Computer Aided Power System Analysis
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Lecture - 06
Power Flow Equations and Classification of Buses

Welcome to this module of computer aided power system analysis. In the last few lectures we have discussed about the formation of the Y Bus matrix where there can be a case that there is no mutual coupling among any of the elements or (b) there can be some mutual coupling between two of the elements. Now today we would be talking about this power flow equation, basic power flow equation as well as the classification of buses. So let us start.

So what we are trying to do is that how to analyze a general N Bus M-machine power system.

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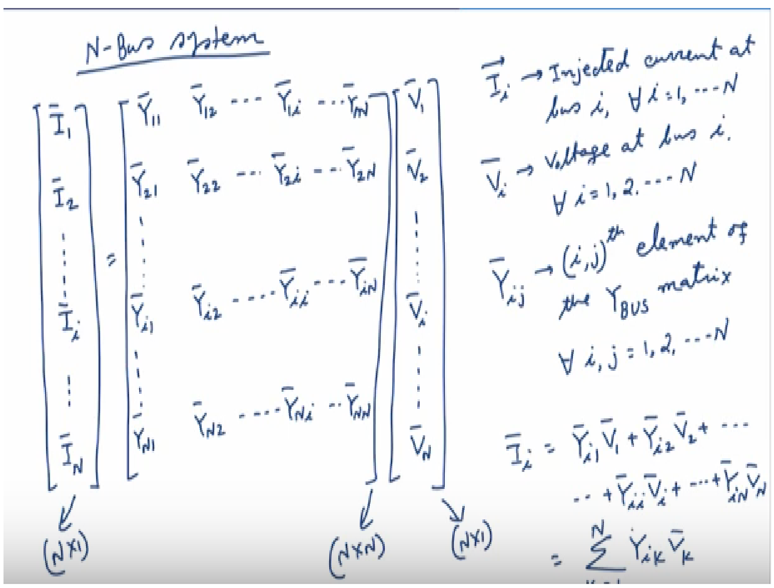

So now suppose that there are N buses in the system, N-Bus system and there can be some mutual coupling between any of the two elements, that is also equally possible. But then even then whether there is any mutual coupling or not, we know that any system can be represented in terms of its bus admittance matrix. So then therefore we define that so then we say that if there are injected currents at buses are I 1, I 2, and I N.

Please note that these currents are injected currents at bus, at all the buses. Then V 1, V 2, V i, V N. So here I i is injected current at bus i for all i 1, 2, N. V i voltage at bus i for all i = 1, 2, N. and this would be Y 11. This would be Y 12. This would be Y 1i, Y 1N. These are all complex quantities. So this is a N X N matrix as we have already seen and we also know that this is also a symmetric matrix. So this is N X 1 vector. This is also N X 1 vector.

And this is also N X N matrix. And we write that Y ij is (i, j)th element of the we write is Y Bus matrix. This is for all i, j. So from here we can write down that I i is Y i 1 V 1 + Y i 2 V 2 + …Yii Vi + Y iN VN. All these are complex quantity. So we can write it down in second form K = 1 to N. Y i K V K.

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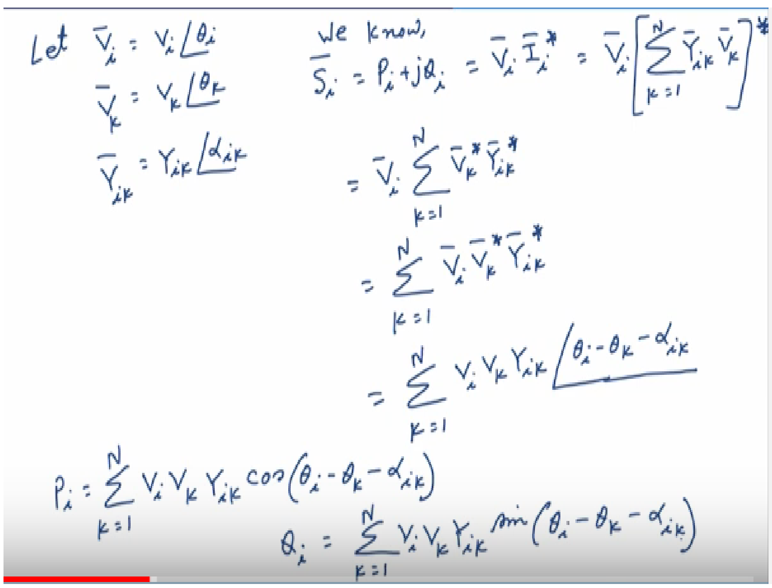

