Computer Aided Power System Analysis
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Lecture – 48
Power System State Estimation (Contd.)

Hello friends. Welcome to this lecture on computer aided power system analysis. We have been looking at the structure of the Jacobian matrix. So let us look at that now with a more detail.

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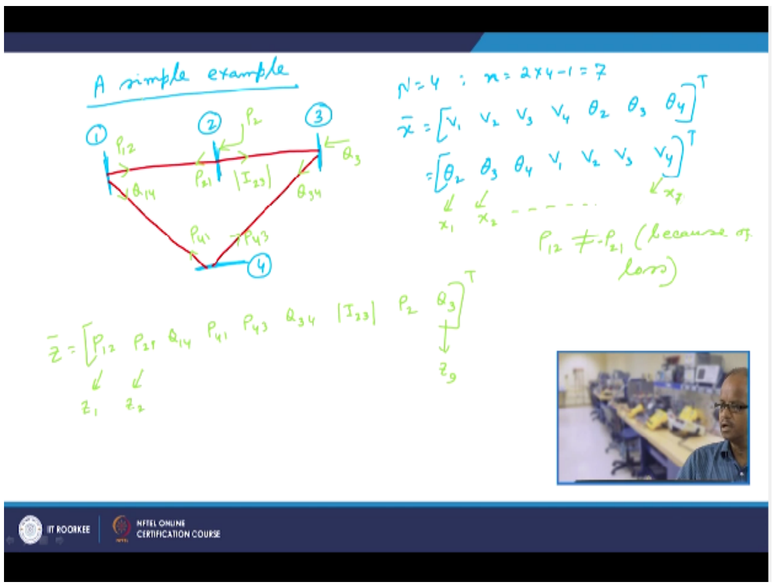

So now just to take a simple example. This is just to show that what will be the structure. Suppose I do have a bus, I mean I do have a system just the 4 bus system. These blue lines are buses. So this is bus 1, bus 2, bus 3. So these are all lines. So this is bus 1, 2, 3, 4. So N=4. So the number of strips 2*4-1 that is 7. So now what is the state vector. State vectors would be nothing but V1, V2, V3, V4, theta 2, theta 3, theta 4.

We generally prefer to write the theta before and then after this V just to, I mean it is just a matter of notation. I mean, just so we usually like to write like this theta, usually. So this is 7*1 vector. Now suppose my measurement quantities are, measured quantities are P12, let us say P21, let us say Q14, let us say QP41, let us say PQ34, let us say P43, 1, 2, 3, 4, 5, 6. Let us say P2, let us say Q3.

And we also measure, let us say, here in this I23 magnitude. So my measured quantities are Z are P12, P21, Q14, P41, P43, Q34, then I23, P2, Q3. So say that these are the my measured. So 1, 2, 3, 4, 5, 6, 7, 8, 9. So I have got 9 measurements, 7 states. Please note that that because of the losses, P12 is not equal to -P21 because of loss. So now what this Jacobian matrix would look like?

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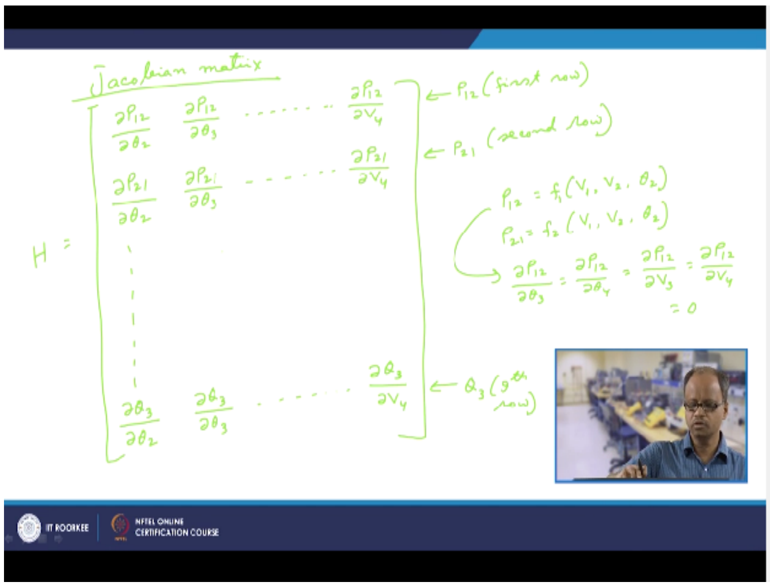

