

**Computer Aided Power System Analysis**  
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**Lecture – 54**  
**Fault Analysis (Contd.)**

Hello friends, welcome to this lecture on computer aided power system analysis, we have been looking at the process of find the Y bus matrix of a three phase unbalanced system in the presence of a transformer, so towards that goal, we have first derive the Y bus matrix of a simple 5 bus system which also includes a transformer but then we have seen that from that it is not very clear that how the matrices is corresponding to the transformer can be embedded into the overall Y bus matrix.


So for that the reason you are now in the process of considering the same system but by removing the transformer from the circuit.

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*In matrix form*

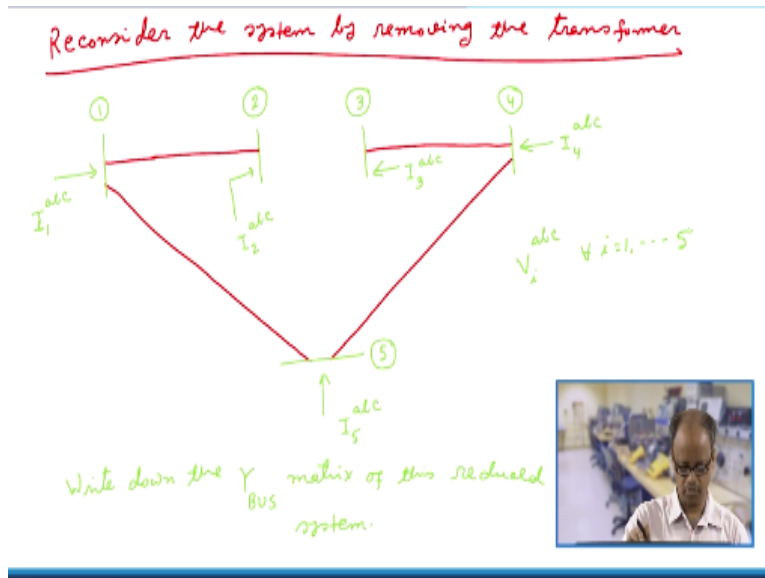
$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ I_3^{abc} \\ I_4^{abc} \\ I_5^{abc} \end{bmatrix} = \begin{bmatrix} y_{11}^{abc} & -y_{12}^{abc} & 0 & 0 & -y_{15}^{abc} \\ -y_{12}^{abc} & Y_{II}^{abc} + y_{12}^{abc} + y_{13,23}^{abc} & 0 & 0 & 0 \\ 0 & Y_{III}^{abc} + y_{34}^{abc} + y_{34,23}^{abc} & -y_{34}^{abc} & 0 & 0 \\ 0 & 0 & -y_{34}^{abc} & y_{44}^{abc} & -y_{45}^{abc} \\ -y_{15}^{abc} & 0 & 0 & -y_{45}^{abc} & y_{55}^{abc} \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \\ V_5^{abc} \end{bmatrix}$$

*Y Bus*



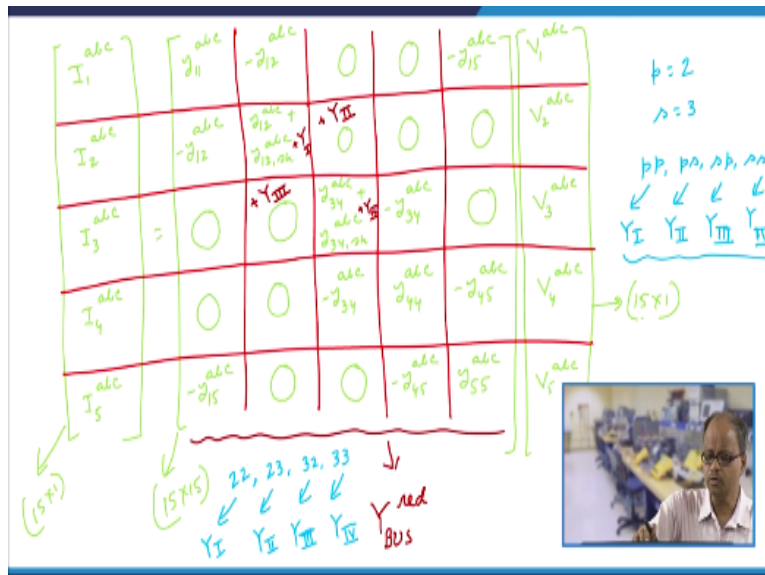
So then therefore, what we have done is that I mean this is the overall Y bus matrix which we have already derived in the last lecture which includes also the transformer.

