Advance Power Electronics and Control
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Lecture – 38
Linear and Non Linear Control in Power Electronics - II

Welcome to our lectures on the advance power electronics and control. We were discussing about state space averaging modelling in our non-linear and the linear control. Let us go to the slide actually which I was discussing. So this is, basically we have to go back little bit. So we were here.

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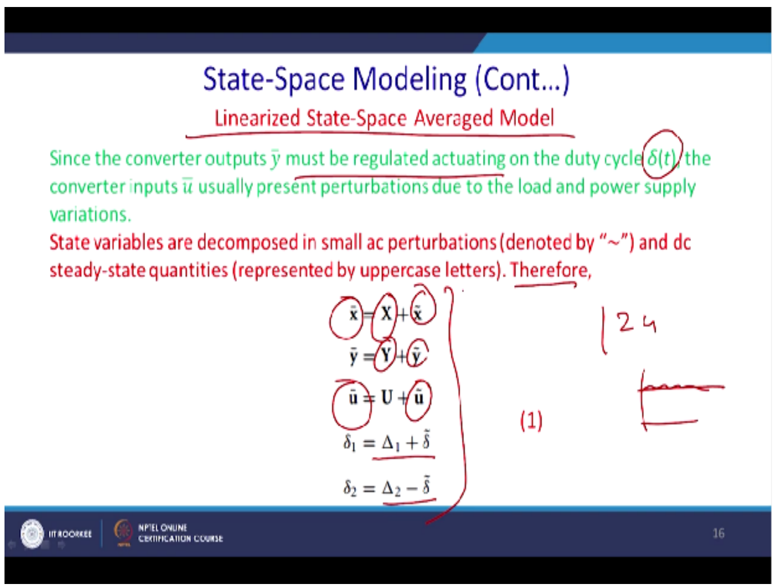

Basically we were discussing about the state space average modeling. So state space average modeling are decomposed in small AC perturbations denoted by ~ and the steady state quantity. It is represented by the uppercase letter.

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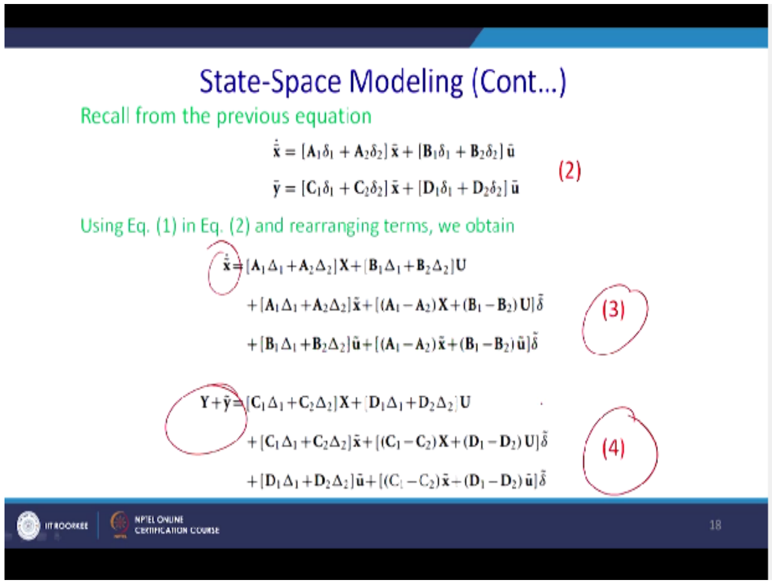

So let us come to this point. So similarly we have discussed about actually x~. This will have the derivations. Similarly, we will have a y~.

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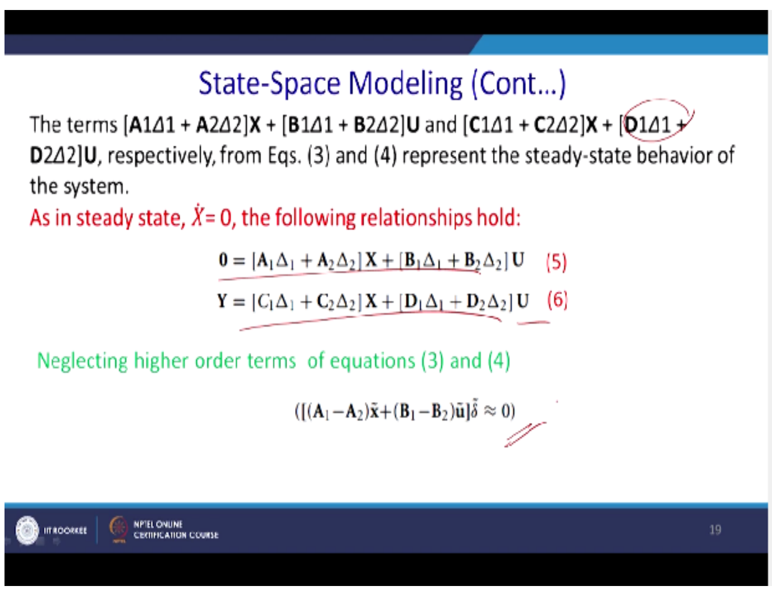

