

Computer Aided Power System Analysis
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Lecture – 05
Bus Admittance Matrix with Mutual Impedance(Contd.)

Welcome to this lecture of this course computer aided power system analysis. In the last lecture we have started with the concept of building the Bus admittance Matrix. While there is a mutual impedance present between any two lines. We have shown that if we consider a two elements connected between 4 buses which are also connected with each other through some mutual impedance.

So then we can actually represent this entire circuit by an equivalent current voltage relationship. Where these currents are nothing but the bus injected currents at this 4 buses and this voltages are nothing but the bus voltages of this 4 buses and the connected matrix or rather the connecting matrix connecting this Bus injection current or these bus voltage vectors is actually nothing but a 4*4 admittance matrix.

From that we have also shown that the injected current at bus 1 or let us say bus u can be represented by an equivalent circuit. So then we will now continue from that part from that point onwards. So what do we have that we have already represented in the last class that we had?


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Therefore,

$$\begin{bmatrix} \bar{I}_u \\ \bar{I}_v \\ \bar{I}_x \\ \bar{I}_y \end{bmatrix} = \begin{bmatrix} \bar{Y}_c & -\bar{Y}_c & \bar{Y}_m & -\bar{Y}_m \\ -\bar{Y}_c & \bar{Y}_c & -\bar{Y}_m & \bar{Y}_m \\ \bar{Y}_m & -\bar{Y}_m & \bar{Y}_d & -\bar{Y}_d \\ -\bar{Y}_m & \bar{Y}_m & -\bar{Y}_d & \bar{Y}_d \end{bmatrix} \begin{bmatrix} \bar{V}_u \\ \bar{V}_v \\ \bar{V}_x \\ \bar{V}_y \end{bmatrix}$$

$$\Rightarrow \bar{I}_u = \bar{Y}_c \bar{V}_u - \bar{Y}_c \bar{V}_v + \bar{Y}_m \bar{V}_x - \bar{Y}_m \bar{V}_y$$

$$= \bar{Y}_c \bar{V}_u - \bar{Y}_c \bar{V}_v + \bar{Y}_m \bar{V}_x - \bar{Y}_m \bar{V}_y$$

$$= \bar{Y}_c (\bar{V}_u - \bar{V}_v) + \bar{Y}_m (\bar{V}_x - \bar{V}_y) - \bar{Y}_m (\bar{V}_u - \bar{V}_v)$$


Buses u v x and y between bus u and v there is an element connected having impedance z_c between bus x and y there is an element connected having an impedance z_d and these 2 elements are also interconnected with each other by an equivalent mutual impedance z_n . I_u I_v I_x and I_y those are nothing but the bus injected currents at bus u v x and y respectively. V_u V_v V_x and V_y are nothing but the bus voltages at this buses uv x and y respectively.

And we have shown that this bus injection current as well as this bus voltage vector can be interconnected with each other by this 4*4 admittance matrix and from this admittance matrix we can represent that this current I_u which is nothing but the injected current at bus u can be represented by an equivalent circuit as shown here. So we will also know follow the same procedure for the other 3 currents.

So we will form 4 different equivalent circuits and then we will try to form a composite equivalent circuit which will represent this particular matrix relationship. So now let us do.

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$$\begin{aligned} \bar{I}_0 &= -\bar{Y}_c \bar{V}_u + \bar{Y}_c \bar{V}_v - \bar{Y}_m \bar{V}_x + \bar{Y}_m \bar{V}_y \\ &= -\bar{Y}_c \bar{V}_u + \bar{Y}_c \bar{V}_v - \bar{Y}_m \bar{V}_x + \bar{Y}_m \bar{V}_y + \bar{Y}_m \bar{V}_v - \bar{Y}_m \bar{V}_v \\ &= \bar{Y}_c (\bar{V}_v - \bar{V}_u) + \bar{Y}_m (\bar{V}_v - \bar{V}_x) - \bar{Y}_m (\bar{V}_v - \bar{V}_y) \end{aligned}$$

