

Computer Aided Power System Analysis
Prof. Biswarup Das
Department of Electrical Engineering
Indian Institute of Technology - Roorkee

Lecture – 47
Power System State Estimation (Contd.)


Hello friends, welcome to this lecture on computer aided power system analysis, we have been discussing about power system state estimation, so let us process this this, so we are talking about the algorithm.

(Refer Slide Time: 00:43)

Algorithm for PSSE

m → no. of measurement
n → no. of states

1. Take initial guess $\bar{x}^{(0)}$ and choose $\epsilon(k, \alpha)$ for a given α (we choose α)
2. Set $k=0$ → iteration count and set $H=R^{-1}$ matrix
3. Evaluate $h_i(\bar{x}^{(k)})$ and $\Delta z_i = z_i - h_i(\bar{x}^{(k)}) \quad \forall i=1, \dots, m$
4. Evaluate $H_{\bar{x}^{(k)}}$ → Jacobian matrix → $H_{\bar{x}^{(k)}}$ is the Jacobian matrix evaluated at $\bar{x} = \bar{x}^{(k)}$.
5. Calculate $\Delta \bar{x}$
6. Calculate $\bar{x}_i^{(k+1)} = \bar{x}_i^{(k)} + \Delta \bar{x}_i \quad \forall i=1, \dots, n$
7. Calculate $\epsilon = \max_i |z_i^{(k+1)} - z_i^{(k)}|$
8. Check $\epsilon < \epsilon(k, \alpha)$
 - Yes → $\bar{x}^{(k+1)}$ is the solution and go to step 9
 - No → $k = k+1$ and go back to step 3.



So, let us just very quickly recapitulate the algorithm and then we will algorithm for very quickly we have already seen it but just to recapitulate algorithm for PSSE, so we first have taken initial guess, x_0 and choose ϵk alpha, ϵk alpha is nothing but a threshold as obtained from the chi squared distribution for a given value of alpha; for a given alpha, we should also say that I mean this alpha is also we choose, we also choose actual alpha, we actually choose alpha, so we must also know that we choose alpha.

So then after that set $k = 0$, this is the iteration count, then what do we do; we evaluate h_i at x_k , right and from that and z_i ; Δz_i ; $\Delta z_i = z_i - h_i x_k$ for all $i = 1$ to m , m is the number of measurements, then what do we do; evaluate H_{x_k} , now what is H_{x_k} that is the Jacobian matrix,

this is the Jacobian matrix, so and then after that we and we also set $W = R$ inverse matrix; $W = R$ inverse matrix is nothing but the weight matrix.

So, I mean in this particular W matrix is this weight matrix calculate delta x vector, then we check calculate $x_{i, k+1} = x_{i, k} + \Delta x_i$, this is for all $i = 1$ to n , n is the number of states, so n is the number of states, then calculate max of absolute $x_{i, k+1} - x_{i, k}$ for all i , let us say this is epsilon, let us say this is e , we should write it more clearly, $e =$ this, then what we do; check whether e is $<$ epsilon.

If yes, states are estimated, $x_{i, k+1}$ is the solution, if not; no, if not, then what we do; we simply $k = k + 1$ and then go back to step 3, so this is the basic algorithm.

(Refer Slide Time: 05:59)

Algorithm (contd.)

9. Calculate $\hat{z}_i \quad \forall i=1, \dots, m$
10. Calculate $\hat{e}_i = z_i - \hat{z}_i \quad \forall i=1, \dots, m$
11. Calculate \hat{f} .
12. If $\hat{f} < c(k, \alpha)$
 - Yes \rightarrow no bad data and therefore final solution is obtained.
 - No \rightarrow Go to step 13.
13. Calculate R^i matrix
14. Calculate $\hat{\sigma}_i = \frac{\hat{e}_i}{R_i^i} \quad \forall i=1, \dots, m$
15. Calculate $p = \max_{i=1, \dots, m} |\hat{\sigma}_i|$ and find the index of 'p' in the vector $\hat{\sigma}$. Let this index be 'i'. and hence, i^{th} measurement is bad.