So then therefore, so the Jacobian matrix would be essentially, so then here we say that this is Z1, this is Z2 and dot, dot, dot. This is Z9. And this is X1, this is X2, dot, dot, dot. This is X7. So then therefore, the Jacobian matrix would be H would be for example, del P1, sorry del P12/del theta 2, so del P12/del theta 2. Then del P12/del theta 3, dot, dot, dot, del P12/del V4. Similarly, the second one would be del P21/del theta 2, del P21/del theta 3, dot, dot, dot, del P21/del V4.

And up to this, we will go and the last one is del Q3, so del Q3/del theta 2, del Q3/del theta 3 and dot, dot, dot, del Q3/del V4. This will be the Jacobian matrix, right. So the first row would be corresponding to P12. So first row is P12. Second row is P21 and similarly, this is the ninth row is Q3. This is the ninth row, right. This is the first row. This is the second row.

And similarly the first column is theta 2, this is the first column theta 2, second column theta 3 and so on and so forth. Now we have already seen, now we know that P12 is a function of only V1, theta 1 is 0. So then therefore, theta 1 is not an unknown quantity. So it will be V1, V2 and theta 2. P21 is also another function of also only V1, V2, theta 2.

So then therefore, I mean, which quantities would be only basically non-0? So essentially here, so then essentially here, del P12/del theta 3=del P12/del theta 4=del P12/del V3=del P12/del V4 would be all equal to 0. Only non-0 partial derivatives would be only corresponding to del P12/del V1, del P12/del V2 and del P12/del theta 2. All the other quantities would be 0, right.

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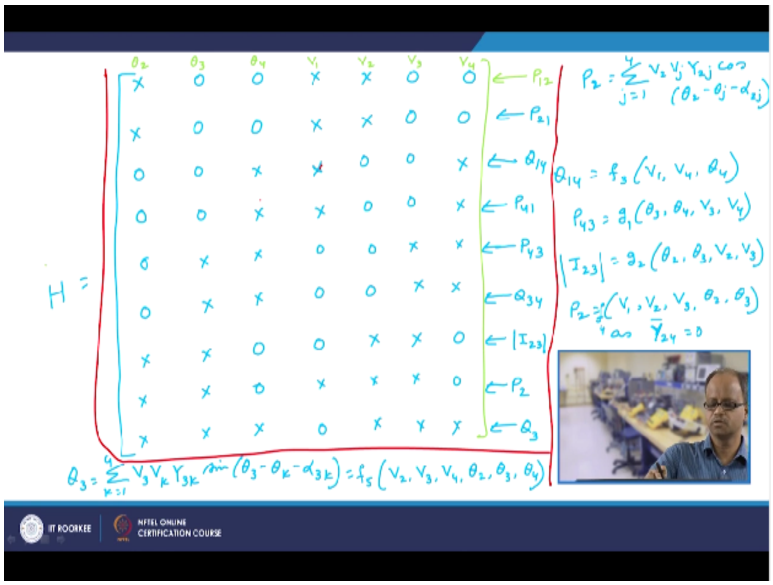

Similarly also for del P21, so then therefore, we can write down, so then here, I can write down the H matrix is something like this, so then H matrix, so first one is del P12/del theta, so this is non-0. Then this is 0, this is 0. Then also this is non-0, non-0, this is non-0, this is non-0, this is 0, this is 0. We note that, we just note that here it is, this column is corresponding to theta 2, this is theta 3, this is theta 4, this is V1, this is V2, this is V3, this is V4, right.

