Computer Aided Power System Analysis
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Lecture - 43
Error Analysis (Contd.)

Hello friends. Welcome to this lecture on the course on computer aided power system analysis. We have been discussing the error analysis regarding the weighted lease square method. So what we have been doing is so far is this. So we have been trying to find out that what is the expected value of the estimated errors and we did and we have seen that this expected value of the estimated error can be given by this expression.

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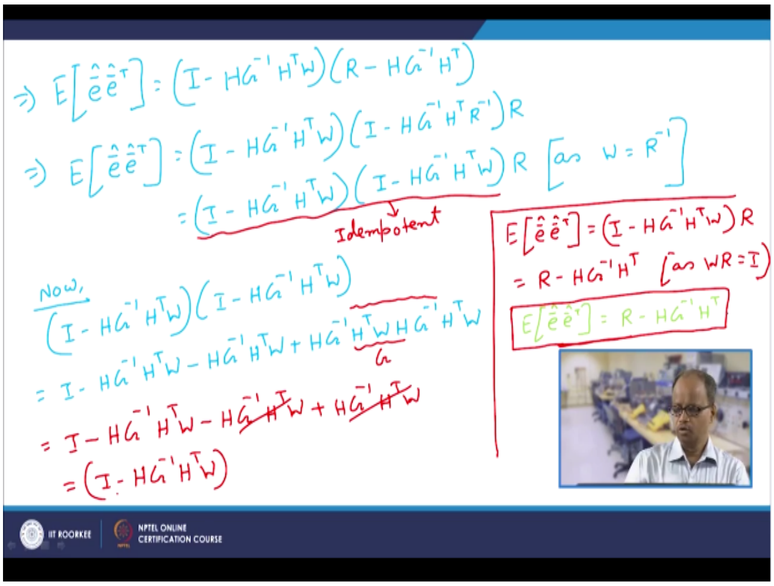

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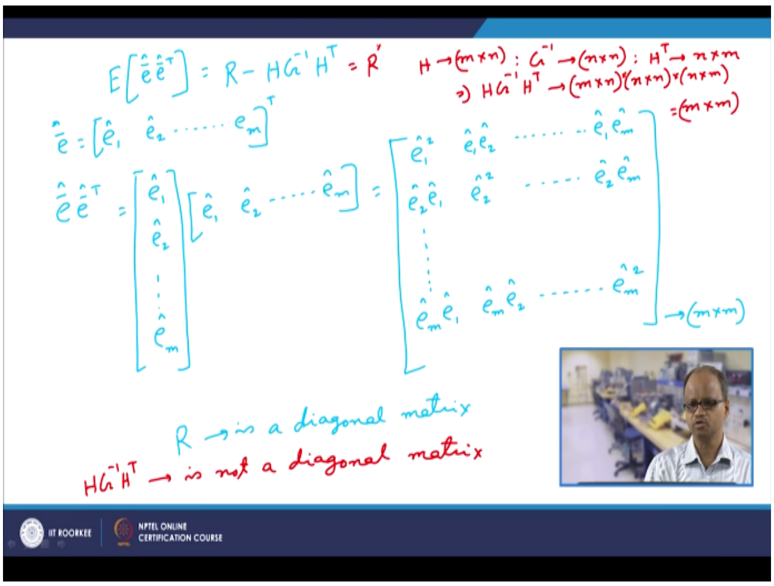

So now we will actually start from this point. So we write that E of e is=R-HG inverse HT. Now e transpose I mean e hat is actually the e1 hat, e2 hat dot dot dot em hat transpose. So then therefore e hat and e this thing would be essentially e1 hat e2 hat dot dot dot em hat*e1 hat e2 hat dot dot dot em hat so that can be written as e1 hat square e1 hat e2 hat dot dot dot e1 hat em hat e2 hat e1 hat e2 hat square dot dot dot e2 hat em hat and dot dot dot em hat e1 hat em hat e2 hat em hat square.

So this is again a square matrix m x m so this is also a square matrix m x m. Now R is a diagonal matrix that we know that we have already seen. What about HG inverse H transpose? Now H we know that H is actually H is m x n, G inverse is n x m and H transpose is actually n x m. So then therefore HG inverse H transpose would be m x n *n x n*n x m so it would be m x m.

Now depending upon the entries of H matrix as well as the G matrix, so then therefore this matrix HG inverse H transpose matrix we know that HG inverse H transpose matrix is not a diagonal matrix. That is important to know, so then therefore because it is not a diagonal matrix.

