Computer Aided Power System Analysis
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Lecture - 56
Fault Analysis (Contd.)

Hello friends, welcome to this lecture on Computer Aided Power System Analysis. We have been discussing the representation of any unsymmetrical fault and we have seen that any symmetrical or unsymmetrical fault can be represented as, we have already seen in the last class, can be represented as:

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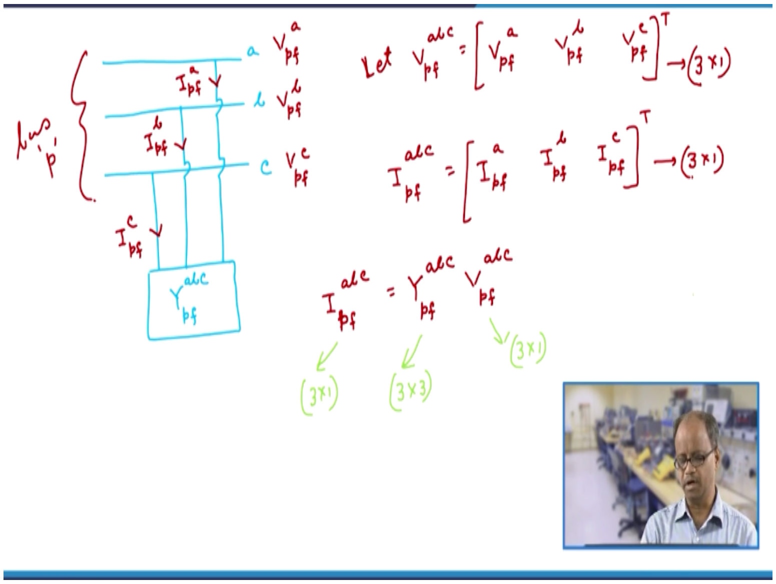

Let $V_{pf}^{abc} = \begin{bmatrix} V_{pf}^{a} & V_{pf}^{b} & V_{pf}^{c} \end{bmatrix}^T \rightarrow (3\times1)$

$I_{pf}^{abc} = \begin{bmatrix} I_{pf}^{a} & I_{pf}^{b} & I_{pf}^{c} \end{bmatrix}^T \rightarrow (3\times1)$

$$\underset{(3\times1)}{I_{pf}^{abc}} = \underset{(3\times3)}{Y_{pf}^{abc}} \; \underset{(3\times1)}{V_{pf}^{abc}}$$

Say that this, so phase A, phase B, phase C and there we have got fault. We have said that this Ypfabc and we have already noted what is Ypfabc and then we said that this, and then what we are saying that this voltage has got Vpfa, Vpfb, Vpfc, this is phase P, this is bus p. Bus p has phase abc and we also said that these are the currents, these are currents and this we have said that this is Ipfc, this is Ipfb and Ipfa and we have already defined all these quantities in the last lecture.

So now let defined, please note that all these quantities are essentially complex quantity, but scalar quantity. So we will define that pfabc has 3x1 vector as Vpfa, Vpfb, Vpfc transpose. So this is a 3x1 vector. Similarly, we defined that Ipfabc is Ipfa, Ipfb, Ipfc. This is also transposed and this is also 3x1 vector. So then therefore from our early analysis, so then therefore we can simply write down that Ipfabc is nothing but Ypfabc*Vpfabc.

So let us again look at just for the sake of clarity. This is 3x1 vector, this we have seen. This we have already seen that is the 3x3. This is actually representing any type of fault and it is 3x1 and therefore these are all confirming. So these are all nothing but confirming matrix operation. So this is the final representation of the fault current, fault voltage on the fault admittance.

