Computer Aided Power System Analysis
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Lecture - 51
Fault Analysis (Cont)

Hello friends. Welcome to this lecture on Computer Aided Power System Analysis. We have been discussing the process of building the Y bus matrix for an unbalanced 3 phase network so let us continue.

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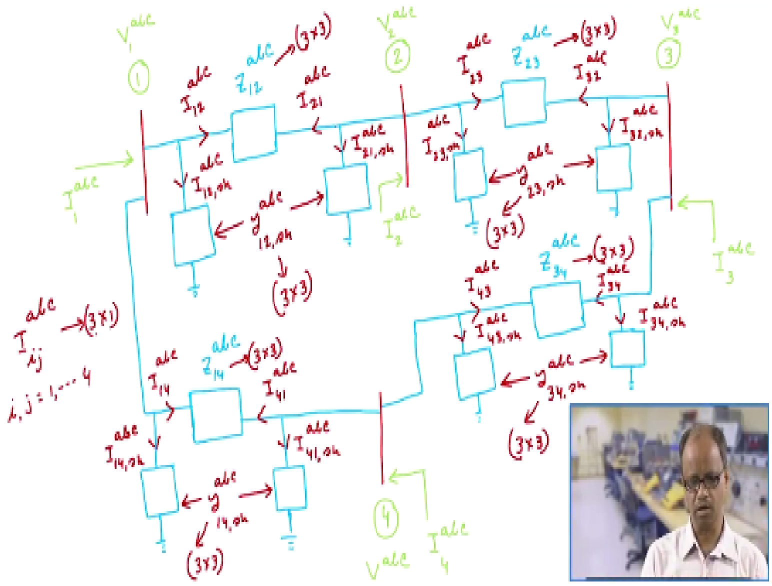

So what we have done is that we have taken a simple 4 bus, 4 line system and we have also drawn it equivalent circuit as shown. Now with this equivalent circuit now in this equivalent circuit we have already integrated all this voltages as well as the currents. And also we have talked about the various impedances as well as the admittances and now we are ready to start writing the equations.

But before we start writing the equation for this entire network we should first talk about that how to write down the equations for individual current in the series branch as well as the individual current in the shunt branch because once we understand that how to write down the equations for the individual currents in the series branch as well as individual current in the shunt branch then we would be able to write down this equations for this entire network.

So let us consider again as usual which we have already shown that a line between bus m and n and that particular and in that line bus voltage at bus m is given by Vmabc as well as bus n Vnabc and here Imnabc and here let us say that we said it is Imn, shunt abc. So now let us try to try to write down the equation for this 2 current.

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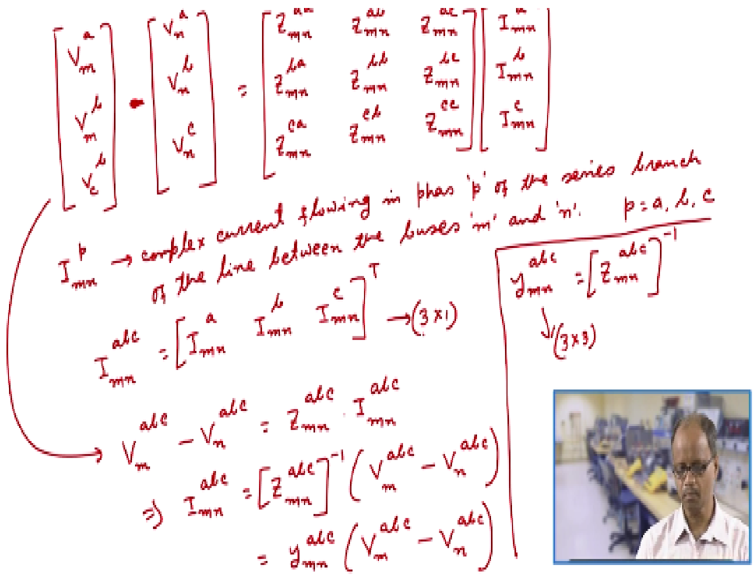

So now what do we have is we have Vma if I write down in the matrix Vmb, Vmc= or rather if I apply this KVL- Vna, Vnb, Vnc that is obviously would be given by. So this voltage- this voltage would be obviously given by this total drop in this series branch and total drop in the series branch would be given by Zmnabc so that is= essentially Zmnaa, Zmnab, Zmnac then Zmnba, Zmnbb, Zmnbc that is the convention we have taken yes.

