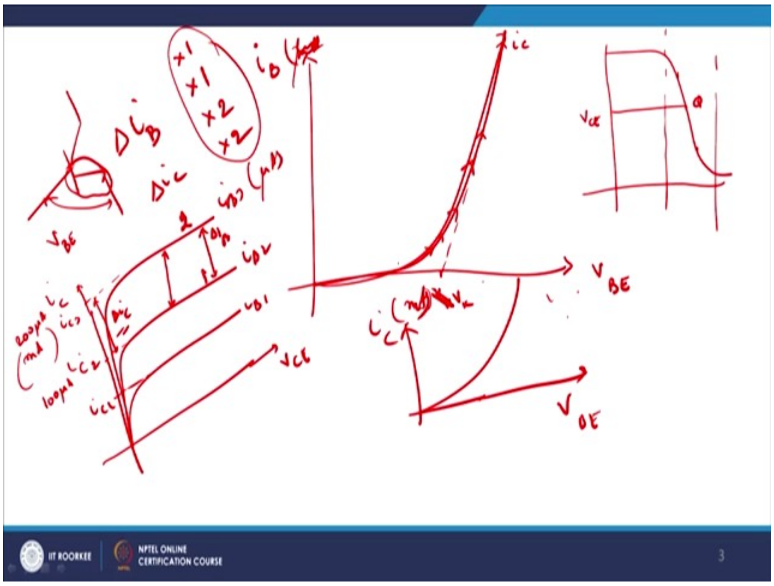

So for common emitter configuration if you plot say V_{BE}, right? And you plot say, maybe an i_B, let us suppose, right? Let us see how you plot i_B. If you plot i_B it will look something like this, right? We have discussed this in our previous discussion, this is the threshold voltage or the knee voltage for the PN junction diode which is the 0.7, right? So for i_B if you have a common emitter configuration in this case then, this is the value of V_{BE}, right?

And when this V_{BE} crosses 0.7, right? This fire this junction, switches on this PN junction and as a result the current starts to rise, right? Below that particular point the current is very -very low or very small and drops very drastically. So i_B with V_{BE} is in this manner. We have also seen from our previous discussion that if you bias your device in such a manner that your Q-point is here, so this is your active region.

If you bias your device in the Q-point here V_{CE} let us suppose, right? Then I can safely say that for the same change in the value of i_B, I would expect to see same change in the value of i_C. So remember when we are plotting the graphs, right? For various values of i_B when we are plotting I_C vs V_{CE}, right? The common emitter configuration, for various values of i_B.

So it is i_{B1}, i_{B2}, i_{B3}, so you see this is i_{B1}, this is i_{C1}, this is i_{B2}, i_{C2}, this is i_{B3} , i_{C3}, so this i_B is of the order of few µA and this is of the order of few mA, so thousand times increases the approximately between base current….. but..but please understand that for the same increase, so if this is say i_{B2} is say 1 micron, this is 2 micron let us suppose, then this current will be actually if suppose 100 times larger, so I get 100 micrometer(9.31 mins) (µA) here or 0.1 mA and I get 200 µA.

So you see for the same change in ∂i_B, right? I get the same change in ∂i_C, so ∂i_B, right? And ∂i_C if you want to find out the rate of change is always the same. So for the same increment in ∂i_B, I get equal increments in ∂i_C, right? The increment themselves might be different obviously 100 times larger so quantitatively they are different but I am 100 percent sure that if I increase the base current by once my collector current will also increase by once.

If I increase the base current twice my collector current will also increase by twice, why? Because I am in the linear region of operations, right? And therefore I wanted to tell you is, that this is i_B vs V_{BE} exactly the same graph is for i_c vs V_{CE}. So if you plot i_C vs V_{BE}, you will get the same curve again, right? Only thing will be that this i_C will be mA which was here it was in µA or may be mA as well but this will be more on mA, right?

