Computer Aided Power System Analysis
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Lecture - 08
Gauss - Seidel Load Flow (GSLF)

Welcome to this module of computer aided power system analysis. So far till the last lecture we have discussed about the basic Gauss – Seidel numerical method. In this lecture as well as in the next lecture we would be looking into the application of that basic Gauss – Seidel numerical method for the solution of the power flow equations of any power system to compute the voltage magnitudes as well as the angles at all the other buses.

So to start with we first recapitulate the basic Gauss – Seidel numerical method briefly so that we can immediately connect those equations to the problem of solving the power flow equations at hand. So we start. So basic Gauss – Seidel load flow I need to give a very short introduction again. I mean it is just a kind of recapitulation. So what we have?

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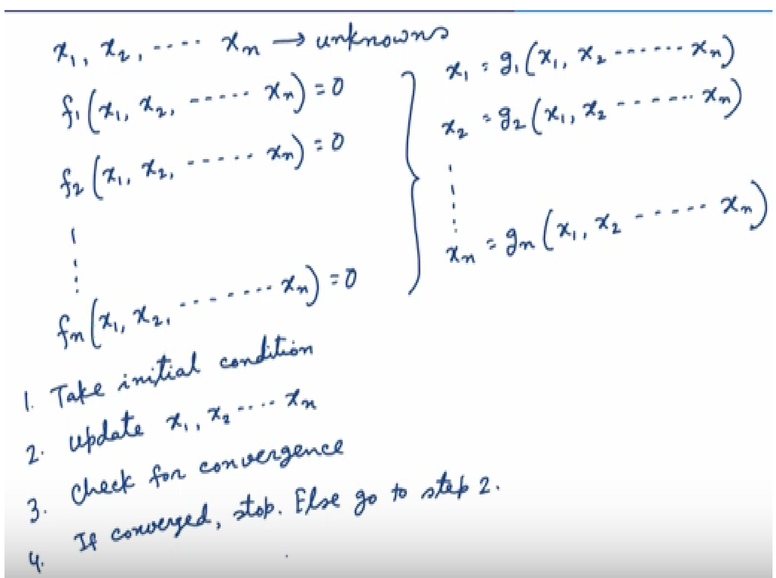

We had that our unknowns there are n unknowns x 2 to x n. these are unknowns. Then we have n equations. These are all nonlinear equation. No particular form of this equations have been assumed. So no particular form of this equations had been assumed. Now we have seen that from these equations we would be explicitly representing x 1 in terms of some other function g 1 of all the variables x 1, x 2 to x n.

Similarly, x 2 would be g 2 (x 1, x 2, to x n)… x n = g n(x 1, x 2 to x n). So after that, what we do we take initial condition. So the steps are take initial condition. So we take initial condition. Then what we do we update x 1, x 2 to x n. Then what we do, we check for convergence. If yes, then we stop. Otherwise, we go to if yes if converged, stop else go to step 2. Of course, this is a very brief recollection of this basic procedure of the Gauss – Seidel numerical method. So we would now apply this in our problem. So what we do? So now we consider an N- Bus system.

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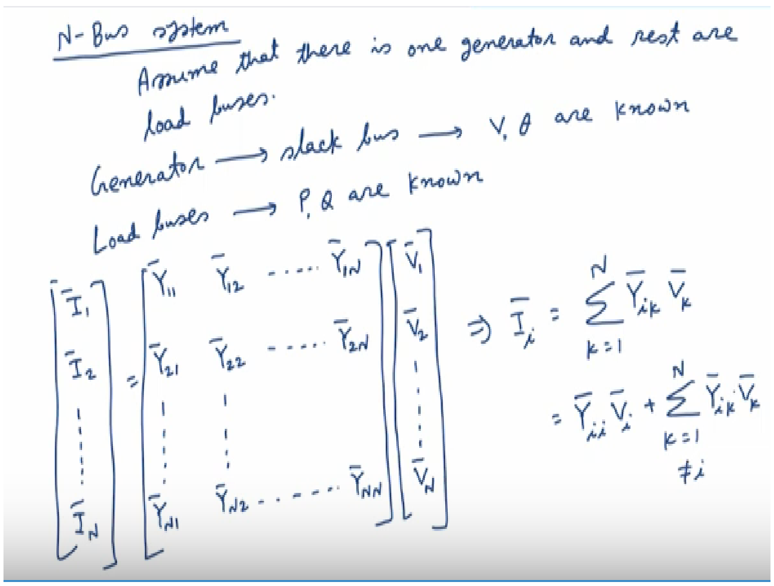

So we consider an N- Bus system and to start with we assume that there is only one generator. Of course we would be relaxing this, but we would, so there is only one generator and rest are load bus and rest are load buses. Of course we would be relaxing all these assumptions in a little later stage but just now we are making this assumption just to introduce the basic Gauss – Seidel load flow method.

