Microelectronics: Devices to Circuits
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Lecture - 49
Bode Plots and Frequency Response

Hello everybody and welcome again to this NPTEL knowledge certification course and on Microelectronic Devices to Circuits. In our previous module we had looked into the stability criteria from the point of view of poles and zeroes, poles specially, and we saw that the poles should be on the left half plane of the system and the phase margin as I discussed with you in the previous turn, should not exceed 180 degree for the proper stability. And we will discuss that in detail as we move along. So we will actually look into the today's discussion is on bode plots and frequency responses.

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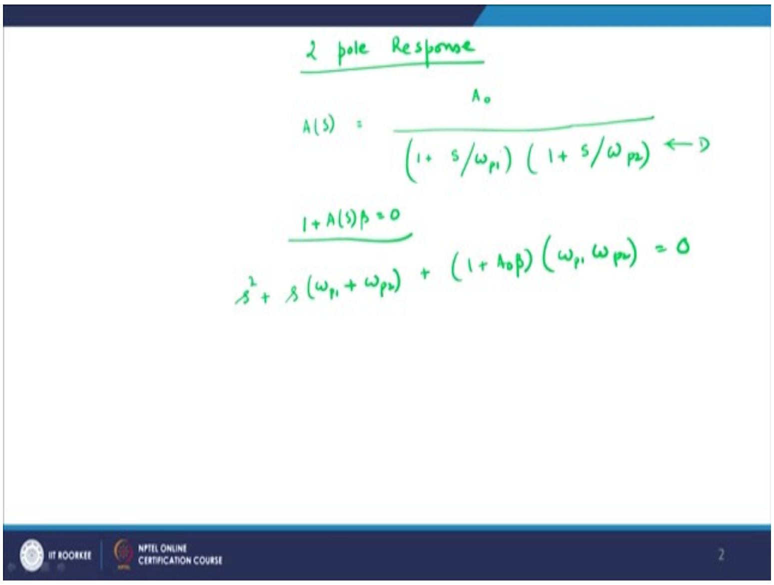

So before I move forward, let me just discuss in detail slightly about the two pole networks. Two pole network primarily means that the 2 pole response actually which we did in our previous turn also, but I just wanted to just to refurbish again the whole thing, in 2 pole response the general your response transfer function looks like this S upon ω_{p1}. Right? And this is 1 plus S upon ω_{p2}.

With this if you solve it, I get 1 plus A(S) times β equals to 0 is the condition when you get a sustained oscillations under such a criteria if you solve if you breakdown this denominator. Right? So the denominator broken up into S square plus S times ω_{p1} plus ω_{p2} plus 1 plus A_0 time β. Right? Into ω_{p1} into ω_{p2} this must be equals to 0.

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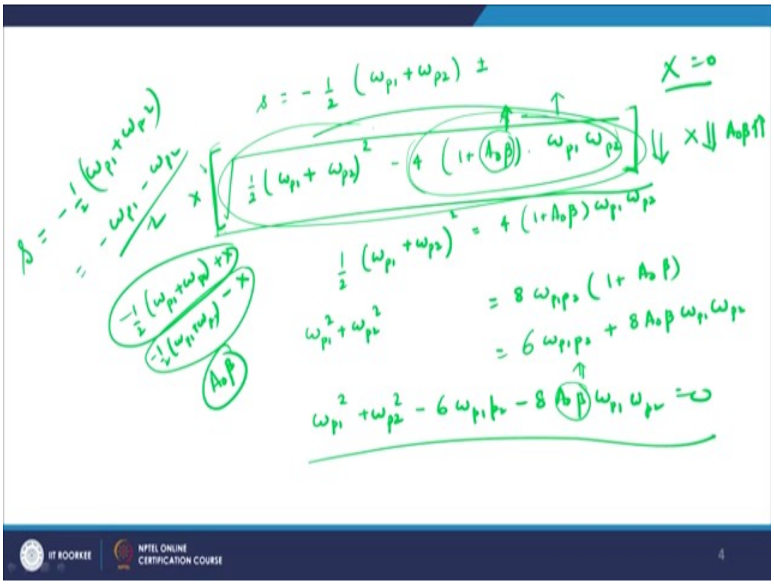

Which effectively means that s will be equal to plus minus. Right? Or sorry s will be equal to minus half ω_{p1} plus ω_{p2}. Right? And then plus minus taking sign of root over 1 by two ω_{p1} plus ω_{p2} whole square minus four times 1 plus A_0 β into ω_{p1} into ω_{p2}. So this are the two roots of the equations, one root is with the positive sign another root will be with respect to the negative sign.

If you look carefully the whole quantity within this root this under root sign can be made to 0 then you only you will naughts have A_0 or β coming in the picture, in that scenario I get 1 by 2 ω_{p1} plus ω_{p2}. Right? Whole Square if that equals to 4 times 1 plus A_0 times β times ω_{p1} ω_{p2} that makes these two sides equal and therefore thus the sustain this to be whole this let us suppose this is X then X will be equal to 0.

If you solve it I get ω_{p1} square plus ω_{p2} square plus 2 ω_{p1} ω_{p2}. Right? Will be equals to 8 times ω_{p1} ω_{p2} into 1 plus A_0 time β. So if you solve it I get 8 times $\omega_{p1}\omega_{p2}$ plus 8 times A_0 times β times ω_{p1} ω_{p2}. Right? So with this so if you look if you see carefully the two sides of the equation then this will cancel with this and make it 6 so this will this will go and this will become 6 times so this will go and therefore what I will get from here is this will becomes and I can vanish this. Right? And I can get I can get 6 ω_{p1} ω_{p2} plus 8 times this thing into ω_{p1} ω_{p2}. Right? And this what I get overall picture for this whole design.

So or in other sense I can wright down this to be as ω_{p1} square plus ω_{p2} square minus 6 ω_{p1} ω_{p2} minus 8 times A_0 β ω_{p1} ω_{p2} must be equal to 0. When you get this into the consideration I get

S equal to minus half of ω_{p1} plus ω_{p2} which is equal to minus ω_{p1} minus ω_{p2} by 2. So which is therefore the left half can plane so it is stable.