So from this relation \bar{I}_v we can write down as $-\bar{Y}_c \bar{V}_u + \bar{Y}_c \bar{V}_v - \bar{Y}_m \bar{V}_x + \bar{Y}_m \bar{V}_y$ all these are complex we must be careful. So here also we add and subtract from this expression $\bar{Y}_m \bar{V}_v$ and \bar{Y}_m so we add $\bar{Y}_m \bar{V}_v$ and subsequently also subtract $\bar{Y}_m \bar{V}_v$ such that this expression remains the same. So from here we can see that \bar{Y}_c can be written as $\bar{V}_v - \bar{V}_u$ by taking these two terms together $+\bar{Y}_m \bar{V}_v - \bar{V}_x$ by taking this term and this term together.

$-\bar{Y}_m \bar{V}_v - \bar{V}_y$ by taking this term and this term together. So this is the expression so again so now what we have so now we have bus u bus v bus x bus y and this is bus u this is v this is x this is y. \bar{I}_v is the injected current at bus I and the first term is \bar{V}_v so this is $\bar{Y}_c \bar{Y}_m \bar{V}_v - \bar{V}_x$ so let us see that there is an $\bar{Y}_m \bar{Y}_m \bar{V}_v - \bar{V}_x - \bar{Y}_m \bar{V}_v - \bar{V}_y$ so this is $-\bar{Y}_m \bar{V}_v - \bar{V}_y$. So therefore this current injected current is constituting of this current plus this current plus this current.

This first current is $\bar{Y}_c \bar{V}_v - \bar{V}_u$ that is this current? The second current is $\bar{Y}_m \bar{V}_v - \bar{V}_x$ that is this current and the third current is $-\bar{Y}_m \bar{V}_v - \bar{V}_y$ so this is the second equivalent circuit. Let us come to the third $T_x = \bar{Y}_m \bar{V}_v - \bar{Y}_m \bar{V}_u - \bar{Y}_m \bar{V}_v$ so let us do.

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$$\begin{aligned} \bar{I}_x &= \bar{Y}_m \bar{V}_u - \bar{Y}_m \bar{V}_v + \bar{Y}_d \bar{V}_x - \bar{Y}_d \bar{V}_y \\ &= \bar{Y}_m \bar{V}_u - \bar{Y}_m \bar{V}_v + \bar{Y}_d \bar{V}_x - \bar{Y}_d \bar{V}_y + \bar{Y}_m \bar{V}_x - \bar{Y}_m \bar{V}_x \\ &= \bar{Y}_d (\bar{V}_x - \bar{V}_y) + \bar{Y}_m (\bar{V}_x - \bar{V}_v) - \bar{Y}_m (\bar{V}_x - \bar{V}_u) \end{aligned}$$

$I_x = Y_m v_u - Y_m v_v + Y_d v_x - Y_d v_y$ these are all complex quantities as before we you know here from this expression we add and subtract something. Okay add $Y_m v_x$ and $-Y_m v_x$. So from here we get $Y_d * v_x - v_y$ so taking these two current these two term $Y_m v_x - v_v$ taking these two term $-Y_m v_x - v_u$ so again. So and therefore here also I_x can be represented in terms of these three currents. So again let us try to derive this equivalent circuit.

So, this is u v x y and the injected current is I_x would be considered 3 currents 1 current is I_d second is $v_x - v_v$ so $v_x - v_v$ third is $v_x - v_u$. So, this current is constituting of this current this current and this current. This current is the so this is Y_d so $Y_d * v_x - v_y$ that is this current. This current is Y_m so this is $Y_m * v_x - v_v$ so the first current is $Y_d * v_x - v_y$ so this current this is this term second current is $Y_m * v_x - v_v$ so this is term $v_x - v_v$ this is this term and third term is $-Y_m * v_x - v_u$ this is this term.

So, this is the third equivalent circuit let us go to the last equivalent circuit. So I_y so $-Y_m v_u + Y_m v_v$.