So now because we have got only one generator so that generator is a slack bus. So this generator is slack bus. So V and theta are known. So at this bus as we have already collected, because this is a slack bus voltage magnitude and its angle are known. In fact angle of this particular bus is taken to be 0. And because these are all load buses so all load buses, so load buses all load buses P and Q are known.

So injected power at this load buses both real and reactive they are all both known because these are load buses. Now we have already seen in the earlier lectures that any N- Bus system whether there is any mutual coupling between any two elements or not any N- Bus system can be represented by its Y- Bus matrix. So this Y- Bus matrix relations can be represented as I 1, I 2, I N. So this is Y 11, Y 12, Y IN. Y 21, Y 22, Y 2N…Y N1, Y N2…YNN.

And here we have got V 1, V 2 to V N and these are all complex quantity. These are all complex quantity. As we already know that this vector I 1, I 2, I N is nothing but the bus injection current vector. V 1, V 2, V N is nothing but the bus voltage vector and this matrix Y 11, Y 12, Y NN etc. This is nothing but the bus admittance matrix. We have already seen that how this bus admittance matrix is formed and we have also looked at some its properties.

Now from this for any Ith bus we can write down that Y ik V k. So we take the ith index not equal to i Y ik V k. So what we did here we have done a very simple thing. We have simply taken out the ith index here from here and then we have simply put the other terms together.

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$$\bar{Y}_{ii}\bar{V}_i = \bar{I}_i - \sum_{\substack{k=1\\ \neq i}}^{N} \bar{Y}_{ik}\bar{V}_k$$

$$\Rightarrow \bar{V}_i = \frac{1}{\bar{Y}_{ii}}\left[\bar{I}_i - \sum_{\substack{k=1\\ \neq i}}^{N} \bar{Y}_{ik}\bar{V}_k\right]$$

$$= \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{\substack{k=1\\ \neq i}}^{N} \bar{Y}_{ik}\bar{V}_k\right]$$

$$= \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{k=1}^{i-1} \bar{Y}_{ik}\bar{V}_k - \sum_{k=i+1}^{N} \bar{Y}_{ik}\bar{V}_k\right]$$

We also know, that

$$\bar{V}_i \bar{I}_i^* = P_i + jQ_i$$

$$\Rightarrow \bar{I}_i^* = \frac{P_i + jQ_i}{\bar{V}_i}$$

$$\Rightarrow \bar{I}_i = \frac{P_i - jQ_i}{\bar{V}_i^*}$$

$$\Rightarrow \bar{V}_i = g_i(\bar{V}_1, \bar{V}_2 ---- \bar{V}_N)$$

So then therefore we can write down that Y i V i = I i – K = 1 to N not equal to i Y ik V k. So then therefore we can write down V i = 1/Y i I i – K = 1 to N this is not equal to i Y ik V k. Now we know we also know that at any bus V i that is the bus voltage multiplied by the complex conjugate of the injection current that is equal to P i + jQ i where P i is the injected real power at bus i and Q i is the injected reactive power at bus i.

So then therefore from here we can write down I i * = P i + j Q i/V i or in other words I i = P i – jQ i/V i*. So then therefore we replace this in this equation. So from here we can write down 1/Y ii P i – jQ i/V i* - K = 1 to N not equal to i Y ik V k. So then therefore this is the equation for V i. So you can see that this equation of V i is actually in the form of that V i equal to some function g i of the function V 1, V 2 up to V N.

So then therefore we can write down that as if that V i is some function of g 1 of sorry it should be g i V 1, V 2 to V N. So it is exactly in the same form as we have considered in the case of basic Gauss – Seidel numerical method. Now here before we apply the basic Gauss – Seidel numerical method here we need to do something more. Now here we have to see that when we are writing down the expression of V i here at the right hand side for this summation this V i term does not exist.

So then therefore we can simply write it down as Y i P i – jQ i to V i* minus, so you would be simply breaking this into two parts. In the first part we would be taking k = 1 to i – 1 then it is Y ik V k and in the second part we would be taking k = i + 1 to N Y ik V k. What we have done here, because this term does not have the term V i here so then we have simply taken the term from 1 to i -1 together and then the terms i + 1 to N together. So this is the final equation.