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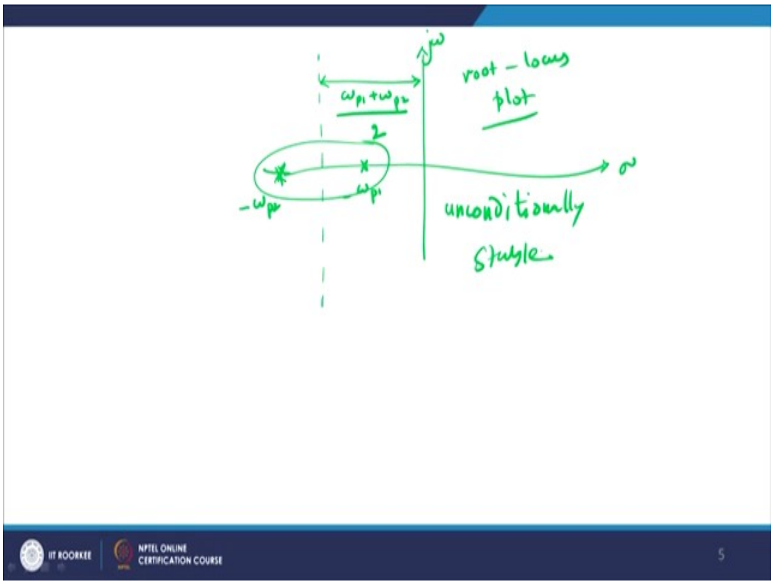

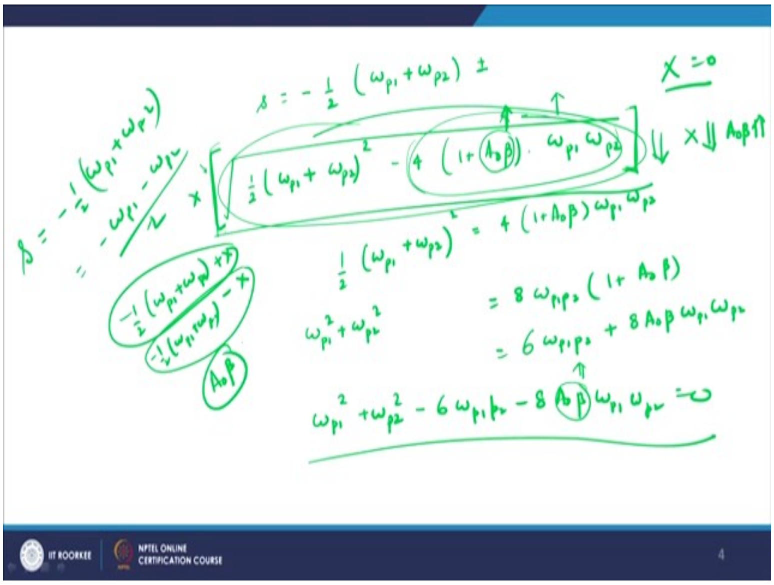

So with this condition let me plot draw the plot for you and show to you, suppose this is my this is my $j\omega$ axis and this is my σ axis and I have got here this is this is ω_{p1} and this is your minus ω_{p2} and this is your point which is the half line point between the two, and this is actually your ω_{p1} plus ω_{p2} by 2. Right? So the distance of the half line from this is what you get it.

Now an interesting idea if you look very carefully from all this discussion is as you start increasing your frequencies which means that or if you go on increasing gain A_0 times β if you go on increasing it, if you let us look at this characteristics and if you go on increasing A_0

times β then this quantity goes on increasing and therefore this quantity goes on decreasing. Right? So I get suppose so I get 1 by 2 ω_{p1} plus ω_{p2}. Right? I get plus X_1 once and then I get minus X_1 once. Right?

Now, when this A_0 times β goes on increasing then you will see that this X quantity because this increases means this whole quantity will increase. Right? Which means the whole quantity will actually go on decreasing. So X quantity so X will decrease when A_0 times β goes on increasing. Right? So when A_0 times β increases X increases so this quantity will go on increasing whereas this quantity will go on decreasing because this is a minus sign here.

And as a result what will happen is that with increase in A_0 times β. Right? These two location of these two poles on this two occasions they will start shifting away from each other and therefore that is what happens when they shift away from each other. This is also known as a root locus plot in the X axis and this is the condition so this whenever the two on left half plane we defines this to be an unconditionally stable condition. Stable condition. Right? So this is for 2 pole response which we get in reality. Right?

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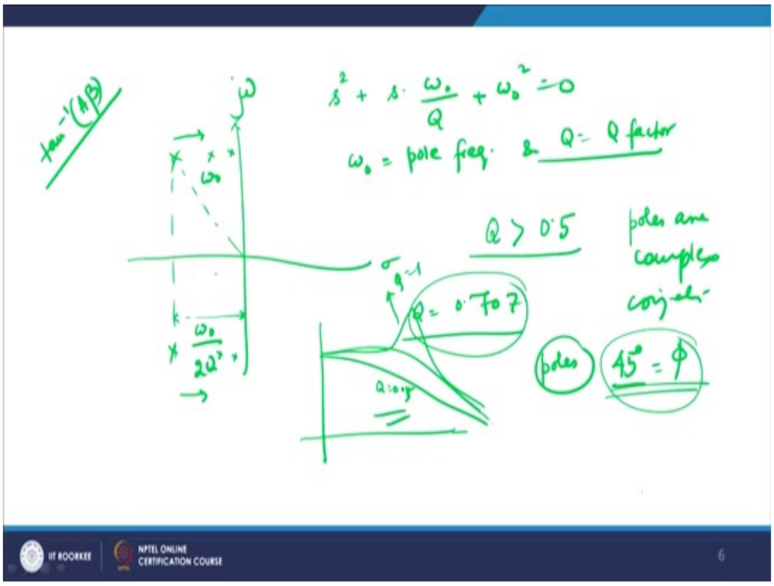

Now, let me explain you the Q factor because that was left out in the previous turn. So let us suppose the characteristic equation in again a 2 pole network is given as ω_0 by Q plus ω_0 square is equal to 0. So ω_0 is defines as the pole frequency and Q is defined as the Q factor. Q factor or quality factor. Right? Typical rule of thumb is that if Q is greater than 0.5. Right? The poles are a complex conjugate. Right?