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So, now we are considering the same system but this transformer between bus 2 and 3 have now; has now been actually eliminated and now, we have also now denoted all these injected currents and obviously, this voltages are beyond abc,  $V_2^{abc}$ ,  $V_3^{abc}$ ,  $V_4^{abc}$  city and  $V_5^{abc}$ , so the voltages are  $V_i^{abc}$  for all  $i = 1$  to 5, so we will be now writing down the Y bus matrix of this system following the usual procedure, so for that we will not write down the equations as we have done.

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We will now just follow this simple algorithm for performing the Y bus matrix, so the Y bus matrix will be looking like this, I1 abc, I2 abc, I3 abc, I4 abc, I5 abc, so this is how it will look like, so and then of course we have here this, and then of course here this and then of course here

this  $V_1$ ;  $V_5$  abc, so if we write down this equations, so there will be looking like, so we will do simply, so  $y_{12}$ , so it will be  $y_{11}$ , so again as usual first draw the grid, and then we do this, this, this and this.

So, why, so then now, this one would be  $y_{11}$  abc, then  $-1/2$  abc,  $-y_{15}$  abc from the circuit very easily because this is only connected between bus 2 and bus 3 and this is of course 0, these are all 3 cross 3, for  $I_2 -1/2$  abc and this is  $I_2$  abc +  $y_{12}$  shunt abc, this is only connected to bus 1, it is not connected to anywhere else directly, so all of them would be 0, 0, 0, 0, all of them would be 0.

Now bus 3; bus 3 is only connected to bus 4 but to any other bus, it is not connected to 1 and 5, it is not connected, so between bus 3 and bus 4, so it would be 1 and 2, this is 0, bus 5 it is also 0 and at bus 3, it would be  $y_{34}$  abc, so it is  $-y_{34}$  abc, it is  $y_{34}$  abc +  $y_{34}$  shunt abc, bus 4 would be usual, bus 4 is  $y_{44}$  abc and this  $y_{11}$  abc and  $y_{44}$  abc, I mean this is the same thing which you have already done and  $-y_{43}$  abc,  $-y_{34}$  abc and  $-y_{45}$  abc.

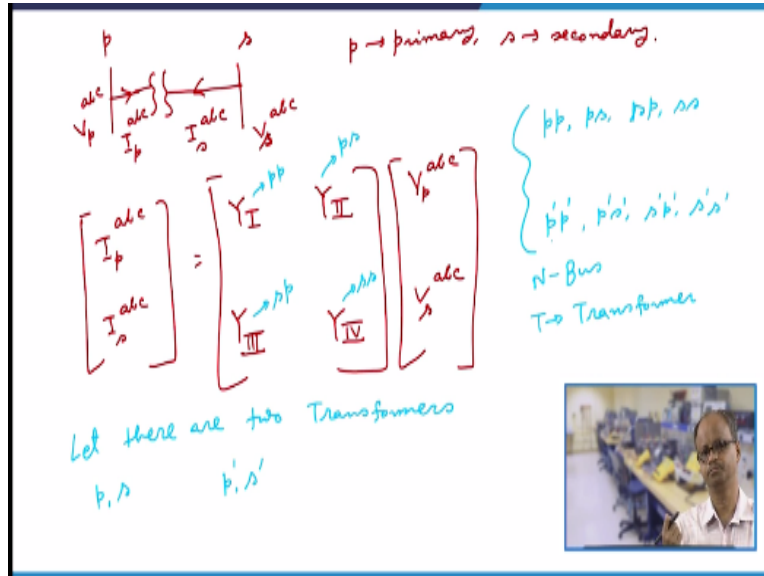
Because it is connected directly to bus 3 and bus 5, this would be 0 and then last one is bus 5, it is connected to bus 4 and bus 1, so this is  $y_{55}$  abc –  $y_{45}$  abc, this is  $-1/5$  abc, 0, 0 again this is a 15 cross 1, this is a 15 cross 15, this is a 15 cross 1, so this is the Y bus matrix, so we can write down that this is as if the Y bus matrix, so this is we can say this is the Y bus, let us say reduced, this is the reduced in a sense that essentially that all this transformer have been eliminated from the circuit.