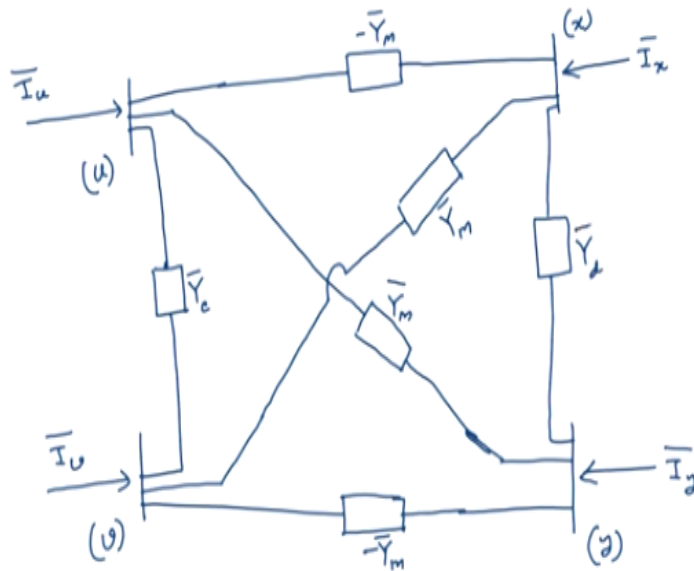
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$$\begin{aligned}
 \bar{I}_y &= -\bar{Y}_m \bar{V}_u + \bar{Y}_m \bar{V}_v - \bar{Y}_d \bar{V}_x + \bar{Y}_d \bar{V}_y \\
 &= -\bar{Y}_m \bar{V}_u + \bar{Y}_m \bar{V}_v - \bar{Y}_d \bar{V}_x + \bar{Y}_d \bar{V}_y + \bar{Y}_m \bar{V}_y - \bar{Y}_m \bar{V}_y \\
 &= \bar{Y}_d (\bar{V}_y - \bar{V}_x) + \bar{Y}_m (\bar{V}_y - \bar{V}_u) - \bar{Y}_m (\bar{V}_y - \bar{V}_v)
 \end{aligned}$$

So, $I_y = Y_m v_u + Y_m v_v - Y_d v_x + Y_d v_y$ so $-Y_d v_x + Y_d v_y$ all these are complex. So we $Y_m v_u + Y_m v_v - Y_d v_x + Y_d v_y$ and we add and subtract $Y_m v_y - Y_m v_y$ these are complex. So we take these two terms $Y_d * v_y - v_x + Y_m * v_y - v_u - Y_m$ so we have taken for this we have taken this term and this term and the last $v_y - v_v$. So this term and this term so again we have $u \ v \ x \ y$. This is I_y this is x this is y this is u this is v $Y_d * v_y - v_x$.

Then $Y_m * v_y - v_u$ so here would be another Y_m this is Y_d and $-Y_m v_y - v_v$ so this is $-Y_m$. So, this current injected current = this current plus this current plus this current. If I apply KCL so first current is $Y_d * v_y - v_x$ that is this and second current is $Y_m * v_y - v_u$ this is this current and third current is $-Y_m * v_y - v_v$, so this is this current. So, I have got four equivalent circuits and now we have to club them together to get the final equivalent circuit.. So let us do that.

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So what do we have is so final equivalent circuit bus u bus v let us make it b1 bus x +bus y so this is bus x this is bus y this is v this is u. Injected current here is I_x injected current here is I_y injected current here it I_u injected current here is I_v ? Let us go to the first equivalent circuit first. Okay u and v so between u and v there is an element connected Y_c . So between u and v there is an element connected Y_c .

So this is Y_c then between u and x there is an element called $-Y_m$ u and x $-Y_m$. So this is $-Y_m$ and third one is between u and y there is an element Y_m u and y there is an element Y_m . So between u and y there is an element Y_m so this is Y_m let us see that it is Y_m so first so we have put the first circuit in this equivalent circuit. So let us go to the second one so we have put the first one now let us go to the second one.

