

Computer Aided Power System Analysis
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Lecture - 55
Fault Analysis (Contd.)

Hello friends. Welcome to this lecture on computer aided power system analysis. In the last lecture, we have started the derivation of the admittance matrix correspondingly different types of symmetrical and unsymmetrical fault. In the last lecture, we have looked into 3 different types of fault; one is L-G and then L-L and then L-L-G. Today, we would be looking into the other remaining two types of faults that is L-L-L and L-L-L-G.

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L-L-L fault:

Applying KCL at point 'N'

$$I_a^f + I_b^f + I_c^f = 0$$

$$\frac{V_a - V_N}{z_f} + \frac{V_b - V_N}{z_f} + \frac{V_c - V_N}{z_f} = 0$$

$$\Rightarrow \frac{1}{z_f}(V_a + V_b + V_c) = \frac{3}{z_f}V_N \quad \Rightarrow V_N = \frac{1}{3}(V_a + V_b + V_c)$$

$$I_a^f = \frac{1}{z_f} \left[V_a - \frac{1}{3}(V_a + V_b + V_c) \right] = \frac{2V_a - V_b - V_c}{3z_f}$$

$$I_b^f = \frac{1}{z_f} \left[V_b - \frac{1}{3}(V_a + V_b + V_c) \right] = \frac{-V_a + 2V_b - V_c}{3z_f}$$

So let us start so fault analysis, so what we are doing is so now we are considering L-L-L fault. So this L-L-L fault is we have a, b, c we have so this is an L-L-L fault please note that so this is I_a^f , I_b^f , I_c^f and these are all z_f , z_f , z_f . So let us see that these are all given as z_f . So now let us so that this point is N neutral points, so then therefore applying KCL at N, applying KCL at point N that $I_a^f + I_b^f + I_c^f = 0$ so we get $V_a - V_N / z_f + V_b - V_N / z_f + V_c - V_N / z_f = 0$.


So then therefore, $1/z_f * V_a + V_b + V_c = 3/z_f * V_N$ or in other words V_N is nothing but the average of $V_a + V_b + V_c$. So then therefore, I_a^f would be $1/z_f * V_a - 1/3 * V_a + V_b + V_c$ so that is $= 2V_a - V_b - V_c / 3z_f$. Similarly, I_b^f would be $1/z_f * V_b - 1/3 * V_a + V_b + V_c = -V_a + 2V_b - V_c / 3z_f$.

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$$I_c^f = \frac{1}{z_f} \left[V_c - \frac{1}{3} (V_a + V_b + V_c) \right] = \frac{-V_a - V_b + 2V_c}{3z_f}$$

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \frac{y_f}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad ; \quad y_f = \frac{1}{z_f}$$

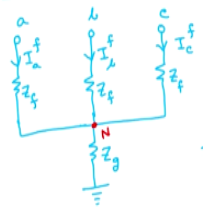
Y_{fault}



And I_c^f is $1/z_f * V_c - 1/3 V_a + V_b + V_c$ so it is $-V_a - V_b + 2V_c / 3z_f$. So then therefore, if I put all of them in a matrix form, it would be $y_f/3 \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$ where y_f is $= 1/z_f$. So then therefore, this quantity is nothing but Y_{fault} right. So this is Y_{fault} , which is easier. So now let us look at L-L-L-G fault.

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L-L-L-G fault




Applying KCL at point N

$$\frac{V_a - V_N}{z_f} + \frac{V_b - V_N}{z_f} + \frac{V_c - V_N}{z_f} = \frac{V_N}{z_g}$$

$$\Rightarrow \frac{1}{z_f} (V_a + V_b + V_c) = \left(\frac{3}{z_f} + \frac{1}{z_g} \right) V_N = \frac{3z_g + z_f}{z_g z_f} V_N$$

$$\Rightarrow V_N = \frac{z_g}{3z_g + z_f} (V_a + V_b + V_c)$$

$$I_a^f = \frac{1}{z_f} \left[V_a - \frac{z_g}{3z_g + z_f} (V_a + V_b + V_c) \right]$$


So now we are looking at L-L-L-G fault. So what we have, we have is and then we have so what we will have is so if a, b, c all are $z_f, z_f, z_f, z_g, I_c^f, I_b^f, I_a^f$. Again, as usual we take this point to be let us say N, this point is N, so applying KCL at point N, what we can write down, we can write down that $V_a - V_N / z_f + V_b - V_N / z_f + V_c - V_N / z_f = V_N / z_g$. So then therefore, we have $1/z_f * V_a + V_b + V_c = (3/z_f + 1/z_g) * V_N$.

So it is $3z_g + z_f / z_g z_f^* V_N$, so then therefore V_N would be simply $z_g / 3z_g + z_f^* V_a + V_b + V_c$. So this would be the V_N . So then therefore, I_{af} would be $1/z_f V_a - V_N$, $-V_N$ is $z_g / 3z_g + z_f^* V_a + V_b + V_c$.

