Microelectronics: Devices to Circuits
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Lecture 41 - S-Domain Analysis, Transfer Function, Poles and Zero- II

Hello everybody and welcome to the NPTEL online certification course on Microelectronics Device Circuits. We start from where we left in the previous module and this time we will be studying again the S-Domain Analysis, Transfer Function, Poles and Zeros, we will do the part II.

Yesterday, we have seen, in the previous turn we have seen that how do you define a pole, a pole is basically a value of frequency at which the transfer function which is basically the gain function goes to infinity and 0 is that frequency at which the transfer function goes to 0.

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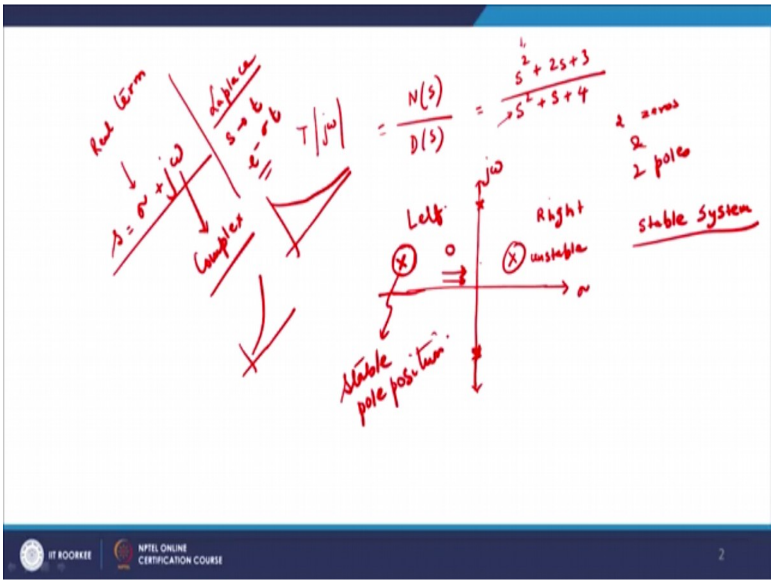

So I can have multiple zeroes and multiple poles with me. We also, we did not learn previous turn but we will be learning it later in our course structure, is basically that if you have a say numerator by denominator in both S term, this is the transfer function. Let us suppose we are talking about say j omega. Then the maximum order of your S will determine the number of zeroes, so if you have got S square plus 2S plus 3 divided by S square plus S plus 4 let us suppose, then since the maximum order is 2 here and the maximum order is 2 here, I will get two 0s, and I get two poles.

So whenever we plot these poles and zeroes, generally we plot it in what is known as an S curve or S circle. So generally we plot it in this manner that this is my sigma and this is my j omega. So S is written as a mixture of sigma plus j omega where sigma is basically the real term, real term and j omega is basically the complex quantity available with me.

Whenever we therefore try to find out poles for example, it is been seen, we will not derive it in this classroom or in this module that if the poles are on the left half plane which so this is your right half plane and this is the left half plain of j omega plot. If your poles are generally shown by this crosses and zeroes are shown by such elements, so if the poles are on the left half plain of the j omega axis, then we define the system to be a stable system, which means that you will never the system to actually be oscillatory or even if it is oscillatory it will be infinitely oscillatory.

And stable means that the transfer function will go on will not go on increasing in differently onwards. So therefore your left half plain is one of the reasons why we require to do it and there are certain reasons for that if you we will learn it later on if the time permits when we do a Laplace transform and we covert from frequency space to time domain, we get e to the power minus sigma t. So if it is minus sigma t, it will always be exponentially decaying function and therefore that is a stable function which you see.

If it is an exponentially increasing function like this it will always be a non-stable function with me. So typically we what is the importance of getting poles and zeroes is that the poles helps you to find out whether the system is stable or not. So as long as they are on the left half plane of the system, you will automatically get a stable system.

This is basically an unstable pole position, this is basically a stable pole position and this gives you a stable pole and this gives you an unstable pole. Where you place your poles? The poles can also be placed on j omega axis here as well as here, no problem, it can be placed on these j omega as well and as a result you might have a oscillatory behaviour of the circuit. For example pure oscillation, RC oscillations which are there in which the poles falls on the j omega axis.

The second point which you should remember at this stage only and we might not be revisiting it again once again is that more the pole is near the j omega axis more better the more stable the system is. So more pole closer to the j omega axis on the left half plane better it is and these are the few just small nuances of this sigma j omega plot and therefore just by looking at these positions of poles and zeros we can predict whether the system is stable, whether the system is stability is in the system or not.

We come back to, with this basic knowledge which we left I think yesterday we did not do this, we come back to yesterday’s talk again and let me show to you where we left in the yesterday.

