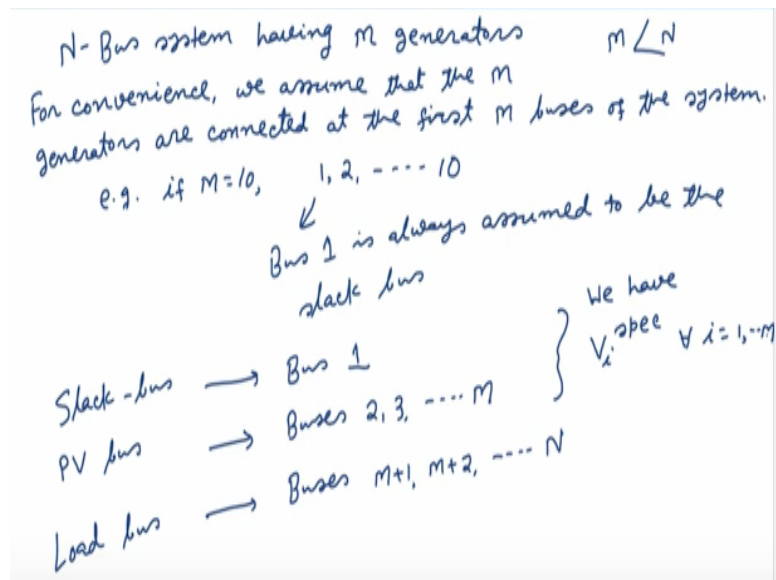


**Computer Aided Power System Analysis**  
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**Lecture - 09**  
**GSLF with Multiple Generators**

Welcome to this module of this course computer aided power system analysis. Now in the last lecture we have discussed about the GSLF load flow, Gauss - Seidel load flow for a power system which has only one generator. Now in this lecture we would be looking at the procedure for Gauss - Seidel load flow in which there are multiple generators. Basically, which is essentially we would be discussing the practical case of a power system and we will see how this Gauss - Seidel load flow equations can be applied for solving this power system network. So let us start.

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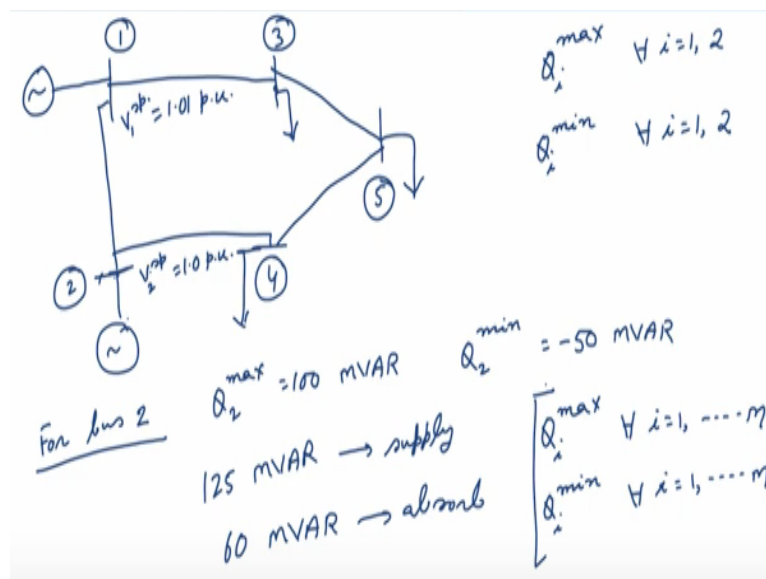
So what we have, we have got on  $N$ - Bus system and having  $M$  generators. So there are  $N$ - Bus system having  $M$  generators. Obviously, it means much less than  $N$ , should be. And for convenience, for convenience we assume that the generators, the  $M$  generators are connected at the first  $M$  buses of the system. So then therefore for example e.g. if  $M$  is let us say 10 so then these generators are connected at bus 1, 2, 3 to 10.

And out of this bus 1 is always assumed to be the slack bus. Bus 1 is always assumed to be the slack bus. So then we have now 3 types of buses. One is slack bus that is bus 1. We have got PV bus. We have already seen what is meant by PV bus. PV buses are nothing but those buses at which generators are connected physically and this buses are buses 2, 3, up to M and then we have load buses, load bus.

