Computer Aided Power System Analysis
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Lecture – 45
Power System State Estimation

Hello everybody, welcome to this lecture on the course of computer aided power system analysis, in the last lecture we have looked into the aspect of bad data detection, we have looked into the entire procedure with an example but they are one particular issue, did not explain in detail, so let us look at that detail today. First and then we will after that we will talk about the issue of power system state estimation which is the basic purpose of this entire unit.

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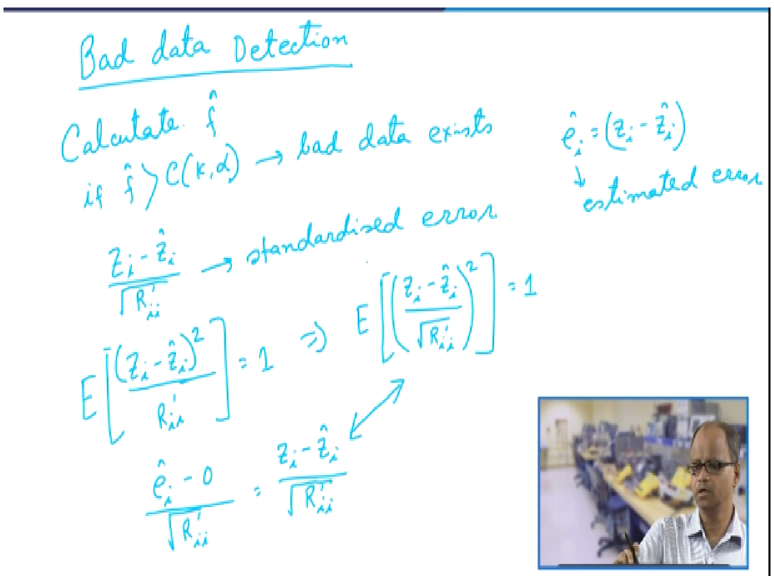

So, what we are doing is that bad data deduction, when we are doing bad data detection, what we did is that we calculate and if f hat is > C k alpha, then we said that bad data exists, when you say that bad data exists, now to find out that which one is the bad data, now to find out which one is the bad data, we did, what we did is; we calculated some quantity called zi – zi hat/ Rii dash root, we call it at standardised error.

And then we said that whichever measurement has got this error to be maximum that is basically the maximum magnitude, so that particular measurement is actually taken to be a bad data and then we simply discarded and then we again do it, now the question is that what is the justification for calculating this? Now, to do this, what we calculate, please note that when you said we have calculated expected value of zi – zi hat/ square/ Rii dash, you said that it is = 1.

So, this can be written as simply zi/ zi – zi hat/ root Rii dash whole square = 1 that can be; now, suppose now we also note that ei hat that is estimated value of error is zi – zi hat, so this also we know, this is the estimate error, now, if we do define one random variable, now please note that I mean this is a random variable, now, if I do define one random variable such that which is let us say ei hat – 0/ Rii dash.

So, this is nothing but zi – zi hat/ Rii, so root over, so now this random variable has got the property that this has got a variance = 1, so then therefore this quantity is nothing but the standardised data with a mean = 0 and the variance = 1, so then therefore that is why we call this quantitatively the standardised error and so then therefore we first calculate this particular standardised error, and then we simply take the absolute magnitude of this standardised error.

And then we reject that particular measurement which has got the absolute maximum value of the standardised error, now one important point needs to be understood.