$\hat{\sigma} = [\hat{\sigma}_1, \hat{\sigma}_2, \dots, \hat{\sigma}_m]^T$

Now, after this also, so algorithm continued, so here also is the solution and from then and we should write here and go back to step 9, so this is actually go to step 9, what do you do in step 9, we calculate, in step 9 calculate z_i hat overall $i = 1$ to m , then step 10 calculate e_i hat = $z_i - z_i$ hat, this is for all $i = 1$ to m , then we calculate f hat, then what do we do; if f hat is $<$ $c_k - \alpha$ is no bad data and then therefore, final solution is obtained.

If no; if it is no, so then what happens; if no go to step 13, what do you do in step 13; calculate R dash matrix, we have already seen how to calculate R dash matrix, then what do we do; calculate

$y_i = e_i \text{ hat} / R_{ii}$ dash for all $i = 1$ to m , then we calculate position P as max of y_i , this is for all i , for all $i = 1$ to m and find the index of P in the vector y . What is this vector y ; this vector y is actually, this vector y is nothing but y_1, y_2 to y_m transpose, so this is the vector y .

Now, after that what you do; we find the location, let this index be l , so therefore and hence l th measurement is bad, so after that what do we do; index and vector, let this index be l and hence l th measurement is bad.

(Refer Slide Time: 10:47)

Algorithm

16. Remove the l th measurement from the set of measurements.
17. Go back to step 1.

Structure of the Jacobian matrix

Measurements are:

$$z_i = h_i(x_1, x_2, \dots, x_n) \quad \forall i=1, \dots, m$$

$$H = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \dots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \frac{\partial h_m}{\partial x_2} & \dots & \frac{\partial h_m}{\partial x_n} \end{bmatrix} \rightarrow (m \times n)$$

Measurements can be $P_{ij}, Q_{ij}, V_i, P_i, Q_i, |I_{ij}|$

So, then therefore what do we do; remove the l th measurement from the set of measurements, right, go back to step 1, so this is a complete algorithm of the state estimation which includes also the process of bad data deduction now, here, now we have to look into this basically, the structure of this Jacobian matrix, so now let us look at the structure of the Jacobian matrix, so now we are looking at the structure of Jacobian matrix.

Now, here also we must do one thing, now this Hx_k we must note that Hx_k is nothing is actually the Jacobian matrix, you must note it, evaluated at $x = x_k$, so that is what is Jacobian matrix, so now what is the structure of the Jacobian matrix; so now let my measurements are let us $z_i = h_i * x_1 x_2$ up to x_n for all $i = 1$ to m , right, we just for the sake that m is the number of measurements, n is number of state.

So, now therefore, what is this H matrix; so then therefore H matrix is nothing but $\frac{\partial h_1}{\partial x_1} \frac{\partial h_2}{\partial x_2} \dots \frac{\partial h_m}{\partial x_n}$ and this will go up to $\frac{\partial h_m}{\partial x_1} \frac{\partial h_m}{\partial x_2} \dots \frac{\partial h_m}{\partial x_n}$, so this is an m cross n matrix, this is straight forward m cross n matrix, now as we said that our measurements can be now we said that our measurements can be let us say P_{ij} , I have already defined what are these quantity Q_{ij} , V_i , P_i , Q_i , $\text{mod } I_{ij}$, anything can be this measurement.

So our overall set of measurements, set is like this, we already know what is the; what are the expressions of P_i and Q_i that we already know and similarly, this V_i is also already know, so now let us look at that I mean what are the expressions of P_{ij} and Q_{ij} .

(Refer Slide Time: 15:51)

Expressions of measured quantities

$$P_i = \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$Q_i = \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$N \rightarrow$ no. of bus in the system
 $n = 2N - 1$

$\bar{V}_i, \bar{V}_j \rightarrow$ complex voltages of bus i and j
 $\bar{Y}_{ij}, \bar{Y}_{sh} \rightarrow$ are complex admittances of the line
 $\bar{Y}_{sh} \rightarrow$ half-line charging susceptance of the line.