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$$\Rightarrow I_a^f = \frac{1}{z_f} \left[\frac{(3z_g + z_f)V_a - z_g V_a - z_g V_b - z_g V_c}{(3z_g + z_f)} \right]$$

$$\Rightarrow I_a^f = \frac{1}{z_f} \left[\frac{(2z_g + z_f)V_a - z_g V_b - z_g V_c}{3z_g + z_f} \right] \dots \text{--- (1)}$$
 Now, Let $y_0 = \frac{1}{3z_g + z_f}$; $y_f = \frac{1}{z_f}$
 Then, $y_0 + 2y_f = \frac{1}{3z_g + z_f} + \frac{2}{z_f} = \frac{z_f + 6z_g + 2z_f}{z_f(3z_g + z_f)} = \frac{3(2z_g + z_f)}{z_f(3z_g + z_f)} \dots \text{--- (2)}$
 and $y_0 - y_f = \frac{1}{3z_g + z_f} - \frac{1}{z_f} = \frac{z_f - 3z_g - z_f}{z_f(3z_g + z_f)} = \frac{-3z_g}{z_f(3z_g + z_f)} \dots \text{--- (3)}$
 From eq: (1)

So now what we have, we have so then therefore I_{af} is $= 1/z_f * 3z_g + z_f^* V_a - z_g V_a - z_g V_b - z_g V_c$ which we will see into $3z_g + z_f$. So then therefore, I_{af} would be $1/z_f 2z_g + z_f^* V_a - z_g V_b - z_g V_c / 3z_g + z_f$. Now suppose we define now let y_0 is given by $1/3z_g + z_f$ and y_f is given by $1/z_f$. So then therefore then $y_0 + 2y_f$ what I get, y_0 is $1/3z_g + z_f + 2y_f$ is $2/y_f$ so that would $y_f + 6z_g +$ sorry this should be z_f .

So $6z_g + 2z_f / z_f * 3z_g + z_f$ so this can be written as $3 * 2z_g + z_f / z_f * 3z_g + z_f$ that we can write so this is equation 1 and $y_0 - y_f$ will be $1/3z_g + z_f - 1/z_f$ is $= z_f - 3z_g - z_f / z_f * 3z_g + z_f$ it is $-3z_g / z_f * 3z_g + z_f$.

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$$I_a^f = \frac{2z_g + z_f}{z_f(3z_g + z_f)} V_a - \frac{z_g V_b}{z_f(3z_g + z_f)} - \frac{z_g V_c}{z_f(3z_g + z_f)}$$


$$= \frac{y_0 + 2y_f}{3} V_a + \frac{y_0 - y_f}{3} V_b + \frac{y_0 - y_f}{3} V_c \quad \text{[utilizing eqns (2) and (3)]}$$



$$\dots (4)$$

$$I_b^f = \frac{1}{z_f} \left[V_b - \frac{z_g}{3z_g + z_f} (V_a + V_b + V_c) \right]$$

$$= \frac{1}{z_f} \left[\frac{-z_g V_a + (2z_g + z_f) V_b - z_g V_c}{3z_g + z_f} \right] = \frac{y_0 - y_f}{3} V_a + \frac{y_0 + 2y_f}{3} V_b + \frac{y_0 - y_f}{3} V_c$$

$$\dots (5)$$

$$I_c^f = \frac{1}{z_f} \left[V_c - \frac{z_g}{3z_g + z_f} (V_a + V_b + V_c) \right]$$


So then therefore now from equation 1 I_a^f is $= \frac{2z_g + z_f}{z_f(3z_g + z_f)} V_a - \frac{z_g}{z_f(3z_g + z_f)} V_b - \frac{z_g}{z_f(3z_g + z_f)} V_c$. Now here if you do utilize these two relations, it can be written as this is $y_0/2y_f$ so it is essentially $y_0 + 2y_f/3 * V_a$ and this says $y_0 - y_f/3 * V_b + y_0 - y_f/3 * V_c$. Let us say this is equation 2, this is equation 3. So then write down utilizing equation 2 and 3. So this is the expression of I_a^f . So let us say this is equation 4.

So similarly if we look that the expression of I_b^f it will be $1/z_f * V_b - z_g/3z_g + z_f * V_a + V_b + V_c$. So if you repeat all these derivations as compared as you have done here so we will find that it is nothing but $1/z_f * \text{this is } -z_g V_a + 2z_g + z_f * V_b - z_g V_c / 3z_g + z_f$. So it would be also similarly it turn out to be $y_0 - y_f/3 V_a + y_0 + 2y_f/3 V_b + y_0 - y_f/3 V_c$. This is equation 5. Similarly, for the expression I_c^f would be $1/z_f * V_c - z_g/3z_g + z_f * V_a + V_b + V_c$.