And thus there are the terms, actually the A1delta1 A2delta2, and the average value X are the steady state values. So it does not feature out as a perturbed quantities. Similarly, we want delta1, B2delta 2 and the u are also the DC values and C1delta1, C2delta2*X, this is also (()) (01:51) values and this, it is not there, that is a different issue in our condition. In most of the cases that the D matrix was 0, D1delta1 and D2delta2*U are the other DC quantities.

Thus eliminate those DC quantity from it and you will have these overall equations. And thus actually from this equation 3 and 4, you have to short it out the DC portion. So you can rewrite since that X dot should be equal to 0. So it is a steady state condition. So ultimately 0=A1delta1+A2delta2X+B1delta1+B2delta2*U and Y=, similarly, C1delta1+C2delta2*X, definitely, +D1delta1+D2delta2*U.

So we can actually, neglecting this higher order terms in the equation 3 and 4, see there will be a multiplications of actually delta 1 delta, so this kind of term has been actually no point of multiplying those terms since this gives you a very small values. And thus what we can rewrite is that again that A1-delta1*delta x B1-B2*delta U*del U should be equal to 0. So these are the 3 actually conditions. This can be rewrite as equation number 7 also.

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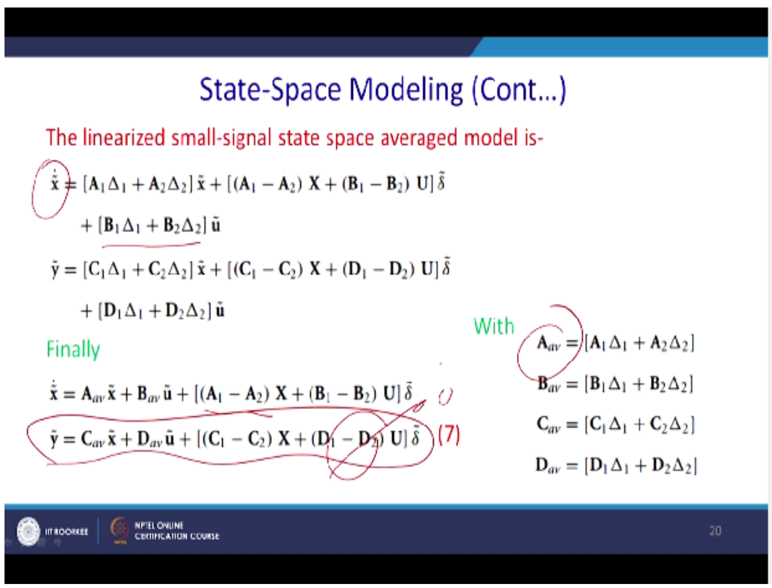

Thus we can linearize with that help and ultimately x~ can be actually substituted, those terms which can be equated to 0, ultimately A1delta 1+A2delta 2+x A1-A2X B1-B2*U to delta and B1*delta 1+B2*delta 2*u. So there should not be any term without ~. So all the DC value has been eliminated from the overall equation.

Similarly, y~ will have this conditions, you can actually derive these equations. After deriving these equations since actually A average we can write, that is A1delta1+A2delta2. B average is B1delta1+B2delta2. Similarly, C1 and C2. So thus we can write x~ dot=A average*x~+B average*U~ A1-A2*X+B1-B2*U*delta. Similarly, the output matrix that is given by the y. Most of the cases, this is equal to 0.