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$$\bar{V}_i = \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{k=1}^{i-1}\bar{Y}_{ik}\bar{V}_k - \sum_{k=i+1}^{N}\bar{Y}_{ik}\bar{V}_k\right].$$

Let $N = 6$, Bus 1 → slack bus and buses 2, 3, ... 6 are load buses.

$$\bar{V}_4 = \frac{1}{\bar{Y}_{44}}\left[\frac{P_4 - jQ_4}{\bar{V}_4^*} - \bar{Y}_{41}\bar{V}_1 - \bar{Y}_{42}\bar{V}_2 - \bar{Y}_{43}\bar{V}_3 - \bar{Y}_{45}\bar{V}_5 - \bar{Y}_{46}\bar{V}_6\right]$$

$i = 4$

$$= \frac{1}{\bar{Y}_{44}}\left[\frac{P_4 - jQ_4}{\bar{V}_4^*} - \sum_{k=1}^{3 \to (i-1)}\bar{Y}_{4k}\bar{V}_k - \sum_{k=5\,(i+1)}^{6 \to N}\bar{Y}_{4k}\bar{V}_k\right]$$

We wish to maintain the voltage magnitude profile in the system at the nominal level ⇒ In p.u. we wish to maintain the voltage magnitudes at 1.0 p.u.

So we again write down this equation just to recollect so that so we have so finally we have V i = 1/Y i [P i – jQ i/V i* - k = 1 to i – 1 Y ik V k – k = i + 1 to N Y ik V k]. Now to illustrate this let us take a very small example. Suppose N = 6. So let N = say 6 and we are trying to write down and bus 1 is the slack bus and bus 2 to 5 2, 3 to 6 are load buses. And we are trying to find out, we are trying to write down the expression of V 4.

So in the original expression of V 4 what we can write down is this is Y 44 P 4 – jQ 4 V 4*. Then –Y 41 V 1 – Y 42 V 2 – Y 43 V 3. This fourth term of V 4 will not come here. Then –Y 45 V 5 – Y 46 V 6, all are complex quantity. So this equation we have written following this expression. Now we take here, so it is Y 44 P 4 – jQ 4 V 4*. Now we take these 3 together. So it is we can write down that it is K = 1 to 3 Y 4k V k.

And then if we take these two together we can write down k = 5 to 6. This is Y 4k V k. So all these are complex. Now here we need to notice that, so here in this expression i = 4. So this is i. So this is i = 4. This 3 is actually is i -1. So this is i - 1. This 5 is actually i + 1 and this 6 is = N. So this is exactly this form what you have got. So then therefore we can write down this original equation in the more second form by this.

So we have simply decoupled the terms which is coming before the ith bus in the sequence and we are taking together other terms which are coming after the ith bus. Now as we have already said when we are trying to solve any set of nonlinear simultaneous algebraic equations we need to take an initial guess. Now here our unknowns are all voltages and essentially all voltage magnitudes and all voltage angles except the slack bus because at slack bus everything is known.

Now what should be the initial guesses? So that is very important. Now fortunately in power system what we wish to do is the power system we wish to maintain the voltage profile in the system at the nominal value because if we can maintain the voltage profile of the system at the nominal value that is the most optimum case because if it is higher than the nominal value, so in that case there will be more stress.

And again if this is lower than the nominal value so then in that case there will be other problems such as under voltage problem. Loss would be increased etc. So then we always wish to maintain the voltage profile of any system at the nominal level and voltage magnitude, I should write here voltage magnitude at the nominal level. So that means in per unit, we wish to maintain it, we wish to maintain the voltage magnitudes at 1.0 per unit.

So then therefore because we wish to maintain the voltages at 1.0 per unit so it makes sense that we should take the voltage magnitude initial value as equal to 1.0 per unit for all the buses. But then the question comes what should be the angle. Now we know from our basic understanding of the power system operation we know that if we have a line between any 2 bus, bus let us say i and bus j.

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$\angle\theta_i - \angle\theta_j > 30^\circ$ to 35°

We take initial guess as $\bar{V}_i^{(0)} = 1.0\angle 0^\circ \quad \forall\, i = 2, 3, \cdots N$

Flat start

Algorithm steps

1. Assume flat start as the initial guess: $\bar{V}_i = 1.0\angle 0^\circ \quad \forall\, i = 2, \cdots N$
2. Set iteration count $j = 1$.
3. update all bus voltages as;

$$\bar{V}_2^{(j)} = \frac{1}{\bar{Y}_{22}}\left[\frac{P_2 - jQ_2}{\bar{V}_2^{(j-1)}} - \sum_{k=3}^{N} \bar{Y}_{2k}\bar{V}_k^{(j-1)} - \bar{Y}_{21}\bar{V}_1^{(j)}\right]$$

So then therefore and if this bus has got angle theta i we are not right now bothered about this voltage magnitude. So if this angle is theta i and if this bus j has got an angle of theta j and if the difference between theta i – theta j difference is let us say more than 30 degree to 35 degree. So then usually we face the problem of the transient stability now. This we know from our basic understanding of the power system and we can also infer this from the understanding of our equal area criteria.