And this is corresponding to P12. Similarly for P21 also, it would be V1, V2 and theta 2. So for P21 also, it would be this 0, 0, 0, 0, this is non-0, this is non-0, this is 0, this is 0. So this would be P21. Next is Q14. Now we know that Q14 is a function of, some function of only V1, theta 1 is 0, it is only V1, V4, theta 4. So Q14 would be V1, so this is a non-0. So V1, then V4, then theta 4, all are 0.

Then we have, let us write P41, P43. P41 also, P41, then P43, then we have Q34, I23. Then it is

Q34, then it is magnitude I23, then it is P2, Q3. Then it is P2, last is Q3, okay. P41 also would be the function of V1, V4, theta 4. So it would be also V1, V4 and theta 4. So this 0, 0, 0. P43. P43 would be a function of, it is some function g, it will be theta 3, theta 4, V3, V4, that we have already seen. So P43 would be theta 3, theta 4, V3, V4, V3, V4, all are 0.

Q34 also would be something like this. This, this 0, 0, this, this, so this is Q34. And I23, I23 would be again a some function g2, say this is g1. It will be theta 2, theta 3, V2, V3. So V2, so theta 2, theta 3, V2, V3 and all would b 0. P2. Now P2 is a function of, now P2 is actually j=1 to 4 V2VjY2j cos theta 2-theta j-alpha 2j. Now here, if we look at the structure of this, so then therefore, in the Y bus matrix, which quantities would be 0?

Now when there is no direct connection between bus 2 and bus 4 and also there is no direct connection between, so now when we are looking at P2, so then therefore, there is only directly connection between from bus 2 is bus 1 and bus 3. So there is no direct connection between bus 2 and bus 4. So then therefore, P2 would not be a function of, P2 would be only a function of V1, V2, V3, theta 2 and theta 3.

Because it would not be a function of V4 and theta 4 because of this simple reason that Y24 would be 0 as Y24, whereas Y24=0, as Y24=0. So P2 would be simply a function of V1, V2, V3, theta 2 and theta 3. So P2 would be theta 2, theta 3 and V1 and V2, V3, 0, 0. And last but not the least, Q3 is again we write it here, k=1 to 4 V3VKY3K sin theta 3-theta K-alpha 3k. Now if we look at the structure, bus 3 is only directly connected to bus 2 and bus 4.

So then therefore, bus 3Q3 would not be a function of V1. Theta 1 is already 0. So then therefore, it would be V2, V3, V4, theta 2, theta 3, theta 4. So then it would be V2V3V4 and so that is how it would look like. So then you see, so then therefore, depending upon the, so then here in this case, we have looked into all kind of measurements. And we can see that depending upon the type of measurement and the interconnectivity, there would be also, I mean, this particular Jacobian matrix would not be also a full matrix.

It will also have many 0. It means here, in this case, we have taken a very simple example, only a

4 bus system but if we take a large system, so then therefore, this matrix also would be a sparse matrix. So then therefore, we can also apply here this sparse matrix techniques what we have already discussed earlier.

But then this particular simple example gives us the structure that depending upon the type of measurement that how this particular Jacobian elements would be computed. So obviously, I mean, which are already 0 0, so then therefore, only this non-0 elements would have to be computed at each and every iteration. So this is just a theoretical example. But now let us look at a numerical example.