Now because all these are complex quantity so then let V i = V i angle theta i. V K = magnitude V K and angle theta K and Y iK is magnitude Y iK angle alpha i K because these are all complex quantities. Now we know S i that is the injected complex power at bus i = P i + jQ i that is = V i . I i *. We have already discussed in some previous class why do we take this operation I i *. I i * is nothing but the complex conjugate of the current I i.

So it would be V i then Y iK V K. K = 1 to N whole of it *. So therefore it would be V i V K * Y iK * K = 1 to N. So then therefore it would be now because this i is common to all so I can take it as K = 1 to N V i V K * Y iK *. So we can write it down as K = 1 to N V i V K Y iK angle theta i – theta K – alpha i K by substituting V K * from here as well as Y K * from here.

So then therefore taking the real and imaginary component of this we can write down P i = K = 1 to N V i V K Y i K cos (theta i – theta K – alpha iK) and Q i = K = 1 to N V i V K Y iK sin(theta i – theta K – alpha i K). So these two equations P i and Q i they do relate the complex injected power at any bus with the bus voltage magnitudes and angle of all the other buses.

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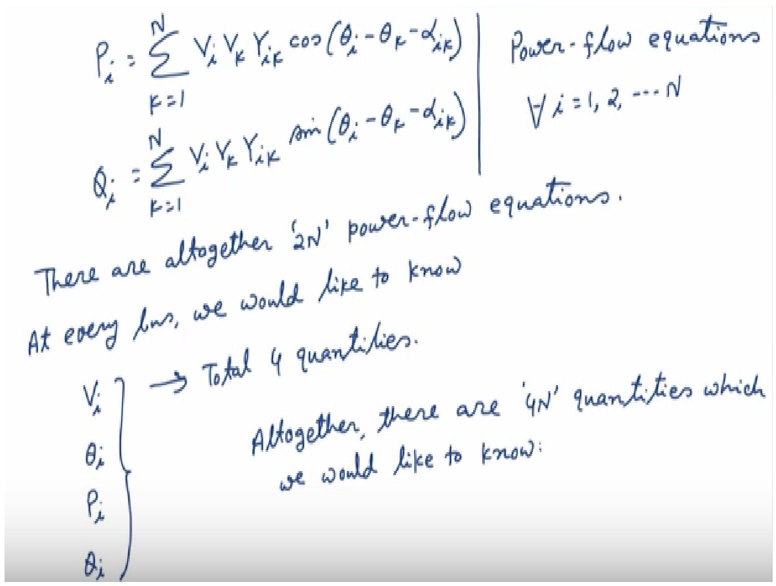

So then therefore, so we again write, so then therefore these two equations we again write for just repeat it just for the sake of clarity V i V K Y i K cos (theta i – theta K – alpha i K). Q i = K = 1 to N V i V K Y i K sin(theta i – theta K – alpha i K). Remember V i V K Y i K are nothing but the magnitudes of the bus voltages as well as the element of the bus admittance matrix respectively. So these two equations are known as the power flow equations.

Now at every bus i we can write down this equation and here because we do have total N number of buses so then i varies from 1, 2 to N. So then therefore there are altogether 2N power flow equations. Because you see at each and every bus i we do have these 2 equation P i and Q i and now we have got a total of N number of buses. So then therefore we have got total number of equations which is equal to 2N.