So then therefore essentially what we are trying to say is that if the angle difference between any two buses over a line is large so then therefore that particular line will be more prone to the problem of transient instability. So then therefore to make the system more stable we would always like to maintain the angular difference between any two buses as small as possible. Now in the case of slack bus we have already assumed that our theta is equal to 0 degree.

So then therefore by the same logic because we want to maintain the stability of the system essentially, so then by the same logic all the bus voltage, other angles also should be as close to 0 degree. I mean they can be little more positive, they can be little more negative but then they should be all around 0 degree within a certain zone so then therefore it basically makes a lot of sense if we take the initial value of all the bus voltage angles as 0 degree.

So then therefore by this logic when we take the bus voltage magnitude initial value as 1.0 and the bus voltage angle initial value as 0 degree this is called flat start. So then what we are doing, we are taking, so we take initial guess as V i (0) = 1.0 angle 0 degree for all i = 2, 3 to N. Please note that this 0 suffix are within the parenthesis denote that this is the initial voltage. Now this case when we are taking the bus voltage magnitude initial guess to be the same as well as the bus voltage angle initial guess to be the same for all the buses we call it flat start.

Flat means the same. So then therefore we say that it is a flat start. That means that we are taking the bus voltage magnitude as well as the angle initial assumptions to be the same at all the buses and this magnitude is 1.0 per unit for all the buses and this angle is 0 degree for all the buses. So then therefore with this, so now we can start writing our algorithm. So then what do we want to do? So the algorithm steps. So then algorithm steps.

So the first algorithm step is assume flat start as the initial guess. Then we set iteration count say j = 1. Then what we do? Then we update, update the update all bus voltages as now you see in this case we have assumed that the bus 1 is the slack bus. So then therefore the bus voltage magnitude and angle at this bus is known. So then therefore you do not have to solve for it. You have to only solve for the other buses.

So then we will start our calculation from bus 2 and then bus 3 and then we will go up to bus N. So we start with bus 2. So then bus 2, the voltage at bus 2 updated value V 2 (j). Now remember here when I am writing j, so this j stands for the iteration number j. Now please note that we have already set j = 1 here. So then essentially it is actually V 21. So it would be 1/Y 22 P 2 – jQ 2. Now here I have to also put some V 2 value.

Now when I am trying to put some value of V 2 at the right hand side what value I know? I only know the initial guess. So that means that is 0. So then what I know is, so what I write is V 2 (j – 1). Now because here j = 1. So then therefore j - 1 = 0. So that means it is actually V 2 0, right? So we should write that V i = 1.0 angle 0 for all i = 2 to N fine. Then minus, now because we are now talking about V 2 and then we have got V 2 and we do not have to solve for bus 1.

So then therefore in this expression this term does not exist because here in this case i = 2 so then therefore it is k = 1 to 1. But then I do not have to solve for 1 because bus 1 is already the slack bus. So then therefore I have to only take this part. So then it would go as K = 3 to N V 2 sorry it would be Y 2k V k. It would be (j -1). It would be j – 1. J – 1 means that here also we are simply taking the initial value, j – 1 = 0. So here also we are taking the initial value. Now after that when we are trying to solve for V 3 j.

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$$\bar{V}_3^{(j)} = \frac{1}{\bar{Y}_{33}}\left[\frac{P_3 - jQ_3}{\bar{V}_3^{*(j-1)}} - \sum_{k=1}^{2} \bar{Y}_{3k}\bar{V}_k^{(j)} - \sum_{k=4}^{N} \bar{Y}_{3k}\bar{V}_k^{(j-1)}\right]$$

$$\vdots$$

$$\bar{V}_i^{(j)} = \frac{1}{\bar{Y}_{ii}}\left[\frac{P_i - jQ_i}{\bar{V}_i^{*(j-1)}} - \sum_{k=1}^{i-1} \bar{Y}_{ik}\bar{V}_k^{(j)} - \sum_{k=i+1}^{N} \bar{Y}_{ik}\bar{V}_k^{(j-1)}\right]$$

$$\vdots$$

$$\bar{V}_N^{(j)} = \frac{1}{\bar{Y}_{NN}}\left[\frac{P_N - jQ_N}{\bar{V}_N^{*(j-1)}} - \sum_{k=1}^{N-1} \bar{Y}_{Nk}\bar{V}_k^{(j)}\right]$$

