


Computer Aided Power System Analysis
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Lecture – 52
Fault Analysis (Contd.)

Hello friends we had been talking about the y bus matrix formulation for non-balance 3 phase system. So, let us continue so what we did is we have taken this simple system and we have written all this equations for all this injected currents I1 I2 I3 and I 4 all these 4 buses and this equations are given by equation 1 equation 2 equation 2 and equation 4. So, now let us write down these equations in matrix form.

(Refer Slide Time: 01:02)

In matrix form

$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ I_3^{abc} \\ I_4^{abc} \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \end{bmatrix}$$


So, in matrix form so I can write down I 1 abc I2 abc I3 abc I4 abc = something we can write something here what we do is V1 abc V2 abc V3 abc V4 abc so now before I write it let us make this grid so that it will be helpful for us. So we can make grid and I am just making it grid and I am just making these grids right so now let us see how it would look like so we will look at.

(Refer Slide Time: 02:50)

$$\Rightarrow I_2^{abc} = -y_{12}^{abc} V_1^{abc} + (y_{12,2h}^{abc} + y_{12}^{abc} + y_{23,2h}^{abc} + y_{23}^{abc}) V_2^{abc} - y_{23}^{abc} V_3^{abc} \quad \dots (2)$$

Bus 3

$$I_3^{abc} = I_{32,2h}^{abc} + I_{32}^{abc} + I_{34,2h}^{abc} + I_{34}^{abc}$$

$$= y_{23,2h}^{abc} V_3^{abc} + y_{23}^{abc} (V_3^{abc} - V_2^{abc}) + y_{34,2h}^{abc} V_3^{abc} + y_{34}^{abc} (V_3^{abc} - V_4^{abc})$$

$$\Rightarrow I_3^{abc} = -y_{23}^{abc} V_2^{abc} + (y_{23,2h}^{abc} + y_{23}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc}) V_3^{abc} - y_{34}^{abc} V_4^{abc} \quad \dots (3)$$

$$y_{34}^{abc} = (Z_{34}^{abc})^{-1}$$



Bus-4

$$I_4^{abc} = I_{41,2h}^{abc} + I_{41}^{abc} + I_{43,2h}^{abc} + I_{43}^{abc}$$

The expression of let us start.

(Refer Slide Time: 02:53)

$$I_4^{abc} = y_{14,2h}^{abc} V_4^{abc} + y_{14}^{abc} (V_4^{abc} - V_1^{abc}) + y_{34,2h}^{abc} V_4^{abc} + y_{34}^{abc} (V_4^{abc} - V_3^{abc})$$

$$= -y_{14}^{abc} V_1^{abc} - y_{34}^{abc} V_3^{abc} + (y_{14,2h}^{abc} + y_{14}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc}) V_4^{abc} \quad \dots (4)$$




(Refer Slide Time: 02:58)

Ym matrix form

$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ I_3^{abc} \\ I_4^{abc} \end{bmatrix} = \begin{bmatrix} y_{11}^{abc} & -y_{12}^{abc} & 0 & -y_{14}^{abc} \\ -y_{12}^{abc} & y_{22}^{abc} & 0 & 0 \\ 0 & -y_{23}^{abc} & y_{33}^{abc} & -y_{34}^{abc} \\ -y_{14}^{abc} & 0 & -y_{34}^{abc} & y_{44}^{abc} \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \end{bmatrix}$$

$y_{11}^{abc} = y_{12,2h}^{abc} + y_{12}^{abc} + y_{14,2h}^{abc} + y_{14}^{abc}$
 $y_{22}^{abc} = y_{23,2h}^{abc} + y_{23}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc}$
 $y_{33}^{abc} = y_{34,2h}^{abc} + y_{34}^{abc}$
 $y_{44}^{abc} = y_{11,2h}^{abc} + y_{14}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc}$

$N=4$
 (12×1)
 (12×2)
 (3×3)
 (3×1)
 (3×1)
 (3×1)
 (3×1)
 (2×1)



So, it is $-y_{14} V_1^{abc}$ sorry it is $y_{14} V_1^{abc}$ and $-y_{34}^{abc}$ and it is $-y_{14}^{abc}$ shunt abc $+y_{14}^{abc}$ $+y_{34}^{abc}$ shunt abc $+y_{34}^{abc}$ and this will be a 0 and I_3^{abc} would be $I_3^{abc} - y_{23} V_2^{abc}$ it is $-y_{23}^{abc}$ and it is $-y_{34}^{abc}$ and this is y_{14}^{abc} shunt abc $+y_{14}^{abc}$ $+y_{34}^{abc}$ $+y_{34}^{abc}$ shunt abc and this is being 0 and if to make it neater we can possibly write y_{44}^{abc} and it is y_{33}^{abc} where y_{44}^{abc} is essentially this y_{14}^{abc} shunt abc $+y_{14}^{abc}$ $+y_{34}^{abc}$ shunt abc $+y_{34}^{abc}$.