These buses are nothing but all the non-generator buses so that is M + 1, M + 2 up to N. Now because we have got M generator so each and every M generators has got their own voltage magnitude specified. So then therefore all this M generators have got  $V_i$  specified for all we have  $V_i$  specified for all  $i = 1$  to M. And now because these are generator buses and each and every generator can only supply or absorb a certain amount of reactive power.

So then therefore each and every generator has also got its reactive power generation absorption limit. Now before we write this, let us first try to understand what is meant by this.

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Suppose I do have a very just a simple system let us say a 5 bus system, just simple 5 bus system; just making an arbitrary connection. So this is bus 1 and this is let us say bus 2. So this is bus 2, this is bus 1 and 3, 4, 5. At this bus there are loads, there are loads, there are loads. And at this bus there are generators. Now at this bus as well as this bus so then therefore I have got  $V_1$  specified. I have got  $V_2$  specified.

Now when I am saying that for this generator I have got something called  $V_1$  specified and for this generator I have got something called  $V_2$  specified, what does it mean? It means that there are excitation system for this generator as well as this this generator which will try to maintain the voltage at this bus at this value and actually at this value for at this bus. For example if let us say  $V_1$  specified is 1.01 per unit and let us say  $V_2$  specified is let us say 1.0 per unit.

So then therefore the excitation system at generator 1 will try to always maintain this bus voltage magnitude at 1.01 per unit and the excitation system of generator 2 will try to maintain this bus voltage at 1.0 per unit. Now what happens? Now suppose there are some heavy loads in the system. For example if this load changes or if this load changes or if this load changes so then what will happen?

Because this load are increased so then therefore they will draw much more amount of current and because they will draw much more amount of current this current have to be supplied from this generator so as a result there will be more drop in all the line. So then consequently these bus voltages will drop. So then therefore these bus voltage also will be dropped. Now if this bus voltages are going to be dropped so then what will happen?

This excitation system for this generator as well as this generator will try to maintain this bus voltages by injecting more reactive power to the system, right? Now if this reduction in the bus voltages are within certain limit so then possibly then these excitation systems will be able to maintain this bus voltages at this particular value let us say 1.01 per unit and 1.0 per unit.

But if this loading conditions becomes much more severe such that the entire bus voltage profile have got very much reduced so then what will happen? This excitation systems will try to maintain this voltage by injecting more and more reactive power. But because of this as well as the physical rating current carrying capacity of all the windings both this field windings as well as the armature winding, each and every generator can only supply up to a max value of reactive power.

So then therefore each and every generator can only supply a maximum amount of reactive power which we denote as  $Q_i \text{ max}$  that is the maximum value of reactive power which they can supply for let us say  $i = 1$  to  $2$  in this case. So then  $Q_i \text{ max}$  denotes the maximum amount of reactive power which this generators can supply because of their physical rating other conditions. Similarly, on the other hand if some loads are thrown off. So then what will happen?

All this bus voltages in the system will try to rise, will rise because these loads have been thrown off. So then in that case this excitation systems will again try to maintain this bus voltages at 1.01 per unit and at this bus at 1.0 per unit by absorbing reactive power from the buses, right? Now if the amount of load thrown is within a certain limit so then therefore possibly this excitation system will be able to maintain this bus voltages at this particular specific values by absorbing adequate amount of reactive power.

But then if the amount of load thrown is more so then what will happen? This bus voltage profile will be quite high. So in that case these excitation systems will have to absorb more and more reactive power to maintain this bus voltage magnitude at 1.01 per unit as well as at 1.0 per unit. Now again each and every generator because of its physical limit, because of its current carrying capability constraint of the field windings as well as the armature windings can only absorb a certain amount of reactive power, not more than that.

So then therefore we also say that for each and every generator there is also some quantity called  $Q_i \text{ min}$  for all  $i = 1$  to  $2$ . Now  $Q_i \text{ min}$  means that essentially that it is the maximum amount of reactive power which it can absorb and  $Q_i \text{ max}$  means maximum amount of reactive power which can deliver. Now obviously if the amount of actual reactive power absorbed or delivered by any generator is within its respectively made so then we can safely say that the bus voltage at its terminal will be maintained at the specified value.