Now let us see at each and every bus what are the electrical quantities which we would like to know. So at each and every bus, we would like to know the following electrical quantity. So at every bus we would like to know following quantities V i that is the magnitude, angle, injected real power, injected reactive power. So then therefore at every bus, so there are total 4 quantities.

So then therefore we would like to know total for N number of quantities to completely define our power system. So then therefore altogether there are 4N number for N quantities which we would like to know or which are actually necessary for completely defining the power system. So now what we have, we have essentially there are altogether number of quantities is 4N which we would like to know. But we have got only 2N equations.

So then here obviously my number of unknown is certainly just the twice that of the number of equations. So then obviously we cannot solve for all these 4 quantities. So out of these 4N number of quantities we have to pre-specify 2N number of quantities so that the remaining 2N number of quantities can be solved by solving these 2N number of equations.

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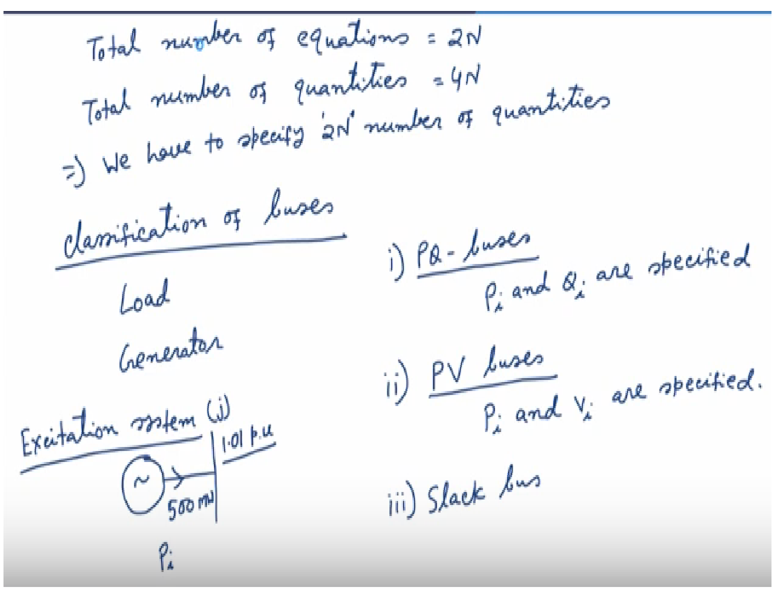

So then we write, so I have got total number of equations; so I have got total number of equations is 2N. But total number of quantities is 4N. so then therefore we have no other choice but to pre-specify 2N number of quantities such that we can calculate the remaining 2N number of quantities by solving these 2N number of powerful equations. So what we have, so we have to specify, so therefore we have to specify 2N number of.

Now here what we have, we have got total N number of buses and we have just now argued that we have to specify 2N number of quantities. So then therefore it makes a lot of sense that we would like to pre-specify 2 quantities per bus. So then therefore if we can do that so in that case we would be able to pre-specify 2N number of quantities very easily. Now depending upon on the physical situation we do define different quantities of different types of buses.

So then therefore for the purpose of pre-specifying the electrical quantities at different buses we first really classify the buses into 3 categories depending upon their physical characteristics as well as the requirement of our calculation. So now let us look at the concept of classification of buses. So what we have, we actually do define, now when you have some bus so then what we have we got at some bus either we have some load or some generator connected in shunt.

Or maybe that there is nothing connected in shunt. Now if there is any load connected at any bus, so then therefore we know that at that particular bus what is the total amount of load. For example if I say at bus 10 the load is 10 megawatt and 5 ampere. So then in that case we know that my injected real power would be -10 megawatt and my injected reactive power would be -5 ampere. So then therefore at each and every load bus we would be able to pre-specify what is the value of injected power.