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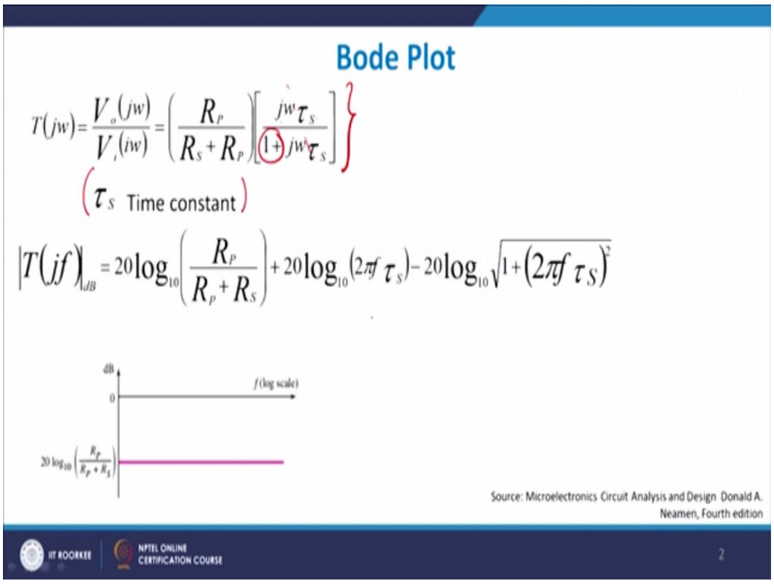

So if you remember yesterday we tried to find out this TJ of J omega and we found out to be R_P upon R_S plus R_P into j of omega tau $_S$ upon 1 plus j omega tau $_S$ where tau $_S$ is defined basically the time constant of the circuitry and omega is the frequency which is you get and which is equals to 2 Pi f.

Therefore if you look very carefully, even if omega is very large quantity and if it is very large as compared to 1 then this will get cancelled out and you are left with the R_P upon R_P plus R_S. Assuming that R_S is very very small I will get overall gain to be equals to unity and that is quite an interesting phenomenon.

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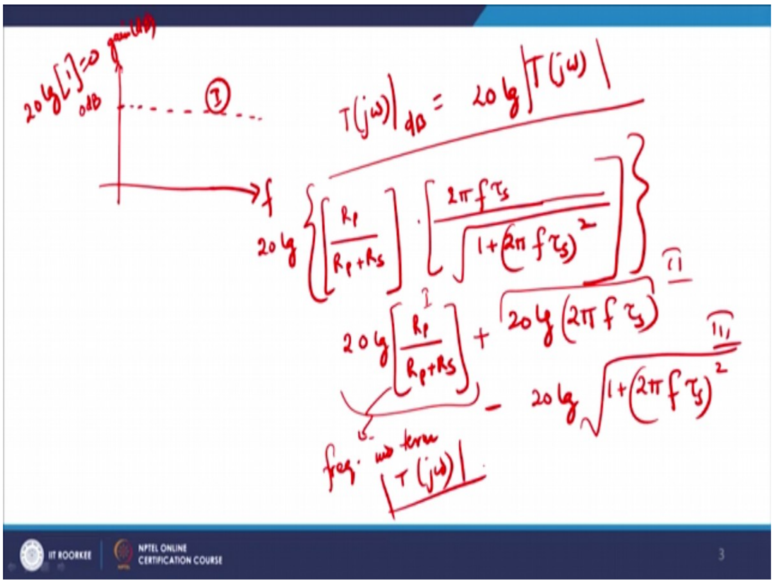

But as we discussed in our previous turn, we told you that we generally find out the we define we gain in terms of dB, in terms of dB as 20 log of T of J omega, this was the definition which we did. So when we find out this definition and we placed the 20 log of this quantity, so what we found out in the previous or yesterday what we did was something like this that we got 20 log of R_P upon R_P plus R_S and multiplied it by 2 pi f tau $_S$ upon 1 plus 2 pi f tau $_S$ whole square into square root. This is your log of this whole quantity is basically your transfer question.

So if you break it open, you get three terms which is with you. One is 20 log of R_P upon R_P plus R_S, this is the first term, the second term is of course 20 log of 2 pi f of tau $_S$ and the third term will be the negative sign because this is the denominator, will be given by this thing, 20 log of 1 plus 2 pi f of tau $_S$ whole square. So this is the term number one, this is term number two and this is term number three and the overall therefore the plot which is basically the gain versus frequency in dB we can do it by principle of superposition of this, number 1 plus number 2 plus number 3.

So let us look at one first, it is very straight forward and simple. If you look very carefully this quantity R_S is very small as compared to R_P and therefore log this becomes log of 1 and therefore 20 log of 1 will be equals to 0. So if you plot a dB plot you get something like this, so it is your this is your 0 dB and this is your gain in dB and this is your frequency in log scale and this is what you get, that you will get always a straight line because of term number 1 and it will be independent of frequency. As you can see here it is almost independent of frequency and therefore I can safely write down this to be as a straight line moving from 0 dB onwards to higher and higher dB, sorry higher and higher frequency and the dB is 0 dB remains constant.

