Microelectronics: Devices to Circuits

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Lecture - 48
Stability and Amplifier Poles

Hello everybody and welcome to the this edition of NPTEL online certification course on Microelectronics Devices to Circuits. We will be now starting an important part of amplifier design and that is basically understanding the stability and the stability will be understood from the position of the poles in the S-plane right. So, we will first understand how do you will design S-plane and then there how do you, what is a pole and which we have already in our earlier section and then using that pole, or the location of that pole we will try to analyze whether this amplifier is stable or not.

What do I mean by stability? Stability primarily means that if you have a sustained oscillation in the output and the peak to peak of the output voltage is constant with respect to time then we define that to be sustained oscillation. If the voltage starts to fall with respect to time or rise with respect to time, we define that to be as an unstable element specially if the voltage rise with respect to time you enter into a loop which is never-ending and therefore, the overall gain, for example, if you have a positive feedback right, you will always go on adding voltage to the input and therefore the output voltage will always be going to be greater in time domain. So, with this basic background let me start today's topic. Let us see how it works?

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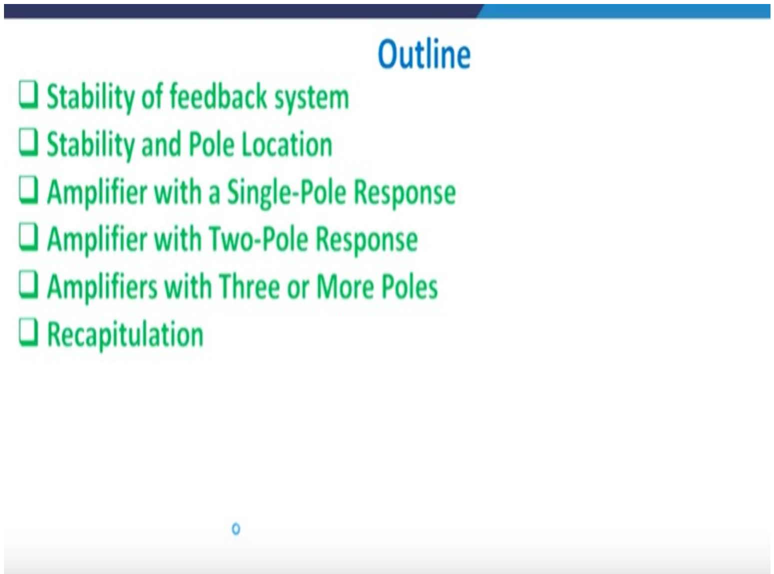

So, with this basic background let me start today's topic. Let us see how it works? So, we will be looking at what is known as the stability of the feedback system. So what are the criterias? We will study the criterias for stability in an amplifier design, right. We will also look into the position of stability and pole location. So, how do I relate the fact that where the poles are located and how stability can be asserted from the position of those poles in the S-plane?

Now, amplifier will look at the single-pole response, two-pole response and multiple pole response right. So, this single pole response, multiple or three-pole, single-pole primarily means that you have only single pole to the left half-plane. Two poles means there are two poles on the left half-plane, three or more poles is the number of poles three or more poles again on the in the left half-plane. So, I am assuming that all the poles are on the left half-plane of the axis. Now, let me explain to you where this concept comes into picture?

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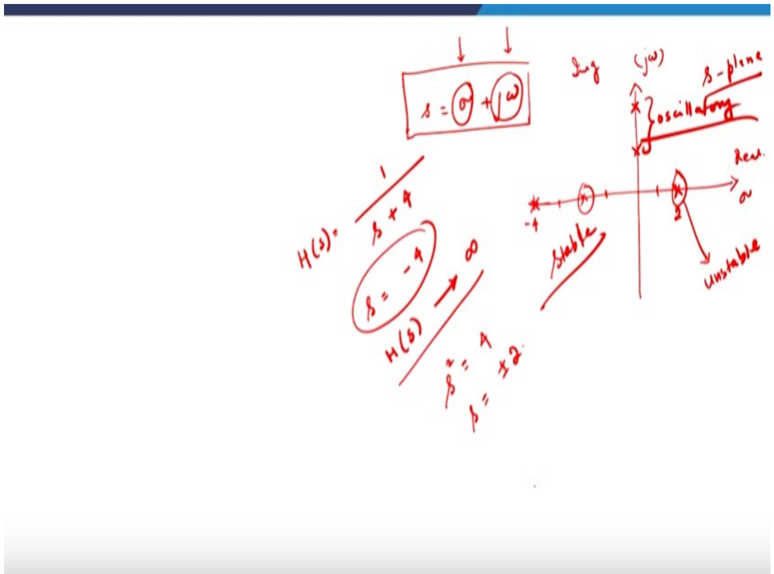

Generally, S is equal to right (σ + jω) right. So, this was general overall any complex quantity will consist of a real quantity plus imaginary quantity right. So, whenever you plot, this plot where this is a real part on the X-axis and this is the jω part and this basically the imaginary part, imaginary part and this is a real part here. Then we define this to be as a S-plane. We will see that if I remember in your previous discussion we were defining the transfer function as for example 1/(S+4) right.

Now, if you see very carefully at s is equal to -4. The value of H(S) will actually go to infinity right and therefore I can one of the poles of this session is S is equal to minus 4. So, where is -4? This is -1, -2, -3, and -4. So, if you place it here I will get a very large output amplitude available to me.

Similarly, let us suppose that S square is equal to some say again say some value say 4, then S will be equal to ±2 right. But, for +2 this is +2 here one pole here and one pole on this side and -2 one pole on this side. This will be stable but this will be unstable. So, on the right-hand side, this will be unstable on right side and everything on left-hand side will be stable in nature.

We will come on this in detail later but just for information sake at this stage whenever you refer to S-plane it is basically σ versus jω, σ is the real part of S and jω is basically the imaginary part and when you place it in this manner, you get this. If the poles are available onto your jω axis this will let you what is known as the oscillatory wave. So, I will have an oscillatory wavefront here sustainable oscillations are available here.