$\bar{V}_i = V_i \angle \theta_i$
 $\bar{V}_j = V_j \angle \theta_j$
 $\bar{Y}_{ij} = Y_{ij} \angle \beta_{ij}$

$\bar{I}_{sh} = \bar{Y}_{sh} \bar{V}_i$
 $\bar{I}_{sh} = j \bar{Y}_{sh} \bar{V}_i$

$\bar{I}_{ij} = \bar{I}_1 + \bar{I}_2$

So, now let us look at some expressions, so now look expressions for different quantity, expressions of the measured quantities, so we first note that P_i is simply $i = j = 1$ to capital N $V_i V_j Y_{ij} \cos \theta_i - \theta_j - \alpha_{ij}$, similarly Q_i is $j = 1$ to N $V_i V_j Y_{ij} \sin \theta_i - \theta_j - \alpha_{ij}$, you should note here that N is the number of bus in the system and we also note that n that is the number of state would be $2n - 1$, V_i is already known, so $V_i = V_i$, now let us look at P_{ij} Q_{ij} and I_{ij} .

To know that let us consider a transmission line, this is bus i , this is bus j , in between there is a line, let us say this has got an admittance Y_{ij} , this is a complex quantity, this is y shunt, this is

also y shunt, so it is a standard pi model, these are all complex admittances, so we must write that these are not vectors, Y_{ij} , y shunt are complex admittances of the line, must write admittances of the line and what is y shunt?

And y shunt is actually half line charging susceptance of the line, so if that is the case, so then this current I_{ij} , so we are not trying to find out what is the current I_{ij} , so this current is nothing but this current plus this current, so $I_{ij} = I_1 + I_2$, so this is I_1, I_2 .

(Refer Slide Time: 19:28)

$$\begin{aligned} \bar{I}_{ij} &= (\bar{V}_i - \bar{V}_j) \bar{y}_{ij} + \bar{V}_i \bar{y}_{sh} \\ &= \left(\frac{V_i}{\angle \theta_i} - \frac{V_j}{\angle \theta_j} \right) \bar{y}_{ij} + \left(\frac{V_i}{\angle \theta_i} \right) (j \bar{y}_{sh}) \\ &= \left(\frac{V_i}{\angle \theta_i + \beta_{ij}} - \frac{V_j}{\angle \theta_j + \beta_{ij}} \right) \bar{y}_{ij} + \bar{y}_{sh} \frac{V_i}{\angle \theta_i + 90^\circ} \\ &= \left[\frac{V_i \cos(\theta_i + \beta_{ij}) - V_j \cos(\theta_j + \beta_{ij})}{j} \right] \bar{y}_{ij} + \bar{y}_{sh} V_i \sin \theta_i \\ &= \left[\frac{V_i \sin(\theta_i + \beta_{ij}) - V_j \sin(\theta_j + \beta_{ij})}{j} \right] \bar{y}_{ij} - \bar{y}_{sh} V_i \cos \theta_i \\ &\Rightarrow \bar{I}_{ij} = a_{ij} + j b_{ij} \end{aligned}$$

Complex operator $\Rightarrow j = \sqrt{-1}$

Complex operator

Two index

$$(P_{ij} + j Q_{ij}) = \bar{V}_i \bar{I}_{ij}^* = (V_i \cos \theta_i + j V_i \sin \theta_i) (a_{ij} - j b_{ij})$$

So, then therefore I_{ij} is actually $V_i - V_j * I_{ij}$, V_i, V_j are the complex voltages, V_i ; so it has got a voltage V_i , this is got a voltage V_j , and we must also write that V_i, V_j complex voltages of bus i and j , and we also should write that $V_i = V_i \angle \theta_i$ and $V_j = V_j \angle \theta_j$ and let us say Y_{ij} is $Y_{ij} \angle \alpha_{ij}$, we should not write α allowed because we are already utilising α , so something else we should write, should write β_{ij} .