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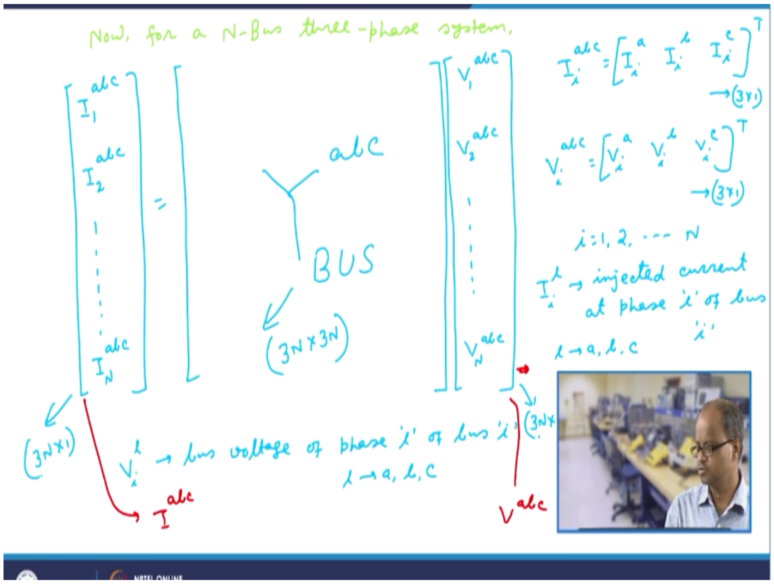

Now, for a N-Bus three-phase system,

$$\underbrace{\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ \vdots \\ I_N^{abc} \end{bmatrix}}_{I^{abc}\ (3N\times1)} = \Big[\ \underset{(3N\times3N)}{Y_{BUS}^{abc}}\ \Big] \underbrace{\begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ \vdots \\ V_N^{abc} \end{bmatrix}}_{V^{abc}\ (3N\times}$$

$I_i^{abc} = [I_i^a\ I_i^b\ I_i^c]^T \rightarrow (3\times1)$

$V_i^{abc} = [V_i^a\ V_i^b\ V_i^c]^T \rightarrow (3\times1)$

$i = 1, 2, \cdots N$

$I_i^l \rightarrow$ injected current at phase 'l' of bus 'i'

$l \rightarrow a, b, c$

$V_i^l \rightarrow$ bus voltage of phase 'l' of bus 'i'

$l \rightarrow a, b, c$

Now for an n bus system, for an n bus three phase system, we have already seen that I1abc, I2abc, Inabc can be written as let us say, at the right hand side it is V1abc, V2abc, Vnabc. So what is I1, I2, I2abc, essentially Iabc is actually Iia, Iib, Iic transpose 2x1 vector and Viabc is also Via, Vib, Vic transposed. This is also 3x1 vector. I goes from 1 to 2n and Iia and Via, Iia is nothing, but the injected current at phase of bus I. So Iia is injected current at phase L of bus I.

L is abc, similarly Vil is bus voltage of phase L of bus I. L goes to abc, right. So then therefore and this is the big Y bus matrix. So this is the big Y bus matrix and in fact you are writing that this is an Y bus abc matrix. So we already noted that this is 3nx1. We have also noted that this is 3nx1. So then therefore we also noted that this is 3nx3n, so that we have already seen. Now if I do just for the sake of brevity, if I do denote this entire vector as Iabc.

This is the vector where Iabc is this vector, so this is 3xn vector and if we denote this as Vabc, so then therefore we can write down that Iabc.

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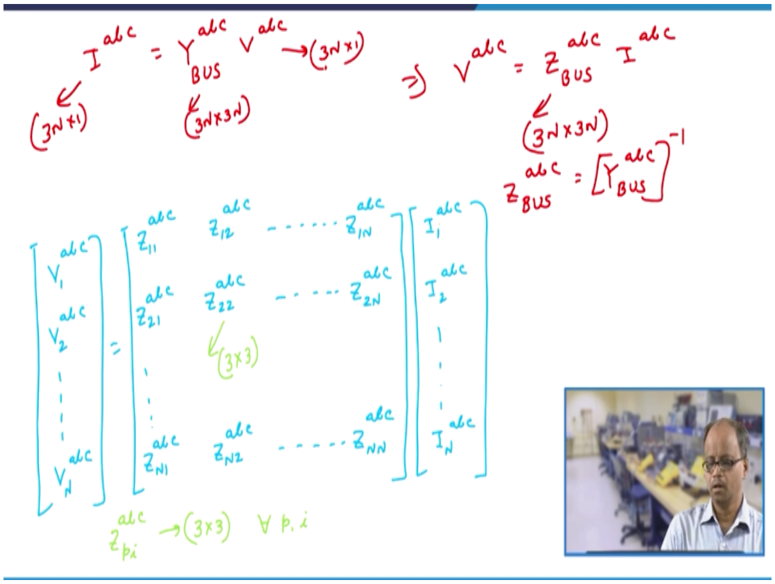