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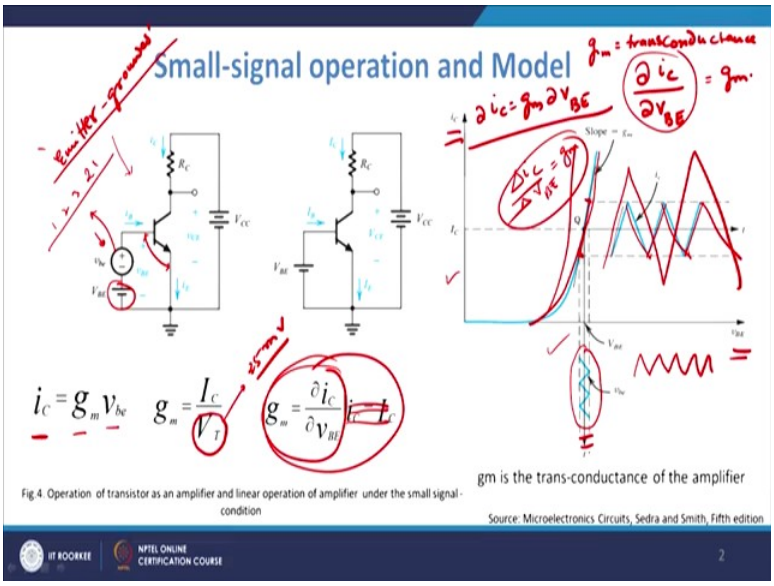

So let me show you therefore if you plot i_C vs V_{BE} here and then in input side if you vary this is your input curve V_{BE} is varying, it is a triangular waveform which you have given, this triangular waveform again small signal assuming so that, the Q is just shifting from this side and coming to this side. So the maximum value is Q is this and this. So we are still in the linear region of operation of the transistor. I get this to be the output, right? Triangular output waveform will be there, right?

Now, the slope of this curve which you see, this curve, the slope of this curve, sorry, the slope of this curve, right? Which is $\partial i_C, \partial V_{BE}$, this is basically your g_m transconductance. So if this slope is not steeper, I would expect to see a much larger variation in the output waveform, for the same change in the value of input waveform, fine. So that’s the small signal model and in the small signal model therefore we have learned one important point and that important characteristics is transconductance (g_m) and g_m is therefore defined as the transconductance which in this case happens to be $\partial i_C, \partial V_{BE}$, right?

And it gives me an idea about sensitivity. So if I know the base to emitter voltage I can predict the value of the collector current directly by multiplying g_m with the rate of V_{BE}, right? And that's quite an interesting phenomena which is there with us.

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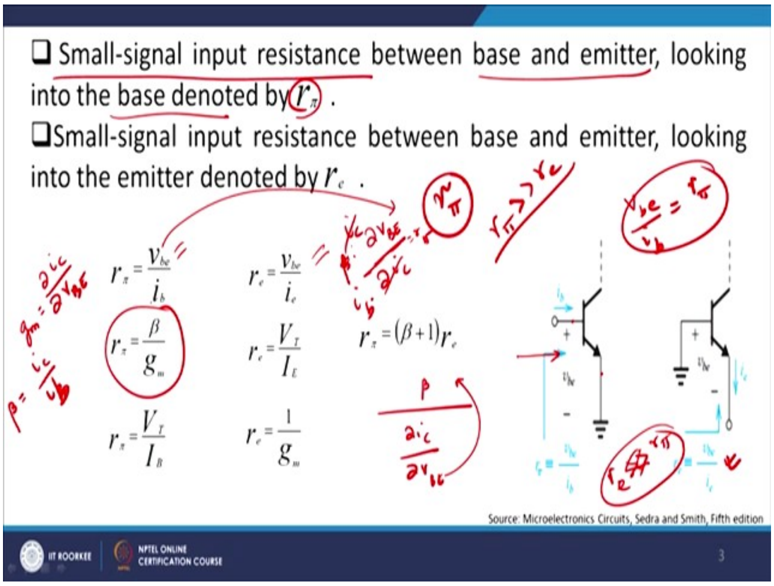

And now we define another resistance known as the small signal input resistance and it is referred to as r_π, right? And it is nothing but the resistance looking from the base side where between base and emitter, right? So looking from the base side, if you're looking from the base side the resistance offered by this is basically V_{be} by i_B. So V_{be} is the voltage which you see between these two points and i_B is the current by virtue of this voltage.

So V_{be} by i_B is equals to r_π, right? So it is basically the forward biased on resistance of a BJT, right? It is an on resistance which means that the base emitter junction is forward biased it is getting on and therefore the resistance offered is defined as r_π, right? Sorry! But plus understand, this you are looking from the base side, right? So you are looking from this base side and you are trying to find out.