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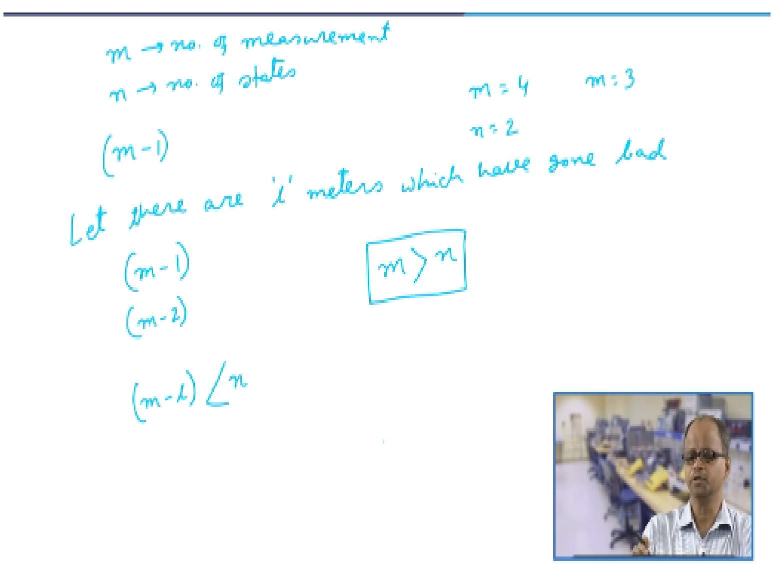

Suppose, I do have m is the number of measurement and n is the number of states, now in case now and suppose that we first do and exercise of the state estimation and then we do the cross check that whether any bad data exists or not, if any bad data exists, so then what happens that we simply discard that particular measurement, so when you discard that particular measurement, so then in that case my effective number of measurement which we use becomes m – 1.

For example, in our example which we have shown in the last lecture that earlier m was 4 and n = 2 but when the fourth measurement was detected to be a bad one, so then we have simply neglected or rather discarded that particular measurement and then again we have done this state estimation using m = 3, number of measurement, so then therefore, total number of measurement utilised has reduced by one.

Now, here in this example, we have found that well, when we have taken let us say a three measurements, we have got our estimated states to be acceptable because in that case we did not detect any kind of bad data but it may happen that in a system, there are multiple meters which have gone bad, say for example, say let there are multiple; let there are l metres which have gone bad, so then what happens?

We start with this m meters, then we first a detect one meter that is basically which is got the highest value of the standardised error, then we simply discard it, so then after that my total number of effective measurements becomes m -1, again when we do our state estimation using this m -1 measurements, again we find out the next meter which is bad, so then after that my effective number of measurements becoming m - 2.

So, if you do; so then therefore if you do follow this process ultimately, my effective number of measurements will be m – l, now if my effective number of measurements m - l becomes less then n, so then in that case this state estimation problem would be an indeterminate one because a number of equations available is < number of states to be estimated, so then therefore, to crosscheck or a so to therefore to take into account the possibility of existence of bad data.

What happens is that we usually take number of measurement to the reasonably large as compared to the number of states to be estimated, so then therefore, the one of the most important conclusion is that for any state estimation exercise the number of measurement is always taken to be reasonably large as compared to the number of states to be estimated, so we always say that m is always > n, in it is always > n.

So that is the very important conclusion, so this computes the discussion of the basic theory of the state estimation and we have explained this basic theory of state estimation by taking a very simple DC circuit but our objective as we have stated in our initial lecture of this module on state estimation, our objective is to ultimately to monitor the power system, monitor the transmission and distribution grid, so then therefore, we need to discuss about the power system state estimation.

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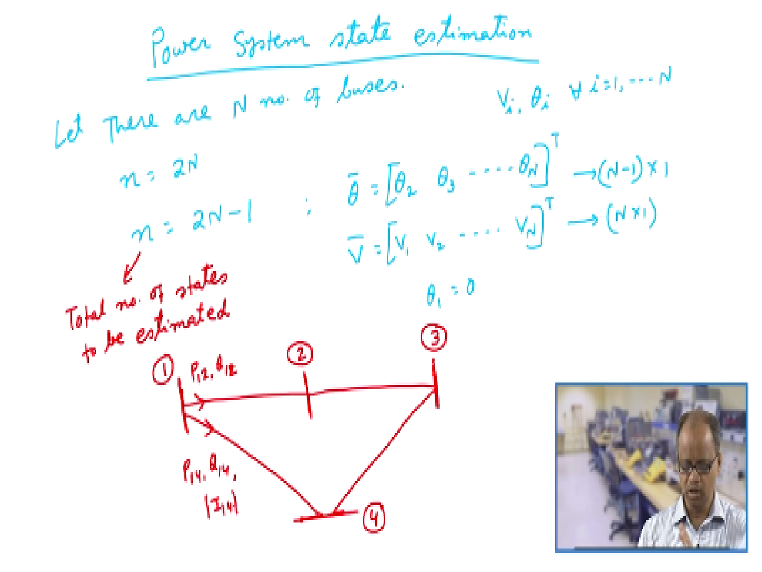