On the other hand if the actual reactive power which is to be supplied by each and which is to be supplied by a generator becomes more than  $Q_i \text{ max}$  so then physically this generator would not be able to supply that amount of reactive power. Let us take a simple example. For example let

us for bus 2, say for bus 2 at some loading condition, say for bus 2  $Q_2$  max is say 100 MVAR suppose and let us say  $Q_2$  mean is let us say -50 MVAR.

This minus stands for that it is absorption and plus means that it is generation. It is actually supply. Now suppose at some particular severe loading condition right it is needed that this particular generator too should supply 125 MVAR, should supply 125 MVAR to maintain the voltage at its terminal at 1.0 per unit, should supply. But then because this particular generator has got a maximum limit of 100 MVAR.

So then obviously this generator would not be able to supply 125 MVAR. It would be only able to supply 100 MVAR. So then therefore what will happen? Because this generator is supplying less than what it should supply so then in that case this particular voltage will not be maintained at 1.0 per unit, it will be maintained at some lower value, correct?

So then therefore when the actual generation of the generator is less than the value which is desired to be supplied by the generator then the terminal voltage magnitude at that generator terminal will not be maintained at the specified value. Similarly, suppose for example that some loads are thrown off so as a result the voltage profile of the systems becomes pretty high. So then therefore in that case, so we should write this is supply.

So now when there is a there is load thrown so then suppose that under that condition this generator should absorb say 60 MVAR, say should absorb 60 MVAR, we should write absorb 60 MVAR such that this voltage is maintained at 1.0 per unit. But then this generator can only absorb up to 50 MVAR. So then therefore it cannot absorb 60 MVAR because of its own limit. So it would be only able to absorb 50 MVAR.

Now because it would be only able to absorb 50 MVAR so then therefore this bus voltage will not be maintained at 1.0 per unit. This bus voltage would be maintained at some higher value than 1.0 per unit. So then therefore again if the actual amount of reactive power absorbed by any generator is less than the amount to be absorbed by that generator ideally then the terminal

voltage of this generator will not be maintained at its terminal voltage, it would be maintained at a higher voltage.

So then therefore when we are doing this load flow solution we should also take into account this limits of reactive power limits for all this generators. So then therefore what we say is that for all generators in general we do have something called  $Q_i^{\max}$  for all  $i = 1$  to  $M$  and for all  $Q_i^{\text{equal}}$  to and we have got  $Q_i^{\max}$  for all  $i = 1$  to  $M$  and we have also got  $Q_i^{\text{mean}}$  for all  $i = 1$  to  $M$ . So then therefore when we would be doing our load flow analysis we also have to take into account these two limits.

Now here before we go for the actual algorithm we need to also discuss another thing. We have already said that we do not have to solve for the voltage magnitudes and angles for the slack bus because at slack bus the voltage magnitudes and angle are already specified. So then therefore we do all calculations only starting from bus 2 and onwards. But then again we have just now said that bus 1 is also a generator bus.

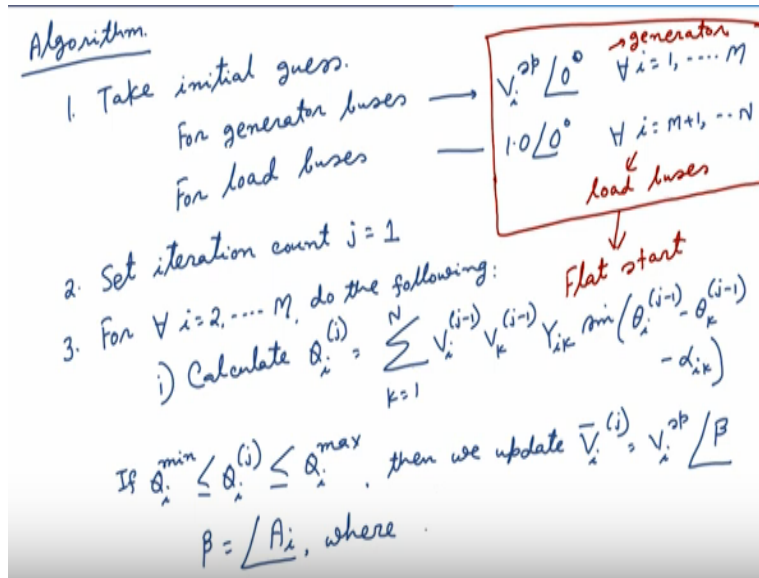
So then therefore if we are not calculating anything for bus 1 so then therefore it may happen that for this bus 1 there is a violation of its limit either for the reactive power generation limit or for the reactive power absorption limit. To avoid this situation although it is theoretically true but then to avoid this situation what we assume that we take this slack bus generator as that bus generator or rather at that generator which has got the maximum capacity.