$$\underbrace{I^{abc}}_{(3N\times1)} = \underbrace{Y^{abc}_{BUS}}_{(3N\times3N)} V^{abc} \rightarrow (3N\times1) \quad \Rightarrow V^{abc} = \underbrace{Z^{abc}_{BUS}}_{(3N\times3N)} I^{abc}$$

$$Z^{abc}_{BUS} = \left[Y^{abc}_{BUS}\right]^{-1}$$

$$\begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ \vdots \\ V_N^{abc} \end{bmatrix} = \begin{bmatrix} Z_{11}^{abc} & Z_{12}^{abc} & \cdots & Z_{1N}^{abc} \\ Z_{21}^{abc} & \underbrace{Z_{22}^{abc}}_{(3\times3)} & \cdots & Z_{2N}^{abc} \\ \vdots & & & \\ Z_{N1}^{abc} & Z_{N2}^{abc} & \cdots & Z_{NN}^{abc} \end{bmatrix} \begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ \vdots \\ I_N^{abc} \end{bmatrix}$$

$$Z_{pi}^{abc} \rightarrow (3\times3) \quad \forall\, p, i$$

It is noted that it is 3x1 vector. So we can say that it is Y bus abc*Vabc. Again we note that it is a 3nx1 vector and this is 3nx3n and this is 3nx1. So then therefore from here, Vabc=Z bus abc*Iabc where Z bus is a 3nx3n matrix and Z bus abc is nothing but Y bus abc inverse. So now I do write them in matrix forms, so what I get. I actually can get that V1abc, V2abc…Vnabc. So I can write it down as I1abc, I2abc, …Inabc.

Because I have already find and let this Z bus abc is let us say that it is Z11abc, Z12abc,… Z1nabc. Similarly, this Z21abc, Z22abc, Z2nabc…Zn1abc, Zn2abc and znnabc. We again note that all these matrices are 3x3. So then therefore we note that Zpiabc are all 3x3 matrices for all pi. So for example what does this matrix Z12abc denote, that these 3x3 matrix is actually connecting the 3 phase voltage of bus 1 to the 3 phase injected current at bus 2.

Similarly, this Z1abc matrix is connecting 3 phase voltage of bus n to the 3 phase injected current at bus 1, similarly all the others. So then therefore I can write down that.

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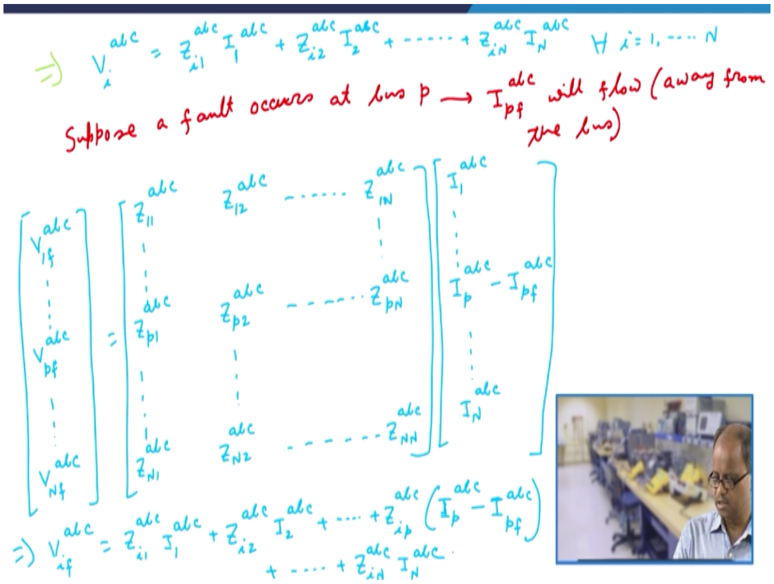

$$\Rightarrow V_i^{abc} = Z_{i1}^{abc} I_1^{abc} + Z_{i2}^{abc} I_2^{abc} + \cdots + Z_{iN}^{abc} I_N^{abc} \quad \forall i = 1, \cdots N$$