Now this definition which means that this is basically a frequency independent term in your T of j omega, let me come to the second term now and explain to you how a second term will look like.

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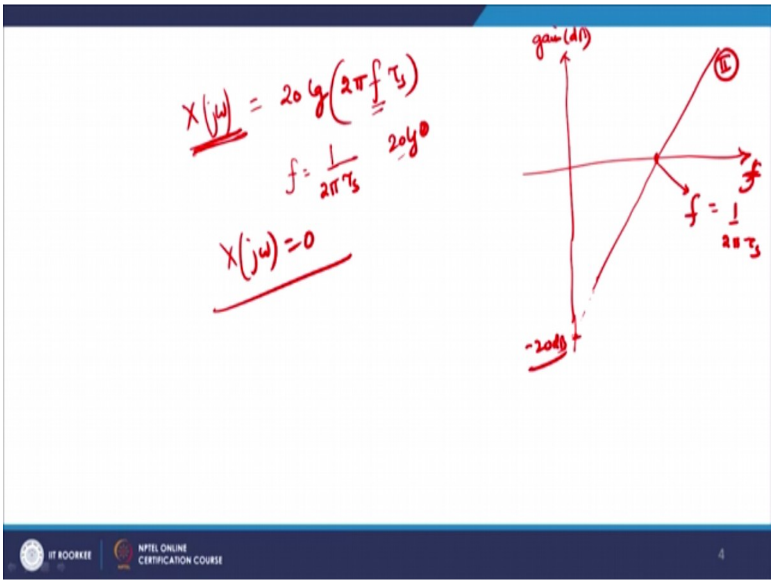

If you look at the second term, it is basically 20 log of 2 pi f of tau $_S$. So if you plot again a gain in dB versus frequency then I can be amply sure that at f equals to 1 by 2 pi tau $_S$. If I put f equals to 1 by 2 pi tau $_S$ I get this to be, suppose this is X of j omega then X of j omega will be equals to 0. And then therefore this is the, sort of a corner frequency or cut-off frequency, and at this value the f value is defined as 1 by 2 pi tau $_S$. Now you see X of j omega is actually a linear function of f. So as you increase f, X of j omega will go on increasing. As you decrease f, X of j omega will go on decreasing, right. X of j omega will go on decreasing.

So this is what you get, similarly if you extend it backward, and this is where it cuts, this is the place where f equals to 0, so when f equals to 0 I get 20 log of 0. Log of 0 equals to 1 and therefore I get minus 20 dB here, fine. So, this will be a straight line, so this is part two of your network. It is part two of the network and therefore that is what you see.

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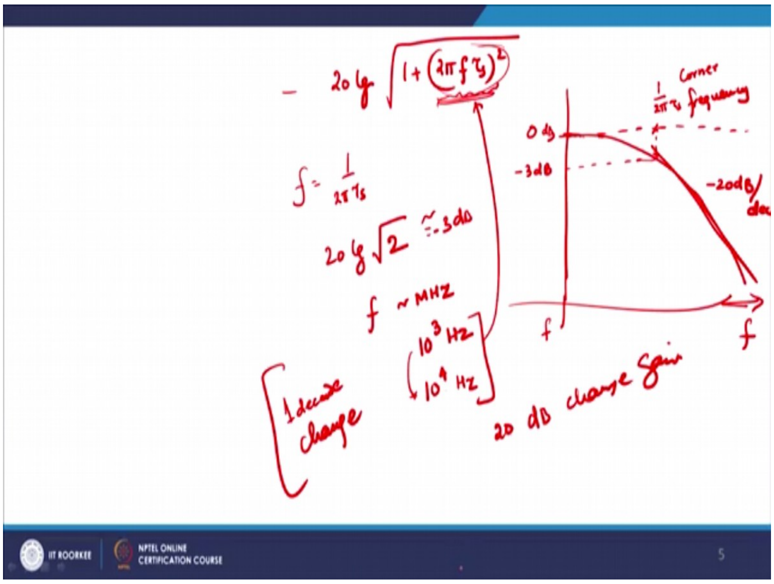

Let us go to part number three and that is basically a negative sign, so it will be 20 log of 1 plus 2 pi f of tau $_S$ whole square and then square root of this whole quantity. If you plot again the same thing, the same concept here that is the 0 degree here and this is your frequency in log scale. Then when f equals to 0, I get again, f equals to 0 means this will be whole term will be 0, implying that log of 1 will be equals to 0 and therefore at f equals to 0, I will get 0 dB, so I will get 0 dB here.