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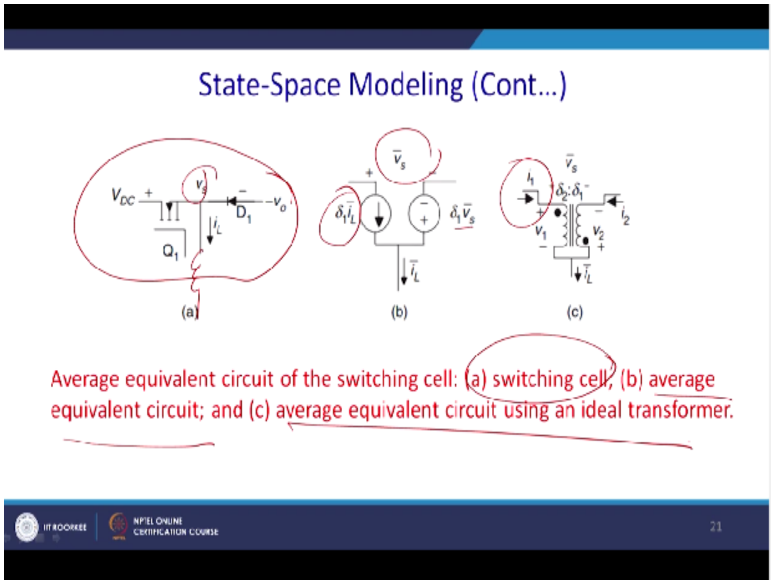

So ultimately you go back to your buck-boost converter. This is actually the switching configurations. So you got a switch and thereafter diode and you got an inductor here. So this should be on and off depending on the duty cycle and that actually the duty cycle is delta 1 and is the current control source, so that will be equal to I*del 1. Similarly, it will be a delta1*Vs but potential difference at this point is Vs, it is blocking the voltage Vs.

And thus we can actually model it as a transformer having actually trans-ratio, delta2 and delta1 because it is a reflected trans-ratio. So you are talking in terms of the current. So it will have that is the inverse. So current I1 is flowing and current I2 is flowing and we can have actually a transformer model where I1 will have a trans-ratio delta1 and I2 have a trans-ratio delta2. So average equivalence circuits, this is for the switching cell.

So we can 0ed down to the equivalent average model is this and we can visualize an ideal transformer as like this. So we are telling several times that we cannot use transformer in DC but with the DC to DC converter, we can have all characteristics may be like an AC transformer. So that is the beauty of AC to DC transformer.

Ultimately polarity is reversed because you are getting that polarity reverse there. If the polarity is same, that dot would have been the same point. So see that how we can derive the transformer, similar transformer in AC in case of the DC. So it is a DC to DC converter. Now let us go beyond it. So hope that you are getting the juice of the subject right now.

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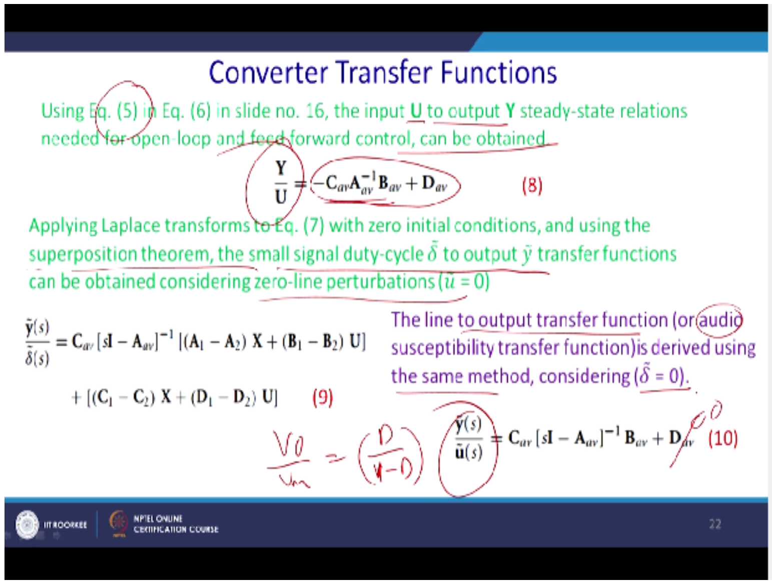

So using this equation 5 and 6, please refer to this equation 5 and 6 because I will be recalling. So this is the 5 and 6 because this is the steady state DC values. The slide number 6, the input U to the output Y steady state relations needed for the open loop and feed forward control, can be obtained. That is the transfer function, that is the output/input. So same thing we can get if we have, if you know this basically the A matrix C matrix and the B matrix.