And lastly Zmnca Zmncb, Zmncc * Imna, Imnb, Imnc. What is this quantity Imna, Imnb, Imnc. So we will write Imnp is actually the complex current flowing in phase P of the series branch of the line between the buses m and n and where of course P= phase a, phase b, phase c. So here when we are writing that this is Imnabc so we must note now probably we are now sure that when I say that Imnabc is actually a vector and this is Imna, Imnb, Imnc transpose.

So this is again a 3 cross 1 transpose. So then therefore this equation can be written as so this equation can be written as Vmabc- Vnabc=Zmnabc * Imnabc. So then therefore Imnabc= Zmnabc inverse because it is a 3 cross 3 matrix inverse * Vmabc- Vnabc. So we call it let us say ymnabc *Vmabc-Vnabc so that is the expression of the current Imnabc. We note that ymnabc is ymnabc we note that ymnabc is Zmnabc inverse.

So this is again 3 cross 3 matrix right. So this is the expression of Imnabc. Now what is the expression of current Imn, sh.

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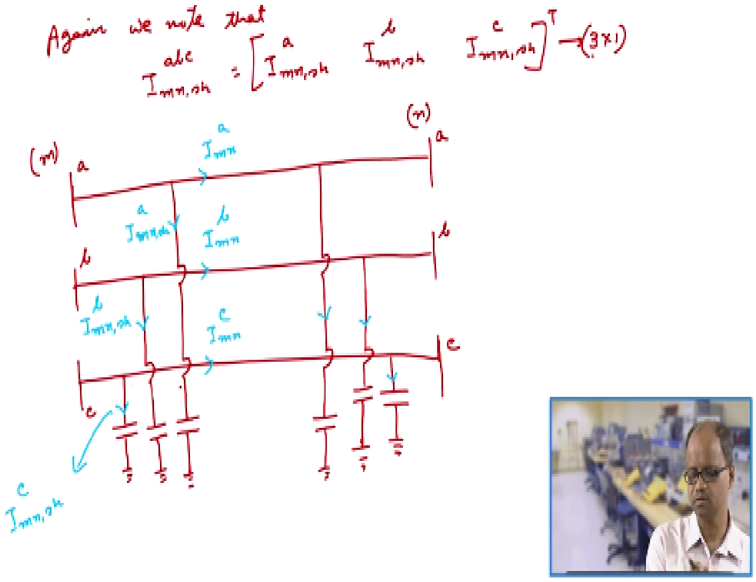

Again we note so basically we note that Imn, shunt abc =Imn, shunt phase a, Imn, shunt phase b, Imn, shunt phase c. So these are the 3 individual currents transpose this is a 3 cross 1. So essentially what is happening essentially physically what is happening that I do have a line between bus m and n bus m and n. So physically there are 3 lines right. So this is phase a, phase b, phase c this is bus m, this is bus n.

And we have already denoted that well there are impedances individual (()) (08:45) they are impedances mutual impedances between any 2 phases. On top of that there are also shunt line charging susceptance or shunt half line charging susceptances between any phase at the ground between any phase in the ground because of their proximity I mean they will have their individual admittance and also because of their proximity.

They will also have their mutual admittance between these 2 and this 2 as well as this 2. So this and again so this is it. So when we are saying Imna, so Imna is actually this current. So when you are saying that it is Imna is actually this current Imnb is actually Imnc is actually this current and when we are talking about Imn, sh a this is Imn, sh a so this is the shunt. This is Imn, shunt for b and this is similarly Imn, shunt at phase c.

So these are the 3 currents we are talking about. Similarly, also there are 3 currents here similarly over there also current (()) (10:34). So basically or physically this is the case right. We have just now derived the expression of these 3 currents in terms of these 3 voltages and as well as this 3 voltages. Now we have to derive the expression of these 3 currents in terms of this voltage and as well as the (()) (10:58) matrix.

Now if we look at this obviously the current here because this is the shunt current it would be something like this voltage multiply by this admittance, but because this is a 3 phase voltage so this is the 3 cross 1 this is the 3 cross 3.