And they will be lying it will be something like this so it will be lying suppose this is $j\omega$, this is σ they will be lying somewhere complex conjugates. So this will be the path which they will take, this is your ω_0 the distance is ω_0 and this is ω_0 and this if we join these two curve then this distance is referred to as ω_0 divide by 2Q.

So if you make your Q higher and higher this distance goes on reducing this distance and therefore these two poles start to shift towards the $j\omega$ axis. Right? And Therefore so at a higher Q this will be the new position of pole at further higher Q this will be further closer to $j\omega$ axis and therefore you when you go on making Q larger you are actually adding a sinusoidal component to it and therefore you will have sustain oscillation as these two fall on the $j\omega$ axis in reality.

Further if Q is equals to 0.707 I am not proving it here then you will get maximally flat response which means you will get something like this and maximally flat. If Q is equal to 1 then I get something like this and then it increases like this so this is for Q is equal to 1. For Q less than 0.707 I get something like this, so this equal to 0.5 or let us suppose something like this. Right.

And therefore, 0.707 is maximally flat at this position at this point the poles should be around 45 degree. The phase margin phi should be at 45 degree. Right? What is the phase which you see? Phase is basically the tan inverse of $A\beta$. Right? If it is a function of this thing and therefore whenever phase margin is equal to 45 degree I will get the most stable option available with me in reality.

So that takes care of our understanding in reality and let me therefore come back to the this module's major portion and let us see what we will be studying here.

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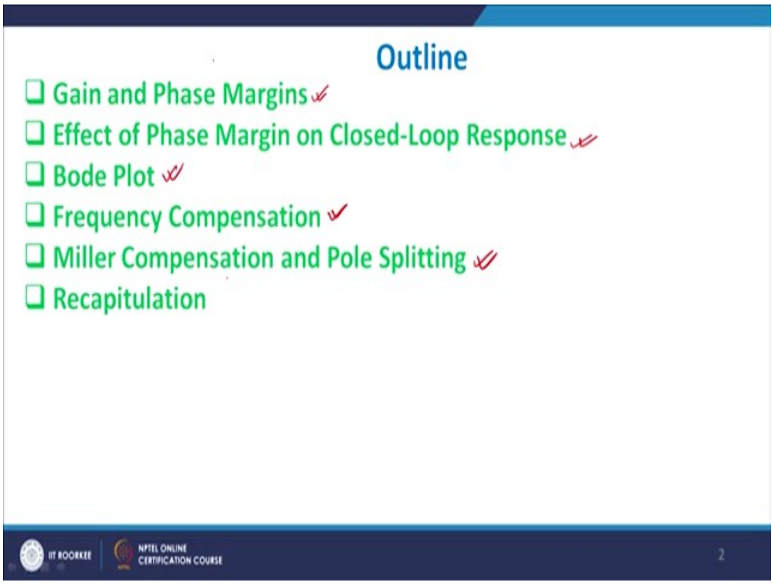

We will look at gain and phase margins. Right? Phase margin, then effect of phase margin on closed loop response, we have already look into the bode plot earlier but with respect to amplifier we will look into the bode plot. Then we will go for frequency compensation and miller and poles splitting we will see. Right? We will go in details of this first 4, but next last two we will keep in a bit low key of air.

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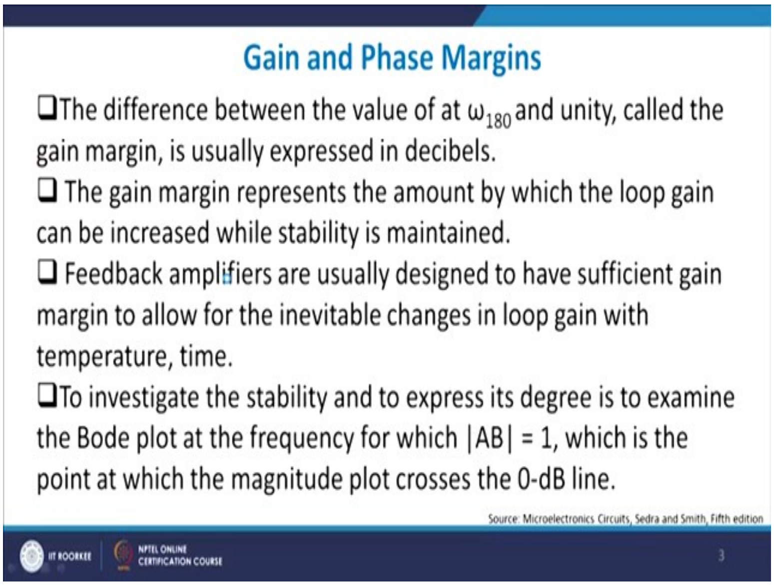

Let us look at the gain and phase margin the difference see as I discussed with you in a bode plot whenever you have whenever you are plotting it.