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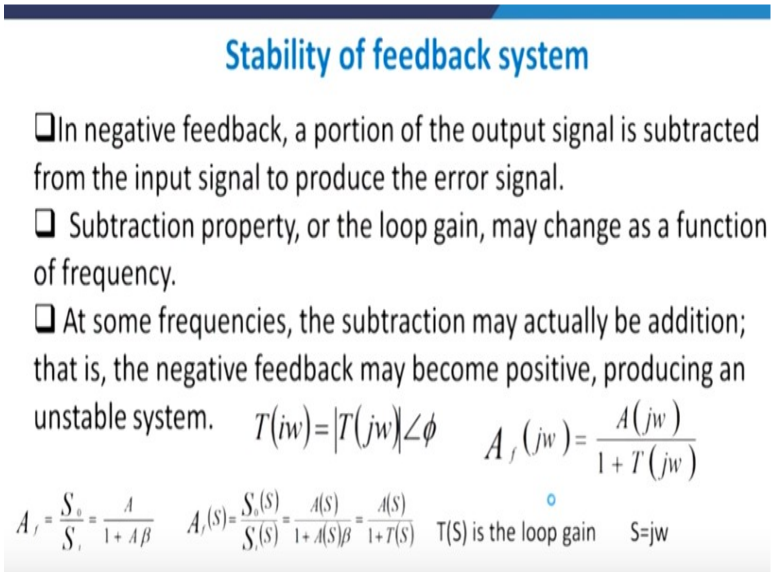

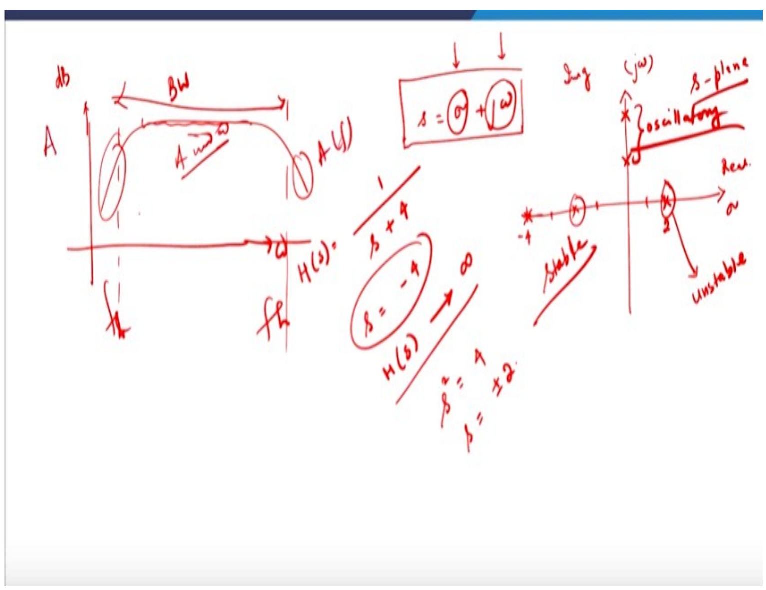

So, with this basic knowledge of S-plane, let me explain you the feedback like feedback. Now, with respect to the feedback system. We have already seen in our previous discussions that the negative feedback primarily means that the output signal is subtracted from the input signal right to produce an error signal. So, not all output signal is subtracted. A part of the output signal is subtracted from the input signal and an error is taken care of. This error is then added up with input or this error signal which is available to you is fed into the amplifier. The amplifier again amplifier is a signal and this process is iterative process through the feedback loop. We have already studied that portion earlier.

Now, as I discussed with you that if you remember how I define my loop gain. Loop gain was basically equal to Aβ and this Aβ will be the function of frequency, right. Assuming initially that they are not, but in reality they might be a function of frequency right.

As we have discussed in our earlier terms that if you plot A verses gain normalized gain in dB versus frequency let us say ω then typical values which you get is something like this. So, this is a mid-frequency gain is almost constant from this point to this point and this is a bandwidth actually and this is f_L and this is your f_H right. So, this part is the part where A is independent of ω and these are the portions where the gain this is your gain the gain is a function of frequency right.

So, with this knowledge, I can safely say that for a large change in the frequency you might have the change in the value of your amplification factor. At some frequencies the subtraction may actually be the addition, the negative feedback may actually be look as positive feedback producing an unstable system. I will explain it to you how I am talking to it.

See, as I discussed with you T is transfer function will consist of its mode value and its phase right and if you have a generic diagram of A_f with feedback is equal to A, without feedback is equal to 1 plus transfer function of feedback then in our case as in our previous discussion A_f is equal to A/(1+Aβ) then you see that af with respect to β comes out to be in S terms comes out to be this where ts is the loop gain so A β is the loop gain which you see right.

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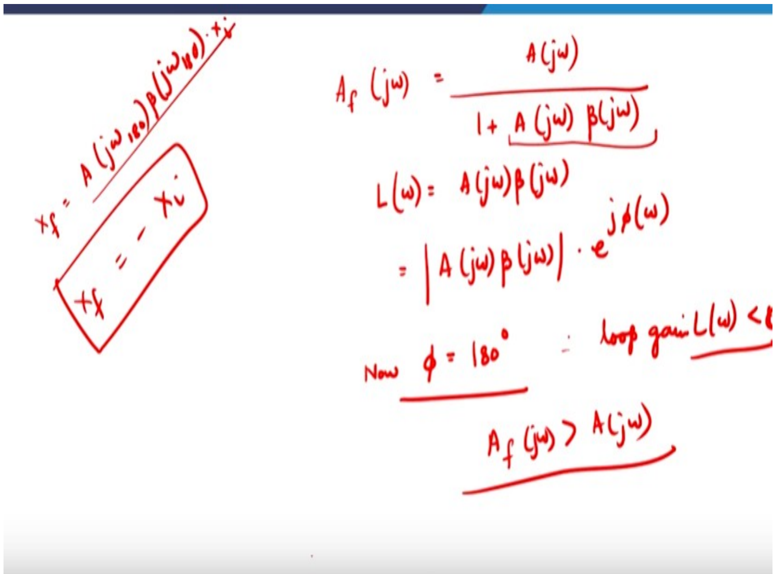

Now, if you look carefully again get back to the whole discussion. What we can write down from all this discussion is that, that $A_f(j\omega)$will be equal to A(jω) right divided by 1 plus A(jω) into β(jω) right. So the loop gain L(ω) is equal to A(jω) into β(jω) right. Now, this can also be written as if you want to find out its in the form which is most understandable to most of us that we find out its mode value so the magnitude multiplied by $e^{j\phi(\omega)}$ right where ϕ(ω) is the phase which you get.