And fortunately, we cannot have a transfer functions because this one is time variant as well as non-linear. But once you have made an average, of course, it is linear. Thus this is the transfer function. So you can come down to the transfer function and once you get a transfer function, all those actually, all those things you have learnt in your transfer function technique like the Bode plot, polar plot, root locus, all those analysis can be done here.

So see that how we can march from the actually state space to the transfer function. So applying if the transfer function to the equation 7 with 0 initial condition, these are the condition, using the superposition theorem, small signal duty cycle of the output transfer function can be obtained considering 0 line perturbations that is u=0, that is you have considered no input variations that is the load variation.

Of course, you can also take a consideration that because but that is also quite natural like in buck-boost converter that source itself is varying that is let us say solar panel. But in all present analysis, we have negated it. So ys/delta s, you can write C average Is-A average A1-A2*X B1-B2*U. Similarly, you can have C1-C2*X+D1-D2*U. Thus what happens? The line to output transfer function or the audio, we sometime say it comes out in the frequency domain in the audible susceptible limit because we can hear the range of 20 KHz and for this, we sometimes say that it is audio susceptible limits.

But nothing to do and much so we can take this is, for this overall the result when we take or try to take our switching frequency above to 20 KHz that, we will get rid of the noises, sound noises. But of course, you will increase the EMI EMC problem. So susceptibility of transfer function is delivered using the same method and considering that delta~=0, that is variation of the duty cycle is 0, that is in a steady state only.

So you know that actually V0/Vm is something D/1-D, we have studied it for the, actually this kind of converter and this holds that when delta~=0. Thus we are coming to the same thing on the state space analysis. Similarly, ys/us that is essentially the perturbed quantities, so you can write it actually C average sI-A average/this one, generally this value is 0.

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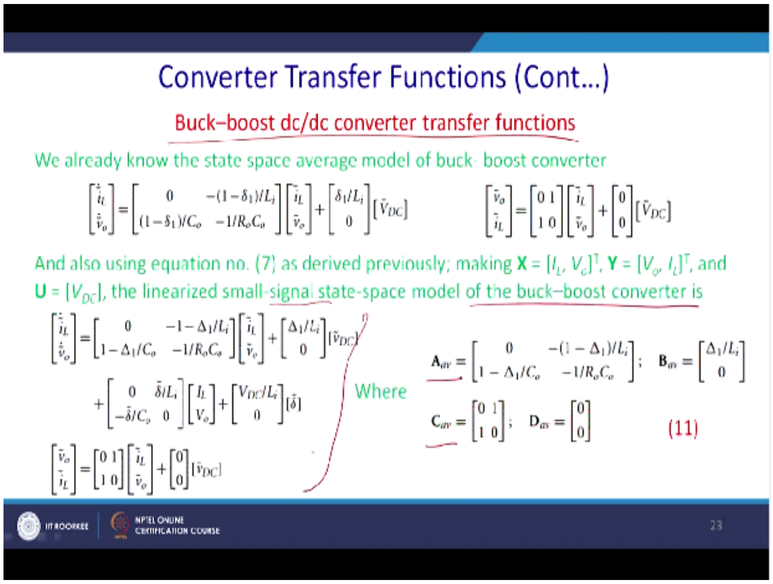

Now let us see Buck-boost converter transfer function. So you have written this matrices previously. So you can combine by this equations, also using this equation 7 derived previously making X is basically the current through the inductor and the voltage across the capacitor as V0 and essentially y is you can take it V0 as well as IL and U=of course, it is the input VDC.

And linearize the small signal state space model of the buck-boost converter. You can rewrite is the substitution and this is basically the A average and this is the basically the C average and D matrix is telling it should be equal to 0 and thus you get these equations. You can substitute here. From there, we can get the transfer function.

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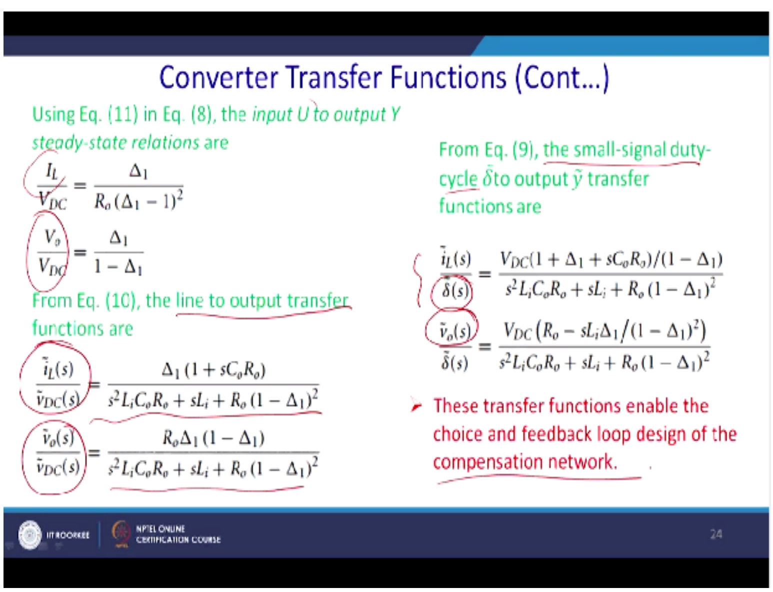