And this is the actual Y bus and this is the actual Y bus, now if we just compare these Y bus matrix and Y bus reduced matrix, I find that this Y bus matrix is actually the Y bus reduced matrix only with the, I mean all the other I mean almost all the elements are identically the same, all elements are identically the same except at bus 2, except at these 4 elements, this is here this is  $y_1$  is added, here  $y_2$  is added, here  $y_3$  is added, here  $y_4$  is added.

So, then therefore, if in this Y bus red, if I add here  $+y_1$ , if I add here  $y_2$ , if I add here  $y_3$  and if I add here  $y_4$ , if I add these 4 matrices or rather these 4 sub matrices, I will just simply get this

original Y bus matrix, so then our simple algorithm is so then, essentially now basically, why it is happening?

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So, let us see that again, so then essentially in general, let us say this is bus p, this is bus s, I mean p stands for primary and s stands for secondary, p, primary, s, secondary, so and if there is a transformer connected here, so this  $I_p^{abc}$ , this  $I_s^{abc}$ , this is  $V_s^{abc}$  and this is  $V_p^{abc}$ , so we said that  $I_p^{abc}$ ,  $I_s^{abc}$  is given by  $Y_1, Y_2, Y_3, Y_4$  and  $V_p^{abc}, V_s^{abc}$  and this positions we can say that this is essentially pp position that is basically, it is connecting the primary current with primary side voltage, we should complete here.

So and this is essentially, ps position because it is connecting primary side current with secondary side voltage, this is an essentially as p position that because it is connecting secondary side with the, I mean it is basically connecting secondary side current with the primary side voltage and this is a s position, this is because this is connecting secondary side current with the secondary side voltage.

Now, here in our case, what was p;  $p = 2$  and this is  $= 3$ , so then therefore the algorithm is very simple that we will first find out the Y bus matrix of the system by removing all, I mean removing the transformer and then we will simply put this 4 matrices;  $Y_1, Y_2, Y_3$  and  $Y_4$  at pp,

ps, sp and ss, you will simply add, so at pp location, we will add Y1 at ps location, we will add Y2, at sp location we will add Y3 and at ss location we will add Y4 matrix.

We will just add these 4 matrices or rather these 4 sub matrices for the transformers at this 4 locations; pp, ps, sp and ss, right and so then after we add these 4 sub matrices to the Y bus reduced matrix, we get the actual system Y bus matrix. Now, here in this case our p is 2 and s is 3, so then therefore here we can see pp is 22, ps is 23, sp is 32 and ss is 33, say I mean 22 would be added at Y1, this is exactly what you have done to at the location 2 3, we would be connect; we would be adding Y2 that is exactly we have done.

At the location 32, we would be adding Y3 matrix that is what exactly we have done at location 33, we would be adding Y4 matrix and that is the and basically that is what we have done, now suppose instead on one transformer, now let us say I do have let us say 2 transformer, now let there are 2 transformers, one is between bus p and s and another is between was bus p dash and s dash.

So, then therefore what we will do; we will simply remove these 2 transformers first and then we will find out the Y bus reduced matrix by the usual process and then we will add these 4 sub matrices at the location the pp, ps, sp and ss and basically, for this transformer we would be adding this but and we would be adding these transformers sub matrices at the location p dash p dash, p dash s dash, s dash p dash, s dash s dash.