Similarly, y_{33}^{abc} will be y_{23}^{abc} shunt abc $+y_{23}^{abc}$ $+y_{34}^{abc}$ shunt abc $+y_{34}^{abc}$ so now let us write down the equation for $I_2^{abc} = -y_{12}^{abc}$ then $-y_{23}^{abc}$ and it is 0 and this let us say y_{22}^{abc} is given by this one so y_{12}^{abc} shunt abc $+y_{12}^{abc}$ so y_{12}^{abc} shunt abc $+y_{12}^{abc}$ $+y_{23}^{abc}$ shunt abc $+y_{23}^{abc}$. similarly, it would be y_{11}^{abc} and then here for bus number 1 $-y_{12}^{abc} - y_{14}^{abc}$ so $-y_{12}^{abc}$ and this it is 0 and $-y_{14}^{abc}$ and $y_{11}^{abc} = y_{12}^{abc}$ shunt abc $+y_{12}^{abc}$ $+y_{14}^{abc}$ shunt abc $+y_{14}^{abc}$.

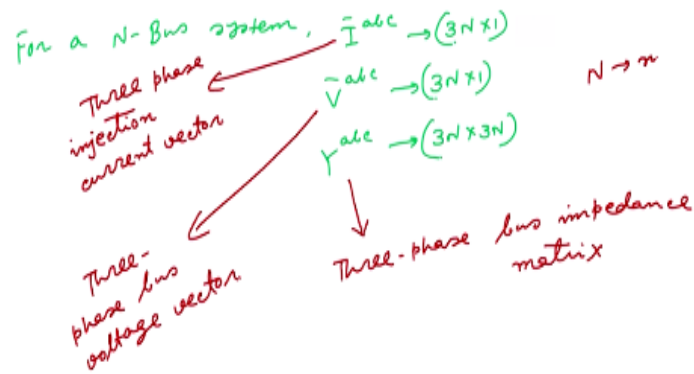
So, this is the case and so this is the entire matrix now let us look at the certain other details each of them is 3×1 . So, then they are food it is total is 12×1 similarly each of them here is 3×1 so then therefore this is 12×1 . Similarly, here all these mattresses everything is 3×3 . Everything is 3×3 all of them are 3×3 similarly please note that the 0 is actually 3×3 null metrics.

All these 0s are 3×3 null matrix all our 3×3 null matrix. Similarly, this is 3×3 this is 3×3 so then therefore this is 12×12 now here we have $n=4$ so then therefore here in this case so now so then therefore this one is nothing but the bus injection vector we call it I^{abc} vector we call it I^{abc}

vector. This is the bus voltage vector we call it v_{abc} vector is a vector and this is nothing but the why ABC, right.

So, then therefore I_{abc} is 12×1 which is nothing 3×4 and y_{bus} is 12×12 which is nothing but $3 \times 4 \times 3 \times 4$ and V_{abc} is 12×1 which is a worksheet $3 \times 4 \times 1$ so then therefore for N bus system.

(Refer Slide Time: 10:32)



So, for N - bus system I vector abc vectored is $3N \times 1$ V_{abc} vector is $3N \times 1$ and Y_{abc} matrixes $3N \times 3N$ so we should say that this is 3 phase injection current vector. This is 3 phase injection current vector. This is 3 phase bus voltage vector and this is 3 phase bus impedance matrix. So, this is the first observation where N is the number of bus so this is the first observation. Now what about the entries are actually you see that.

Whenever there is a direct connection between any 2 bus at the out diagonal term essential the negative of the admitters matrices there. For example, here there is a direct connection between bus 1 and 2 and all and as well as there is another direct connection between bus 1 and 4. So then therefore in this if I say that this is 1 2 positions. So, then here it is basically negative of this series admittance matrix is there.