So what else? So see that what are the beauties of this transfer function which you might have done. The derivation has been done in other way from the differential equations. Same thing can be derived from the transfer function directly. So that is the beauty of the transfer function analysis. So using the equation 11 in 8, input U to the output Y that is IL/VDC, that is delta1/R0 delta1-1 square.

Similarly, V0/VDC that is output by the input transfer function that is you know that is, I was writing D/1-D or delta1 1-delta. So see that how we have derived the same thing from the state space equations. In the meantime, we have commented over the switching frequency. In the meantime actually if you are actually familiar with the state space equations, we can derive also the transfer functions.

All the transfer functions are been actually been put her. Similarly, the line to output transfer functions were actually with the ~ will have basically this/this and with v0/vDC, will be this and we may actually go for actually Bode plot. We may go for the polar plot and see that this is stable or not. And accordingly, we design the controller and we have a different kind of controllers.

We can start with the PI controller. They are the PID controller. There are many other form of controller. Equation 9, the small signal duty cycle of delta 0 to the output y transfer functions are that is basically you can rewrite it that in terms of the delta and we shall try to find it out whether the system is stable with this delta and this is basically the same way for the output voltage versus delta. So this basically help us to choose a feedback, what you want to feed it back.

There is a different actually transfer function. And so designer can choose either one of or two of it. This transfer functions enable the choice of the feedback loop design of the compensation network. We shall see just now.

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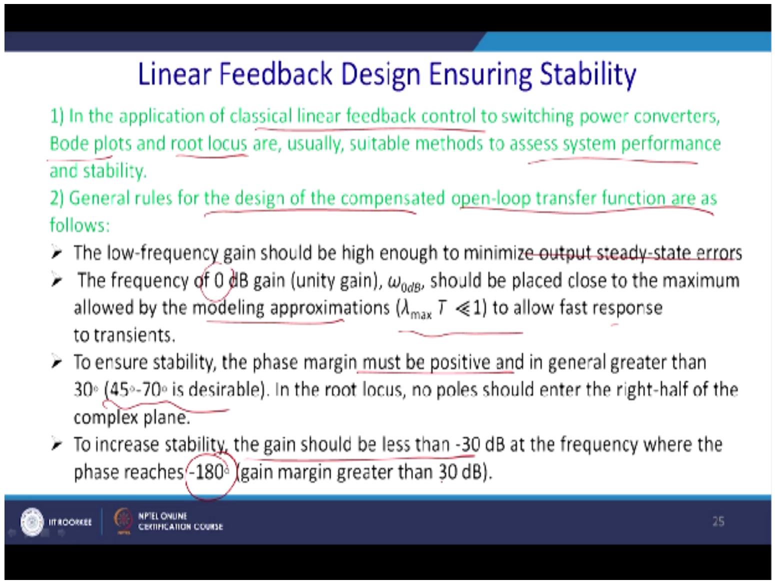

Now what we want basically the system to be stable first of course? And that is what we do. In the applications of the classical linear feedback which you have studied in your control system to switching the power converters, so we have while he was telling us the Bode plot, root locus, polar plot, Nyquist are usually suitable method to assess the transfer function. Now you got a transfer function and you have a Matlab, so it is very easy to analyze now a days.

The general rules to design the compensation if you are familiar with this basically on the Bode plot domain, then only it is applied. Otherwise, if you are familiar with the transfer functions of course, you have better method but we just like to link actually transfer functions and the state space. Open loop transfer function as follows. The low frequency gain should be high enough to minimize the output steady state errors, that is the ideal op amp characteristics.