And same procedure would be followed even if they are 100's of transformer, so then therefore given an N bus system, so then therefore if there are let us say N bus and let us say T transformer, so then what we will do; we will first simply remove this T transformers from these Y bus matrix reduce and then corresponding to each transformer we will look at the 2 buses where they are connected and then we will simply add the 4 transformers or matrices as those locations as discussed here.

So, by this process, so we now in the stage where we can find out the Y bus matrix of any system in which they are all lines and also as well as they are transformers, so now we have reached a

state where we now would be looking into the actual process of fault analysis, now to start with the actual process of fault analysis, we also; we will now first actually, now first find out the Y bus representation of different types of faults.


Because here in this case, we have simply represented the entire system by its equivalent Y bus matrix, so then therefore if we wish to include the effect of any particular fault, so then therefore we also should have equivalent Y bus matrix or rather the equivalent bus impedance matrix corresponding to that particular faults and then therefore as we will see in the subsequent lectures by incorporating that fault admittance matrix.

We call this fault admittance matrix by incorporating this fault admittance matrix into the system bus admittance matrix, we would be able to calculate the all the quantities which are of our interest.

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*Fault admittance matrices of different types of fault.*

$L-G$	} $a-g, b-g, c-g$	$a-g$
$L-L$		$b-c$
$L-L-G$	} $a-b-g, b-c-g, c-a-g$	$b-c-g$
$L-L-L$		$a-b-c$ ✓
$L-L-L-G$	} $a-b-c-g$ ✓	$a-b-c-g$



So, then therefore so now, our attention is to find out the fault admittance matrices of different types of fault, so there are fault, so now what we are trying to do; we are now trying to find out the fault admittance matrix of different types of fault which would be ultimately embedded into the system bus admittance matrix to calculate the different quantities of our interest. Now, we know that we have different types of fault as let us say, LG that is basically line to ground.

And then we have LL that is essentially, line to line, then we have LLG that is line to line to ground, then we have LLL that is line to line to line but this ground is not present and then we have LLLG that is line to line to line to ground, so we have to find out the fault admittance matrices of different, so all these different types of faults, so then therefore altogether there are 5 types of faults; 5 types, yes of course for LG, there would be ag, bg, cg.

And for LL, there would be ab, bc, ca, for LLG, there would be abg, bcg, and cag, for LLL, of course there is only one abc, and for LLLG also abcg, so you may, so then we may think that there altogether 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; 11 types of fault but then what we will do for these and these there are only unique type and from these 3 sets, we will be only considering let us say ag, bc and bcg, right.

And if we know that how to find out the fault admittance matrix for a particular type of faults in any set for example, if I know how to find out the fault admittance matrix, for let us say fault ag, so then immediately would be knowing that I mean immediately we would know that how to find out the fault admittance matrix for this fault bg and cg, similarly if we know that how to find out the fault admittance matrix corresponding to bc fault, then of course similarly by following the same procedure, we would be able to find out the fault admittance matrix for this fault ab and ca.

Similarly, here also, we would be looking into the fault admittance matrix corresponding to bcg fault and by following the same procedure, we would be able to find out this fault admittance matrix for abg and cag, so then therefore, here we would only considering the fault admittance matrix for this 5 types of fault; one is ag, one is bc, one is bcg, then abc, and abcg, so we looking into this 4; 5 types of faults and we would be finding out their fault admittance matrix for this 5 types of fault.

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$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$I_{bf} = 0; I_{cf} = 0$$

$$I_{af} = \frac{V_a}{z_f} = y_f V_a$$

$$y_f = \frac{1}{z_f}$$

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} y_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Fault admittance matrix

So, first fault we are talking is the ag fault, for ag fault what you have is that we have let us say this is bus a, bus b and let us say point bus c and then between a and g, so this is zg, this is a, b, c, so then therefore there is a fault in fact we should use fault impedance, so now for any particular type of fault, when we are talking about the fault admittance matrix, what we are actually trying to say.