Now, let us suppose ϕ happens to be equal to 180 degree then my loop gain loop gain which is basically my L(ω) will actually be negative and therefore, if it is negative so this whole quantity becomes negative this whole quantity and therefore $A_f(j\omega)$ is greater than A(jω) right and that is the problem area that generally you do have your with feedback gains are increases as compared to without feedback. So X_f in general sense. I can write $A(j\omega_{180})$ right into β $(j\omega_{180})$ into X_i. This will be equal to $-X_i$. So, X_f is equal to $-X_i$. So where A and β are the basically the feed-forward feedback factor at ω equals to 180 degrees.

So, what I am trying to tell you is that when the phase angle is equal to 180 degrees. Your negative feedback actually converts into a positive feedback. So, till 180 you will have a negative feedback and your system will be relatively stable as you cross 180 degrees, your system becomes unstable. That is what I wanted to prove from the statement as far as this is concerned.

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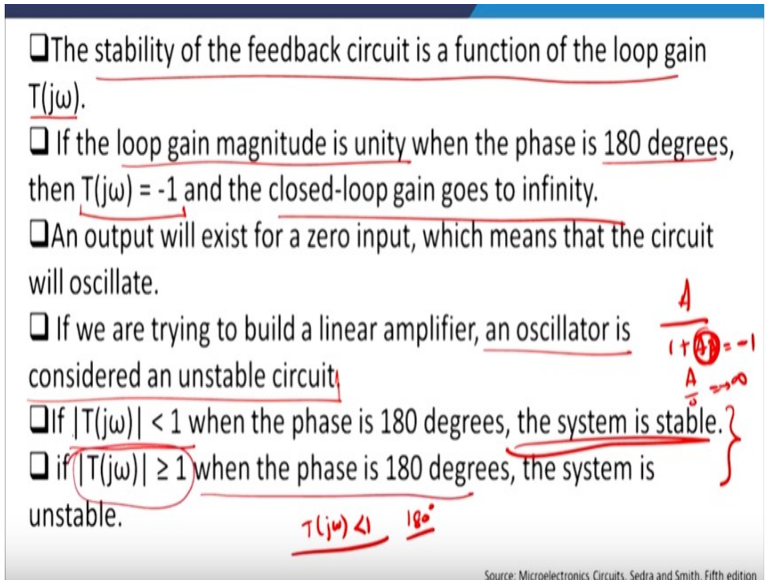

So, therefore as I discussed with you the stability of the feedback system in the circuit is a function of the loop gain T(jω) right. Now, if the loop gain magnitude is unity when the phase is equal to 180 degrees then as I discussed with you T(jω) will be equal to -1 and the closed-loop gain goes to infinity, why? Because if you remember it was A/(1+Aβ). So, now if A β is happens exactly to be equal to -1 then A upon 0 is basically tending to be equal to infinity to a rather extent.

Idea is how do you define therefore oscillation? Oscillation is defined therefore as even with the small input on to the input side of the system or the oscillator I should be able to get an external oscillations available to be right that is what an output will exist as 0 input right. Why this 0 input? 0 input means you do not have any input right given as a signal state, but you might have some small noise signal, some thermal signal is available to you which tries to make it available it to circuit state right.

Now, if you try to have a linear amplifier obviously an oscillator is considered to be an unstable circuit. I hope you understand why is it like that. Linear circuit primarily means that with respect to frequency there will be a linear rise in the gain let us suppose. But that is not true when you have oscillation gain into consideration right. Your oscillator, sustained oscillation will give you a gain independent of frequency, mid-frequency gain and therefore linear amplifier designed with an oscillator is a difficult task almost impossible task.

Now, therefore to sum up all these discussions these two points are very very important that whenever my T(jω) is less than 1 so I get a negative quantity and when the phase is 180 degrees right, then the system is stable right so whenever it is less than 1, A/(1+Aβ) will be A will be therefore less than A and the system will be stable as when the phase is 180 degree at that point of time if your T(jω) is transfer function is greater or equals to 1 then in the denominator you get T(jω) to be negative and as a result automatically you will get 1 minus small quantity will give you a small output in the denominator and therefore you will get the system to be unstable in nature right because it will be rising wave function as such for your case right. So, this is what you get as far as switching is concerned.

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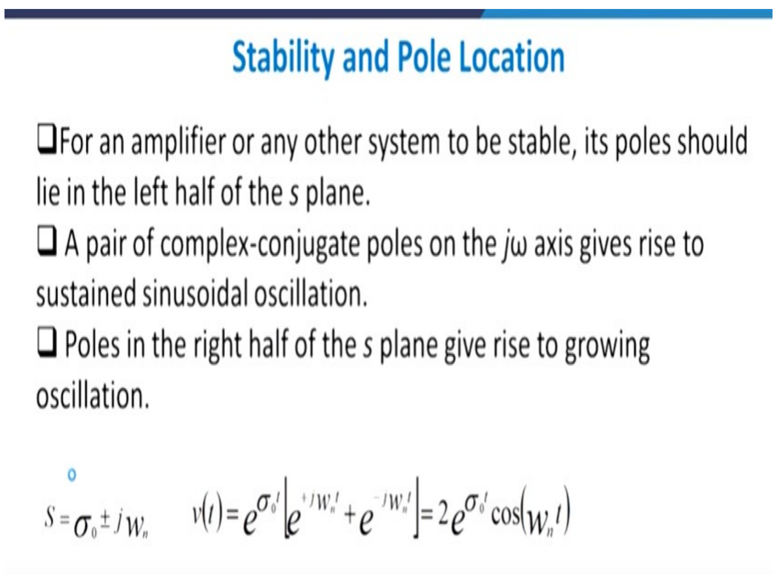