Now, as f goes on increasing, for low values of S f, this quantity will be very very small because square of that quantity and as a result it will still remain 0, right. But as f goes on increasing, this quantity starts to become higher and higher and therefore 1 plus that quantity goes on increasing because it is a negative sign attached to it, this will show a something like this, something like this drop will be shown to you and this will be the overall shape of the curve which you will see.

We define a new term here what is known as a corner frequency and it is given by 1 by 2 pi tau S and again you can understand the reason why, when f equals to again, as you can see, when f equals to 1 upon 2 pi tau$_S$, then I get log of root 2, right and this comes out to be approximately equals to 3 dB. So this is your 3 dB point, minus 3 dB. So this is therefore referred to as a corner frequency. So how do you define a corner frequency?

Corner frequency is the point where the frequency is equals to 1 by 2 pi tau S and where the gain has fallen to 3 dB, minus 3 dB from its actual value, so the original value was 0 degree, now it has fallen to minus 3 dB, so you have minus 3 dB, this thing is there, a gain is there. And therefore as you can, so this is what you get, now as f goes on increasing drastically higher and higher somewhere here, see if you give f a decade increase, for example if f was equal of the order of few megahertz and then you make it say it was 10 to the power 3 hertz, let us suppose and then you make it 10 to power of 4.

Then how does it influence your overall picture, right? That you must be aware of. Generally it is seen that if you feed these two for example figure into this formula, then for every so this is one decade change in the frequency, because it was 10 to the power of 3, it goes tend to power of 4, so this is basically a 1 decade change. So a 1 decade change in frequency should result in a 20 dB change in the gain. If you put it you will find these values here and therefore if you try to find out the slope of this graph, this graph here, it will basically be minus 20 dB per decade, which means that the gain actually falls down at the rate of 20 dB per decade, fine and that is quite interesting that for every 1 decade rise in the frequency, I would expect to see a 20 dB fall in the gain in the voltage gain in the output side.

So, this is what you get when you do a 20 dB, when 20 dB drop is available to me, this is for 20 dB per decade. So decade means you are increasing it by a factor of 10, 10 times increase or 10 times decrease is there in terms of frequency. So now I have got 3 mechanisms, so let me switch back to the basic concept here.

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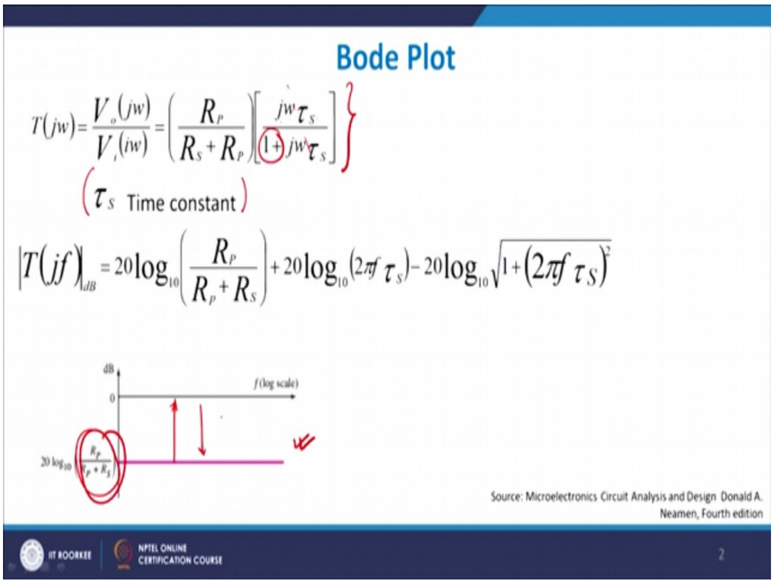

As you can see here the same graph which I was plotting for you, I got this as my frequency scale, this is 20 log R_P, assuming that R_P and R_S is very small, I can, this could be safely assumed to be almost equals to 0, if not it will be lower than this because obviously this quantity will always be smaller than 1 and therefore this will be slightly less than zero.

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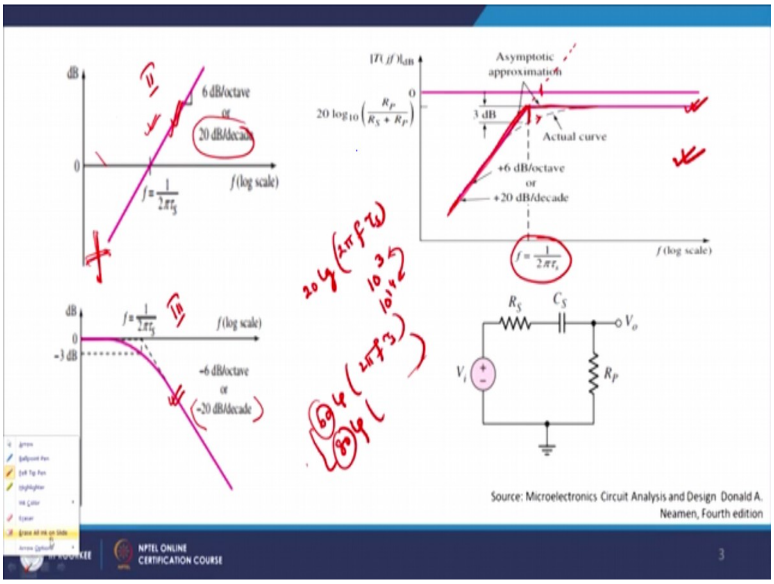