So, for example if I say that the current flowing through the fault is let us say  $I_a$ ,  $I_b$ ,  $I_c$ , we should say that  $I_{af}$ ,  $I_{bf}$  and  $I_{cf}$ , so then, so  $I_{af}$ ,  $I_{bf}$ ,  $I_{cf}$  are the current flowing through the fault, so these would be connected by a 3 cross 3 matrix such that they can be expressible in terms of  $V_a$ ,  $V_b$  and  $V_c$ , so these voltages are  $V_a$ , so this voltages let us say  $V_a$ , so this voltage is  $V_a$ , this voltage is  $V_b$ , this voltage is  $V_c$ .

And this current is let us say  $I_{af}$  and here of course because there is no fault is there, so then therefore we can say that  $I_{bf} = 0$ ,  $I_{cf} = 0$ , from then so then therefore because it is ag fault, so then I can say that  $I_{bf} = 0$  and  $I_{cf}$  is also = 0 and  $I_{af}$  would be =  $V_a/z_f$ , sorry  $V_a/z_f$ , so it is  $y_f * V_a$ , where  $y_f = 1/z_f$ , so then if that is the case, so then therefore we can write down the following relation between  $I_{af}$ , let us say  $I_{bf}$ ,  $I_{cf}$ , so it would be simply  $y_f, 0, 0$  and all the other terms are 0.



So, obviously and indeed that these matrix relation indeed give this relations,  $I_{af} = y_f * V_a$  and  $I_{bf} = 0$ , this is  $0 * this + 0 * this + 0 * this$  and  $I_{cf} = 0$ , so then therefore this is the fault admittance matrix.

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$I_{af} = 0$   
 Applying KCL at point N  
 $I_{bf} + I_{cf} = 0$   
 $\frac{V_b - V_N}{Z_f} + \frac{V_c - V_N}{Z_f} = 0 \Rightarrow \frac{\partial_f (V_b - V_N) + \partial_f (V_c - V_N)}{Z_f} = 0$   
 $\Rightarrow \frac{\partial_f (V_b + V_c)}{2} = 2 \frac{\partial_f V_N}{2} \Rightarrow V_N = \frac{1}{2} (V_b + V_c)$   
 $I_{bf} = \frac{\partial_f (V_b - V_N)}{Z_f} = \frac{\partial_f}{Z_f} \left[ V_b - \frac{1}{2} (V_b + V_c) \right] = \frac{\partial_f}{2} (V_b - V_c)$   
 $I_{cf} = \frac{\partial_f (V_c - V_N)}{Z_f} = \frac{\partial_f}{Z_f} \left[ V_c - \frac{1}{2} (V_b + V_c) \right] = \frac{\partial_f}{2} (V_c - V_b)$

Now, let us consider the fault bc, so now we are considering the fault bc, so we are now considering a fault bc, so what I have, so it is again bus a, bus b, bus c and for bc fault, what we have is; what we simply have is that we have some fault impedance, we have fault, some fault impedance  $z_f$  and this, so it is a bc fault, so then therefore there is a fault between bus b and c and the fault impedance is; we assume that this are basically  $z_f$ .

So, then again as we have said that this is  $V_a$ , this is  $V_b$  and this is  $V_c$  and we have  $I_{bf}$ , this is  $I_{cf}$  because phase a is not involved in the fault, so  $I_{af}$  would be obviously 0, now we have to find out the expression of  $I_{bf}$  and  $I_{cf}$  in terms of  $V_b$  and  $V_c$ , now suppose that this is the point let us say, neutral, say this is the point neutral having a voltage  $V_N$ , now if I apply KCL at this point, so then therefore  $I_{bf} + I_{cf} = 0$ .