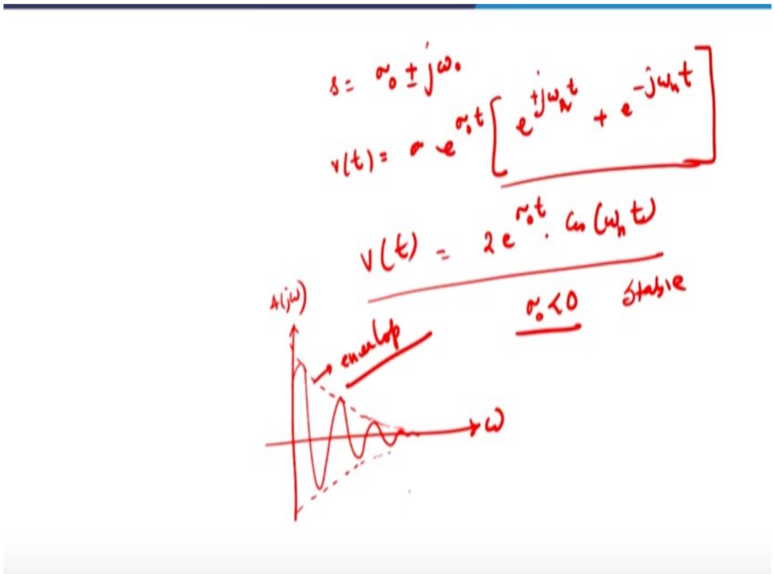

Now, let me come to the concepts of poles and zeroes. As I discussed with you just in the starting of this module that for an amplifier to be stable or any system to be stable its poles should lie on the left half of the s plane. So, as a rule of thumb across the board and we will see just now a numerical reasoning for that that if you have s plane within the S plane you want to generate a stable system then try to ensure that the poles are at actually situated on the left half side of the plane right. That is pretty important where your σ is less than 0. That is very very important where your σ is less than 0.

As I discussed with you earlier also a pair of complex conjugate poles on the $j\omega$ axis gives you a sustained oscillations. It will rather extend right. Now, poles on the right-hand side will always give you an oscillations which are growing with respect to time. So, it will be a primarily unstable design for for any oscillator or for the amplifier. The reason being on the right-hand side you will always get the output will be always gaining with respect to time and therefore it will be a never-ending process as far as this is concerned.

Now, let me let me come to the explanation here that if you see this signal s is equal to $\sigma_0 \pm j\omega_n$ right and therefore if this is the value of s which you see then we can write down voltage.

I just write down for you that let me say let me say is given S equal to $\sigma_0 \pm j\omega_0$ and therefore V(t) voltage is given as $e^{\sigma_0 t}[e^{+j\omega_n t} + e^{-j\omega_n t}]$. So this is the sign of $\omega_n t$ multiplied by $e^{\phi(t)}$ gives you these value. Now if you solve this I get $2 * e^{\sigma 0t} * \cos(\omega_n t)$. This will be your V(t).

Now, if the poles are situated to the left half-plane which means the σ_0 is less than 0 right then we get a stable system otherwise we will get an unstable system. Now, if it is less than 0 you will actually see something like this that the output will be like we will go on decreasing as you move with respect to time. So, if you plot a graph here which is the envelope of the whole design you will see that the envelope is actually a falling envelop. So, this is your gain in $j\omega$ versus ω right and this gives you the decay right.

So, this basically is your envelop, envelop of the cell and you can see where σ_0 is less than 0 so you will get e to the power σ_0 coming into picture which means σ_0 is less than 0 means σ will be negative e to the power negative will actually be a falling function and therefore you will automatically get a smaller and smaller function in general right. So, this is the basic concept which you see here. As I discussed with you $2*e^{\sigma 0t}*\cos(\omega_n t)$.

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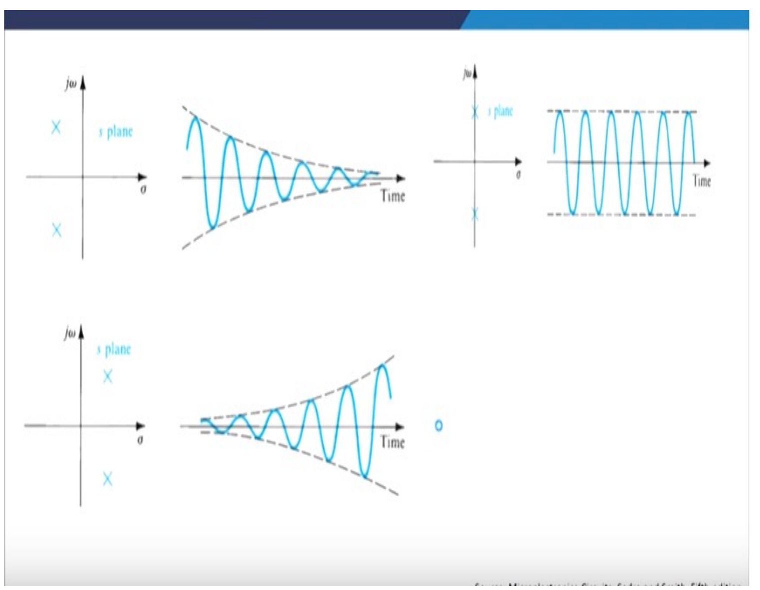

Now therefore, if the two therefore let us suppose you have got two poles and they are on the left half-plane and they are complex conjugate. Conjugate means one is a real part and another is an imaginary part and therefore they are referred to as complex conjugate. So, if you have such types of planes you will automatically get a function which is e to the power ($j\omega_n t$) and then there will be term β here. If β is negative this function will always be a falling function as such and this is falling now right and therefore the envelope is also falling down which means that you are restricting the output to a very very low peak to peak voltage.

Similarly if is in the S plane the design is such that that the S falls to the right for example 1 upon S mimus 4 then I get equals to hs then I get s equal to plus 4 for the poles so which means that it falls on the right-hand side of the plane and therefore it becomes sort of a rising function which you see the envelope is rising envelop this is a falling envelop of the output side right. If they are exactly on the plane as you can see on this graph then it will always give you a sustained oscillations to a larger extent.

So, the job of a designer when designing an oscillator primarily is oscillator or amplifier is that you try to keep your poles exactly on the jω axis that makes a sustainable sustainable growth in terms of peak to peak voltage. So, that is a basic idea that is all about and then we come to the fact that let us suppose that we do have a single-pole response which means that I do have a single design or a single-pole and the single pole will have on the left-hand side.