Please recall the ideal op amp characteristics. We have the statement there. The frequency of 0 dB gain or unity gain or actually omega 0dB should be placed close to the maximum allowed by the modeling approximation that is we have said lambda max should be less than 1 and to allow the first transient response. So we have same transient condition, overshoots, all those criteria of the system but all our knowledge was restricted to the second order system.

But here it does not put any limitations of the dimension of the matrix, though we have a second order system anyway. So this will ensure that the fast response. And if you talk about Bode plot, then gain margin and phase margin will be there. To ensure stability, the phase margin must be positive.

Otherwise, we have to put a lead lag compensator. And in general, it is greater than 30 degree, 45 to 70 is desirable. Otherwise, what will happen, if it is actually 90, then system is very sluggish. So we want also the fast response, right. So it has to be greater than 30 and we try to tune it in a range of the 45 to 70. So that is the task of the compensator.

So once you design and play controller, so you should have the gain margin and phase margin within these conditions to operate this converter successfully. So the root locus or no poles should enter into right half of the x plane or if you are taking of the root locus as a tool to analyze.

To increase stability, the gain should be less than -30 dB at the frequency where the phase reaches -180 degree or we can say in other words that his gain margin is greater than 30 dB. So that is something the criteria we should obtain to have a satisfactory operations of this converter, of any converters. But we have taken an example of a very common topology. So now you see that how you can do that. Now linear feedback design ensuring the stability.

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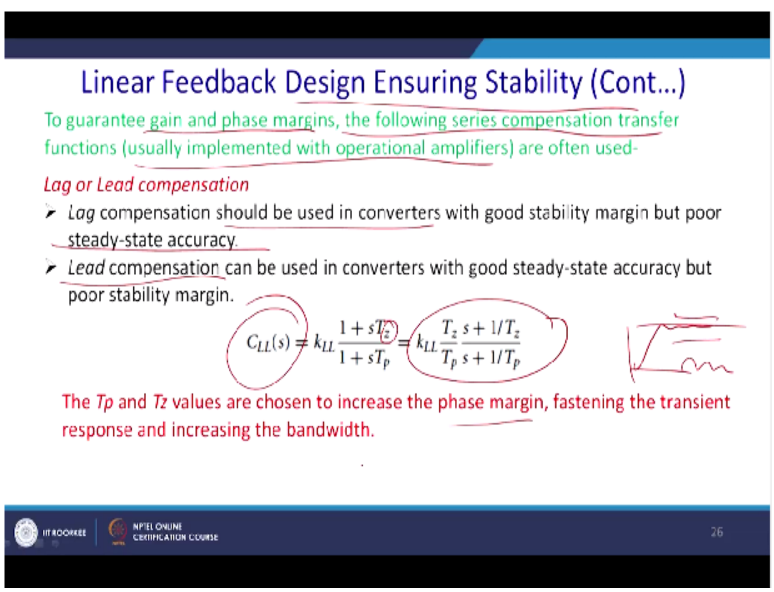

So this is basically now when you are in a transfer function domain, you talk about specifically the Bode plot, Nyquist plot, polar plot. These are essentially you have studied into the linear control system. And essentially this is an average model. And it also works but it has a limitation. For this reason, we require to switch over to the non-linear control. But let us take all the juice available with the linear control.

Then only we shall see that, actually we see the limitations of the linear control and go to the non-linear control. So to guarantee the gain and the phase margin which was the said before, that should be actually, phase margin should be around 45 to 70 and the gain margin should be 30 dB. The following series compensators, actually compensation transfer function usually implemented with the op amp or you can now a days, you can do it with the digital domain and is often used as a lead lag compensator.

What happen actually if you chose an op amp and you set up the circuits and its value is fixed. But sometimes what happens, if you set by programming and you can dynamically set this gain margin and phase margin then that has a big advantage. And for this reason, we are gradually switching to the digital domain. The lag compensation must be used or should be used in the converter with good stability margin but poor steady state accuracy.

So your steady state error will be there. So please recall your this kind of thing. So it will reach the stable limit after sometime. Lead compensator can be used in converter with a good steady state accuracy. So to reach and it will be contained with the, will not be much repel in the DC output voltage. So what does it physically signify these terms? The lag compensation should be used in the converter with a good stability margin but poor steady state accuracy.

If load has changed, it will recover it faster. Let us say load has been changed to 1 amp to 2 amp within a limit of (()) (22:51), then voltage has sacked. But will recover very fast. But this lead compensation will have, what we will do? Lead compensation will actually recover it slowly but it will ensure that it is ripple in the DC voltage is within a limit, it is less than the lag compensator.