We come to the next case and as I discussed with you, this will be a straight line which will be a linear curve with a 20 dB per decade, you will have an increase and here you will have a 20 dB per decade decrease. So there is a negative sign at this particular point. Now you can see also why is it like this, this one, this is 20 log of 2 pi f of tau $_S$. So this was a definition which people saw, if I got 10 to the power 3 then this comes out, if I do 10 to the power 3, then this 3 will come out and come out to be 60 log of something in terms of 2 pi f of tau $_S$.

Now if it is 4, I will get what? 4 will come outside I will get 80 log of something, so you see there is a 20 dB change which you see here, 60 and 80, so for every decade change in the frequency from 10 to the power of 3 to 10 the power of 4, I expect to see a 20 dB per decade drop. So I get 20 dB per decade drop here, now if you add number 1 drop, number 2 and number 3, this is number 2 and if you add number 3 here and then put it all together at one graph then this is what you see here.

So what you see here is basically, since assuming that 20 log of 10 R_P upon R_S plus R_P is a figure which is slightly smaller than 1 by virtue of the fact that R_S is not negligibly small, then its constant value will be less than 0 dB and it will be a constant value independent of frequency, which is this curve which you see. And then if you superimpose on that this curve, the second one, then this is the second curve which you see in front of you. This crosses the 0 dB point at 1 by 2 pi tau S and then the third part is from this point.

So if you look very carefully, if you add this to this then you can see because of this linear fall here, right, you actually see a linear fall in the output voltage, output gain here. And as you move further in the frequency domain, it goes on increasing somewhere around this corner frequency here, you actually start to see constant value because the rate of rise here and the rate of fall here are exactly equal. And therefore if you add graph number two and graph number three, I will almost get a constant profile independent frequency, right.

Where this 1 by 2 pi tau S is basically my corner frequency, also referred to as frequency. Now if you look very carefully, this is basically a graph of low pass filter, sorry, high pass filter, why? Because at high frequencies you are allowing the gain to be very high, at low frequency the gain is very very small in dimensions.

So this is what we have learned or we know how to therefore deal with Bode's plot, how to deal with Bode’s plot, how to make the Bode's plot work for me. And this works because of the fact that you do have a frequency dependent term. I have to ensure that at every decade rise or fall in frequency, I have a 20 dB change in the value of your gain. If you consider that as a basic one you can carry forward. Similarly, you will always get at the corner frequency a 3 dB drop, right. This is the 3 dB drop which you will get at corner frequency. So this the actual curve, this is the idealized curve and this is the actual curve which you see.

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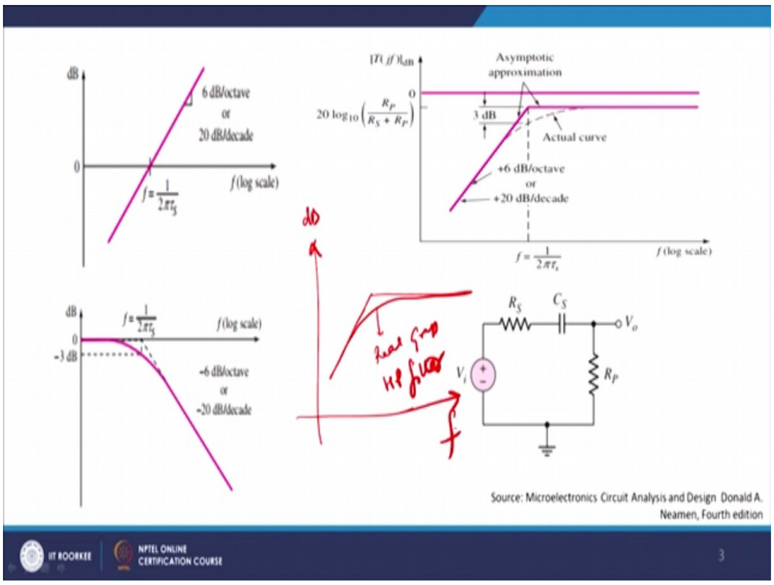

So if I remove all these things, then this is the actual curve, the dotted one is the actual curve, this one is the actual curve. So you see it is something like this, so it is like this, this and then the ideal one should be like this and this and the real one will be asymptotic here and then it will become asymptotic across this.