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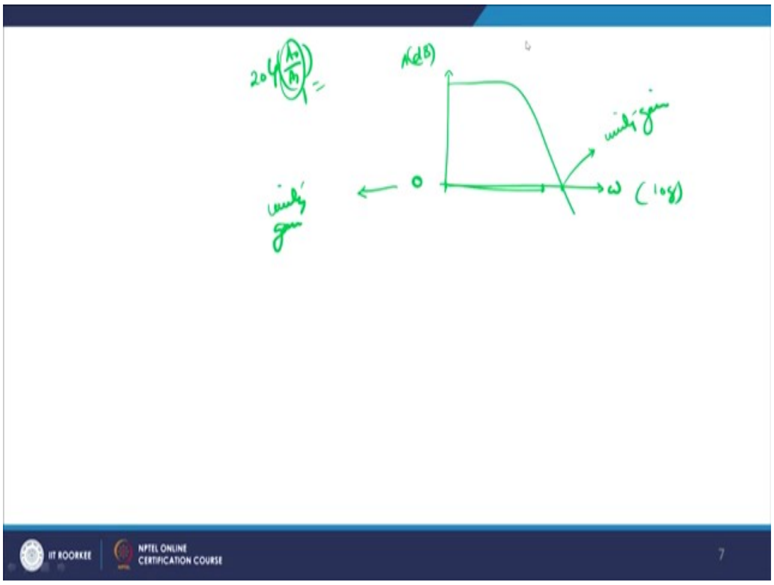

Suppose you are plotting from plotting a gain. Right? So gain is dB, gain is in dB and then you are plotting here ω in basically log scale. Right? So if you look very carefully if you take 0 dB this is where you get a unity gain this is unity gain. Because if you remember it is 20 log of output by input. Right? So if output by input is equal to 1 log of 1 equal to 0 therefore in dB scale you will get 0 here. So whenever my logs scale comes and crosses here let us suppose then this corresponds to unity gain. Right? Unity gain which you get.

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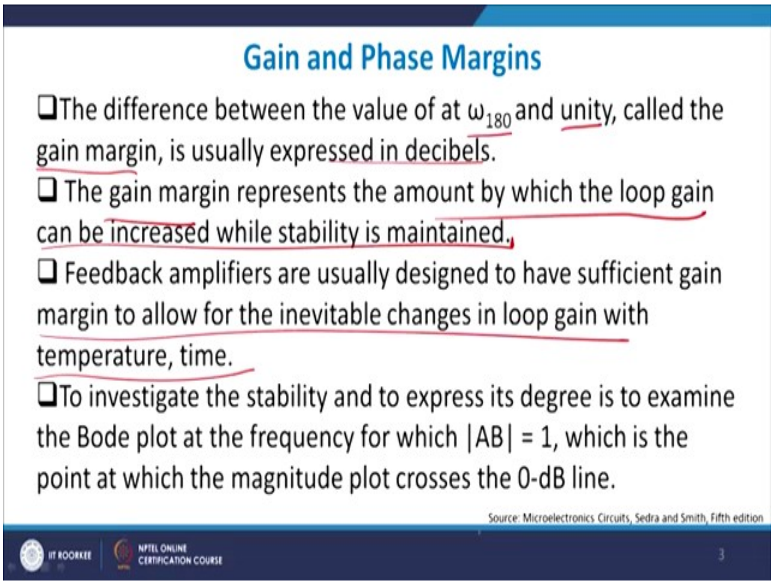

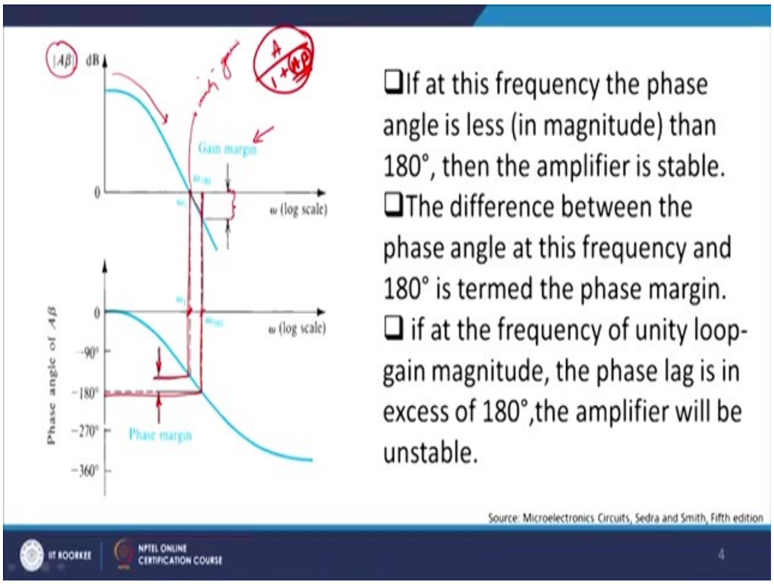

Now, so with this difference between the values of the gain at ω_{180} degree unity is called the gain margin and usually express in decibels. I will explain to you from the diagram I do not know I do have a diagram may be let me check.

Yes I have a diagram here. And if you look very carefully this one on the y axis you do have the Aβ term basically A actually because β is very small let us suppose, then we get loop gain verses ω then as I increase the frequency loop gain starts to fall down be on the particular region, it crosses this point ω1 where gain equals to 0 and therefore this point basically your unity gain.

Now, we define and then we start let it move on the negative side so the dB goes to negative side, as it goes to the negative side it crosses so this ω_1 let us suppose some values of ω, say 130, and then as it crosses this particular point and comes to this point where you have ω_{180}. Right? At 180 degree then we define this difference between this point and this point as a gain margin. Right? So how you define the gain margin?

The difference between the gain from which point to which point? From the point of unity gain to a point where you shifted to ω equals to 180 degree. Right? So if you do its corresponding region so if you see this curve goes to 180 degree. Right? Phase margin and this is the point where you have gone to some 130, 140 whatever, then this is defined as my phase margin. Right? This is been defined as my phase margin. The difference between when unity gain to ω equals to 180 is defines as my phase or the gain.

So I have got two margins here one is known as a gain margin and one is known as a phase margin. If you will go back therefore the gain margin therefore represents the amount by which the loop gain can be increased while the stability is maintained. What does it mean? It means that if you look very carefully that this difference is if the gain margin is very high then I can increase the loop gain because loop gain is Aβ, so I can still have loop gain go higher and I do not have the stability problem issue coming into picture, remember? What was the stability problem? Stability problem was that A upon 1 plus Aβ was there.