So your emitter is grounded, please understand. Another is r_e which is basically the base to emitter resistance looking from emitter side. So you give a potential on the emitter side and you ground the base side and then looking at that, you will get the value of r_e to be equals to

V_{be} by i_e, right? So it is V_{be} by i_e, right? Obviously as you can find out i_e is much-much larger as compared to i_B. So I would expect to see r_e to be much-much smaller as compared to r_π.

So r_π will be relatively larger as compared to r_e, fine. So I get r_π is equals to therefore V_{be} by i_B this is also equals to V_{be} by r_e, right? So r_π can be written as, so what we can write down is that, it is equal to β/g_m, right? How β/g_m we will just show it to you. If you remember g_m, what was its definition? It was, $\frac{\partial i_c}{\partial V_{BE}}$, right? So if you put $\frac{\partial i_c}{\partial V_{BE}}$ here, so I get what?

So I get β divided by ∂i_c by ∂V_{BE}, so this V_{BE} goes to the numerator I get ∂V_{BE} at the top, right? Divided by into β divided by ∂i_c, this you get, right? This you get. So this V_{be} is coming from here, so now you see remember the definition of β from a previous discussion, β was equal to your i_c by i_b, remember. So if you put i_c by i_b here I get i_c, so I replaceβ and make it i_c by i_b, right?

This i_c, i_c gets cancelled out I get V_{BE} by i_b, so this is equals to r_π. So r_π equals to β by g_m also we can write.

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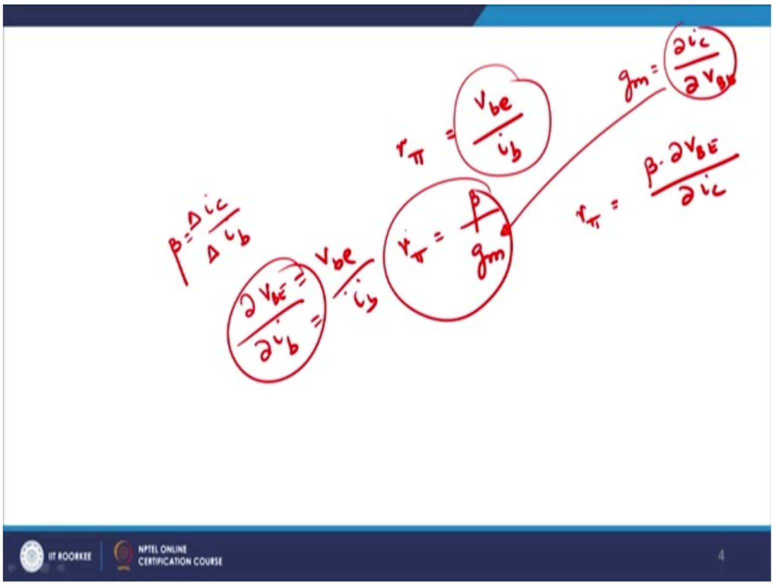

So I will do it once again if you want to do. r_π is given as V_{be} by i_b, remember. So we also know r_π has been also given as beta by g_m. We know g_m to be equals to what? g_m to be equals to ∂i_c, ∂V_{BE}. So if you put the value of g_m I get r_π. If you at place this equation here, I get β times ∂V_{BE} divided by ∂i_c, right? Now we know the definition of β as the current gain in common base configuration and therefore I can write β to be equals to i_c by i_b.

So ∂i_c, ∂i_b, let me write down. Place this value of β here and therefore this ∂i_c and ∂i_c gets cancelled out, I am left with ∂V_{BE} upon ∂i_b, right? And that's what you are getting here, fine. Since it is a small signal therefore ∂V_{BE} equals to V_{be} and ∂i_b equals to i_b and therefore this is valid assumption which we get. So I get β by g_m as the r_π value, fine. So β is relatively very high value and therefore r_π will be very-very high, so this is what we get from our discussion as far as this is there.