So this is my real graph and this is basically a real graph of HP high pass filter and this is your gain in dB and this is your frequency in decade in log scale basically and that gives you quite an interesting phenomena or at least gives me a first-hand implication of how you can design a basic filter also.

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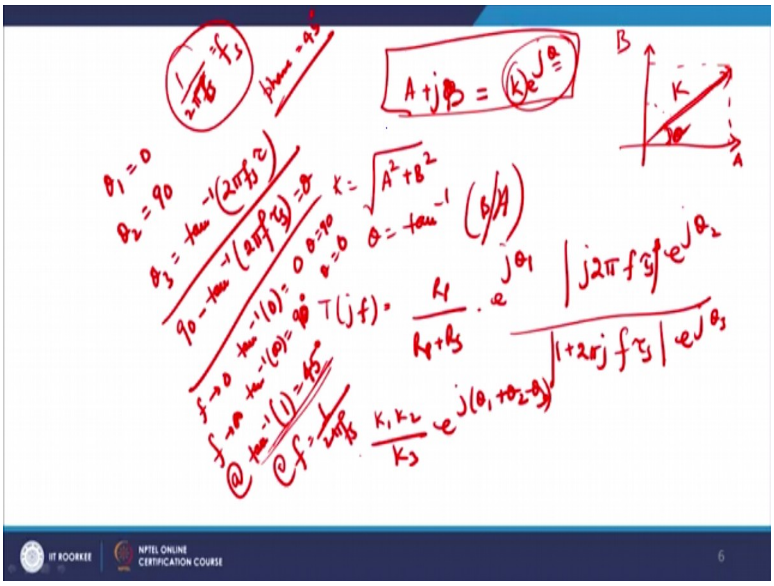

Let me go forward and explain to you but before I move forward let me explain to you about something about polar coordinate system, you can also refer to as A plus j beta right j B as equals to K e to the power j theta. So many a times we represent a Cartesian coordinate system to actually a polar coordinate system. This is A and this is your K theta, this is theta, this is K and therefore I get K e to the power j theta. This is B and therefore K happens to be equals to A square plus B square root over.

Whereas theta is equals to tan inverse B by A. So if you solve it I get T to the power j f was equals to R_P by R_P plus R_S, right e to the power j theta 1 mod of j 2 pi f of tau S, e to the power j theta 2, divided by 1 plus 2 pi j f of tau S, e to the power minus j theta 3. So if you solve it I get K 1, K 2 by K 3 e to the power j theta 1 plus theta 2 minus theta 3, because theta 3 is in the denominator. So I get when theta 1 therefore is equals to 0 and theta 2 equals to 90 degree, I get theta 3 to be defined to be as tan inverse 2 pi f S into tau. This is what we get it.

Similarly and therefore so the theta is basically 90 minus this quantity, this quantity right, so I get 90 minus tan inverse 2 pi f of tau S. This happens to be your theta. Similarly, therefore so I am trying to find the phase margin, when f tends to 0 tan inverse 0 is equals to 0. So theta equals to 90 degree and f equals to infinity I get tan inverse infinity is equals to 1, 90 degree in degree terms it is 90 degree and theta is equals to 0 degree and therefore and at tan inverse let us suppose 1 is given us 45 degree and this will happen at f equals to 1 by 2 pi f S.

So let me just recapitulate what we did by simple mathematical derivation that at corner frequency your phase margin will be approximately equals to 45 degree. The phase margin of the input, you got the phase margin from where I am getting it? See, if you look, see any complex conjugate quantity or complex quantity can be broken down into K e to the power J theta sort of term where theta is basically the phase margin, K is the gain which you see.

So this will be K and if this is theta which you see then I get that 45 degree angle. At 45 angle we will, what is the value of your corner frequency? Corner frequency is place where you get a 3 dB drop in your gain. At that point I get f is equals to 1 by 2 pi f S right and this is tau S sorry, tau S is equals to f S and at this frequency I get my phase margin to be equals to 45 degree and if you go on increasing the value of phase margin if you go on increasing the value of your this thing input, it goes to high value otherwise it goes to a low value.

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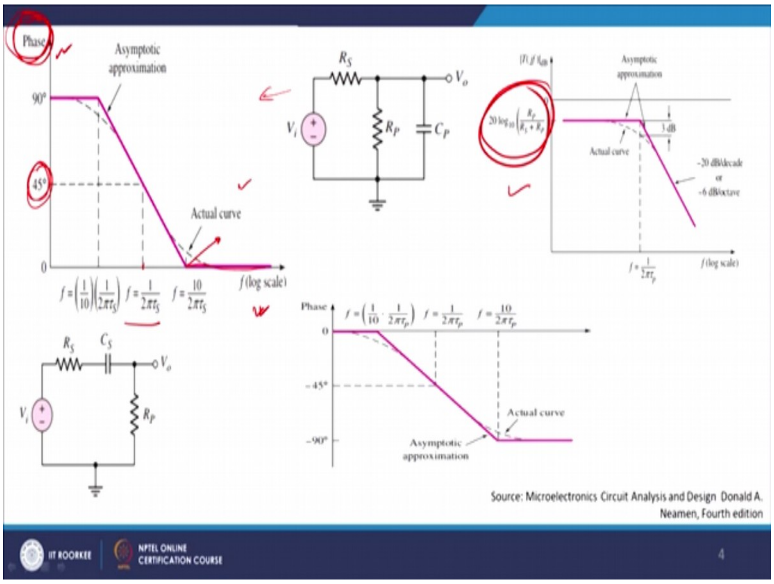