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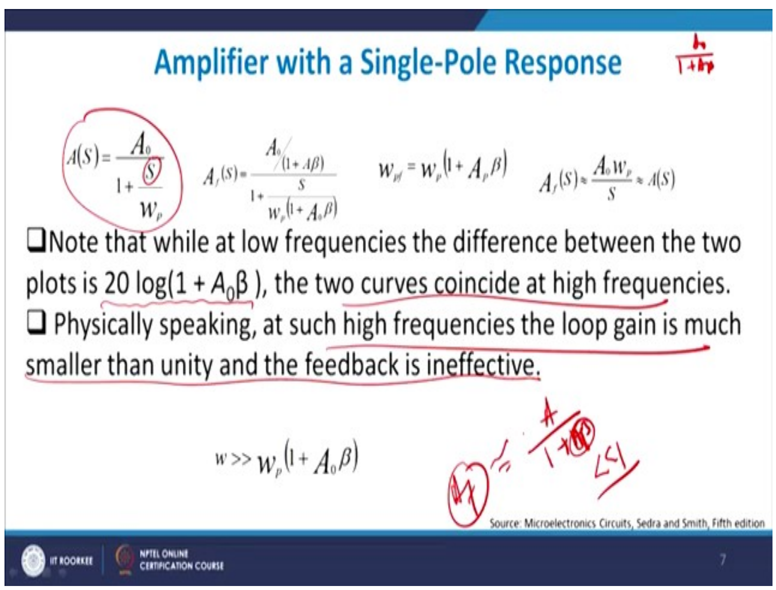

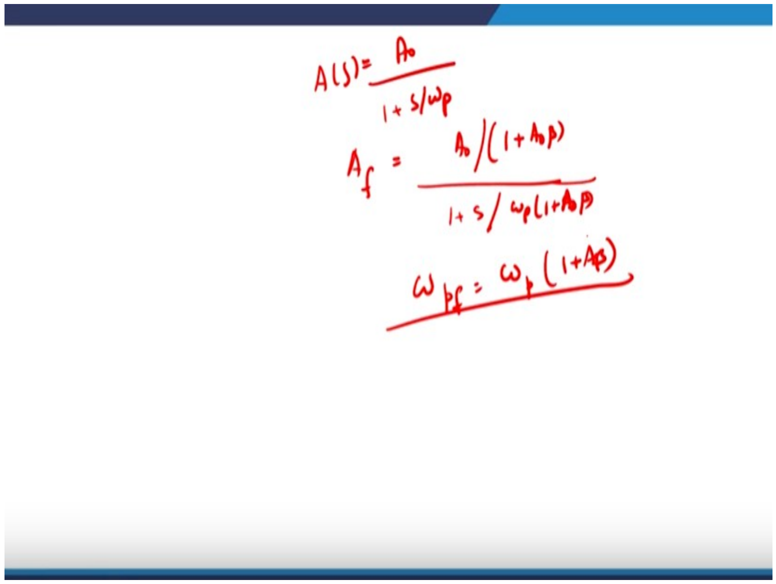

Then we can safely write down A_0 so we can safely write down for a single-pole system that A(S) equals to A_0 upon 1 plus S upon ω_p right single pole you will also get only single S function here. A of s will be feedback A_0 upon 1 plus Aβ so you remember it is basically A_0 upon 1 plus A β so I am replacing this A by A plus 1 β so I get A plus 1 by A β and I get A_0 here so what I get from here is that my with gain the feedback factor increases.

So, I get As is equal to A_0 divided by 1 plus s by ω p right and therefore I get A with feedback to be equals to A_0 divided by 1 plus A_0 into β right divided by 1 plus S divided by ω_p 1 plus A_0 times β fine so I get therefore from here if you solve it I get ω_{pf} with feedback ω_p 1 plus A β. So, you see your gain frequency starts to rise with feedback and that is what you see here. Note that while at a low frequency is a difference between the two plots is 20 log(1+ A_0 β), the two curves coincide at high frequencies right.

Physically speaking at such high frequencies the loop gain is much smaller than unity and the feedback factor is ineffective right. So, if you remember it was A/(1+Aβ) at such high frequencies this value of loop gain which is A β is much smaller to as compare to 1. Therefore, your af which is a feedback factor almost equals to A and it remains A for quite duration of time and therefore feedback is quite ineffective right that is what we have learned through this discussion.

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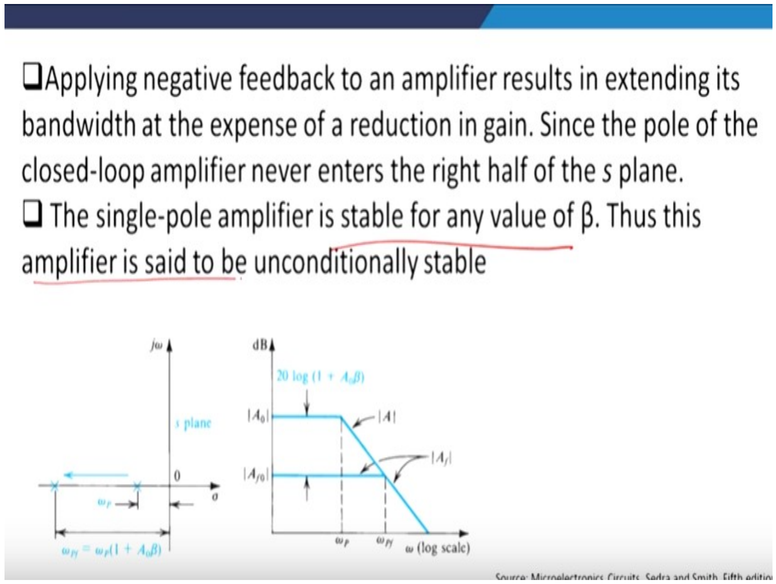

Now, therefore applying a negative feedback as I discussed in your previous turn you are able to increase the bandwidth of the amplifier at the cost of its gain right. Since the pole of the closed-loop of the amplifier never enters the right half of the s plane, you automatically get a dominant much more stable system with you right.

As I discussed with you a single-pole amplifier is stable for any value of β. Thus, this amplifier is said to be unconditionally stable right pretty important statement that this is unconditionally stable which means that if you do not have to put in extra efforts to get its stable unconditionally stable. As you can see in my diagram here on the left half-plane you have got ω_{pf} right and this is ω_p which is basically your left half pole right and this pole is basically on the left half-plane therefore it is stable but by some mechanism or other I am able to shift this by doing negative feedback actually I am able to shift this pole from this point to this point so the access value that you get is basically this much and that is the reason I add ω_p plus ω_p into $A_0\beta$. so, this extra zone which you get is not coming here and you get unstable.