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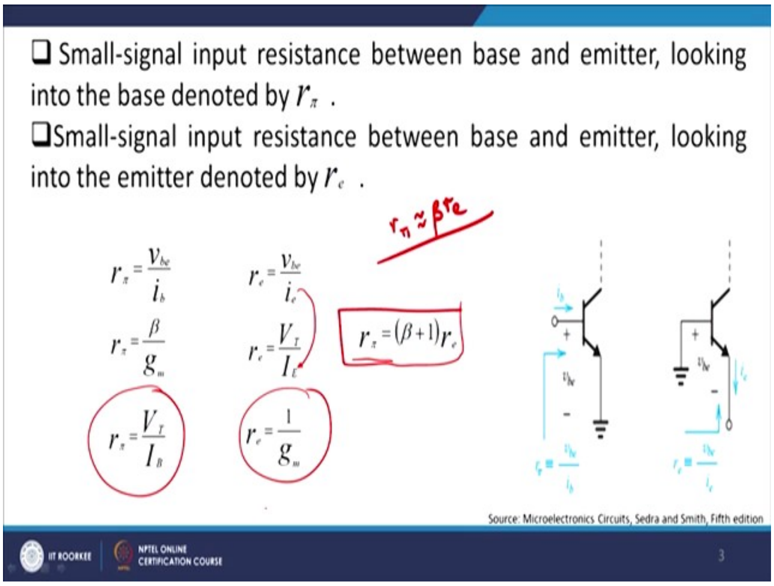

Similarly r_e therefore can be written as i_e is the emitter current and V_T is the base to emitter voltage at 300 mV we will write it as V_T thermal equivalent voltage. Therefore I can write down that r_π could be written as β plus 1 r_e, right? And you can do it yourself to find out this value that means r_π is approximately β times larger as compared to r_e, right?

So…So… Which means that, and it is expected also, if you look very carefully, the reason being, the base is, so looking from the base side the resistance is very large as compared to looking from the emitter side and you can understand why is it? Because base has got relatively low doping, right? And the area is also very small. So in both will result in very - very high resistivity. Resistance will be typically very high and that's the reason r_π is much larger than beta re approximately 100 to 200 times larger, right? And that the reason why you get it.

Therefore r_π can also be written as V_T by I_B in case you want to do it and r_e can be written as 1 by g_m as well, as you mean β is approximately equals to 1, I can get to be approximately these. These approximations can be taken care of or it can be handled easily. So we have understood therefore that for small signal model if you're looking at the base side, emitter side resistance is not equal from the base at resistance is much higher as compared to that from the emitter side.

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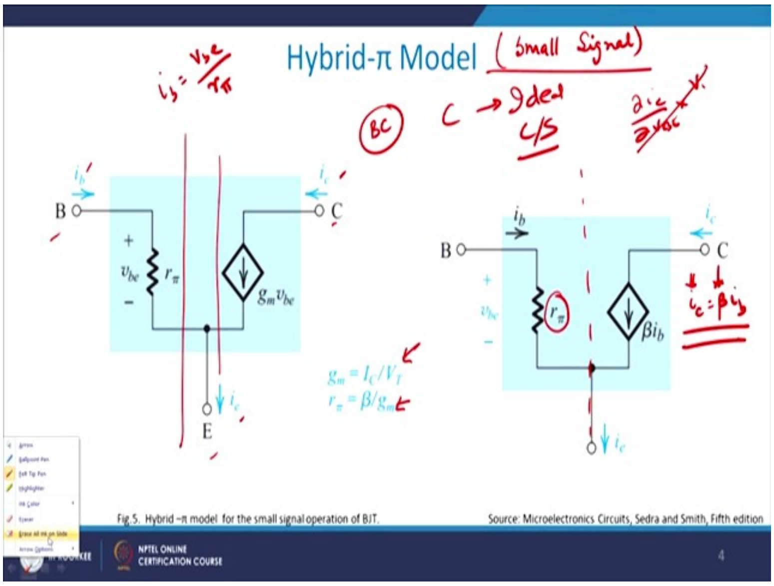

We come to the models, various models of BJT which is available. Small signal model is, these are again small signal models, these are all small signal model which is available, right? And they are quite interesting and very easy to handle from circuit point of view once we know the basic fundamental principles of working of these devices. Look at, so we have already learned that BJT is basically a three terminal device.

That everybody knows that BJT is obviously a three terminal device, so what are those three terminals? Base, collector and emitter, right? So the arrow head which you see, the blue arrow here, the blue arrow here, the blue arrow here shows you the direction of the conventional current, right? And therefore we say that looking from the base side the amount of resistance offered is r_π and across it we have a V_{BE} as the voltage drop available to me.