Now, if Aβ has to be a negative quantity then this quantity will shoot up. Right? And this Aβ will become negative quantity at ω is equal to 180 degree. We have already discussed that point, so the I do have a time of making my Aβ even go to 0 value or very close to 0 value without going to negative value and I can still have a large amount of negative feedback coning into the picture. Right? That is what is defined as my so what is gain margin. Right? So it represents the amount by which the loop gain can be increased without affecting my stability.

Now, feedback amplifiers are usually designed to have a sufficient gain margin to allow for the inevitable changes in loop gain with temperature and time. I will keep this point very clear.

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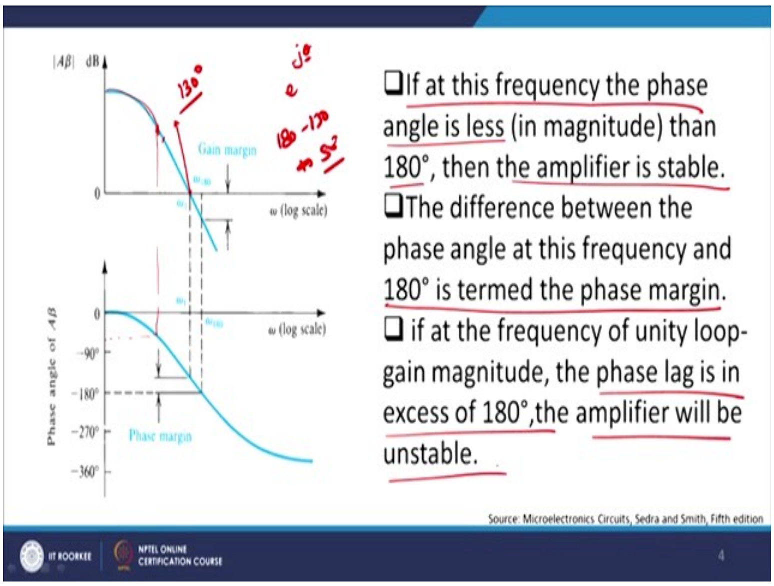

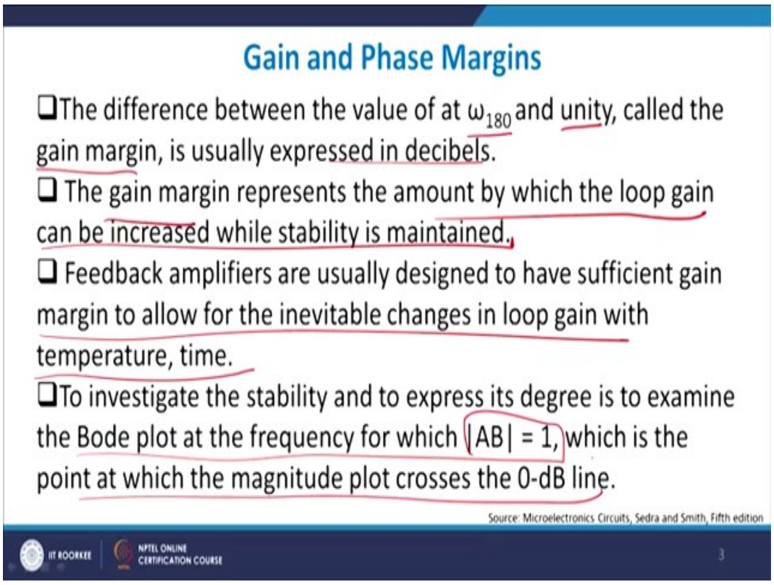

See what happens is that if you suppose let us suppose you take this graph into the consideration. Right? And you have inadvertently done what thing? That you have placed your device somewhere here or you have placed somewhere here. Generally what happens around so this is your 45 degree point. Right? And this is you have placed here, so if you place somewhere here or somewhere here, somewhere you are stable and it is working fine.

But especially in analog design an designs where the currents are high you do have the chip temperature goes high. So the temperature goes high your β your A which is the gain those also change and when they change the Aβ value also changes so I should have a large margin between the point where your unity gain till equals to ω equal to 180. So that will allow me to give a large headroom for the Aβ value so that I am not going away from the stability criteria.

Now, so to investigate stability and to express its degree so we example the bode plot at frequency at which Aβ is equal to 1 which is the point at which magnitude plot crosses the zero dB line. We have just discusses this last point here. Now, if at this frequency which frequency? At this frequency the phase angle is less than 180 degree then the amplifier is stable. And that is what I was saying, that if at any frequency your phase angle is less than 180 degree you are stable if it is more than 180 it is unstable and the reason is you remember e to the power jθ. Right? And therefore at 180 degree this will becomes to minus 1. And as a result you will automatically have a positive feedback.

The difference between the phase angle and at this frequency and 180 degree is term as the phase margin, which means that let us suppose I will give you an example, let us suppose at this angle my phase is 130, then we define phase margin as 180 minus 130 that is 50 degree.

Will be a phase margin. Which means that this much of amount of phase I am able to fore go even if without compromising on the stability criteria of the amplifier. Right? If then again the phase lag is in excess of 180 degree the amplifier will be unstable. We have already discussed this point just now.