So then therefore our first type of bus which we can very easily classify is we call it that PQ – buses. PQ – buses means that at these buses P i and Q i are specified. That means if at any particular bus there is some load connected so then in that case the injected P i and Q i would be nothing but the negative of the actual load connected at these buses. Or if at any bus there is nothing connected, neither load or a generator, in that case we will simply say that at this bus my injected P as well as the injected Q would be 0.

Even then we would be able to specify the value of P i and Q i at these buses. After all 0 is also a known quantity. So then therefore at these buses where we are able to specify the P i and Q i we do call them this PQ buses. Those buses are essentially the load buses and also those buses are the intermediate buses which are simply used to connect two or more different transmission lines without any shunt element connected to these buses.

Then we have other buses at which there are generator connected. So at these buses where there are generator connected we call them as PV buses. Now why do we call them PV buses? Now when we say that we call them PV buses, so here we actually specify P i that is the injected real power and the voltage magnitude are specified. Now the question is what is the physical justification of specifying P i and V i.

Assume that there is a bus, let us say bus j and at this bus there is a generator connected, some generator connected. And let us say that this generator is supplying say 500 megawatt power. So then therefore at this buses what is the actual value of injected power. That is basically nothing but 500 megawatt. So then therefore we know what is the value of P i because we know that what is the amount of real power this particular generator is injecting into the bus.

So then therefore at this bus where there is a generator connected we can pre-specify the injected real power. But then what is the physical justification of specifying this voltage magnitude. Now here we all know that each and every generator is also equipped with something called excitation system, excitation system. Each and every generator is also equipped with something called excitation systems. This excitation systems maintains the voltage magnitude at this bus by suitably changing the field current inside the generator.

So then therefore because we do have an excitation systems at each and every bus or rather at each and every generator so then therefore any generator is capable of maintaining the bus voltage at its terminal at a particular pre-specified value. Of course, any generator cannot maintain the voltage magnitude at its terminal at any value because after all any excitation system has got certain limits because it really cannot produce infinite values of the field current.

So then therefore depending upon the maximum or minimum amount of field current which can be produced by the excitation systems, a generator can maintain the voltage at its terminal within a certain zone. But when within that particular certain zone any generator is able to maintain its voltage at its terminal at a particular value.

So then therefore under steady state condition because of the action of the excitation system we can always safely say that the voltage magnitude of the generator bus is known so then therefore we can pre-specify this particular voltage magnitude at this bus. For example in this case we can perhaps say, perhaps say that this generator is maintaining this particular bus voltage at let us say 1.01 per unit.

So then therefore for this particular generator the value of V i that is the specified value of voltage magnitude is 1.01 per unit because this excitation system is able to maintain this voltage magnitude at 1.01 per unit. Now there is also there is something called a slack bus. Now let us understand the need of the slack bus.

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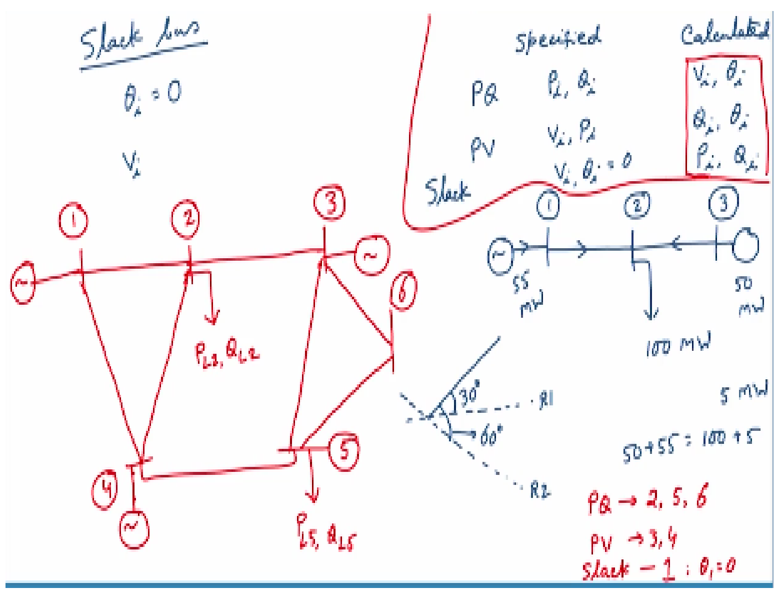