So therefore i_b can be directly written as V_{BE} divided by r_π, right? V_{BE} is always fixed approximately 0.7 is supposed the approximately voltage. If I give you therefore the r_π value you can predict the value of i_b directly, right? Now this is therefore the base emitter junction, right? Let's look at the base collector junction. We have already discussed that base collector junction, the collector starts to behave like an ideal current source, right?

So it gives us ideal current source. How much is the value of current? Will be given by g_m times V_{BE}. Remember g_m was equal to ∂i_c, ∂V_{BE}. So ∂i_c, ∂V_{BE} multiplied by V_{BE}, so V_{BE} gets cancelled out and I get the current available to me, right? So this is the current source with me and this is the direction, the direction of the arrowhead of the current tells me that the current is directed inwards which means, it is basically an NPN transistor and therefore it is directed inwards and therefore you see this directed inwards whereas emitted current is directed outwards, right?

So by the previous definition we know that gm is equal to I_c by V_T which is available here and this is β by g_m is r_π, right? So it's completely analogous to see on the right-hand side, this one, right? It is also a hybrid model, hybrid-π model. Now, it looks like π, so that is the reason it is hybrid-π model. Moreover it is made up of r_π and therefore it is a π model. So you remember, r_π is the resistance offered by the base side, base emitter junction and therefore we have r_π as the resistance here.

Whereas on the collector side it is the current source, wherein current source i_c will be equal to β times i_b, right? So if you have a large value of i_b, I will get a large value of i_c and I can therefore have a constant current source towards the collector side, right? Therefore this model will give you a very first instance the input impedance and output impedance of the device, right?

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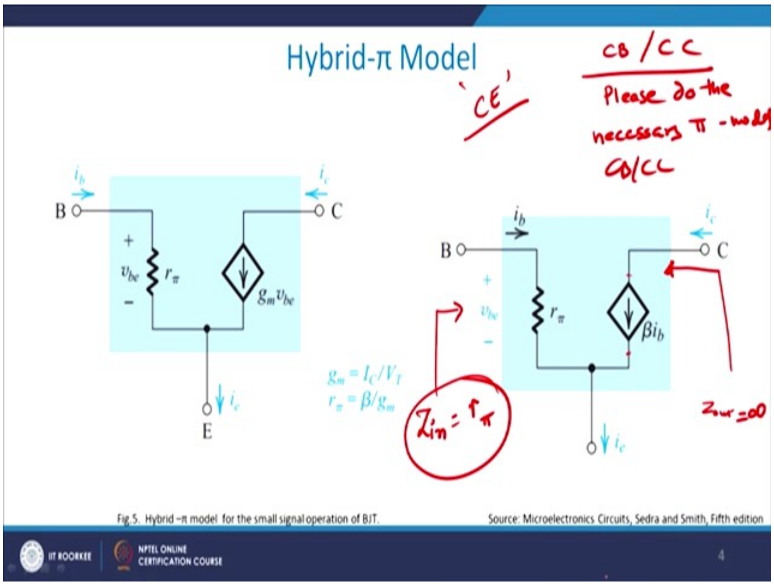

So if you look at the BJT therefore the output impedance is basically mine, Z_{out} if you want to find out, is looking from the collector side this is the output. How much amount of resistance is offered? Of course you can see, from the output side if you look very carefully it is being terminated across a constant current source and therefore Z_{out} will be approximately equal to infinity in idealized condition.

So if you want to follow a hybridized pi model, then Z_{out} will be equal to infinity and Z_{out} will be equal to infinity primarily meaning, the reason being that, we are looking into the current source here, an ideal current source, right? And assuming it to be an ideal current source, we are assuming that the Z_{out} will be the infinity. So whereas looking from the input side Z_{in}, input impedance will be approximately equals to r_{π}, fine.

So this is for the common emitter configuration, right? I leave as an exercise for you to find out common base and common collector, please do it, right? Please do the necessary, right? π model for *CB* and *CC*, common base and common collector, right? In common base, you will see it will be re which will be coming into picture as the input impedance, right? Therefore just be able to handle this hybrid-π model in a much better manner in this case, right? So this is one model.

The second model which is most prevalent but most of the time we use of hybrid-π model for most practical purposes but sometimes we also use what is known as a T model, right? This is a transmission model or T model which you see, this is the T model which is there.