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$$\Rightarrow I_c^f = \frac{1}{z_f} \left[\frac{-z_g V_a - z_g V_b + (2z_g + z_f) V_c}{3z_g + z_f} \right]$$



$$\Rightarrow I_c^f = \frac{y_0 - y_f}{3} V_a + \frac{y_0 - y_f}{3} V_b + \frac{y_0 + 2y_f}{3} V_c \quad \dots (6)$$

From equations (4), (5) and (6),

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \frac{1}{3} \begin{bmatrix} y_0 + 2y_f & y_0 - y_f & y_0 - y_f \\ y_0 - y_f & y_0 + 2y_f & y_0 - y_f \\ y_0 - y_f & y_0 - y_f & y_0 + 2y_f \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$\underbrace{\hspace{10em}}_{Y_{\text{fault}}}$



So it would be I_{cf} would be $1/z_f - z_g V_a - z_g V_b + 2z_g + z_f V_c / 3z_g + z_f$. So in other words I_{cf} is essentially $y_0 - y_f / 3 * V_a + y_0 - y_f / 3 * V_b + y_0 + 2y_f / 3 * V_c$. So from equation 4, 5, 6 we can write down, what you can write down, you can $I_{af} I_{bf} I_{cf} = 1/3$ so it is $y_0 + 2y_f y_0 - y_f y_0 - y_f$ this is $y_0 - y_f y_0 + 2y_f y_0 - y_f$ this is $y_0 - y_f y_0 - y_f y_0 + 2y_f$ and this is into $V_a V_b V_c$. So therefore this is the Y fault matrix.

So this is the Y fault matrix, so then therefore if we know that what is the fault impedance at any phase and what is the ground fault impedance, you can simply find out the Y fault matrix.

So then what does it mean?

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Any fault (L-G, L-L, L-L-G, L-L-L, L-L-L-G) can be represented by a (3x3) fault admittance matrix (Y_{fault})

Let, Y_{fault} be denoted as Y_{pf}^{abc} → denotes that it is a (3x3) matrix involving all phases 'a', 'b' and 'c'.

p → stands for 'bus' at which the fault takes place

f → stands for 'fault'

V_{pf}^x → fault voltage at bus 'p' corresponding to phase 'x' where 'x' = a, b, c

I_{pf}^x → fault current at bus 'p' corresponding to phase 'x' where 'x' = a, b, c

Bus 'p'

a V_{pf}^a

b V_{pf}^b

c V_{pf}^c

Y_{pf}^{abc}

So it means that for any fault either L-G or L-L or L-L-G or L-L-L or L-L-L-G any fault can be represented by a 3 x 3 fault admittance matrix so this is Y fault. In short, when we would be writing as in Y fault let us denote as Y fault as Y let us say pf abc. So here what does this p stands for, p stands for here essentially let Y fault be denoted as we should write that Y fault be denoted as Y_{pf}^{abc} .

So what are the different quantities which they signify p, p stands for the bus at which the fault takes place, f stands for fault and abc denotes that it is a 3 x 3 matrix involving all phases abc. So this is the notation, so then therefore now suppose there is a bus so this is phase a, phase b, phase c and this is bus p and let us say there is some fault and so then therefore there is some fault we can simply represent it as some fault which is let us say Y_{pf}^{abc} , so then this fault can be represented as Y_{pf}^{abc} .

Now if these voltages are $V_{pf a}$, these voltages are $V_{pf b}$ and these voltages are $V_{pf c}$ so why these voltages are V_{pf} let us say l stands for fault voltage at bus p corresponding to phase l where l is a, b or c right. So then therefore, $V_{pf a}$ stands for the fault voltage at bus p corresponding to phase a , this is fault voltage at bus p corresponding to phase b and fault voltage bus p corresponding to phase c .

And let us say these currents are denoted as $I_{pf a}$, this is let us say $I_{pf b}$ and let us say this is $I_{pf c}$ right. So similarly, I_{pf} and let us say these currents are $I_{pf a}$, $I_{pf b}$ and $I_{pf c}$ where $I_{pf l}$ is the fault current at bus p corresponding to phase l where l is again a, b, c . So now so then this is the case, so now what we have, so I have bus p , it has got 3 phases a, b, c and it can have any voltage sorry it can have any type of fault it can occur, let us say L-G or L-L or L-L-G or whatever.

So then therefore as we have already seen that any fault can be represented by a fault admittance matrix which is written as $Y_{pf abc}$ and because of this $Y_{pf abc}$ so then therefore this bus voltages are now let us say fault voltages $V_{pf a}$, $V_{pf b}$ and $V_{pf c}$ and as well as the current flowing to the fault are given by $I_{pf a}$, $I_{pf b}$ and $I_{pf c}$ right. So then therefore, this is the representation of the fault and this representation is perfectly general representation because this representation takes into account any type of fault.

Now with this representation we are now in a position to analyze the effect of the fault accordingly at any bus of any large scale power system. This issue would be taken in the next lecture. Thank you.