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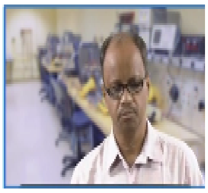
$$\underbrace{\begin{bmatrix} I_{mn,sh}^{a} \\ I_{mn,sh}^{b} \\ I_{mn,sh}^{c} \end{bmatrix}}_{(3\times1)} = \underbrace{\begin{bmatrix} y_{mn,sh}^{aa} & y_{mn,sh}^{ab} & y_{mn,sh}^{ac} \\ y_{mn,sh}^{ba} & y_{mn,sh}^{bb} & y_{mn,sh}^{bc} \\ y_{mn,sh}^{ca} & y_{mn,sh}^{cb} & y_{mn,sh}^{cc} \end{bmatrix}}_{(3\times3)} \underbrace{\begin{bmatrix} V_m^{a} \\ V_m^{b} \\ V_m^{c} \end{bmatrix}}_{(3\times1)}$$

$$I_{mn,sh}^{abc} = y_{mn,sh}^{abc} V_m^{abc}$$



So then we can simply write down the equation is Imn, sh a= Imn, sh b and Imn, sh c so this is a 3 cross 1 matrix and this would be ymn, sh aa, ymn, sh ab, ymn, sh ac, ymn, sh ba ymn, sh bb ymn, sh ca ymn, sh cb ymn, sh cc * Vma Vmb, Vmc right. So this is the 3 cross 1 vector this is the 3*3 matrix this is the 3 cross 1 vector. So we write that Imn, sh abc is nothing ymn, sh abc*Vmabc.

So this is the relation between the bus voltages this bus and this current. So similarly the current here would be if I say that this is Imn, sh abc at this stage it would be and this current would be nothing but ymn, sh abc *Vnabc. So let us say this current would be nothing but so this current I can write that this current is nothing but ymn, sh abc* Vmabc right. So with this background so now we are ready to write down the equation for this entire system. So we write down the equations for this system.

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Equations for the complete system

Bus 1

$$I_1^{abc} = I_{12,sh}^{abc} + I_{12}^{abc} + I_{14,sh}^{abc} + I_{14}^{abc}$$

$$= y_{12,sh}^{abc} V_1^{abc} + y_{12}^{abc}\left(V_1^{abc} - V_2^{abc}\right) + y_{14,sh}^{abc} V_1^{abc} + y_{14}^{abc}\left(V_1^{abc} - V_4^{abc}\right)$$

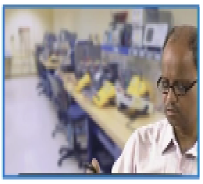
$$= \left(y_{12,sh}^{abc} + y_{12}^{abc} + y_{14,sh}^{abc} + y_{14}^{abc}\right) V_1^{abc} - y_{12}^{abc} V_2^{abc} - y_{14}^{abc} V_4^{abc} \quad \cdots (1)$$

$$y_{12}^{abc} = \left(Z_{12}^{abc}\right)^{-1}; \quad y_{14}^{abc} = \left(Z_{14}^{abc}\right)^{-1}; \quad y_{23}^{abc} = \left(Z_{23}^{abc}\right)^{-1}$$

Bus 2

$$I_2^{abc} = I_{21,sh}^{abc} + I_{21}^{abc} + I_{23,sh}^{abc} + I_{23}^{abc}$$

$$= y_{12,sh}^{abc} V_2^{abc} + y_{12}^{abc}\left(V_2^{abc} - V_1^{abc}\right) + y_{23,sh}^{abc} V_2^{abc} + y_{23}^{abc}\left(V_2^{abc} - V_3^{abc}\right)$$



So equations for the complete system. So now here we have already shown that if I take this injected current this injected current= ultimately this current+ this current+ this current+ this current. So then therefore we can write down that I1abc. So I1abc= you can write down that now I12, sh abc is nothing but y12, sh abc * V1abc. Now here we can write that I12, sh abc+ I12abc+ I14abc+ I14, sh abc+ I14, shunt abc+ I14abc.

So this is from the applying KCL. Now I want to shunt abc is nothing but y12, shunt abc* V1abc. I1abc would be y12abc * V1abc- V2abc right y12abc is similarly I14, shunt abc would be y14, shunt abc * V1abc please look at this if I look at this because this shunt branch is also ultimately connected at this branch. So then therefore I14, sh abc would be this admittance * this I mean this admittance matrix * this particular voltage vector+ it is I14abc is y14abc * V1abc-V4abc.

So then therefore if I now take all this V1abc terms together so y12, sh abc+ y12abc+ y14, sh abc+ y14abc there should not be any comma here y14ab. So this * V1abc- y12abc * V2abc- y14abc* V4abc so this is equation one. Here we note that y12abc is actually Z12abc inverse and y14abc= Z14abc inverse. So this is at bus 1. At bus 2 what would be the current this is at bus 2.