So then therefore what you say is that we choose the slack generator as that generator which has got the maximum capacity so then therefore we assume that after this load flow solution is done over whatever reactive power this particular slack generator will be required to supply that amount of reactive power would be well within the limits. If they are not found to be well within the limits we have to do something else but that is actually beyond the scope of this course.

But for this particular course we will assume that our slack generator is a very high rated generator. So as a result it has got the capacity of supplying whatever reasonable amount of

reactive power is required for this system. So now we start looking at this algorithm. Now for this algorithm as we said so now what would be the algorithm?

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So we have already discussed all this basic philosophy. So we can start what would be the algorithm. So the first algorithm is take initial guess. So the first step of this algorithm is take initial guess. Now what initial guess we will take? For the generator buses, for generator buses we take the initial guess as  $V_i$  specified angle 0 degree. Why do we take this? Because for this generator buses this bus voltage magnitudes are already specified.

And it is desired that this bus voltage magnitudes are maintained at these specified values. So then therefore we should start with this initial condition. And for this angles we have already discussed that all these angles should have an initial value of 0 degree. And for load buses, because there is no such specified value so then we say for load buses our initial guess is 1.0 angle 0. So then for generator buses the initial guess is  $V_i$  specified angle 0 degree.

So this is for all  $i = 1$  to  $M$  and this is for all  $i = M + 1$  to  $N$ . Now for any subsequent discussion in this course, this initial guess we will take as flat start. So this is for load buses and this is for generator buses. So this is for generator buses. So then for all subsequent lectures in this course whenever we will say that we are taking this initial guess as flat start we would be meaning that for all the generator buses the initial guess is  $V_i$  specified an angle 0.

Please note that this  $V_i$  specify need not be equal to 1.0. It can be let us say 1.01, .99, 1.005 anything and everything but it is necessarily not is equal to 1.0. But for all the load buses because there is no such specified voltage magnitude so then therefore for all the load buses we will be taking the initial guess as 1.0 angle 0. So for all subsequent lectures when we would be saying flat start we would be actually meaning this two. So after this now you are ready.

So then we set iteration count  $j = 1$ . Now in contrast to the last case where we have only considered one generator and which was nothing but the slack bus, in this case because we have got multiple generator and so then therefore as we have just now discussed at each and every iteration we have to calculate the amount of reactive power which is being either supplied or absorbed by any generator.

Now based on our discussion which we have just now we can safely say that if the actual amount of reactive power which is being either absorbed or supplied by the generator is within the limit so then therefore we can safely say that this generator can maintain its terminal voltage at the specified value. On the other hand if the calculated value of the reactive power violates either the maximum value or the minimum value, so in that case this particular generator would not be able to maintain its terminal voltage at the specified value.

So then therefore in that case that particular generator bus will be treated as a load bus. Load bus means that at which bus both the voltage magnitude as well as the angles are unknown. So then therefore what happens? If for any generator the actual amount of reactive power is within the respective limit then this particular generator bus will be treated as still the PV bus. PV bus means in which the voltage magnitude is specified.

Because physically this generator excitation system would be able to maintain the bus voltage at its specified value. On the other hand, if the calculated value or the actual value of the reactive power violates any of the limit, either the maximum or the minimum that is either the generation limit or the absorption limit. So in that case this bus voltage magnitude would not be maintained at the specified value.



Because this excitation system would be unable to maintain that voltage so then therefore what will happen? This generator will be either supplying or absorbing that maximum or minimum value of the reactive power but the bus voltage of that particular terminal bus will not be maintained at the specified value and then therefore we have to again recalculate it. So then therefore although this particular generator bus was earlier actually PV bus but now it will be changed to a PQ bus.

PQ bus means at which I have to calculate both voltage magnitude as well as the angle. So this is called PV to PQ bus switching. So all this aspects we have to now take into account in our algorithm. So then what we will do? For all generator bus we do the following calculation. So for all  $i = 2$  to  $M$  that is for generator buses do the following. 1. Calculate the reactive power injected by this bus. So calculate reactive power injected by this bus.