On the right-hand side if you look very carefully it is actually a plot of gain versus frequency and as you can see as the gain falls down its 3dB bandwidth it is basically the higher cut of the frequency which is this one is actually falling down right. So, gain in bandwidth product again is constant. This is the standard rule of thumb which we apply in reality.

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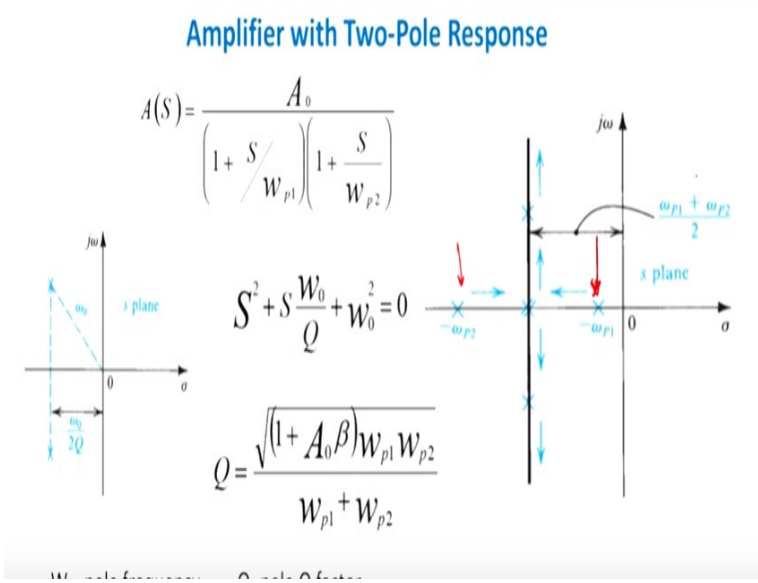

Now, let us look at a two-pole response or a two-pole design. I will explain to you how a two-pole design works typically in this scenario. But before you, ok so as I discussed with you that phase should never go beyond 90 degrees right. It should be between 0 and 180, but if you want to find out a positive feedback it should not be above 90 degrees right.

Let us look at the amplifier with the two-pole response. So two-pole response primarily means that I have one pole here ω_{p1} and I have got another pole here ω_{p2} right and then I start applying frequency. So I do a characteristics equation. This is very simple and straight forward. It is a low-frequency gain or low-frequency parameter which I am fixing up. My my now what is happening is that I am varying my ω_{p1} and ω_{p2} right and we check out how these work out.

So, if you want to if you want to solve it and gets its pole the denominator has to be standardized in the form of $a^2+2ab+b^2$ and they have done that using this formula and from here I can get the value of s to be some value as well as two values of plus and minus sign. Similarly, Q will have Q is basically known as pole a Q-factor or the Q factor here right Q factor. It is typically height by the full-width half maxima that is what you define as so if you have a sharp profile then you have a high Q because the selectivity is very very high. Not only that its height is also very large. So this height by this height happens to be relatively small quantity, but still because of the fact that this gives you an idea about how large is your function which you are dealing with. It is quite an important formula which you see in front of you here. So, I got Q-factor available with me.

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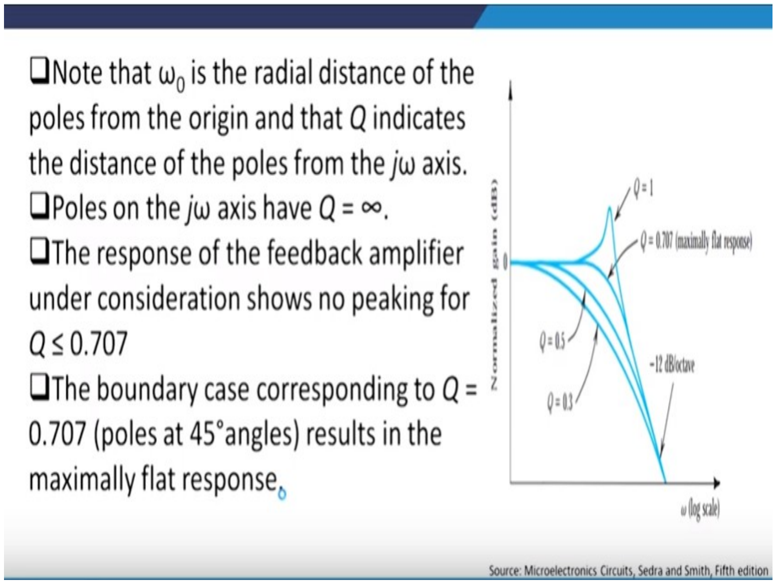

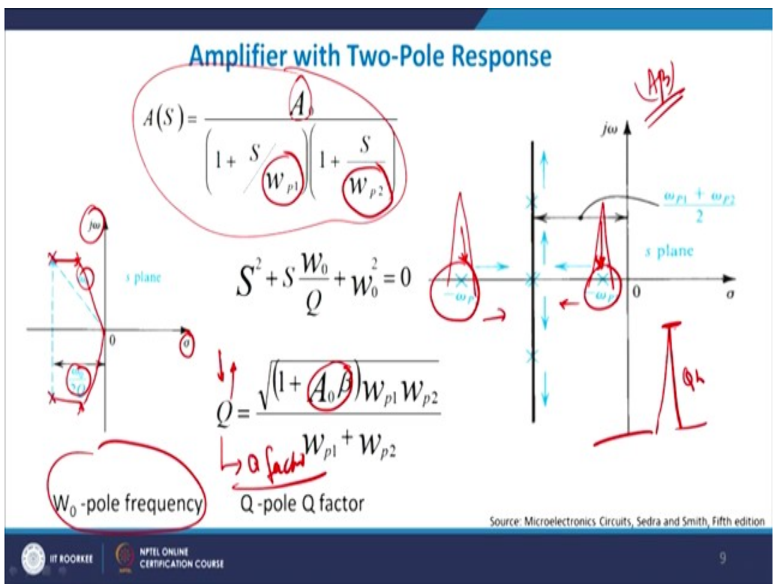

Now, note that ω_0 is the radial distance of poles from the origin and Q indicates distance from the poles from the $j\omega$ axis. So, it is quite interesting that in the previous slide if you look very carefully then you will see that for two-pole response as I discussed with you you have one at ω_{p1} with a negative sign, you have another at minus ω_{p2} on to the negative side here right on the negative side.