So, then now in the case of power system state estimation, what happens now, so now our discussion is; now we start our discussion on power system state estimation, many of this or rather most of this discussion will follow the discussions which were we have done but we have to add something to take into account the basic nature of the power system equations now, first thing that what are the states?

So, for any transmission and distribution with grid, the first thing what are the states now, let that there are n number of buses, N; capital N number of buses, let capital N number of buses, now at each bus, now we have already explain when we have discussed about the load flow analysis, we have already explained that if we know Vi and theta i for all i = 1 to N, so then we would be able to calculate everything else in the system.

So, then therefore it follows that when you are talking about estimating the states of any power system, we can take Vi and theta i for all the buses to be the states, so then therefore it follows that it appears that we have got total number of states n = 2 * N because at each bus we have got voltage magnitude and one angle but then as we have also said in the one we have discussed about power system state estimation that for measuring any angle or rather for calculating the value of any angle, we need a reference value of angle.

Because without any reference value, no calculated value of angle does not make any sense, so then therefore, we have to make one value of angle to be = 0 and once we make one value angle, one value of angle = 0, rest of the other angles would be calculated with respect to this particular angle, so then because we are making one value of angle to be 0 usually, we make it let us say at bus number 1 or let us say at let us say slack bus.

But the central point is that we do make one angle to be = 0, so then therefore, total number of states is = 2N – 1, so and these states are these states are given by theta vector that is theta 2, theta 3 up to theta N transpose, so this is an N -1 cross 1 vector, here we assume that theta 1 = 0 and V vector that is V1, V2 up to VN, so this is N cross 1 vector, right, I have we take theta 1 = 0, so this is N -1, this is N.

So that you have got total number of states, so we write that this is the total number of states, so total number of states, this is the number of states to be estimated, so now this is the basically the total number of states to be estimated, so now we need m number of measurements where m is > n, so now let us look at that what could be the possible measurements, so for that let us say, look at some simple system just 4 bus system, so these are all buses.

So, as far as a measurement is concerned, we can and let us say this is bus 1, bus 2, bus 3, bus 4, so as far as the measurement is concerned, we can think of that we are measuring let us say P12, Q12, right, we can also measure let us say P14, Q14, we can also here can measure I14 that is the magnitude or in other words, we can measure all real power flow over all the lines, we can measure all reactive power flow over all the lines.

We can also measure the current flow magnitude over all the lines, we can also measure some of the voltage magnitudes right, of course you probably cannot measure the angles but you can always measure the some of the voltage magnitudes by just putting a volt meters.

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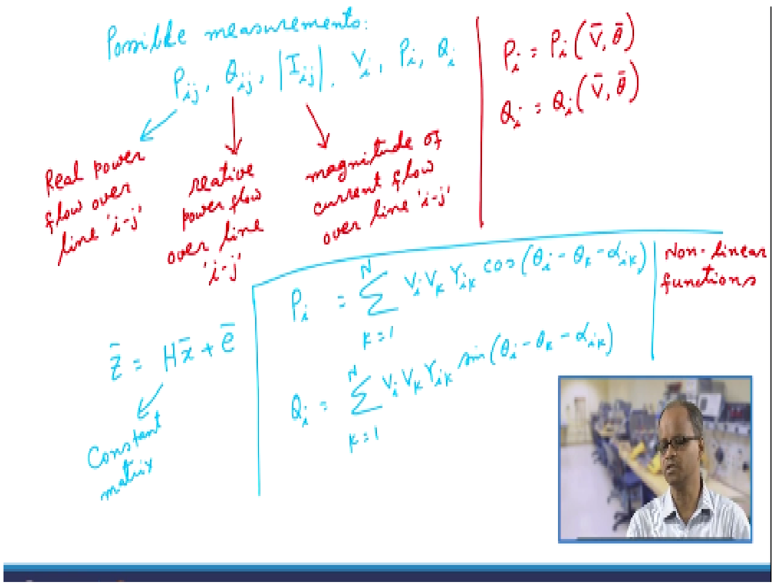