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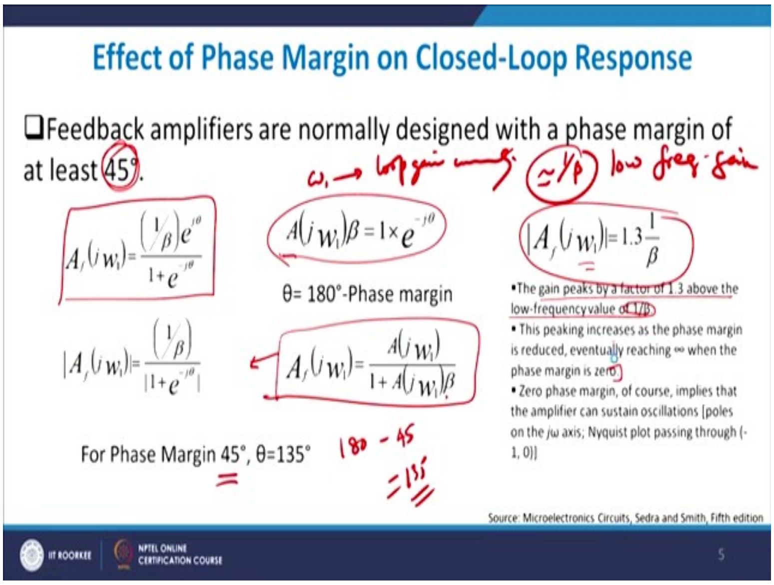

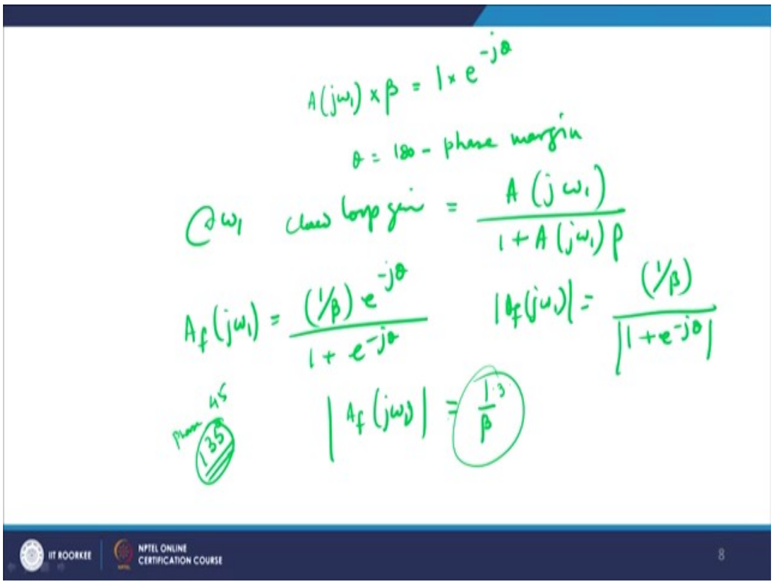

So let us look at the effect of phase margin the close look response, I have the close loop response, close loop means basically I do have feed forward amplifier connected to Aβ which is basically my feedback network and then it is mixed to a mixture or to a sampler and then the error signals goes again to the amplifier. So this is a basic close loop feedback.

Now, generally in most of the cases we defined feedback amplifiers are designed such that your phase margins are approximately equal to 45 degree. Right? At 45 degree you have maximum stability criteria coming into picture. Right? And that gives you a pretty descent values of this thing. Why? Because tan45° if you see it will give you 1. Right? And tan45° 1 primarily means that the most stable criteria will be coming into picture.

Now, if you see that for phase 45° I told you the phase margin will be this minus 45 and that is equal to 135 degree. Right? So I get $A(j\omega_1)$. Right? ω_1 is the point where I am assuming that loop gain is unity. So ω1 is the point where my loop gain is unity. Right? And my low frequency gain will be how much? 1 by β approximately. This will be my low frequency gain. So if you look at this particular issue then I get if you solve it I get low frequency gain is as I said 1 by β so 1 by β e to the power jθ upon 1 plus e to the power minus jθ. Right? Converting into polar of coordinate systems.

So I can wright down therefore that if we take mod value of that I get mod of this equal to 1 by 1 upon because this will go to 1 and I get 1 by β coming into picture, 1 plus e to the power minus jθ comes into picture and I can safely wright down this to be is 1 into the power minus jθ. Right? Therefore I get with feedback to be equals to this consideration is there with me it is always equal to the value.

Now so the closed loop gain, this is my closed loop gain with feedback. Right? Is a quantity which is given by this, now let me do one thing let me show to you that how I am getting it, so if you do a small derivation here I can safely wright down that A of j ω1 which is unity gain into β will be equals to 1 into e to the power minus jθ where θ is phase margin, where θ is equal to 180 minus the phase margin. Right? At ω_1, ω_1 where unity gain this thing is there, the close loop gain. Right? Will be given A j of ω_1 divided by 1 plus $A(j\omega_1)$ into β.

So I get $A_f(j\omega_1)$ with feedback equals to 1 by β as I showed I the earlier slides e to the power minus Jθ upon 1 plus e to the power minus jθ. So if I take the mode value of $A_f(j\omega_1)$ with feedback then this comes around 1 by β because mod value of this will come out to 1 and I can remove it and from here I can get 1 plus e to the power minus jθ mod of that. Right? Mod of that.

Now, mod of that if you solve it this comes out to be equals to 1 and therefore I get $A_f(j\omega_1)$ mod of this happens to be equal to 1 by β times. Right? And I get this to be approximately equals to 1.3. Right? Assuming that phase angle is 45 I get phase margin to be equal to be 135°. Right? So tan inverse of 135° becomes out to be 1.3 by β. Right?

So this is what I want to do here that A of f means that with feedback at unity gain bandwidth I get 1.3 upon β, so the low the gain peaks by a factors of 1.3 above low frequency value of 1 by β. This peaking increases of the phase margin is reduced eventually reaching to infinity when the phase margin is 0. Right? So what happens is with feedback your gain became accessibly high and it becomes infinity at phase margin equals to 0. Phase margin primarily means that your θ is 180 degree. So at 180 degree is the point beyond which if you increase your phase margin you are in a problem that you will enter into a non-zero part. Okay.

So, 0 phase margin of course implies that the amplifier can sustain oscillations. Right? So poles are in jω axis. Right? And that gives you 0 phase margin, so the 0 phase margin is the best one, but if you want the stable oscillation keep it at 45 degree that is what we have learned across this whole thing.