Now, so if as you increase the value of the A β gain into loop gain goes on increasing these two poles start to come closer to each other right. How is it possible? You see it is possible because of this reason. A_0 β will go on increasing and therefore this quantity will go on increasing, as a result, your peaks will be sharp. So, corresponding to this you will get one peak here, corresponding to this you will get one peak here. Bring it closer and you will start getting single peak of a large dimension within the typical value of σ right and that is very important issue which people are looking into it.

ω_0 is referred to as the pole frequency of the design right ω_0 is defined as the pole frequency right and this pole frequency gives you the value of your this thing. So in a jω versus σ plane if you want to find out then we define this to be as the σ plane and this to be jω axis, then if my pole is complex conjugate and lying on the left half-plane we try to first plot them and then check the value of ω_0 and $\omega_0/2Q$. Which is this is distance right.

Now, if you bring closer here if you bring closer here then you will see that let us suppose you want here then this and you will have this right and as a result what will happen is that you are bringing the poles closer to the jω axis so you are allowing it to go for a sustained oscillations right but that will be only possible provided you are able to enhance the values of your or reduce the values of your Q. Then only this will be possible.

Now, let me come to a two-pole system as I discussed with you Q indicates the distance of the poles from the jω axis. The poles on the jω axis have Q equal to infinity and that makes sense also. So at, if you are on jω axis you put a pole, right put a pole here on the jω axis obviously Q has to be infinity because in this case your S is equal to 0 or σ equals to 0 right. So, the real function is gone. Only what you will have is an oscillatory function that is because of the complex conjugate pair of pair of output signal.

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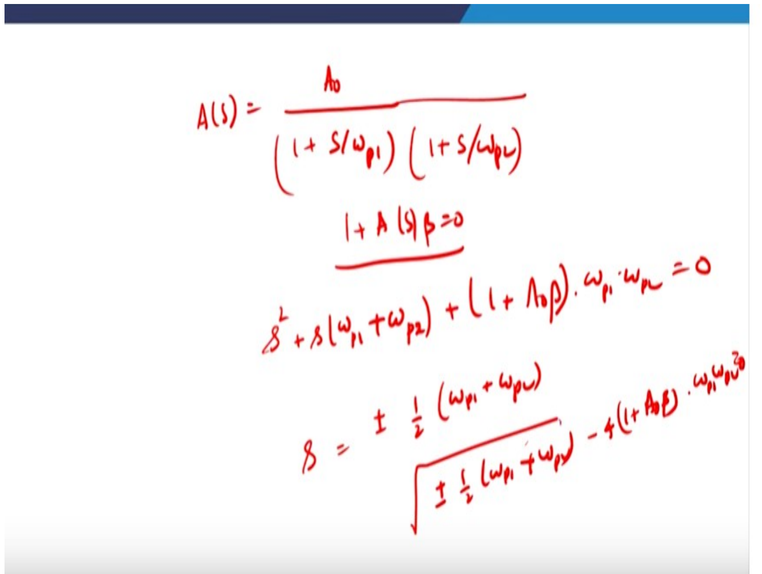

So, what you do is I will give you just a thought and we can move forward that if you have a multiple let us suppose that we have a multiple system then the two-pole response again let me show A_s is given as A_0 upon 1 plus S by ω_{p1} right into 1 plus S upon ω_{p2}. A generalized statement of a two-pole network. Then if I assume that this is the time when I will get the overall picture then again I write down as ssquare plus S into ω_{p1} plus ω_{p2} plus 1 plus $A_0\beta$ into ω_{p1} into ω_{p2}. This must be equal to 0. If it is 0 then you automatically you get this function to be high.

So I get S to be equal to plus minus 1 by 2 right ω_{p1} plus ω_{p2} right. This multiplied by multiplied by obviously plus minus 1 by 2 ω_{p1} plus ω_{p2} right ω_{p2} and then minus 4 times 1 plus A_0 times β multiplied by ω_{p1} and ω_{p2}.

So the same formula which I have been using time and again minus B by A and so on so forth and then you get s to be equals to this quantity here right. So as and as as as therefore as A_0 into β goes on increasing right so what happens is that this quantity goes on decreasing sorry, yes this quantity goes on increasing and therefore this quantity goes on decreasing when this quantity goes on decreasing this quantity goes on increasing. So, I will the poles will start moving to the left of the plane right-left of the plane here right.

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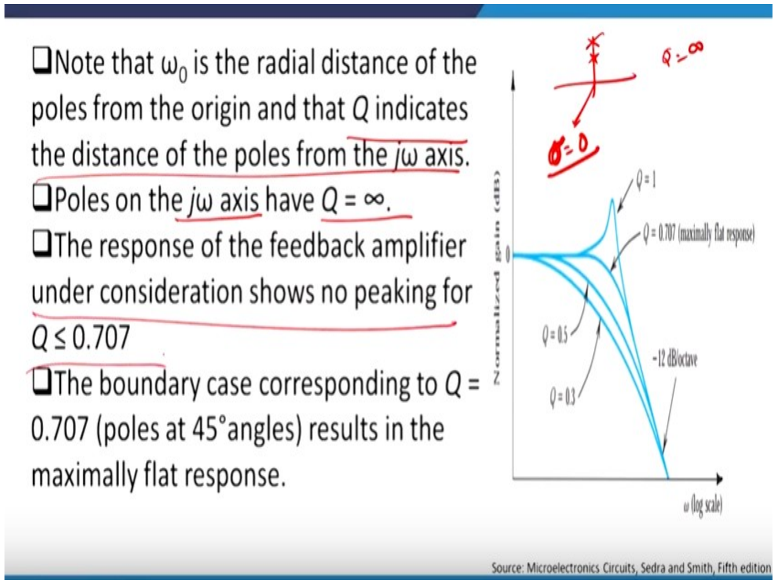

So the response of the feedback amplifier shows no peaking at Q equal to less than 0.707 right and that is the reason you will get a stable system at Q equals to 0.707. That is also referred to as maximally flat response right. If Q is greater than that you might get a peak here which is not a very good sign. If Q is less than this value the 0.707 and 0.03 my mid-frequency gain will be very very low and I will not be able to have a stabilized values of frequencies where they will fit into each other in a proper fashion.

Now, boundary case value is a value is equal to 0.707 which corresponds to a phase margin of 45 degrees. So, 45 degree is the phase margin which you see as there the poles location will be at 45 degrees also referred to as pole position right.