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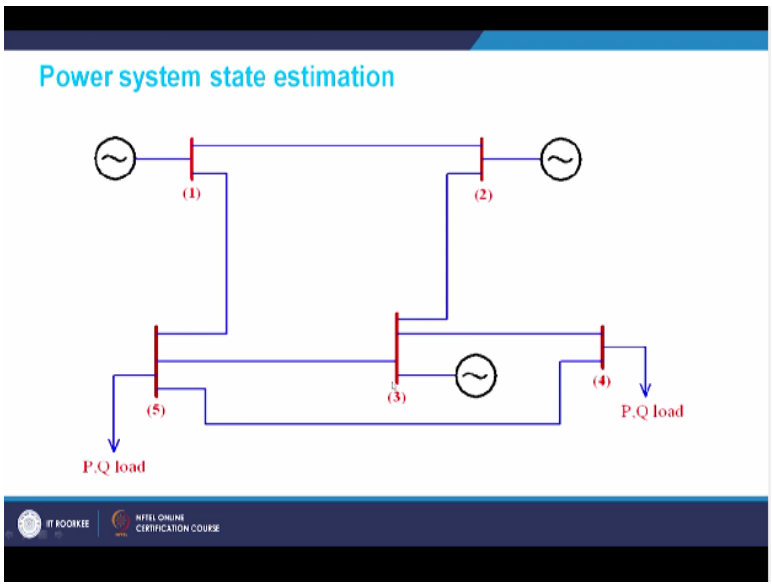

So for numerical example, so let us look at simple 5 bus system and this simple 5 bus system we have already taken earlier when we were discussing about the load flow analysis.

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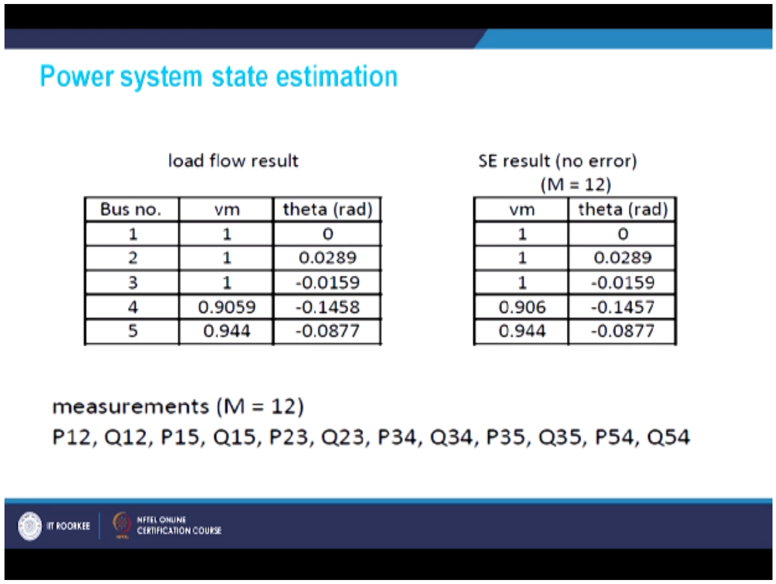

Now in this system, this is the load flow result. So now in this system, this is the load flow result. Please do remember that in this system, bus 1 is this red bus and bus 2 and bus 3 are 2 generated bus and both of them have got voltage specified as 1.0 and then bus 4 and bus 5 are P,Q load. And those same data and etc, whatever we have discussed in the case of load flow analysis, are also utilized here.

So then this load flow analysis is this. Here of course, we have not taken the case of generate a limit valuation. So then therefore, all these voltage magnitudes are maintained equal to 1. So this is the load flow result. Now if I do state estimation without any error, now what we have done, that after this load flow result, we have calculated that what are the power flow through all the lines.

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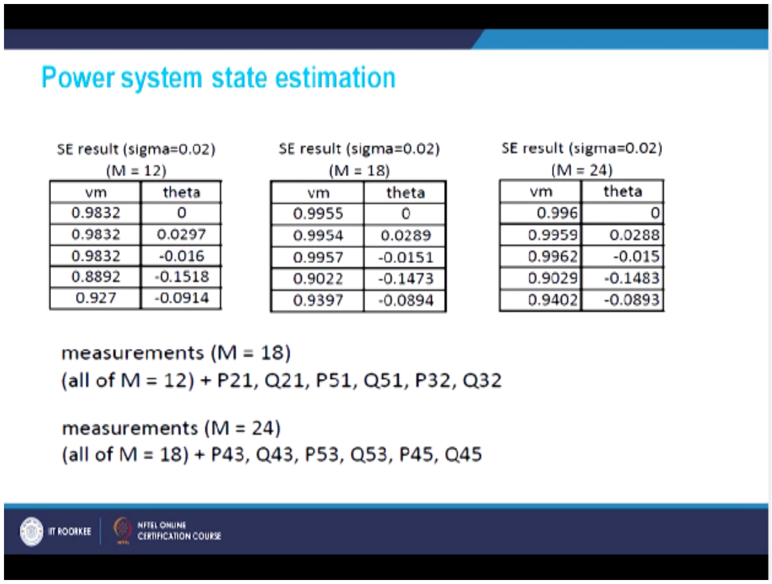