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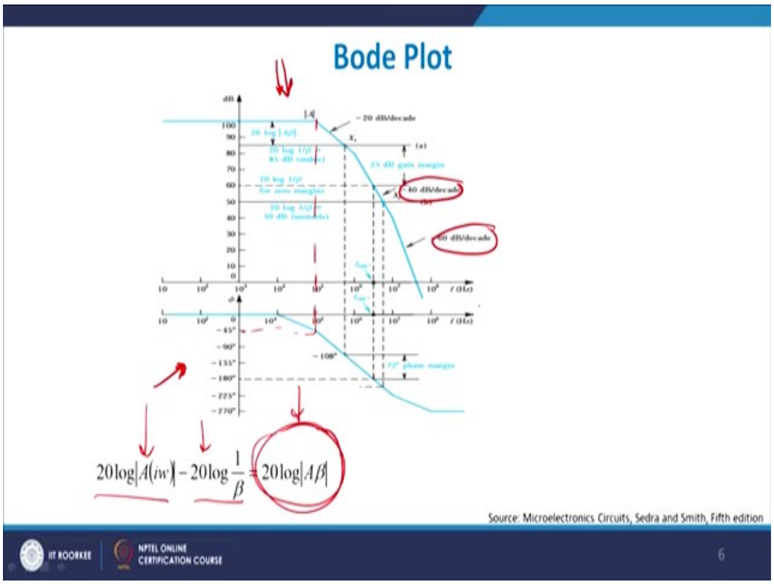

Lets come to the bode plot and we already and from bode plot we can also calculate the stability criteria for the design. Let us see how we can do that. Remember 20 log of jω minus 20 log of 1 upon β will be 20 log Aβ. So, if I subtract this and this I get this, and I will be able to plot this with respect to ω. Right? Therefore if you look at this particular issue and this is my phase margin plot and this is my frequency gain operation plot. Then you see that it is maximally stable at up till this much frequency. Right?

If you see this will be around 45 degree so I get maximum stability at 45, till 45 get a maximum stable, then I get 20 dB per decade drop because this first order system, I get 20

DB per decade drop. It reaches till X_1 which is basically a point where you have got to reach the value of 135. Right? And 135 you have reached.

Further if you go down I reached to a point equals to 180 and beyond this particular point your become unstable. Right? So your gain falls down drastically lower it reduces at the rate of minus 40 dB per decade and here minus 60 dB per decade. So you see even with the one decade change in the frequency you have 60 dB drop taking place here, which means the gain is not at all stable here and it falls very fast beyond a phase margin of 180 degree.

So this plot gives you an idea about the fact that 180 is basically therefore the phase margin where the stability criteria can be violated if you go beyond that particular point and therefore the you can also get the stability criteria understanding from this bode plot as well.

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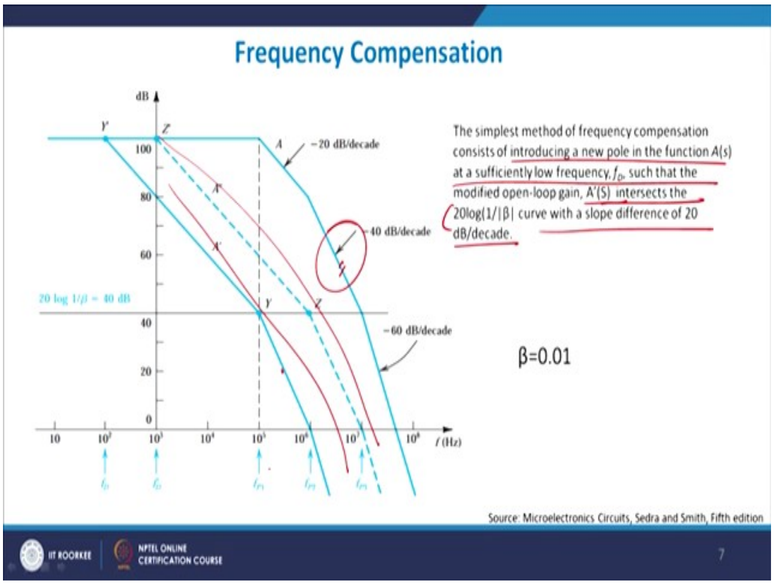

What is the meaning of frequency compensation is that the simplest method of frequency compensation is introducing what is the meaning of frequency compensation? See the problem which we have faced earlier in our earlier discussion that till 180 degree you will have stability and beyond 180 you will your gain will be falling very fast and you will have unstable system with you.

We required to compensate for an increased frequency here, and to do that what we do is that we introduce a new pole at very very low frequencies at sufficiently low frequencies such that this A'(S) intersects 20 log 1 by β curve with the slope difference of 20 dB only. Which means that I allow so I just told you previous slide that it will be 20, 40, 60 dB per decade drop, but if you are able to sustain a 20 drop even beyond 180 degree phase margin then it will be relatively less or it will be relatively more stable. Right? And that gives me better frequency compensation in reality.

As you can see here I only for example I have just remove this 40 dB decade drop here and if you go ahead and do this I get this compensated profile and I get this to be a compensate profile. Which means I have left alone 40 dB here and I only get a 40 dB drop at this particular point beyond a particular point f_{p1}, so this is permanently more stable as compared to previous case.

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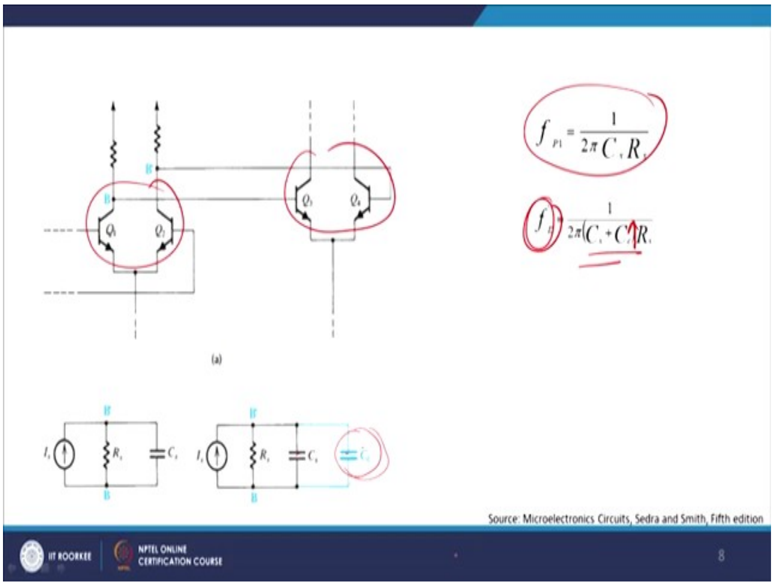