And let us say y shunt = y shunt angle let us say γ_{sh} shunt, actually γ_{sh} shunt would be = 90 degree but just for the sake of it is actually should be = $j \gamma_{sh}$ shunt, it is actually $j y_{sh}$ shunt because I mean γ_{sh} shunt would be = 90 degree, so $j y_{sh}$ shunt, so then therefore, $+ V_i * y_{sh}$ shunt, so this is $V_i \angle \theta_i - V_j \angle \theta_j * y_{ij} \angle \beta_{ij} + V_i \angle \theta_i * j y_{sh}$ shunt, so it would be $V_i \angle \theta_i + \beta_{ij} - V_j \angle \theta_j + \beta_{ij}$ whole of $Y_{ij} + y_{sh}$ $V_i \angle \theta_i + 90$ degree.

So, then therefore it would be $V_i \cos \theta_i + j a_{ij} - V_j$, so it should be V_j ; $V_j \cos \theta_j + j b_{ij}$ whole of $Y_{ij} + j$, now this is the complex operator, this j is complex operator, so that means $j = \sqrt{-1}$. $V_i \sin \theta_i + j a_{ij} - V_j \sin \theta_i + j a_{ij} * Y_{ij} - y_{sh} V_i \sin \theta_i + j y_{sh} V_i \cos \theta_i$, please note here also this j is also the complex operator, so this is also complex operator.

So, then therefore I can write down that $I_{ij} =$ let us say something i real part $+ j$ b_{ij} , again I note that these j is actually the complex operator and these j ; these $2j$'s are the bus index, so then from here, I can write down that $P_i + j Q_{ij}$ that is if I am measuring P_{ij} and jQ_{ij} at this point, so I am actually measuring here, $P_{ij} + jQ_{ij}$, this is at this point at that is at this point I am measuring that means at the outgoing point from bus i .

So, then it would be V_i star, sorry, $V_i * I_{ij}$ star, sorry, $V_i * I_{ij}$ star, so then therefore, this is $V_i \cos \theta_i + j V_i \sin \theta_i * a_{ij} - j b_{ij}$.

(Refer Slide Time: 26:26)

$$\Rightarrow P_{ij} = V_i (a_{ij} \cos \theta_i + b_{ij} \sin \theta_i)$$

$$\Rightarrow Q_{ij} = V_i (a_{ij} \sin \theta_i - b_{ij} \cos \theta_i)$$

$$\Rightarrow |I_{ij}| = \sqrt{a_{ij}^2 + b_{ij}^2}$$

\Rightarrow For known line parameters,

$$P_{ij} = P_{ij}(V_i, \theta_i, V_j, \theta_j)$$

$$Q_{ij} = Q_{ij}(V_i, \theta_i, V_j, \theta_j)$$

$$|I_{ij}| = f(V_i, \theta_i, V_j, \theta_j)$$

So, then therefore, from here, I can write down $P_{ij} = a_{ij} V_i \cos \theta_i$, so $V_i a_{ij} \cos \theta_i + b_{ij} \sin \theta_i$ now, if I look at this expression and similarly, Q_{ij} would be $V_i -; Q_{ij} \sin \theta_i - b_{ij} \cos \theta_i$, so now if I look at that these 2 expressions; a_{ij} is nothing but this plus this and these

are all only, this is j , I am sorry, this is j , this is also j , this is also j , snow $a_{ij} = \text{this} + \text{this}$ and $b_{ij} = \text{this} + \text{this}$ and all these are only a function of V_i V_j θ_i θ_j .

Of course, here we are assuming that this quantity is a β_{ij} and I mean we are assuming that this all this line parameters are known, so then therefore and also I can write down that $I_{ij} \text{ mod} = \frac{\text{root}}{\sqrt{a_{ij}^2 + b_{ij}^2}}$, right, so then therefore I can write down that for all for known line parameters, P_{ij} is some function of V_i θ_i V_j θ_j , similarly Q_{ij} is some other function $Q_{ij} V_i \theta_i V_j \theta_j$ and $\text{mod } i_{ij}$ is also some other function and let us this call this function f .

This is also be would be functions of will be V_i , V_j , θ_i , θ_j , so all these quantities corresponding to line would be only the function of the terminal voltage magnitudes and angle of that corresponding line. So, with this we now ready to look at the structure of the Jacobian matrix which we will do in the next lecture, thank you.