Similarly, here also negative of the series at matrix is here and these diagonal elements this and this. These 4 are called the diagonal elements I mean but again this diagonal elements are

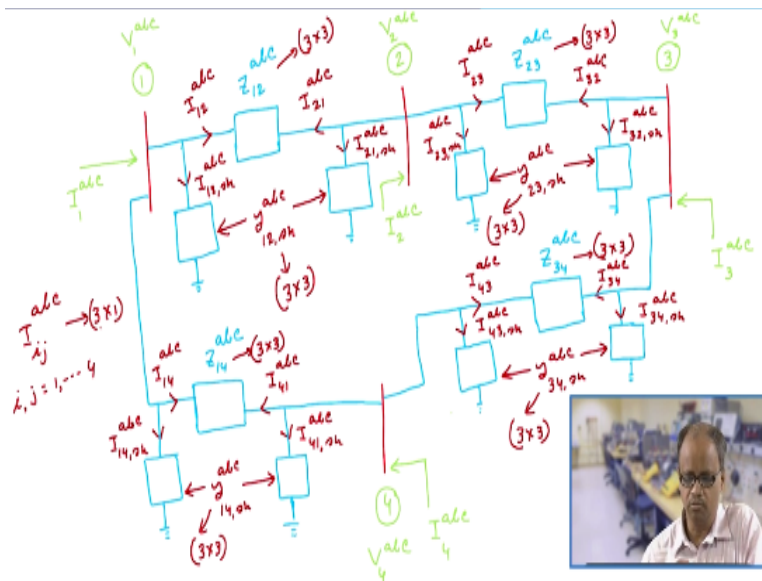
actually essentially these are diagonal 3*3 matrix and these are all actually diagonal but 3*3 matrix. This is also 3*3 matrices so when these diagonal 3*3 matrix is nothing but the sum total of all the diagonal.

Sorry I mean this diagonal 3*3 matrix are nothing but the sum total of all the 3*3 admittance matrix is directly connected to that bus. So, then therefore as we can see the rule for building of the 3 phase admittance matrix exactly follows the rule for building up the single phase. Admittance matrix in the case of single phase admittance matrix is also the diagonal term is nothing but the sum total of their admittance is connected at any particular bus.

And there are out diagonal terms nothing but the negative of the admittance connected between any 2 buses. Here in this case also the diagonal 3*3 matrix is nothing but the sum total of all 3*3 admittance matrixes connected directed to that particular bus. And the off diagonal 3*3 matrix is nothing but the negative of the 3*3 admittance matrix are 3*3 series admittance matrix connected between any 2 buses.

So, by this small example we know that it is very easy to find out the bus admittance matrix of any 3 phase system in which they are wounded lines. In fact, in front of in this particular method can we very easily programmed in a computer.

(Refer Slide Time: 15:01)



But now the question is here so far in this diagram we have not considered the presence of any transformer but in any system there is a transformer. So, then therefore we need to also take into account from the presence of the transformer in this particular waivers formulation so let us look at that how to consider transformers.

(Refer Slide Time: 15:30)

Consideration of three-phase Transformers in Y_{Bus} matrix

$$I_1^{abc} = [I_1^a \ I_1^b \ I_1^c]^T$$

$$I_2^{abc} = [I_2^a \ I_2^b \ I_2^c]^T$$

The diagram shows two buses, (m) and (n), connected by a three-phase transformer. Bus (m) has voltage V_m^{abc} and current I_1^{abc} entering. Bus (n) has voltage V_n^{abc} and current I_2^{abc} entering.

So, the next is consideration of transformer in waivers matrix so the next step is consideration of 3 phase transformer in Y bus matrix so now let us say that we have a bus we have 2 buses. Let us say bus m and n and between please note that this is basically a 3 phase bus. So, then it is voltages again V_m^{abc} and this voltage is V_n^{abc} and between these 2 buses there is one 3 phase transformers connected.

And this 3 phase transformer can have any connection it can have star delta star grounded star grounded delta star grounded delta star ungrounded anything and everything it can have. Now as soon as well we have got so now we have got V_m^{abc} and V_n^{abc} we are modelling the transformer what we do is we denote some current here which is going into the transformer from this 2 buses.

So, let us say that we denote this transformer we denote this 3 phase transformer so then where I mean although we are showing in one single phase line but then it is actually a 3 phase line so then therefore there will be current I_1^{abc} and there also current I_2^{abc} of course I_1^{abc} is $I_1^a \ I_1^b \ I_1^c$

I1c transpose and I2 abc = I2a I2b I2c transpose so these are all 3 + 1 so now what we have is. So, now what I have got actually is 6 voltage quantities.

What are these 6 voltage quantities? It is Vma Vmb Vmc Vna Vnb Vnc and 6 current quantities I1a I1b I1c I2a I2b I2c so therefore if I have to write down the equation.