Suppose a fault occurs at bus p → I_{pf}^{abc} will flow (away from the bus)

$$\begin{bmatrix} V_{1f}^{abc} \\ \vdots \\ V_{pf}^{abc} \\ \vdots \\ V_{Nf}^{abc} \end{bmatrix} = \begin{bmatrix} Z_{11}^{abc} & Z_{12}^{abc} & \cdots & Z_{1N}^{abc} \\ \vdots & & & \vdots \\ Z_{p1}^{abc} & Z_{p2}^{abc} & \cdots & Z_{pN}^{abc} \\ \vdots & \vdots & & \\ Z_{N1}^{abc} & Z_{N2}^{abc} & \cdots & Z_{NN}^{abc} \end{bmatrix} \begin{bmatrix} I_1^{abc} \\ \vdots \\ I_p^{abc} - I_{pf}^{abc} \\ \vdots \\ I_N^{abc} \end{bmatrix}$$

$$\Rightarrow V_{if}^{abc} = Z_{i1}^{abc} I_1^{abc} + Z_{i2}^{abc} I_2^{abc} + \cdots + Z_{ip}^{abc}\left(I_p^{abc} - I_{pf}^{abc}\right) + \cdots + Z_{iN}^{abc} I_N^{abc}.$$

I can write down that Viabc=Zi1abc I1abc + Zi2abc I2abd, …, Zinabc*Inabc. This is for all I=1to n. Now suppose a fault occurs at bus P. because now there is a fault now occurring at bus p, so then therefore a fault current Ipfabc will flow. So therefore the fault current Ipfabc will flow and it will flow where, it will flow away from the bus as we have already seen. It will flow away from the bus like this. So it will simply flow away from the bus.

So then therefore if now at bus p, there was already current Ipabc, which is nothing but the injected current at all the phases of bus p. So now on top of that current, now this current is also now flowing but away from the bus, so therefore net injected current at bus p would be Ipabc-Ipfabc. So then net injected current would be Ipabc-Ipfabc. So then therefore we can write down this equation as.

So therefore this earlier equation we can write down as simply like this V1abc, V2abc… let us say Vnabc. Let us say this is V1fabc, V2fabc, Vnfabc. This f stands for fault voltage because now this fault has taken place at some bus. So we should not use this, should not use this, we should write Vpfabc and …. So then it is Z11abc, Z12abc…, Z1nabc, then …, Zp1abc, Zp2abc…, Zpnabc,… and then there was Zn1abc, Zn2abc…, Znnabc, and here I1abc ….

Here it would be Ipabc-Ipfabc and this is Inabc. So then therefore at any bus, Vifabc would be given by Zi1nabc*I1abc+Zi2abc*I2abc+… Zipabc*Ipabc-Ipfabc+… Zinabc*Inabc. Now this into this and now this plus this plus this plus this is nothing but Viabc. So then therefore we write down that Vifabc=Viabc that is the initial voltage that is the pre fault voltage - it would be Zipabc*Ipfabc. So it would be Zipabc*Ipfabc. So Vifabc would Vi. I mean from this equation, if I expand this, you have got Viabc and -Zipabc*Ipfabc.

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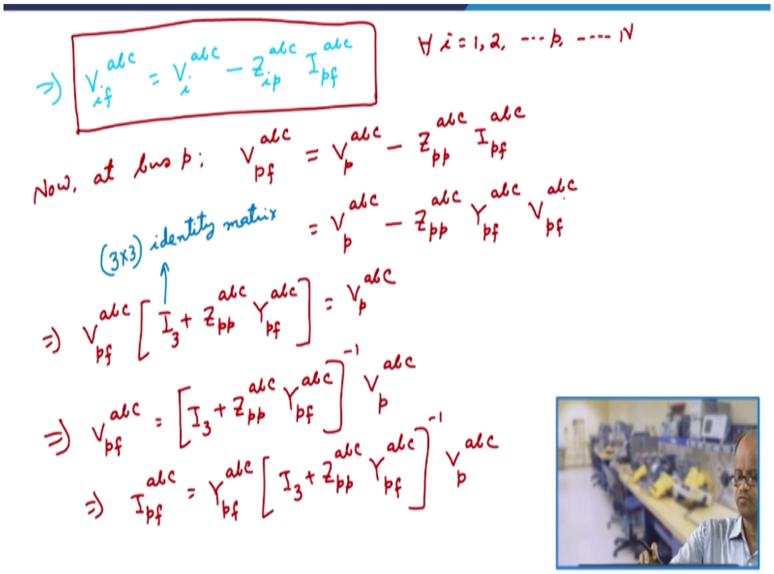