At bus 2 if I look at this picture I2abc would be= this current+ this current+ this current+ this current. So we can write down I2abc= I2abc is I21, shunt abc, I21abc, I23, shunt abc and I23abc. So at bus 2 I2abc= I21, shunt abc+ I1abc+ I23, shunt abc+ I23abc right. An I21, shunt abc would be nothing but y21, shunt abc. In fact, it should be y12, shunt abc because y12, shunt abc because this same admittance is connected.

So it is y12, shunt abc so we should write y12, shunt abc* V2abc+ y12abc* V2abc-V1abc+ I23, shunt abc is y23, shunt abc* V2abc+y23abc *V2abc- V3abc. We again note that y23abc is nothing but Z23abc inverse. So therefore I2abc= -y12abc V1abc. So it is –y12abc *V1abc+ y12, shunt abc+ y12abc sorry.

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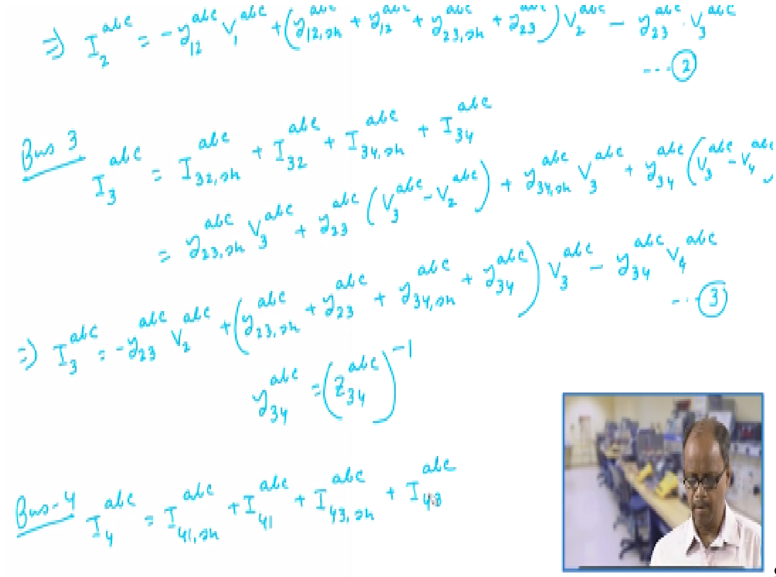

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y12, shunt abc+ y12abc+ y23, shunt abc+ y23abc that is= V2abc- y23abc* V3abc- y23abc * V3abc. So this is equation 2. At bus 3 what we have so at bus 3 if I apply this same principle I3abc= I32, shunt abc+ I32abc+ I34, shunt abc+ I34abc. So it is 132, shunt abc. So it is I32 y3abc it is I32, shunt abc+ I32abc+ y34, shunt abc+ y34abc. Y32abc is so it is y23 shunt abc* V3abc+y23abc * V3abc-V2abc and this is y34, shunt abc*V3abc+ y34abc * V3abc -V4abc.

So then therefore I3abc is essentially V2 V4 so it is –y23abc V2abc+ y23, shunt abc+ y23abc+ y34, shunt abc+ y34abc *V3abc-y34abc *V4abc. So this is equation 4 and we again note that y34abc= Z34abc inverse right. Similarly, at bus 4 very quickly if we write at bus 4. I4abc= I41, shunt abc+ I41abc+ I43abc+ I43, shunt abc. So it is I41, shunt abc+ I41abc+ I43, shunt abc+ I43abc.

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$$I_4^{abc} = y_{14,sh}^{abc} V_4^{abc} + y_{14}^{abc}\left(V_4^{abc} - V_1^{abc}\right) + y_{34,sh}^{abc} V_4^{abc} + y_{34}^{abc}\left(V_4^{abc} - V_3^{abc}\right)$$

$$= -y_{14}^{abc} V_1^{abc} - y_{34}^{abc} V_3^{abc} + \left(y_{14,sh}^{abc} + y_{14}^{abc} + y_{34,sh}^{abc} + y_{34}^{abc}\right) V_4^{abc} \quad \cdots (4)$$

So then therefore I4abc= y14, shunt abc * V4abc+ y14abc *V4abc-V1abc+ y34, shunt abc * V4abc y34, shunt abc *V4abc+ y34abc* V4abc- V3abc. So if I take them together so I get -y14abc V1abc-y34abc* V3abc+ y14, shunt abc+ y4abc+ y34, shunt abc+ y34abc* (()) (28:42). So you see we have got all this 4 equations here. So now what we have to do we have to now write all this 4 equations in a matrix form and then what kind of conclusion we can draw. This, we will do in the next lecture. Thank you.