So we have to play around it. And see that actually you will have a design specifications, this much of the DC ripple, this much is actually the rise time and this much is actually load change settling time, all those quantities will be there, accordingly you are required to design it, right. So ultimately you got basically CLL, that is the output matrix that is KLL1+sTz/1=sTp where Tz are the poles of this transfer functions and the Tps are the, actually where Tzs are the 0's of the transfer function and Tps are the poles of the transfer functions and we are getting this.

So we can rewrite KLL Tz/Tps/1/Tz by so, you get, basically you can design the lag lead compensator like this. Tp and Tz values are chosen to increase the phase margin, faster transient response and increasing the bandwidth of the system. So this is the way you can actually do the compensator. Now of course the typical compensator terms in our mind are the PI controller. Proportional plus integral compensation.

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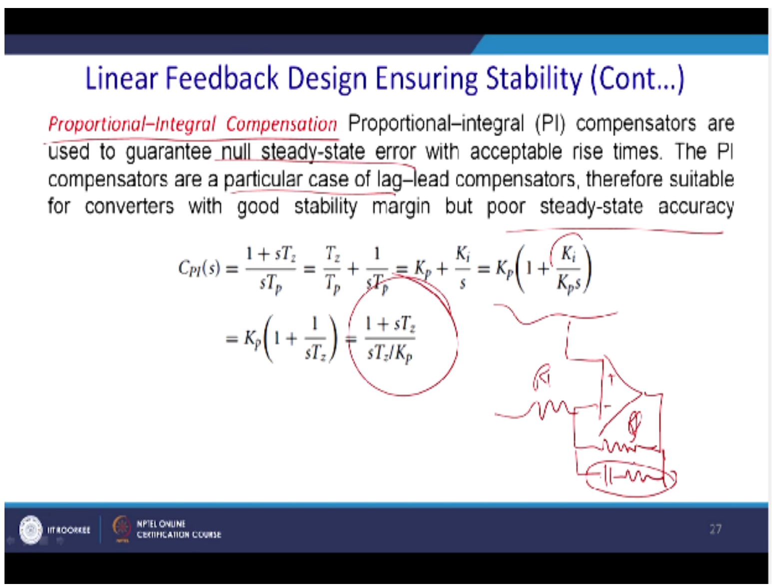

Proportional-integral compensator are used to guarantee steady state error with acceptable rise time. The PI compensator are a particular case of lag-lead compensator. Therefore, suitable converter with good stability margin but poor steady state accuracy can be achieved. So this is actually PI controller. So 1+sTz/sTp. So you can split it like that and you can rewrite as Kp+Ki/Ks. Or you can write it this term.

So this is the simple PI controller and which you can easily design by an op amp. So you can have a proportional controller. So that is basically R1 and that is Rf and thereafter, you will have a capacitor, that will, in a feedback path. So there bandwidth and you have to have a realistic circuit and this will give you basically the Ti. So accordingly basically you can, this ratio can be rewrite as 1/Tz, so this ratio can be Tz. And Kp is the gain of Rf+Ri. So this is a very simple way to design and this compensator can be used.

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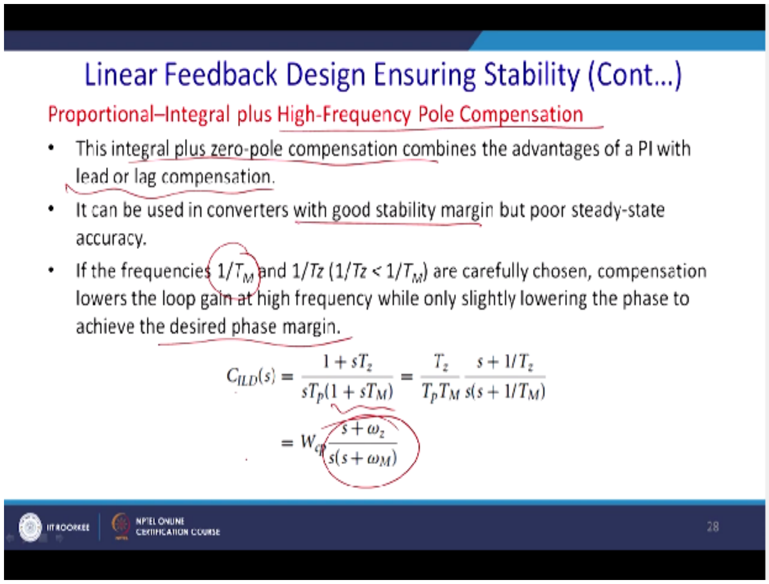

Now we have discussed the limitations and if you are working practically, ultimately we think that PI controller is the panacea that will offers you the solutions. But frankly speaking, no and tuning of the PI controller is quite tough job. And for this reason, we have different controller also. Proportional integral plus high frequency pole compensation. So what does it do? The integral plus 0 pole compensator combines the advantage of the PI controller with the lag lead compensation.