$$\Rightarrow \boxed{V_{if}^{abc} = V_i^{abc} - Z_{ip}^{abc} I_{pf}^{abc}} \qquad \forall i = 1, 2, \cdots p, \cdots N$$

Now, at bus p: $V_{pf}^{abc} = V_p^{abc} - Z_{pp}^{abc} I_{pf}^{abc}$

$$= V_p^{abc} - Z_{pp}^{abc} Y_{pf}^{abc} V_{pf}^{abc}$$

(3x3) identity matrix

$$\Rightarrow V_{pf}^{abc} \left[\underset{\uparrow}{I_3} + Z_{pp}^{abc} Y_{pf}^{abc} \right] = V_p^{abc}$$

$$\Rightarrow V_{pf}^{abc} = \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right]^{-1} V_p^{abc}$$

$$\Rightarrow I_{pf}^{abc} = Y_{pf}^{abc} \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right]^{-1} V_p^{abc}$$

So Vifabc=Viabc-Zipabc*Zipfabc. So this is a very central result. So we would utilizing this. Now at bus p, this is for all I=1-pn. At bus p what is this? Vpfabc=Vpabc. Please note that Viabc or Vpabc or for everything these are nothing but the free fault voltage. So it is Vpabc–Zppabc*Ipfabc. So from this we get this. Now what is Ipfabc? Ipfabc, from this is nothing but Ypfabc*Vpfabc. Ipfabc=Ypfabc*Vpabc–Zppabc*Ypfabc*Ipfabc.

Please note that this is fault admittance matrix. So then therefore, sorry this is Vpfabc. From this relation Ypfabc*Vpfabc, so then from this relation, so then what I have is Vpfabc*identity matrix+Zppabc*Ypfabc=Vpabc. Please note this is essentially a 3x3 identity matrix. We can denote it as I3. So I3 denotes for 3x3 identity matrix, right. So then therefore we can write that Vpfabc=I3+Zpabc*Ypfabc inverse*Vpabc, right.

So then therefore we already know that what is the prefault voltage is. So then if we know this prefault voltages and we already know that what is the bus impedance matrix, this, this, this, Z bus abc is actually called bus impedance matrix. We must note it that this is called bus impedance matrix. This is of course a 3 phase matrix. So this is 3 phase bus impedance matrix. So once you know this 3 phase bus impedance matrix, so then therefore I know this matrix.

For a given type of fault, I also know this matrix. So then therefore I know everything, so I know Vpfabc. So once I know what is Vpfabc, so then therefore I can calculate Ipfabc as, so then therefore Ipfabc as it is nothing but what is Ipfabc is Ypfabc*Vpfabc. So it is Ypfabc*I3+Zppabc*Ypfabc inverse*Vpabc, correct, because Ipfabc=Ypfabc*Vpfabc. So once I know this Vpfabc, so then therefore I know what is Vipfabc.

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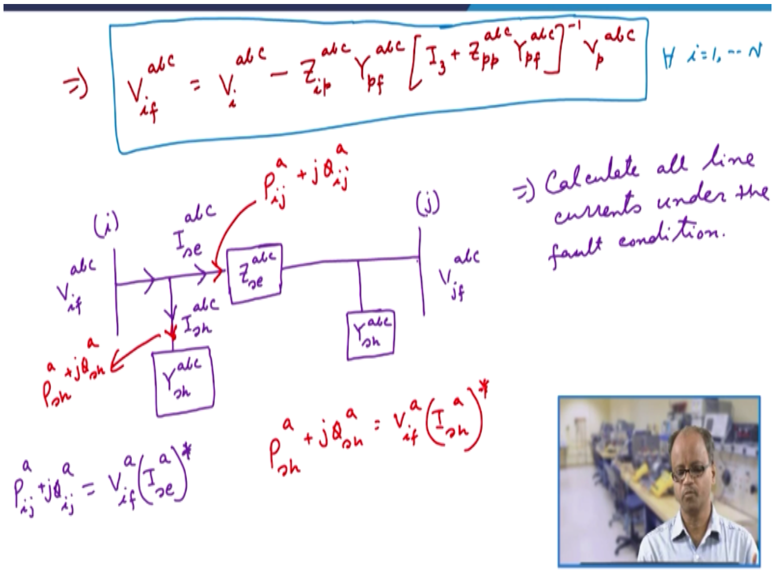