(Refer Slide Time: 18:24)

I1a I1b I1c I2a I2b I2c these are 6 current quantities please note that all these currents are complex current so because so this is a 6*1 and here also V1a V1b V sorry this should be Vn so this should be Vna Vmb and Vmc Vna Vnb Vnc so this is also 6*1 so now obviously this matrix s would be 6*6 matrix so now what happens that are depending upon the transformer connection right.

There will be all together 36 elements these elements may be 0 and these elements may be non0. I mean what this elements would be that would depend on actually the particular transformer connection but there will be all together 36 elements. Again I repeat these elements maybe 0 these elements may be non 0 but they will be altogether 36 elements and these 36 elements are denoted like this.

Now what do we do is that we do partition on this 36 elements like this and we call that it is YI please note that is a 3*3 matrix we call that it is Y II matrix we say call it is YIII and we call it is

YIV please note that these matrices YI YII YIII YIV they would be known. They are known for particular type of connection we will try to see in this picture if we can give one or 2 examples that what would be the matrices.

For any let us one particular type of transformer connection but right now for the purpose for analysis let us assume that for a given particular transformer connection these matrices I1 these matrices YI YII YIII YIV are all known. And you all note that this all 3*3 matrix. These are all 3*3 matrix. So, then therefore now of course now this is as we said that this is I1abc this is I2abc and this is Vm abc and this is Vn abc. So, then therefore this enter relationship can be written.

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$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \end{bmatrix} = \begin{bmatrix} Y_I & Y_{II} \\ Y_{III} & Y_{IV} \end{bmatrix} \begin{bmatrix} V_m^{abc} \\ V_n^{abc} \end{bmatrix}$$

$$\Rightarrow I_1^{abc} = Y_I V_m^{abc} + Y_{II} V_n^{abc}$$

$$I_2^{abc} = Y_{III} V_m^{abc} + Y_{IV} V_n^{abc}$$

Line-models

abc
 Z_{mn}
abc
 $Z_{mn, sh}$

m	n
1	2
3	4
1	5
4	5

Second clear I1abc and I2abc = YI YII YIII YIV and this is Vmabc Vnabc so this is the transformer so then therefore we can write down that I1abc = YI Vmabc +Y II Vnabc similarly I2abc = YIII*Vmabc +YIV Vnabc. So, these are the guiding equations or these are the modelling equations for a transformer so now let us take and small example. Let us take a small example again but some other example let us say now everything.

We are considering as a 3 phase system so now it is a bus 1 it is a bus 2 and it is a bus 3 and let us say bus 4 and let us say there is a bus 5 and so this is bus 1 please note that these are all 3 phase system. So, we are not writing those specifically bus 4 bus 5 in between there is a line in

between. There is a line in between and in between there is a line between there is a line but in between there is transformer.

But now you like to find out what is the bus impedance matrix so again as well we take that this is current is I_{1abc} this current is I_{2abc} and this current is I_{3abc} and this is I_{4abc} and this is I_{5abc} and this voltages are V_{1abc} V_{2abc} V_{3abc} etc and all these lines I am mean they are at 4 lines 1 2 3 4 all these lines are represented by their equivalent pi- models. So, then they are full line models are but the line models are line models or you have just now seen.

That essentially $y_{mn abc}$ y_{mn} shunt abc so a and m would be can be so then here m and n values will be for the first 1 2 2 the second line 3 4 and the third line it is 1 5 and the 4th line is 4 5. So, these are the 4 lines and these are the different values of $Y_{mn abc}$ so then with this. We are now ready to find out that what would be d bus impedance and other the bus admittance matrix so now as we have done earlier so we can write down that.

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$$I_1^{abc} = y_{11}^{abc} V_1^{abc} - y_{12}^{abc} V_2^{abc} - y_{15}^{abc} V_5^{abc} \dots \text{--- (1)}$$

$$y_{11}^{abc} = y_{12}^{abc} + y_{12,sh}^{abc} + y_{15}^{abc} + y_{15,sh}^{abc}$$



I_{1abc} is actually as we know now which is no way simple that it is nothing but $y_{11abc} * V_{1abc} - y_{12abc} * V_{2abc} - y_{15abc} * V_{5abc}$ and y_{11abc} will be nothing but $y_{12abc} + y_{12 shunt abc} + y_{15abc} + y_{15 shunt abc}$ so this is the bus1 so then similarly we will have to write down the equations corresponding to bus 2 bus 3 bus 4 bus 5 and we will be in our writing utilizing this conventions.

So, it would be now written quickly so then after that we write all the equations cushions corresponding to bus 2 bus 3 bus 4 and bus 5 then we will clubbed them together and y bus matrix see that what is that particular conclusion we can take. So, we will do this in the next lecture so thank you so much.