So at  $j$ th iteration so how do I calculate? It is equal to  $k = 1$  to  $N$   $V_i$ . Now which value of  $V_i$  will be taking? When I am trying to calculate  $Q_i$ , I have not as yet calculated the value so then what I have? I have got only the previous value, that is the previous iteration value. So I write  $V_i(j-1)$ . Similarly,  $V_k$  also we write  $(j-1)$  I have got only the previous iteration value.  $Y_{ik}$  it is the magnitude of the element of the essentially it is the magnitude of the  $ik$ th element of the Y-Bus matrix. So it is constant.

So it has got nothing to do with the iteration count. Then sin, please remember we are now using here the normal power flow equation sign.  $\theta_i$ , now what value of  $\theta_i$  do I have? I have not as yet calculated  $\theta_i$ . So what I have? I have got only the previous value. It should be  $(j-1) - \theta_k$ . Again for this also I have got only corresponding to the last previous or rather the corresponding to the previous value and  $\alpha_{ik}$  is the angle of the  $ik$ th element of the Y- Bus matrix. So this is constant.

So with this we first calculate  $Q_i(j)$ . If  $Q_i(j)$  less than equal to  $Q_i \text{ max}$  and greater than equal to  $Q_i \text{ min}$  then we calculate then we update  $V_i$  at  $j$ th iteration as  $V_i$  specified angle say  $\beta$ . Now what is this angle  $\beta$ ? So what we were saying? That if this actual value of reactive power

is well within the limit so then therefore this voltage magnitude will remain at a specified value. So then we are saying that this voltage magnitude remains at the specified value  $V_i$  specified. But we are only calculating its angle. Now what is this angle.

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$$A_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i^{(j)}}{V_i^{*(j-1)}} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k^{(j)} - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k^{(j-1)} \right]$$

If  $\theta_i^{(j)} > \theta_i^{\max}$ , then we calculate,
 
$$\bar{V}_i^{(j)} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i^{\max}}{V_i^{*(j-1)}} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k^{(j)} - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k^{(j-1)} \right]$$

If  $\theta_i^{(j)} < \theta_i^{\min}$ , then
 
$$\bar{V}_i^{(j)} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i^{\min}}{V_i^{*(j-1)}} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k^{(j)} - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k^{(j-1)} \right]$$

*PV to PQ switching* (indicated by a red arrow pointing to the first case)  
*Complex value* (indicated by a red arrow pointing to the second case)

So beta is essentially, beta is the angle of the quantity  $A_i$  where  $A_i = 1/Y_i P_i - jQ_i^{(j)}$ . Remember we would be taking the actual value minus  $V_i^{*(j-1)} - k = 1$  to  $j - 1$   $Y_{ik} V_i$  sorry  $V_k^{(j-1)}$  because we would be only calculated stake. It should be  $i - 1$  as we have said it would be  $i - 1$  and we have already and for as we have done in the last lecture for any bus which is previous to this bus in the sequence we already known their, so this is also another mistake which is previous to this.

We have already updated this value and for the other one  $k = i + 1$  to  $N$   $Y_{ik} V_k^{(j-1)}$ . So this is the  $i$ th bus. This is the  $j$ th iteration. And  $k$  is the iteration count. So this is the  $j$ th iteration and as we have already seen for any bus which is before this  $i$ th bus in the sequence we already know this updated value. So we use this updated value and for any other bus which is after this bus in the sequence we have not yet updated their voltage magnitude and angle.

So then we use their value as the previous iteration value. So then therefore what we are doing? We are first calculating this quantity  $A_i$  as usual we are simply calculating this quantity  $A_i$  as usual as we have done in the last case. But now this  $A_i$  would be a complex quantity because

everything here is complex quantity but we will not be taking its magnitude. We would be only taking its angle and this angle we will be assigning to this bus voltage.

However, we would maintain this bus voltage magnitude at the specified value because we said when this actual value of reactive power is well within the limits, this bus voltage magnitude will remain at the specified value. If  $Q_i(j)$  is greater than equal to say greater than  $Q_i^{max}$  then we set, then we calculate  $V_i$  at the (j)th iteration as  $Y_{ii} P_i - jQ_i^{max}$ . Please remember when  $Q_i(j)$  is greater than  $Q_i^{max}$  what will happen?