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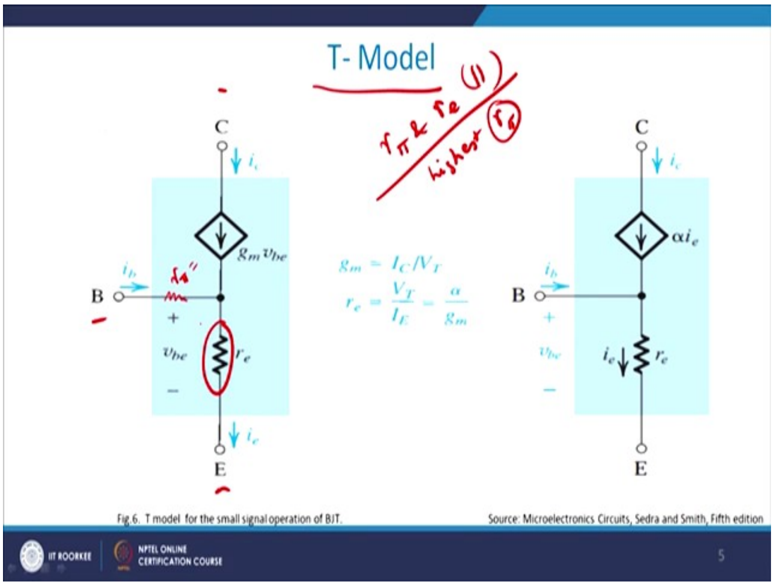

And the T model, still again three terminal device so base, collector, and emitter. We have also seen that the resistance between emitter and base is r_e looking from emitter side. Looking from the base side, it would have been here r_{π} but since we are looking from the emitter side at this case we are giving r_e, you can also have r_π here but they will be in parallel. So being parallel it will be always less than the least, remember? And therefore this has been neglected. So you see r_π and r_e will be in parallel. Since they will be in parallel the maximum, the highest value will get neglected. So highest is nothing but r_π and therefore r_π is not shown in this diagram, right?

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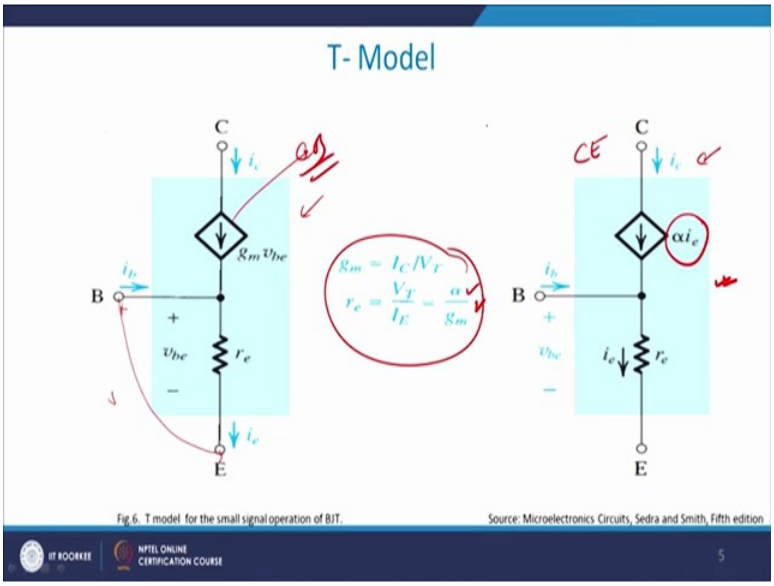

So therefore understand that why r_π is not here or here or here, because it is so large that it does not play a major role, r_e plays a major role. So I get r_e here, the voltage across r_e is basically my V_{BE}, base to emitter voltage and this is the applied voltage which you see the current direction is of course, the conventional current direction as we have been discussing quite often.

And therefore, the collector current will be equal to g_m times v_{be}, right? We have again discussed this point as the input point. So... So… you see, what I'm trying to tell you, is your base emitter junction or your base emitter voltage, right? v_{be} starts to change the value of your current flowing through collector, right? And that is basically therefore a control current source with you, right?

So basically, it is a current control current source because i_b will change the value of i_c and therefore it is basically known as the current control current source. So I get from here that it is α by g_m is the value of r_e which we get. α by g_m you can prove it yourself to get the value of r_e. α is basically the common emitter current gain, right? Common emitter current gain will mean that you will have r_e approximately just to α by g_m, right?