So, then therefore applying KCL at point N, what we have is that  $I_{bf} + I_{cf} = 0$ , what is  $I_{bf}$ ;  $I_{bf}$  is  $V_b - V_N - z_f + V_b -$  sorry,  $V_c - V_N / z_f = 0$ , so it is actually nothing but so we can write down that  $y_f * V_b - V_N + y_f * V_c - V_N = 0$ , so we can write down that  $y_f * V_b + V_c = 2y_f * V_N$ , so

then therefore  $V_N = 1/2$  of  $V_b + V_c$ , so once we get  $V_N = 1/2$  of  $V_b + V_c$ , B so then therefore I can simply find out that  $I_{bf} = y_f * V_b - V_N$ , so it is  $y_f * V_b - 1/2$  of  $V_b + V_c$ .

Or in other words, it is  $y_f/2 * V_b - V_c$ ,  $I_{cf}$  is nothing but  $y_f * V_c - V_N$  that is  $y_f * V_c - 1/2$  of  $V_b + V_c$ , so that is  $y_f/2 V_c - V_b$ .

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$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \frac{y_f}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$Y_{fault}$$

Fault admittance matrix =  $Y_{fault}$

$I_{af} = 0$  at point N


$I_{bf} + I_{cf} = I_g$

So, then therefore the if I write down these equation in matrix form, what we will get is that I get  $V_a, V_b, V_c$ , because  $I_{af}$  is 0, so then it will be 0, 0 and 0 and  $I_{bf}$  is  $y_f/2 V_b - V_c$ , so we can simply take  $y_f/2$  and this is 1-1 and this is 0, -1, and it is  $V_c - V_b$ , so it is -1, 1, so then therefore this is the fault admittance matrix, so this is the fault admittance matrix, so this green coloured, this is the fault admittance matrix, right.

So, so far we would be only denoting this fault admittance matrix as  $Y_{fault}$ , so  $Y_{fault}$  is essentially this, so this green coloured okay, now let us look at the another on, now let us look at b-c-g, if I look at b-c-g, so then therefore I have again phase a, phase b, phase c, now between phase a, phase b and then so we as usual we said that this phase a, this is phase b, this is phase c, we say that this is point N, fault is  $z_f$ , this is  $z_f$ , this is  $z_g$ , this is the ground impedance.

This has got voltage  $V_a$ , voltage  $V_b$ , voltage  $V_c$  and as usual  $I_{bf}$ ,  $I_{cf}$ , and this current is 0, so again  $I_{af} = 0$  and if I say that this is  $I_g$ , so at point N, we can say that  $I_{bf} + I_{cf} = I_g$  by applying KCL.

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$$\begin{aligned} \Rightarrow y_f(V_b - V_N) + y_f(V_c - V_N) &= y_g V_N \quad y_g = \frac{1}{z_g} \\ \Rightarrow y_f(V_b + V_c) &= (y_g + 2y_f)V_N \\ \Rightarrow V_N &= \frac{y_f}{y_g + 2y_f}(V_b + V_c) \\ I_{bf} = y_f(V_b - V_N) &= y_f \left[ V_b - \frac{y_f}{y_g + 2y_f}(V_b + V_c) \right] = y_f \left[ \frac{y_g + y_f}{y_g + 2y_f} V_b - \frac{y_f}{y_g + 2y_f} V_c \right] \\ I_{cf} = y_f(V_c - V_N) &= y_f \left[ V_c - \frac{y_f}{y_g + 2y_f}(V_b + V_c) \right] \\ &= y_f \left[ -\frac{y_f}{y_g + 2y_f} V_b + \frac{y_g + y_f}{y_g + 2y_f} V_c \right] \end{aligned}$$


And  $I_{bf} =$ ; so then therefore, what is  $I_{bf}$ ;  $I_{bf} = y_f * V_b - V_N$  and  $y_f * V_c - V_N$  that  $= I_g$ ;  $I_g$  is  $y_g * V_N$ , where  $y_g$  is  $1/z_g$ , please note that  $y_g$ ,  $y_f$ ,  $z_f$ ,  $z_g$  everything are essentially complex quantities but just for the sake of brevity, we are not really putting this overall, so, then therefore we get  $y_f * V_b + V_c = V_N y_g + 2y_f * V_N$ , so then therefore  $V_N = y_f / y_g + 2y_f * V_b + V_c$ , so  $I_{bf} = y_f * V_b - V_N$ , so it is  $y_f * V_b - V_N$  is  $y_f / y_g + 2y_f * V_b + V_c$ .