So, then therefore our possible number of measurements or rather the possible measurements are; so our possible resentments are Pij, Qij, Iij, Vi, what is Pij; Pi is the; Pij is the real power flow over line ij, this is the reactive power flow over line ij, this is the magnitude of current flow over line ij, we can also measure possibly Pi, Qi that injected power at some bus P and injected sorry, I mean injected reactive power at some bus i.

I mean how is it possible for example, just for example, suppose that I do have a load here, and this has got PQ load and if I do put a let us say, and what meter kind of a thing, so then therefore we would be able to measure that what is the amount of load and so then therefore, our measured value of Pi and Qi would be nothing but the negative of these measurements, similarly if I do have an let us say generator here, I can always put another voltmeter here.

And then we can always measure that what is the value of Pi and Qi from this particular generator, so then therefore our possible set of measurements can be anything, now if we look at now if we do recollect our earlier discussion, in our earlier discussion when we have discussed about the state estimation using a simple DC circuit, we have noticed that the measurement vector Z is given by some constant matrix x + some vector e.

Please note that this is a constant matrix, this is a constant matrix and why this was a constant matrix; because all these equations were linear equations and of course, these equations were linear equations because of the simple fact that we have taken a very simple DC circuit and in any DC circuit, any measure quantity can be expressed as a linear combination of the unknown states.

But what happens in the case of power system state estimation, for example if we look at the expressions of Pi and Qi, so Pi, I know that it is simply k = 1 to N, Vi, Vk, Yik cos theta i – theta k - alpha ik, similarly Q1 is also ki, k = 1 to N, Vi, Vk, Yik sin theta i – theta k – alpha ik, right now, obviously these 2 equations are non-linear in nature, so these two equations are nonlinear in nature, so these 2 equations are nonlinear functions, right.

Now, these 2 equations I can say that as if that Pi, Pi can be said that as if that it is a function of Pi of vector Vi and V and theta, similarly Qi is can be said as a function of V and theta and these are nonlinear functions similarly, if we do write down the expression of Pij, Qij and Iij that is basically the magnitude of Iij, we will find that these expressions are also nonlinear functions of the unknown state vector, Vi theta.

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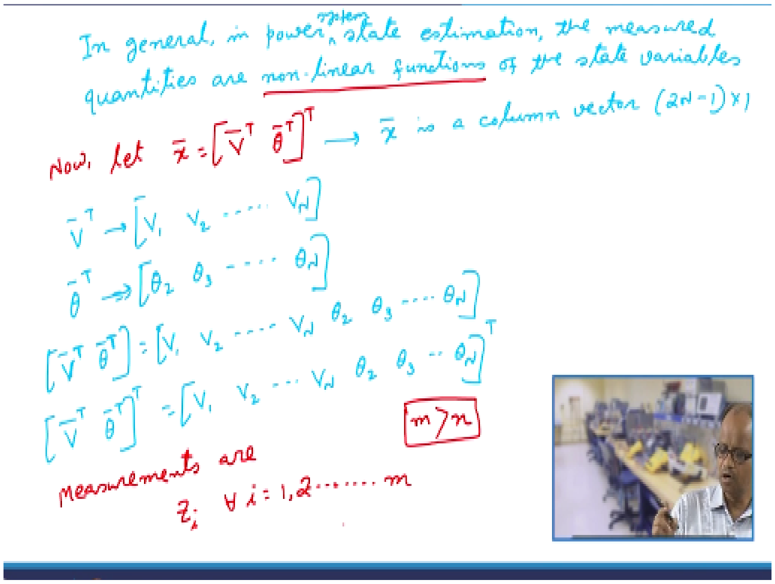

So, we note that in general in power system state estimation, the measured quantities are this we must so, this is a very important observation, the measured quantities are is nonlinear functions of the state variables, so this is a very important to understand that these are nonlinear functions of the state variables now, let I denote my x; vector x as V transpose theta transpose, transpose T, please note that there is too many transpose.