And you can get the value of α and g_m, it is typically very low because, α will be hardly of the order of 0.99 or 0.98 and this g_m will be typically large because sensitivity is very large, so your r_e will be typically very small quantity. So your resistance looking from the emitter side will be very small, right? And that makes it go to a very small state or a very small region.

And therefore as you can see on the right-hand side if you look very carefully from the common base configuration I get r_e I get α times i_e as the collector current and therefore, for a common emitter configuration, so this is common emitter consideration *CE* and this is for the *CB* configuration and therefore I can get the value of this thing. So I will recommend that for both hybrid-π model as well as, T model for all the three configuration *CB*, *CC* and *CE* which means common base, common collector and common emitter please try to find out the value and the model itself please try to get it from any standard books which are available in the market, right? Right?

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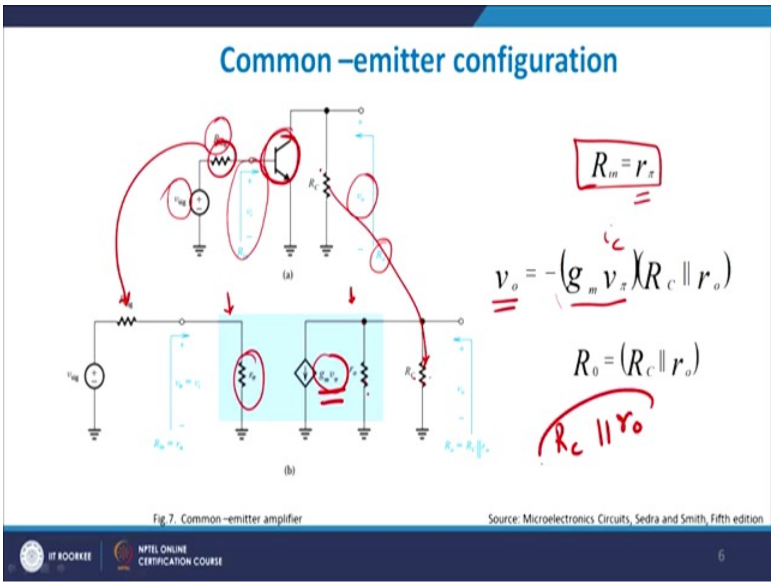

Let me therefore now come to the basic, we will finish with this common emitter configuration. Let me come to the common emitter configuration now and explain to you what common emitter configuration how it works, as far as, how we are inserting the small signal model in a common emitter configuration, right? So this is the application of the small signal model, right? So what is the application?

The application is something like this that I have a signal input resistance, I have an input signal, I have BJT which is there, R_C is the resistance, so if you look r_n is basically, which is this one, is the input resistance looking from the base side and output resistance looking from the collector side is R_o. So what we are doing, we will replace this. So if you look out of the blue box.

This blue box, out if you look, this R_C is this R_C, so this R_C is this R_C, right? This R_{sig} which you see is this R_{sig}, right? Signal. So looking from this side this can be broken down into an r_π, right? Because r_π is already known to you and this will be g_m times r_π, right? And R_o will be the output impedance offered by the device which is there for the device itself, not for the circuit point of view.

So therefore I get R_0 always parallel to my current source here, right? Howsoever large or small we don't know but it will be the value which will be there R_0 here. So we get R_{in} to be equals to r_π, no need to understand why is it, very simple, because looking from this side from the base side I only get r_π as my impedance, so r_π is the Z_{in} value. V_0 is g_m times V_π, V_π is nothing but the voltage across this V_{BE} or V_π whatever you want to name, it is basically the output current.

This multiplied by R_c, so this R_c and R_o in parallel, right? So the affective resistance seen by the current is basically R_c parallel to R_o, this if you multiply with i_c, I will get V_0, right? And I get V_0 here. So R_0 is nothing but R_c parallel to r_0. So the output impedance is basically R_c parallel to r_0, right? So this is the output impedance which you see. So for a common emitter configuration we are able to see the output impedance, right?

Okay, with this knowledge, with this basic knowledge will stop here today and what we have learned is basically the small signal input model for the BJT and in the next turn we will look into how we can apply further for the common base as well as common character configuration design. So we have learned how to bias a device, DC bias, super impose AC signal over it and we have also learned how to use those for the purpose of common emitter configuration using the small signal model, fine. I hope you have understood the issues taken up in this lecture, thank you for your patient hearing.