Now with this knowledge let me come to the short circuit and let me come to the previous slide. So this is what I was talking about, if you look very carefully therefore if you plot phase versus log scale for this filter design then we see that around, when it is corner frequency 1 by 2 pi tau S, your phase is basically 45 degree.

So 45 and then as you move forward the phase falls to 0 approximately and at very high values it is then, so when the frequency is very small you theta is equals to 90 degree. And when the frequency is very large, just now I discussed with you, the frequency theta value, the phase is almost equals to 0 degree and this is what I am getting here.

So it is almost 0 degree at very high frequencies and almost 90 degree at very low frequencies. At corner it is exactly equals to 45 degree which you see, so this is sort of a phase scale which you see and how the phase behaves with respect to frequency. We have just now seen how does a gain behaves with respective frequency. So these both are taken care of in a detailed manner as far as this one is concerned.

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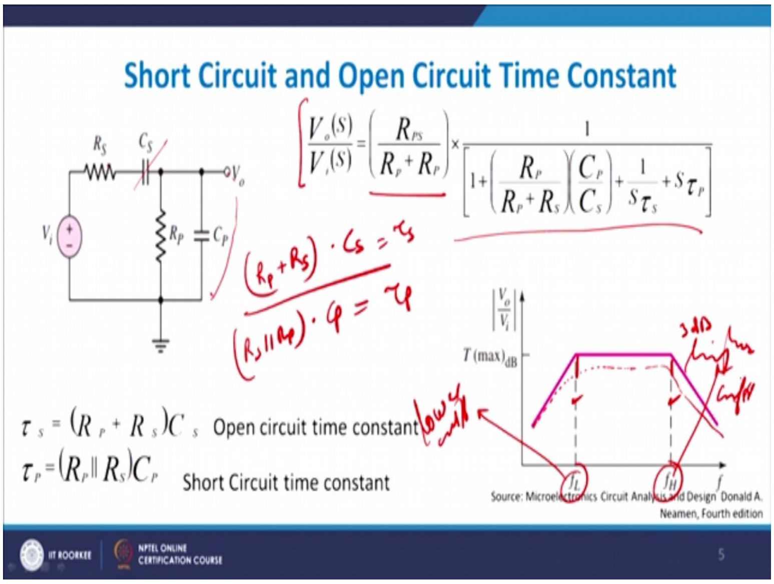

If we plot the graph of output voltage to input voltage and we try to find out the various principles here, we see that we can write down in this manner and given by this function, then we define R_P times R_P plus R_S times C_S to be as the open circuit gain. So if you look very carefully at this place, if it an open circuit means you open up C_P, C_P is opened up, so C_P does not come into picture. These two are in series, so I get R_S plus R_P multiplied by C_S and this happens to be your time constant tau_S.

Whereas if you short it when you are shorting it then C_S vanishes off C_S vanishes off, your C_P stays with you but R_S and R_P are parallel to each other. When you short this output voltage with respect to ground, then R_S is parallel to R_P and then this is in sense dot of C_P will give you value of tau P. So tau_S is basically the short circuit time constant when you shot the output and you do not have any C_P coming into picture and you only have C_S into picture and both R_S and R_P are basically series combination resistances.

Whereas when we talk of tau_P and make sure it is to be as a short circuit. So we short it and we try to find out the value of R_P parallel to R_S. So we define two sort of boundary elements here, one is known as F_L and F_H, F_L and F_H are the points where your 3 dB bandwidths have been maintained. So if I actually plot the graph it will be somewhere like this something like it will come out and it will follow like this asymptotic and it will go down something like this.

So this drop is basically your 3 dB and we define F_L to be as the lower cut-off frequency, this is known as the lower cut-off and this is known as the higher cut-off. So I have a higher cut-off, I have a lower cut-off and this tends to make the difference between the two bandwidth available to us.