The second one between u and v there is Y_c is connected. So between the u and v Y_c already connected so we need not put anything extra here then between v and y there is $-Y_m$ connected so v and y there is $-Y_m$ so and then v and x there is Y_m so we need to this is Y_m . So, we have put the second circuit. Let us go to the third circuit corresponding to I_x . For I_x there is an Y_d between x and y.

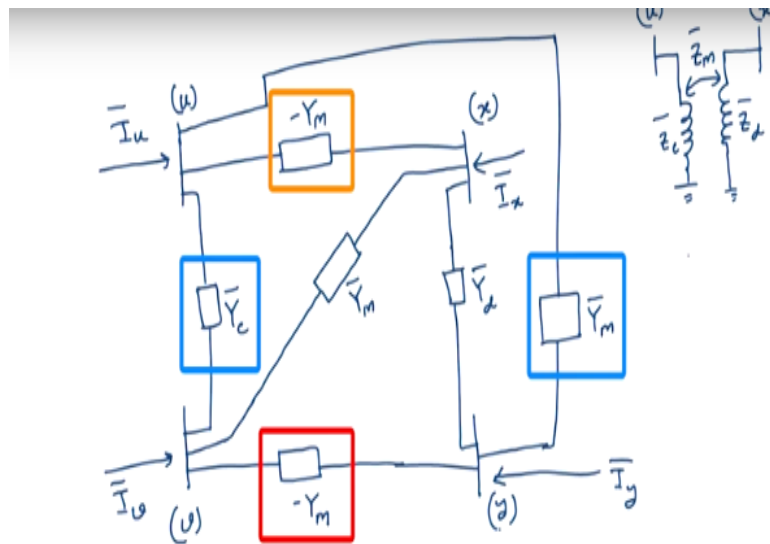
So, there is an element Y_d between x and y so Y_d then between x and u there is $-Y_m$ between x and u there is already put $-Y_m$ so we do not have to add anything and between x and v there is

one Y_m between x and v there is one Y_m . We have already connected that so we also do not have to add anything. So let us go to the last one between y and x there is an Y_d we have already connected them.

Between Y and v there is a $-Y_m$ and between Y and u there is one Y_m . So, then therefore this is the equivalent circuit. So this is the complete equivalent circuit of this mutually coupled network. So now we can see that now these four branches are now this entire equivalent circuit because they are only consisting of admittances or impedances. So then they can very easily be incorporated into the standard bus admittance matrix procedure without any problem.

So we can possibly make it a little neat. So let us try to do that because it is looking very clumsy so we can possibly try to make it a little neat.

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So we again take this as u this is v this is x this is y so $-Y_m$ between Y and u so this is $-Y_m$ this is Y_c u and v is Y_c x and y is Y_d Y and v is $-Y_m$. And these two need to be done so between u and y there is Y_m so to make it between u and y there is $1 Y_m$ and between v and x there is another Y_m this we can put here so this is the circuit and as we said I_u I_v I_x I_y so this is the circuit this is the equivalent circuit.

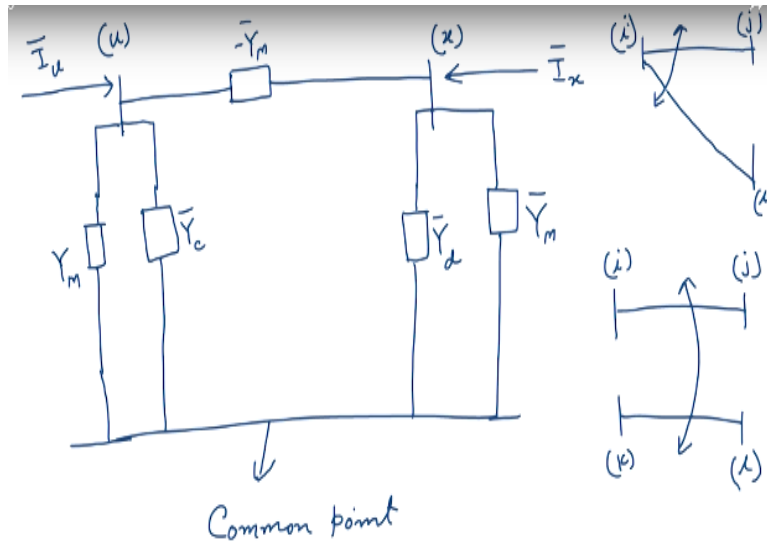
So then therefore if I wish to incorporate this into a bus admittance matrix for example then therefore what I will do when I am writing down the self-admittance term or the diagonal terms I will simply add this this and this. And between u and x there will be a term – of $-Y$ so that is Y_m between u and v that would be an $-Y_c$ between u and y this term would be $-Y_m$. Similarly, when I would be writing down the row corresponding to v bus v.