So talking about slack bus, if we go at this, now here if we look at this PQ bus as well as the PV bus, so then what we are seeing that at this PQ bus P and Q i are specified. So then therefore we have to calculate V i and theta i. And also for the PV bus, we are pre-specifying P i and V i and so then therefore we have to calculate Q i and theta i. So then therefore, so I write here specified calculated and type of bus so for PQ bus the specified quantities are P i, Q i.

And calculated quantities are V i, theta i and for PV bus specified quantities are V i and P i and the calculated quantities are Q i, theta i. So now you can see you have to calculate theta i. Now the question is when we are calculating any angle there has to be some reference. For example if I have a vector like this, some vector, if I take the reference as this line so then with respect to this reference this vector may have an angle of 30 degree.

But now if I do change my reference like this, so then therefore with respect to this reference so let us say this is reference 1, this is reference 2. So then therefore with respect to this reference, this same vector which has not even budged in this space, which has not really moved at all in this space but because you have just now changed this reference so then this same vector with respect to this reference may now make an angle of say 60 degrees.

So then therefore we can very easily see that whenever we are trying to calculate a current there has to be some reference because without any reference, so essentially without any reference the calculation of angle does not make any sense because depending upon the reference the calculated value of angle will change. So then therefore whenever we are calculating the angle at all the buses we have to make one angle as the reference angle that is the 0 angle.

So then therefore there has to be one bus at which we will pre-specify that the angle of this bus voltage would be made 0 and all the other angles either it is of the voltage or current would be calculated with respect to this particular reference angle. So then therefore this particular bus at which we make the angle to be equal to 0 we call it as a slack bus. So at slack bus we make we also specify theta i but we simply specify it to be is equal to 0.

Now physically what is actually a slack bus. Physically slack bus is basically also an generator bus. So then therefore physically at this particular slack bus also there is one generator is connected. So then therefore the question comes if there is already generator connected, why cannot we pre-specify the P i at this bus also as we have done for the other generator. The answer is as follows.

Suppose I do have a very simple system say there are 3, so let us say there is, so this is bus 1, this is bus 2, this is bus 3 and there are 2 lines connected and at bus 2 there is a load of let us say 100 MW and let us say at bus 1 and bus 3 there are 2 generator connected. And let us say bus 3 is producing 50 MW. So then therefore what should be the value of generator at bus 1? Apparently, it appears that it also should produce 50 MW such that this 50 plus this 50 should be equal 100.

But that is not so because, because of this flow of the current from here and here there would be power loses. So then therefore this generation plus this generation should be equal to this load plus this loses. Now the problem is what is the value of loses? If we know this value of loses a priori so then in that case we could have possibly pre-specified the amount of generation at this bus.

For example if we knew that this total amount of loses in this line as well in this line is let us say 5 MW so then therefore you could have said well if this generator is generating 50 MW so in this case this generator would be generating 55 MW. So in that case this generation plus this generation would be equal to, so 50 + 55 would be equal to load + loss. This is perfectly fine. But unfortunately we do not know what is the value of current flowing through this because this value of current would be again dependent on the value of the voltage at bus 1, bus 2, and bus 3.

So then therefore until and unless we are able to calculate the value of voltage, both magnitude and angle at bus 1, bus 2, and bus 3 so then therefore we would just not be able to calculate the value of current. And because we would not be able to calculate the value of current, we would not be able to calculate the value of loss occurring in this 2 lines a priori. So then therefore if we have to calculate the value of loses, we have to first calculate the value of voltages at the 3 buses.

But we are trying to calculate the value of voltages at this 3 buses actual in this case. So then therefore we are actually back to square 1 and because we have not been able to calculate the value of voltages it is just not possible for us to pre-specify the value of loses occurring in the system. So then therefore we really cannot pre-specify the generation at all the generators in the system.