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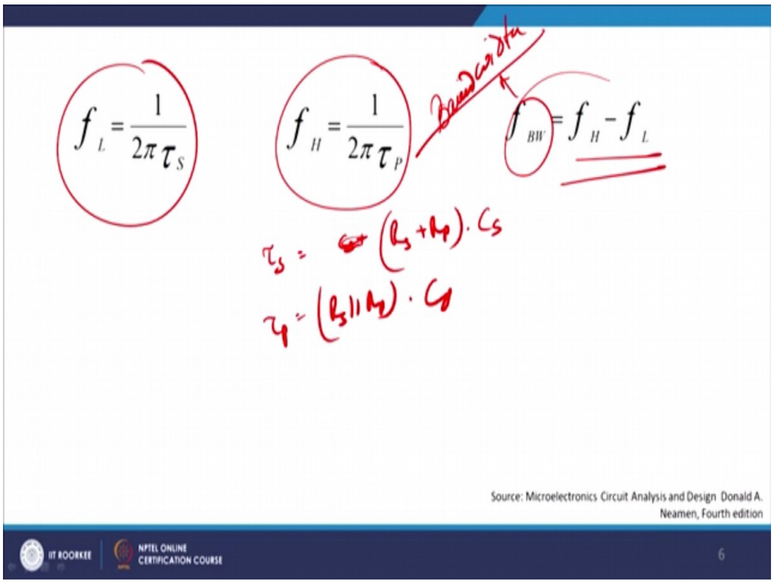

So if you look very carefully therefore what we finally get is that the lower cut-off will be given by this quantity and the higher cut-off is given by this quantity where tau$_S$ is the short circuit gain given by C_S, R_S plus R_P multiplied by C_S .Whereas tau P is given as R_S parallel to R_P multiplied by C_P and this is what you get. And f_H minus f $_L$, high cut-off frequency minus low cut-off frequency is defined as my bandwidth. So this is my bandwidth which I get, this is my bandwidth, so that is the difference between high and low cut-off frequency.

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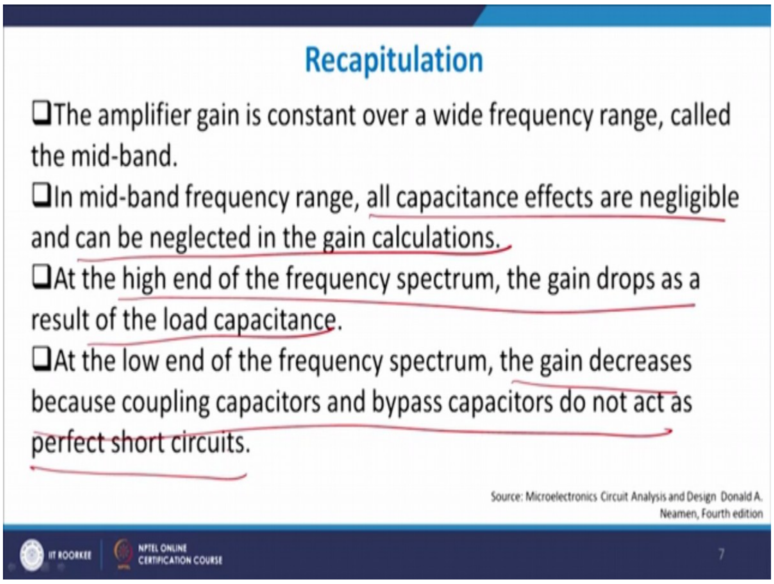

We come to the, let me recapitulate therefore, generally amplifier gain is constant somewhere in the mid frequency band. At very high and low frequencies the gains starts to fall down for high frequencies fall down with increase in frequency. At low frequency it falls down with decrease in frequency. The mid-band frequencies are all the constant and in that case all the capacitive effects are negligible and can be neglected in gain calculation.

So in gain calculation we do not have any capacitive effects coming into picture. At a high end of the frequency spectrum, the gain drops because of the load capacitance, we have discussed this point already. Because of a heavy load capacitance your system has to actually charge or discharge the capacitor at in the output side and it therefore takes time to do it and that is the reason your frequency, the gain drops down drastically. At the low end of the frequency, the gain also deceases because of the coupling capacitor and bypass capacitor do not act as perfect short circuits.

So there are two reasons why a gain is getting lowered with increasing (freq) with decreasing frequency in the input side when the frequency is low. So, if you remember X_C is which is the capacitive reactance is 1 by j omega C. So if your omega is very very small, X_C is typically very large and as you lower your value of omega, X_C still goes on increasing.

As a result what happens is that the resistance offered is going on increasing and it does not let the signal to pass through from point A to point B. As a result, you always have a loss of gain whereas at very very high frequencies typically high frequencies what happens is that as the frequency becomes very large, the 1 by j omega C as usual drops down and therefore it starts to short your output and therefore the gain starts to fall down in that case.

So for both the reasons you do have a gain drop which is there with us for all practical purposes. So in this way we have finished the concept of frequency spectrum, high frequency mid frequency and low frequency spectrum and how we are able to extract that from a system, what is a Bode's plot and how a Bode plot is generally plotted in a log scale. So we have done all these things and the next time we will take up common emitter and common base configurations for high frequency modelling. Thank you very much.