The actual amount of reactive power generated by this generator would be clipped at  $Q_i^{max}$ . So then in that case it would be only able to inject  $Q_i^{max}$  and then therefore after that everything is same.  $V_i^*(j-1) - k = 1$  to  $i-1$   $Y_{ik} V_k(j) - k = 1$  to  $N$   $Y_{ik} V_k(j-1)$ .  $V_i$  is equally the same thing. Only difference is that here we have replaced  $Q_i(j)$  with  $Q_i^{max}$  because this is the max value which is taking and here we are taking this complex value.

So here this is a case, so again if  $Q_i(j) < Q_i^{mean}$  then we again calculate  $V_i(j)$  is equal to same thing. Only difference is that  $P_i - jQ_i^{mean}$  please note when  $Q_i$  actual value of reactive power is less than  $Q_i^{mean}$  the amount of reactive power which is being absorbed by the generator is  $Q_i^{mean}$ . So it would be clipped here and rest of the  $Y_{ik} V_k(j-1)$ . We note here that these 2 cases is the case of PV to PQ switching.

And here in both this cases this and this we take the complex value. So this and this we take the complex value. So then therefore what we have done? It is a very simple thing we have done. We have simply said if the actual reactive power generated by the generator is within the limit then this bus voltage magnitude would be remaining at  $V_i$  specified.

Only its angle would be calculated or if the actual reactive power is crossing the limit so then this actual generation or absorption by the generator would be clipped at that limit and then we would be recalculating both the bus voltage magnitude and the angle. So then therefore we will not be maintaining the bus voltage magnitude at their pre-specified value because this excitation system would not be able to maintain it.

So, so far we have done for this calculation only for the generator buses and for the load buses this is very simple.

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4. For  $i = M+1, M+2, \dots, N$  (load buses), we do

$$\bar{V}_i^{(i)} = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{\bar{V}_i^{(i-1)}} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k^{(i)} - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k^{(i-1)} \right]$$

5. Calculate  $e_i^{(i)} = \left| \bar{V}_i^{(i)} - \bar{V}_i^{(i-1)} \right| \quad \forall i = 2, 3, \dots, M$

6. Calculate  $e_{\max}^{(i)} = \max \left[ e_2^{(i)}, e_3^{(i)}, \dots, e_M^{(i)} \right]$

7. Check whether  $e_{\max}^{(i)} < \epsilon$ . If yes, stop. Else update  $j = j+1$  and go back to step 3.

For load buses for  $i = M + 1, M + 2$  to  $N$  so this is essentially load buses. So this is actually load buses we do the same thing, it is very standard, very simple. So  $V_i(j) = 1$  by which is the earlier, there is nothing into it simple  $P_i - jQ_i V_i^*(j-1)$ . Please remember for all this load buses both the injected power  $P_i$  and  $Q_i$  they are all known. So there is nothing in it. And then it is the standard  $k = i - 1 Y_{ik} V_k$ . It would be  $j$  and then  $k = i + 1$  to  $N Y_{ik} V_k(j-1)$ .

So for the load buses there is nothing to do. It is the standard same expression what we have done earlier because there is no such issue of reactive power generation and absorption is to be considered. So after we are finished with step 4 we have updated the bus voltage, both the magnitude and angle for all the buses. So now it is time to check for convergence. So then what we did, again as before calculate  $e_i(j)$  as magnitude of  $V_i(j) - V_i(j-1)$  for all  $i = 2, 3, \dots, M$ .

Then calculate  $e_{\max}$  at the  $j$ th iteration as maximum of  $e_2(j), e_3(j) \dots e_M(j)$ . Then we check whether  $e_{\max}$  at the current iteration is less than some epsilon or not. If yes, stop else update  $j = j + 1$ . That is we are updating the iteration count and go back to step 3, which step? 3. So this is the entire algorithm of any general power system having  $N$  buses  $M$  generators. Please remember

we have not taken any assumption regarding the value of  $N$  and  $M$ .  $N$  can be thousand, million, billion. So thousand, any thousand,  $M$  can be any value.

And we have also considered the practical aspect of the limitation of the reactive power generation for any generator and we have also looked into that how this limits of the reactive power generation or absorption by the generators are taken into account in the load flow equation of Gauss – Seidel method. So in the next lecture, we would be looking at an example of Gauss – Seidel load flow technique for any general  $N$ - Bus  $M$ - machine power system. Thank you.