4. Calculate error as $e_i^{(j)} = \left|V_i^{(j)} - V_i^{(j-1)}\right| \quad \forall\, i = 2, \ldots N$
5. Calculate maxm error as $e_{max}^{(j)} = \max\left[e_2^{(j)}, e_3^{(j)} \ldots e_N^{(j)}\right]$

So V 3 (j). Now this is interesting V 3 (j). It would be 1/Y 33 P 3 – jQ 3. V 2 we have not as yet solved. So V 3 (j – 1). So V 2 I have solved. So then what we will write is k = 1 to 2 V 3 k this is Y 3k V k (j) remember. For bus 1 it is already known and for bus 2 we have just now calculated. So we would be actually utilizing this bus voltage V 2 j. So we would be utilizing, so then from here when let us say k = 1 it would be Y 3 1 V 1j; j = 1.

But then because this particular voltage of the slack bus is always held constant so then therefore for bus 1 it is always the same and then when k = 2 then it would be actually V 2 (j) and this V 2 (j) we have already calculated in the last step. So then therefore this is and then from k = 4 to N Y 3k V k (j - 1). But for bus 4 and onwards we have not as yet calculated so then therefore for these buses we would be only utilizing the value of the voltage as we have assumed as the initial condition.

Now here I guess 1 is missing. So in fact it also should be it should be Y 21 V 1. So then like this if we go for V i at the jth. So I got Y ii P i – jQ i. V i you have not as yet solved. It should be V i* (j – 1). Here also it should be V 2*, should be here V 2*. Then what we have, we have K = 1 to i – 1 Y ik. Now for all the buses before i that is up to i – 1th buses we have already solved for them at this present iteration.

So we will utilize this present iteration and for the other buses K = i + 1 to N. We would be only utilizing the initial guesses, … for V N (j) it would be Y NN P N – jQ N. V N we have not as yet solved for. So it would be (j – 1), still the previous one. And then what we will do is K = 1 to N Y. Because when I am solving for Nth bus up to all the other buses N – 1 we have already solved. Only the, so only this part would be remaining.

So for up to all this bus we have already solved and for this Nth bus it has already come into picture. So these are the equations. So after this what we will do? Then we go to the next step. So in the next step what we do, calculate error. Error as e i at jth iteration as V i at the jth iteration minus V i into (j – 1)th iteration this mod for all i = 2 to N. Then calculate maximum error as e max at the jth iteration as max of e 2 (j), e 3 (j) … e N (j).

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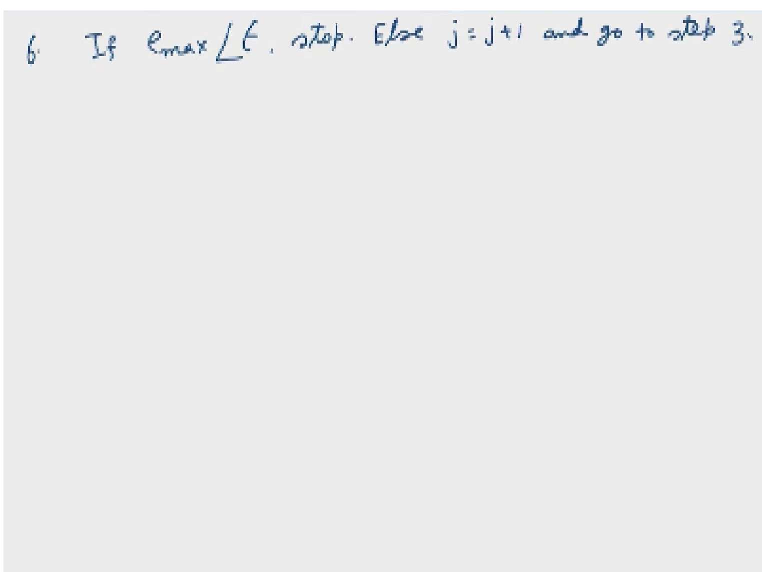

And then if e max is less than some epsilon stop else increment the iteration count and go to step 3. So this is the basic equation and this is the basic approach for solving the Gauss – Seidel load flow within single generator. So in the next lecture we would be continuing this lecture and we will show how this particular method can be used for solving a power system in which there are multiple generators. Thank you.