But what happens is that V transposes actually, just to explain it, V transpose is actually V1, V2, VN and theta transposes actually theta 2, theta 3, theta N, so then therefore V transpose theta transpose is nothing but V1, V2, VN, then theta 2, theta 3, theta N, right, so then when I take V transpose theta transpose of so then transpose of this transpose, we get that V1, V2, VN, theta 2, theta 3, theta N, transpose, so we simply make it as a column vector.

So then therefore x is a column vector, so we say that x is a column vector and the dimension is 2N – 1 + 1, now suppose I do have total number of measurements as m, so my measurements are Zi, now let us say the measurements are let us say Zi for all i = 1 to N, right and these measurements can be either Pij or Qij or magnitude of current Iij or your Vi or Pi of Qi, anything, they can be anything.

Only thing is that m > n, so that is the most important thing we said and this is due to this fact that we are making this constraint to ensure that if there is any bad data which can be detected and removed.

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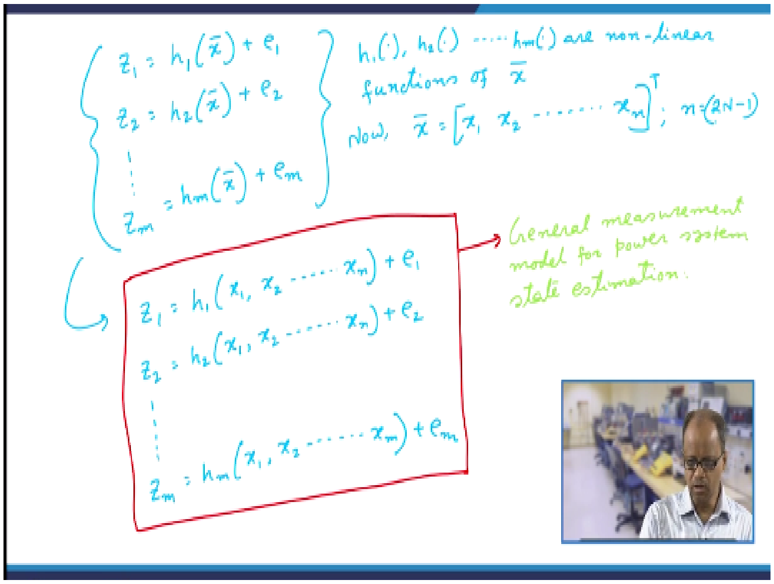

So, then therefore because these are all nonlinear functions, so then in general, we can write down that Hi, in general I can write down that Z1 that is the measured value Z1 is some function h1 of x + e1, Z2 is some function of h2 of x + e2 dot dot dot Zm = some function of + em, where h1 , h2 dot dot dot hm are nonlinear functions of x; vector x, now let now here now we know now let, we say that x vector is given by x1, x2 dot dot dot dot dot xn transpose T.

Of course, we note that; of course we all of us should note that n = 2 * N-1 that you always know but x1, x2 = n, so then therefore these equations can be written as Z1 = h1 of x1, x2, xn + e1, Z2 = h2 function x1, x2, up to xn + e2 dot dot dot Zm = hm * x1, x2 * up to xn + em, so this is the general measurement model, so this is the general measurement model for power system state estimation, so this is the general measurement model, so the and we note that in this general measurement model, all this functions are basically, nonlinear in nature.

So, then therefore the technique whatever we have used earlier cannot be applied immediately directly, so then in the next lecture, we will look at that how this nonlinear functions can be incorporated into our general state estimation process, which we have discussed earlier, thank you.