$$\Rightarrow \boxed{V_{if}^{abc} = V_i^{abc} - Z_{ip}^{abc} Y_{pf}^{abc} \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right]^{-1} V_p^{abc}} \quad \forall\, i = 1, \dots N$$

⇒ Calculate all line currents under the fault condition.

$$P_{ij}^{a} + jQ_{ij}^{a} = V_{if}^{a} \left(I_{se}^{a} \right)^{*}$$

$$P_{sh}^{a} + jQ_{sh}^{a} = V_{if}^{a} \left(I_{sh}^{a} \right)^{*}$$

So then therefore I know from this equation Vifabc would be Viabc and that is =-Zipabc*Ipfabc so then from –Zipfabc–Zipabc and then this Ipfabc is Ypfabc*I3+Zppabc*Ypfabc inverse*Vpabc. So by this, we know that what is the fault voltages at all the buses. So then therefore this expression gives me the fault voltage at all the buses. So therefore this expression gives me the fault voltages at all the buses.

So once you know this fault voltage at all the buses, now let us consider any particular line between bus I and J. So let us consider any particular line between bus I and J. So I have got a bus I and bus J and we know that I have got n series impedance matrix. So let us say this is Z series abc and we have got this Y shunt. So this is bus I. So this is bus J. So this fault voltage is Vifabc, we have just now calculated from here.

This fault voltage is Vjfabc, this also we will be able to calculate from here and let us say, Y shunt abc and let us say this is Y shunt abc. So now we know this and as well as know this voltage, so then therefore I can calculate this current and this current would be nothing but this current plus this current, right. So then therefore once I know this voltage, I would be able this current and once I know this voltage and this voltage I would be able to know this current.

So therefore I know what is the voltage in the line. So then therefore I know what is the voltage in the line. So then therefore once I know all these bus voltages, we would be able to calculate all line currents under the fault condition, right. So then therefore, once we know that I mean what is the fault current, under the fault condition, so then after that we would be simply able to calculate any power flow at the line under the fault condition.

We will also be able to calculate what is basically the reactive power flow in this branch and as well as reactive power flow in this branch. For example, if I say that this current is let us say total current, let us say I series abc, right, so then therefore let us say this current is I shunt abc for example, so then therefore the real power * phase A would be, that is essentially Vifa*Iaca*. It is not real power, it would be basically complex power, let us say Pija+Jqija with is Vifa*IJa*, right.

And what is this Pij+Qij, we are actually calculating here, we are actually calculating here and here it is Pija+Jqija, right. Similarly, for all the other phases, phase b and phase c also can calculate and then similarly I can also calculate Ddl and reactive power flowing in this branch also corresponding to any phase. For example, if I say that here b shunt a+Jq shunt a, so then I can say that b shunt a+Jq shunt a would be nothing but Vifa*I shunt a *.

So then therefore what you can see that, therefore once we have the 3 phase bus admittance matrix of the system as well as the 3x3 fault admittance matrix of the system, so then therefore for any type of fault occurring at any bus of the system , we can simply calculate each and every quantity of our interest, that is the fault current flowing through any line or real power flowing through any line as well as the reactive power flowing through any line as well as the real and reactive power flow through the shunt branches of any line.

So everything I can calculate. So then therefore this particular method is absolutely general and this particular method does not need any kind of assumption. So what we only need to know, we only need to know the prefault bus voltages of the system, which is already known. So once we know that what is the prefault bus voltage of the system, so by this expression everything is known. So this is known, this is known, and everything else is known.

After that we can calculate this and after that we can calculate every other point of interest as we have shown here. So we stop today and we would be discussing some other aspect of this particular fault analysis in the subsequent lectures. Thank you.