Essentially the diagonal terms would be consisting of this plus this plus this and between v and y it would be – of $-Y_m$ that is Y_m between v and x it would be $-Y_m$ and between v and u it would be $-Y_c$. Similarly, for bus x and bus y. Now suppose there is only one point now suppose that v and Y are the same point. That is both v and Y are grounded so then therefore what is happening? So then what is the case?

The case is that I have got bus u and from bus u there is one impedance which is connected to impedance ground then I have got bus x this is bus u this is bus x from there also I have got another impedance connected to ground. So then therefore point v and vy they are actually the same potential same point but they are connected to each other through Z_m and this is Z_c this is Z_d and point v and Y they are at the same potential.

Then therefore v and Y they are actually the same. So then if they are the same so then what would be the equivalent circuit. So then what will happen because v and y are the same. So therefore this will go this will remain this and this will come in parallel as well as this and this will come in parallel. So then let us draw that.

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So then what I have got bus u bus and there is a common point. This is the common point this is bus u bus x so between bus u and bus x I still have got $-Y_m$ that we cannot have avoid between bus u and v that is the common point there is Y_c . Okay and between u and Y also there is another element that is Y_m but because v and Y are the same point. So then therefore Y_m is also in parallel with Y_c .

So then therefore we have got another Y_m this is the common point similarly for bus x what we will get between bus x and y. Y is nothing but the common point now there is Y_d so there is an Y_d and between bus x and v which is also nothing but the common point there is also another element Y_m . So then therefore there also would be another element Y_m this would be in parallel. So then therefore in this circuit there are altogether 1 2 3 4 5 five elements.

Now in our previous circuit how many elements were there? 1 2 3 4 5 6. So you may be wondering what happens to sixth one. The sixth one is eliminated because v and u they are the same points and then therefore this term $-Y_m$ is completely gone. So then therefore if there is a common point between these two elements which are in turn mutually connected with each other so then this would be the equivalent circuit. So in this case this is I_x and this is I_y sorry I_u .

And this situation is actually more common because you see usually our shunt branches or any branch was just connected to each other. They are usually connected to ground so then therefore

this situation is more common right? But of course if let us say there are two transmission lines for example let us say bus 1 let us say bus i and j there is one transmission line. And let us say there is bus k and l.

There is another transmission line and these two transmission lines are mutually coupled to each other. So then of course in that case this particular circuit will hold good in this case. But however if this is the case that this is bus I and bus j bus l and there is a transmission line like this and there is a transmission line like this. So this is bus i this is bus j this is bus l and these two lines mutually coupled with each other.

So in that case this particular circuit will hold good there can also be a third possibility that there is a bus i and bus j and there are two parallel transmission lines between bus i and bus j and these parallel transmission lines are also mutually coupled with each other. That is we cannot neglect their mutual coupling. So then therefore in that case what will happen even these two will go and all these four will come into parallel to each other.

So, that is previous; so I am just leaving it for you. So, this is the entire equivalent circuit of any two mutually coupled elements. So by this we have now finished the topic of formation of bus admittance matrix. Now with this bus admittance matrix at our disposal we will now continue our actual analysis of the power system network which we will do in the next classes. Thank you.