So what are the power flow through all the lines? In this line, in this line. So in all these lines, what are the power flow we have calculated by utilizing these values. These power flow results, we treat as the ideal value of the measurements which are true values, absolutely ideal and true values. Now here of course, now because it is a 5 bus system, so then therefore, the total number of states is 9.

I mean, except this angle at this bus, all the other bus voltage and magnitudes are to be estimated. So then therefore, total number of state variables is 9. And then when we do this state estimation with this 12 measurements, P12 denotes the real power measured between bus, I mean measured on the line bus 12, measured at bus 1. So then here also Q12 denotes that the reactive power flow over line 12 measured at bus 1.

Similarly, P15 is real power flowing over the line bus 15 measured at bus 1. So then therefore, when we are talking about PIJ, PIJ is nothing but the real power of flow measured, I mean, it is basically the real power flow over line IJ measured at bus I. So if we take these 12 ideal measurements computed from these results and they may back calculate all these voltage magnitudes and angles, we get the identical result.

Almost identical result. So then therefore, we can see that when we take the ideal measurement that is the true value of the measurement, state estimation result gives the exactly this load flow result. Suppose now we are taking errors in the measurement. Suppose we are now taking error and we take that all these meters I have got in sigma of 0.02, so that means we are taking all the measurements to be of equal weightage.

So we are taking all the measurements to be equal weightage. So then, so when I take this error and utilize the same 12 measurements, now we get different values. Now we get completely different values. Earlier these 3 were maintained at 1. Earlier these 3 were maintained at 1 but now these values are now completely changed. These values are now completely changed. So then you see that whenever there is an error, so then this estimated values are not equal to the true values.

So we can say that these values are the true value but using those erroneous meters or other erroneous measurements when you do this state estimation result, basically this state estimation results do not give the true measurements or rather do not give the actual value of the states. Now suppose we do increase the number of measurement that we still take sigma as equal to 0.02 and if we have to increase the number of measurements, then we get much better result.

Then we get a little better result. And why we get this little better result? Because more number of meters and if these meters, they do not have bad data in them, so then therefore, more number of meters, more number of measurements will actually improve the estimated states. Will actually improve the estimated states. So then we may see that when we take 18 measurements and these 18 measurements are nothing but all of these 12 measurements said earlier plus these 6 measurements.

Therefore, the sake of simplicity, we have only taken P and Q flow measurements over all the lines. Similarly with the same sigma when we keep on increasing, again these values have also improved but then that improvement is not much. But then still they are improved. And by 24 measurements are nothing but like this. So then all of these 18+this. Now these results are due to the, now these results correspond to the fact that there are no bad data.

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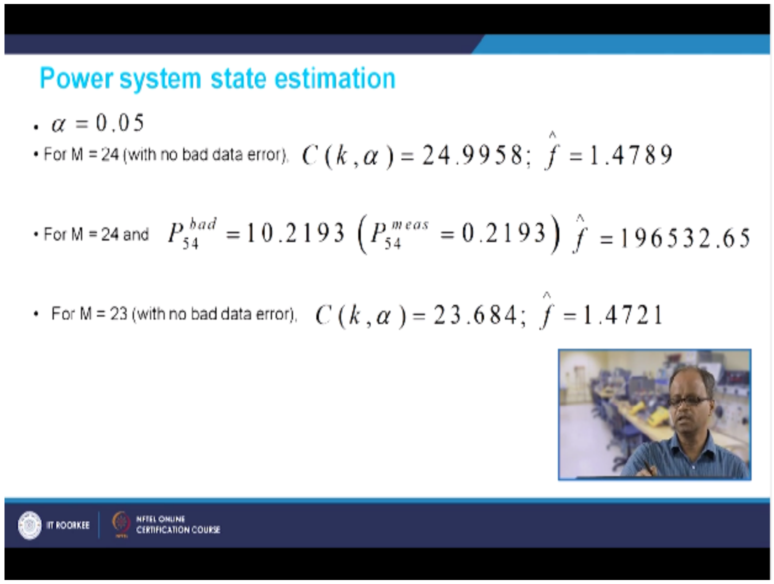