This is another method of showing a frequency compensation, how do you do that you put BJT here, you have BJT complex here and you have a BJT complex here and this is represented by a current source and a resistance current source and a RC resistance then you apply a compensation capacitor here. As you apply compensation capacitor here the frequency is inversely proportional to the capacitance and since these two capacitors this is are in parallel they add up and as a result the f_d prime value actually shifts to the left, because this reduces because of the increased value of the capacitance here. And as a result what will happen is that the point where it cuts the 180 degree curve will shift to the left and you will automatically get a better compensation in this case.

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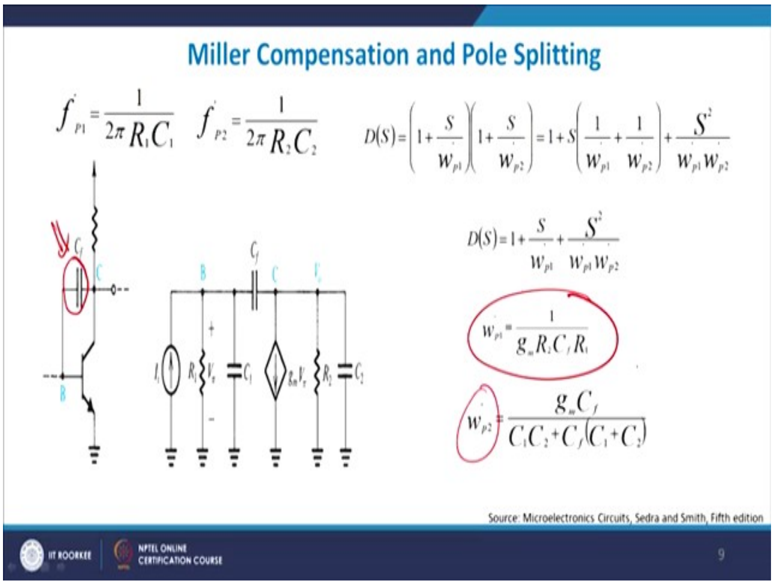

We will not discuss in detail this slightly ahead of its course but just to give regularity and to give you an idea about where you are looking to discuss this point and we will close here. That we also try to do one more important point and that is we try to split the poles into two parts, so rather than having a single from a single order system can be have multiple poles and those multiple poles can add to the stability of the system. Right?

And this is one of the methods of doing it and I will not go in to the details of whole method but typically applied to a BJT, here you apply C_f which is basically my feedback capacitance here and also referred to as a Miller capacitance and if you put an feedback capacitance here my ωp1 actually becomes a function of C_f, similarly ω_{p2} also becomes the function of C_f and simply by making C_f higher and higher I can shift relatively the ω_{p1} and ω_{p2} from its initial values and this is known as pole splitting in analogue design. We will not talk any further than this at this stage because it is slightly ahead of this course. Right?

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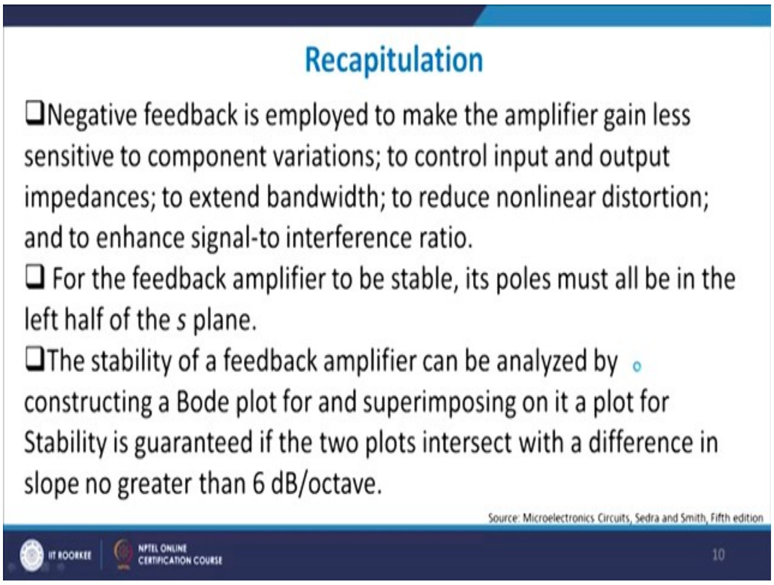

Therefore let me give you recapitulation of what we did in this module and the previous one, we have taken up the fact that negative feedback is employed. Right? And to make the amplifier again less sensitive we have already learned that we have less reduced non-linear distortions enhance signals to noise ratios, the poles as I discussed with you for stability the poles all will be in the left half of the plane S plane. Right?

The stability of the feedback amplifier can be analyse by constructing a bode plot and superimposing a plot for the stability, and we ensure that difference between the two plot is not more than 20 dB per decade or 6 dB per octave difference is not there. So we sustain a 20 dB per decade drop even after 180 degree phase margin and then the stability will be maintain.

Similarly, we can also therefore use frequency compensation for the stability of the amplifier, by choosing an appropriate value of the compensation capacitor and making the poles. We have also learned finally about what is known as the poles splitting but that you need not worry about because it requires further basic knowledge to understand this, if time permits later on we will revisit this part latter on. Right? And that keeps my this thing ready.

From the next module onwards we will actually looking into operational amplifiers and design of operational amplifiers and then utility of operational amplifiers for analog circuits. Right? So till the next week the lectures will be over as far as analog design is concerned and then we will shift to digital combinational block design as per the syllabus of the NPTEL course. Right? Thank you very much for your patient hearing.