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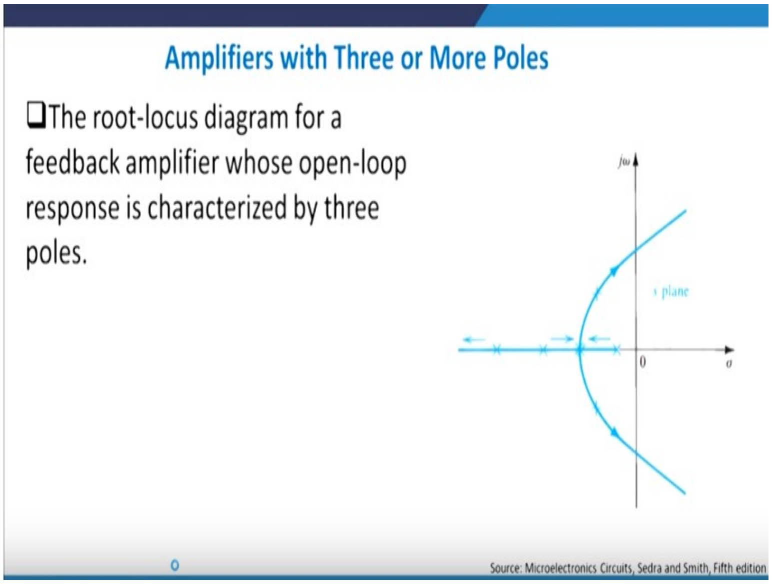

Now, amplifier with three poles is that just explained to you this thing and then we can move forward. What we say is that the last there are three or last one as we move away from most away from the origin axis starts moving to the left away from the pole as the value of the frequency goes on increasing. However, however, the other two poles starts to move closer to each other right. A time will come when they will actually override with respect to each other and then start moving to the left and to the right up and down.

So, till this movement, you will only have a real real frame movement. So σ movement as available to you, but be on this particular point you do have complex conjugate movement of these two poles as you move towards the jω axis. So, that gives you a proper idea about the three-loop system or the three-loop system in a sense is there. It is much more complex conjugate and makes it unstable as such.

So with this let me show to you also as I discussed with you in the previous this thing that if you go back so if you back to your previous discussion then you will see that you require such high end maximally flat amplifiers where your gain is almost independent of frequency.

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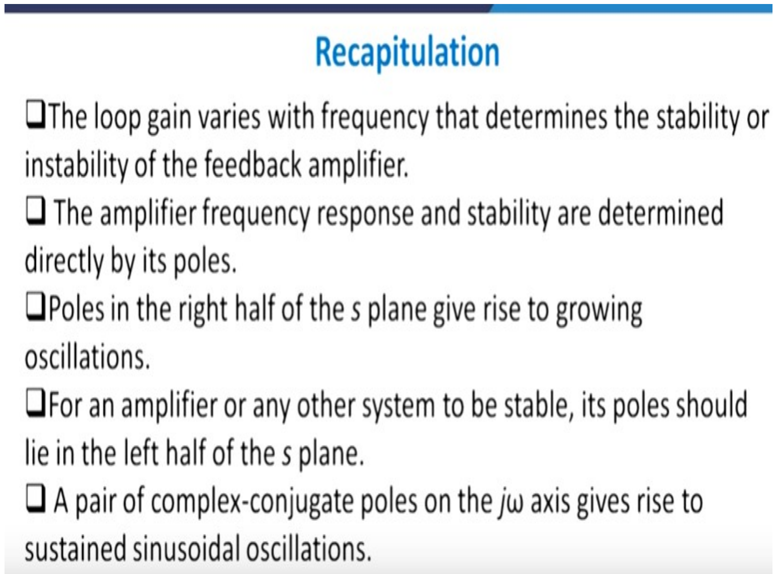

So, let me recapitulate what we studied in this module for stability the loop gain has to be less than greater than 1 and if it is equal to 1 or the phase margin equals to 180 degrees I enter into the region of unstability of (oscillation) amplifier. The amplifier frequency responds and the stability is determined by the poles. So, typically the poles of left half-plane it is much more stable. Poles in the right half-plane gives rise to growing oscillations right and that is quite difficult to achieve. If not difficult it is not desirable to achieve in common physical systems.

For an amplifier or any other system to be stable, its poles should be in the left half of the s plane, we have seen that. A pair of complex conjugate poles on the jω axis gives rise to sinusoidal oscillations in reality right. So, you have will always have sinusoidal oscillations if the two poles are located on the jω axis in a sense right and that gives you an idea about why we are requiring such a such a concept here.

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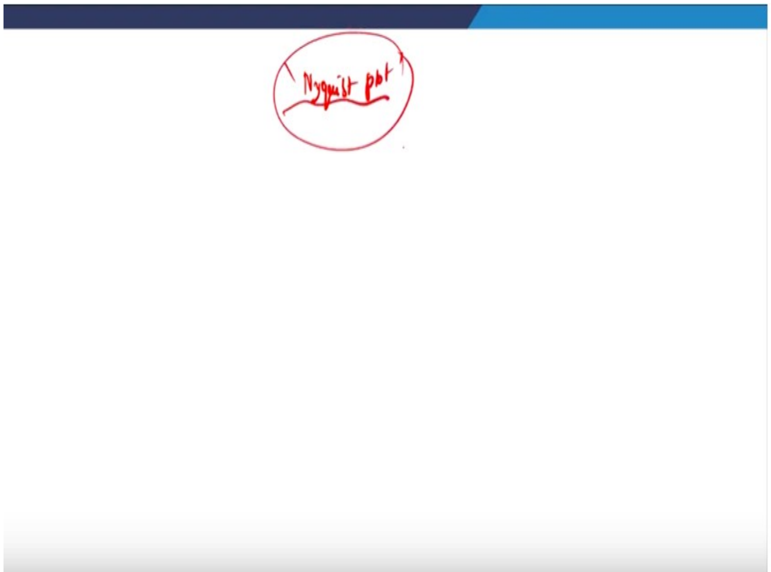

Now, I will just stop this discussion with one small enhancement and that is known as Nyquist plot and I will discuss this Nyquist plot may be in the next turn giving you an idea what is Nyquist plot right. We will discuss this Nyguist plot in the next module. Thank you for your patience hearing.