We have to make free one generator and at this particular generator we will not pre-specify the value of real power injection. It would be actually calculated such that the total generation comprising of this generator and the other generators would be equal to the total load in the system plus the total loss in the system. So then therefore if there is one generator at which we cannot pre-specify the injected real power so then what other quantity we can specify?

Now we have already seen that at all the other buses we have already specified two quantities each but here in the case of slack bus we have only just now pre-specified only one quantity. But here we have to now, therefore we have to now also pre-specify another quantity. So now what that quantity can be? Now please note because this is a generator, this generator is also equipped with an excitation system.

And because this generator is equipped with an excitation system so then therefore this generator is also capable of maintaining the terminal voltage at its terminal at a particular value. So then therefore as we have done for the other generators for this generator also it is possible for us to pre-specify the voltage magnitude. So then therefore for this slack bus we also pre-specify the voltage magnitude. So then therefore for a slack bus, we do pre-specify V i and theta i.

Theta i would be 0 and we calculate P i and Q i. So now let us take a very small example. Suppose we have a five bus system and there are, see this is the connection. This interconnection is not important because whatever is the intra connection that would be captured in bus admittance matrix. So we are not worried about this intra connection. So let us say this is a 5 bus system.

And in this 5 bus system there are 3 generators at bus 1 and let us say at bus 4 and let us say at bus 3. And bus 2 there is some load and bus 5 also there is some load. For the purpose of more clarity, let us add another bus, does not matter. Let us say this is bus 6 and bus 6 there is nothing connected. So we have got a 6 bus system; bus 1, 2, 3, 4, 5, 6. At bus 1, 4, and 3 there are generators connected and bus 2 and 5 there are some physical load connected.

So there is some load connected P L2, Q L2. There is some load connected P L5, Q L5 and at bus 6 there is nothing connected. So then for this case, what are my PQ buses? My PQ buses are bus 2, bus 5 and bus 6. For bus 2, my P 2 = –P L2 and my Q 2 is actually = –Q L2. For bus 5, P5 = –P L5 and Q 5 = -Q L5 and for bus 6 because there is nothing connected here so then therefore P 6 would be = 0, Q 6 also would be = 0.

But even then, so then therefore in this case bus 2, bus 5, and bus 6 are the PQ buses. So here PQ buses are bus 2, bus 5, and bus 6. And my PV buses are let us say bus 3 and bus 4. My PV buses are bus 3 and bus 4. These are basically the generator buses. So at this P 3 what would be the value of P i. P i would be nothing but equal to P Gi. What would be the value of V i? V i is basically nothing but the value of the voltage at which this terminal voltage is maintained by the excitation system.

This value is known to us. And also bus 4 the same thing. The value of P i is = P G4 and the value of V 4 is essentially nothing but the value of terminal voltage at which the excitation system of this generator is maintaining this particular voltage magnitude. And bus 1 we do specify, so and also my slack buses bus 1. At this bus we do maintain theta 1 = 0 and we do also pre-specify V 1 and this V 1 is nothing but the voltage at which this terminal voltage is maintained by the excitation system of this particular generator.

So then therefore depending upon the physical condition and also for the purpose of calculation we do classify all the buses of any general power system into 3 categories which is called PQ bus PV bus as well as the slack bus. So with this classification we are now ready to start our actual calculation procedure of for calculating these quantities. So now our task is to calculate these quantities for each and every bus. So for all this PQ bus I have to calculate V i and theta i.

For all this PV bus I have to calculate Q i and theta i. And for slack bus I have to calculate P i and Q i. Now, if we look at this powerful equations we know that if we know V i and theta i at each and every bus, I would be able to calculate P i and Q i. So then therefore our essentially our objective is to calculate the voltage magnitude and angle at all the buses by utilizing these powerful equations. So how these are done we would be looking into from the next lectures. Thank you.