So, it would be  $y_f$ , it would be  $y_g +$ ; so it will be  $y_g + y_f / y_g + 2y_f * V_b - y_f / y_g + 2y_f * V_c$ , right, similarly  $I_{cf} = y_f * V_c - V_N$ , so it is  $y_f * V_c - y_f / y_g + 2y_f * V_b + V_c y_f - y_f / y_g + 2y_f * V_b + y_g + y_f / y_g + 2y_f * V_c$ . Now, what we will do is; we do a little simplification.

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$$\text{Now, } y_f \cdot \frac{y_g + 2y_f}{y_g + 2y_f} = \frac{1}{z_f} \left[ \frac{\frac{1}{z_g} + \frac{1}{z_f}}{\frac{1}{z_g} + \frac{2}{z_f}} \right] = \frac{1}{z_f} \left[ \frac{z_f + z_g}{z_f + 2z_g} \right] = \frac{z_f + z_g}{z_f^2 + 2z_g z_f} = a_1 \text{ (let)}$$

$$\text{And } \frac{y_f^2}{y_g + 2y_f} = \frac{1}{z_f^2} \cdot \frac{1}{\frac{1}{z_g} + \frac{2}{z_f}} = \frac{z_g z_f}{z_f + 2z_g} \cdot \frac{1}{z_f^2} = \frac{z_g}{z_f^2 + 2z_g z_f} = b_1 \text{ (say)}$$

Therefore,

$$I_{af} = a_1 V_b - b_1 V_c$$

$$I_{bf} = -b_1 V_b + a_1 V_c$$

$$I_{cf} = 0$$



Now,  $y_f \cdot y_g + y_f / y_g + y_f$ , what we will get;  $y_g + 2y_f$ , so what I will get;  $1/z_f \cdot 1/z_g + 1/z_f / 1/z_g + 2/z_f$ , so it is  $1/z_f \cdot z_f + z_g / z_f + 2z_g$ , so it is  $z_f + z_g / z_f^2 + 2z_g z_f$ , so then therefore let this is  $a_1$ , let and also minus and  $y_f \cdot y_f^2 / y_g + 2y_f$ , so what I will get; I will get is  $1/z_f^2 \cdot 1/z_g + 2/z_f$ , so what we will get is;  $z_g z_f^2 / z_f + 2z_g \cdot z_f^2$ , so this would be =; so it will be  $z_g / z_f^2 + 2z_g z_f$ .

So, therefore it is let us say this is  $b_1$  say, so then therefore if I utilise this, so then what we will get is  $I_{bf} = a_1 V_b$ , so then therefore you can say that  $I_{bf} = a_1 \cdot V_b - b_1 \cdot V_c$  and  $I_{cf} = -b_1 \cdot V_b + a_1 \cdot V_c$ , so then therefore utilising these and  $I_{af} = 0$ , so these are 3 equations.

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$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & a_1 & -b_1 \\ 0 & -b_1 & a_1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$Y_{\text{fault}}$

So, then therefore  $I_{af}, I_{bf}, I_{cf} =$ ;  $I_{af} = 0$ , so it is  $V_a, V_b, V_c, I_{af} = 0$ , so it should be 0, so it is 0,  $a1, -b1$ , it is 0,  $-b1, a1$ , so it is  $a1 - b1$  and  $-b1 a1$ , so that is so, this is it, so then therefore this is nothing but Y fault, so this is the Y fault matrix corresponding to the bcg fault, so we are still remaining with 2 types of fault; abc and abcg fault, so we will be deriving their fault admittance matrices in the next lecture, thank you.