Now look at that if there is bad data. Now here we have taken alpha=0.05. Now if I take M=24, so that is this. So then therefore, if my M=24 and my total number of states to be estimated is 9, so then therefore, my K, that is basically the degree of freedom is 15, so K is 15 and alpha=0.05, so if we look at this particular table of chi squared distribution, we find that CK alpha=24.9958. And then utilizing these estimated values, if we now calculate this Z hat vector and then from that we can calculate f hat value, so then this f hat value comes out to be 1.4789.

See that this estimated value of the objective function is much less than this. So then therefore, we conclude that there is no bad data here. Now suppose, now in this analysis, in this case, if we calculate the value of, with this true values, if we calculate the value of real power flow over line 45 measured at bus 5 and if we calculate that particular value of P54, P54 is nothing but the real power flow over line 45 measured at bus 5.

If we calculate that value, the true value comes out to be 0.2193. But now we assume that suppose this particular meter has gone bad. So instead of measuring somewhere closer to 0.193, that is I mean even if this meter has got a sigma, I mean basically error sigma=0.02, so its measured value would not be very far away from this value. Its value would not be equal to this but that particular measured value would be quite close to this value.

But now suppose that this meter has gone completely bad and this error is too much, so now suppose that this instead of measuring any value closer to this, it is now actually measuring a value 10.193. So it is almost 50 times. So it has gone completely bad. So now with this bad measurement, with this single bad measurement, if we now calculate, I mean if we now do this state estimation, then we find that my f hat value is 196532.65 which is much more than this.

Much more than this. So then therefore, we conclude that there is certainly a bad data. So when we conclude that there is certainly a bad data, then we calculate for each of these 24 measurements, then we calculate our dash matrix and then subsequently we calculate the quantity EI dash/RII dash. So then therefore, we get the 24 values of EI dash/RII dash. Then I take their absolute value and I take their maximum value.

And I find out that what is the index of that maximum value. I find that the index of that maximum value is exactly given by this. So then this index of the maximum value is, it is indicating that this particular measurement is bad. So then therefore, in this example, for example if I say that it is P54, for example this one, so then it was giving the index as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

So then after that, what do you do once we identify? So then therefore, this particular measurement has got the maximum value of EI dash/RII dash. So then we remove it. So now my effective number of measured value becomes 23. So now my effective number of measurements become 23. And I still keep this alpha=0.05 and with this C and with this, I again find out that what is the value of CK alpha.

Now please note that now K becomes 14. Because my number of state is 9 and my total number of measurement is 23, so now my K that is the degree of freedom becomes 23-9, that is 14. So then which this 14 CK alpha, so then with this 14.05, I get this value. And when I do this, my state estimation with this 23 remaining measurements, I find that my f hat value is 1.4721. So again we now conclude that there is no bad data in the system.

So then therefore, whatever is the estimated value, those are the acceptable estimates. So this is the basic procedure. Now here one question comes into mind. So we can see that to cater for the presence of this bad data, we have to make that our measurement value, we have to maintain that our number of measurement should be more than the number of states. Now in case, now if we are able to put meters everywhere and bring those values to our control center, so that in that case we will be getting those measurement values.

But if not, so then we use, we have to have some other way of taking this or rather basically some other way of generating this measurement values, right. And these are called the concept of the pseudo-measurements as well as the virtual measurements. So this concepts, we will continue in the next lecture. Thank you.