It can be used in converter with good stability margin but poor steady state accuracy. But steady state accuracy is given by the PI controller. So this basically are the complementary of each other. The frequency of 1/TM and 1/Tzs and these are carefully required to be chosen and the compensations, these will be given into the assignment, please remember that compensation lowers the loop gain at a high frequency while only slightly lowering the phase to the achievement is the phase margin.

So this is the linear feedback design using basically PI controller plus pole placement method we say. So it is CILD, so basically what you have, it is the same plus you have placed the pole. So ultimately, so Tz/pTM, so there will be a little modification. Ultimately it will be a second order system and this pole will ensure that actually with a good stability and there is low steady state error. Now there is another linear technique and which is used very frequently.

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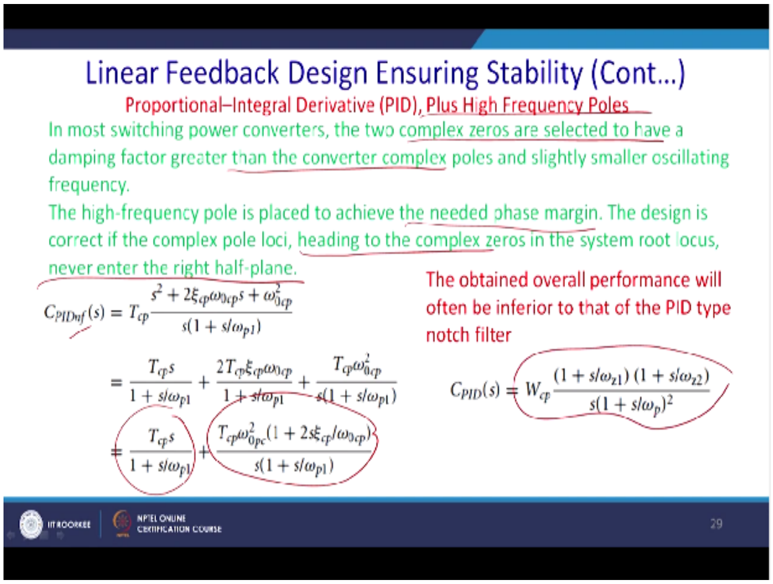

These are the proportional integral plus derivative with the plus high frequency pole. One of the advantage of the derivative controller that it can predict the error because it will actually work on the derivative of the error. And thus it can take the corrective actions while seeing that actually rate of change of error. So thus it can act on the slope and correct that actually the, correct it very faster but there will be a steady state error.

And that required to be eliminated by the PI controller. So most of the switching power converters, 2 complex 0's are selected to have a damping factor greater than the converter complex poles and slightly smaller than the oscillating frequency. So you have a combinations of the LNC and thus you get a natural frequency of oscillations. So what we require to choose? That this actually the converter complex poles slightly smaller than the oscillating frequencies.

The high frequency pole is placed to achieve the needed phase margin. So it will ensure that the phase margin you require around 45 degree or something like that. The design is correct if the complex pole locus, if you have multiple poles then it is loci, is heading to the complex 0 in the system of the root locus. And never enters into the right hand of the x plane. That is actually the optimality and the very fast design and it approaches the steady state, then it smoothens out.

So it is just like a lift, actually going at a very high speed, then stops jerkless. So you can write, we have written actually say PID with the pole factor. So ultimately we will have a quite big term, s square+2zeta omega n, then omega 0 is the natural frequency of oscillation and say pS*omega 0, so you can calculate and ultimately you will have this term as well as this term. This term is the PID and this term is essentially is coming from the high frequency pole placement.

To obtain the overall performance, PID is often inferior than the notch filters. So ultimately you can actually combine them and ultimately you get this kind of thing and then something it can have a response of a notch. We shall continue our discussion in our next class and we shall start from this slide also because this slide require more explanations. Due to lack of time, I have to stop here. Thank you.