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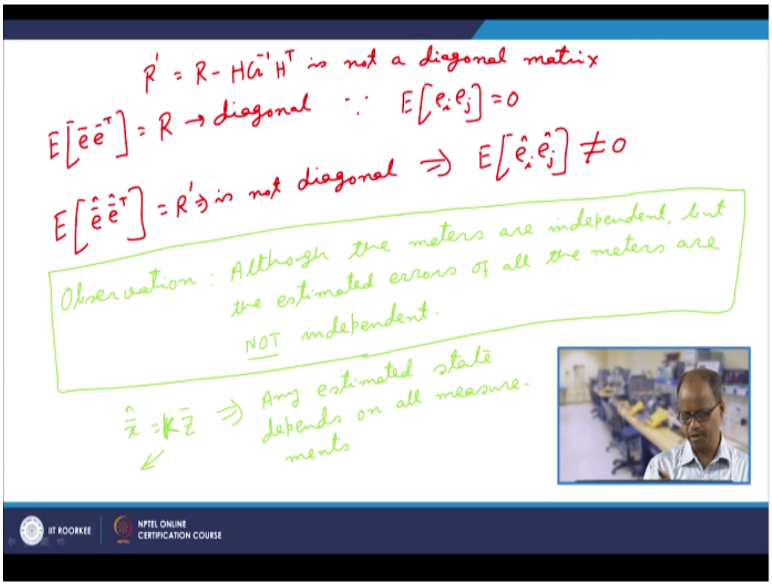

So then therefore now if I say that this matrix is R dash so then therefore R dash is=R-HG inverse H transpose is not a diagonal matrix. Now what does it mean? Earlier if we do remember correctly we said that expected value of earlier expected value of e transpose is a diagonal matrix R is a diagonal and why we would say that it is diagonal because we said that this meters any two meters are independent this is because expected value of ei ej was 0 as these meters are independent, so it is diagonal.

But then this matrix is not diagonal, this matrix is R dash is not diagonal that only means that is not diagonal that only means that expected value of ei hat ej hat is not equal to 0. That is what exactly it means. So that means what you are trying to say that although originally this meters are independent in nature but then the estimated value of errors of the meters they are not independent in nature.

So the first conclusion is a very important observation so then the observation is that although the meters are independent but the estimated errors of all the meters are not independent. The question is why? That although these meters are independent we have assumed because we have worked to it this but then the estimated errors of all these meters are not independent. So why this is so?

Now this is because of this fact that our estimated vector x is actually some quantity*Z something into Z I mean this K matrix we all know so it is basically some matrix K*Z. So then therefore any estimated state vector depends on so then therefore it means that any estimated state depends on all measurements although the meters are independent but any estimated state depends on all measurements right.

And we note that this matrix K is a constant matrix and we know what this matrix says. This is HG inverse H transpose*Z.

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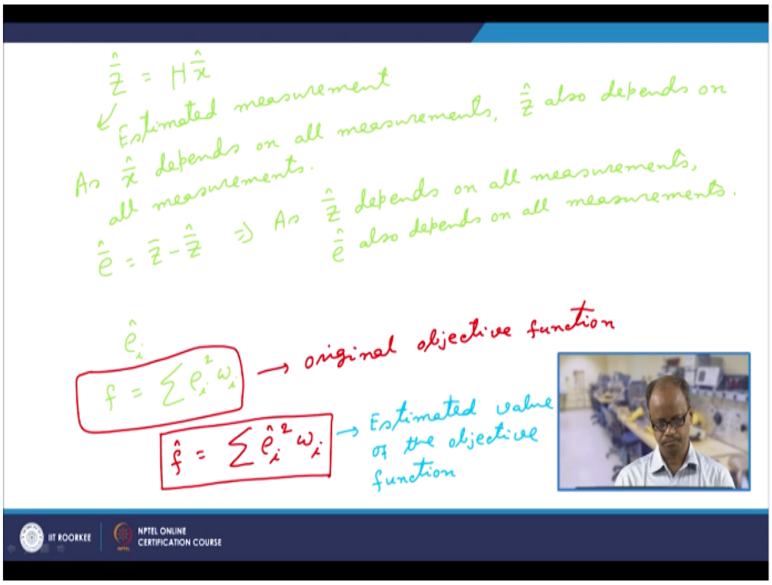

Now with this x hat we find out that this then we calculate z hat matrix and that sorry z hat vector that is nothing but H*x bar hat. So this is the estimated measurement. Now we observe that as x bar depends on all measurements z bar also depends on all measurements and then subsequently we calculate the estimated error as z-z hat although this vector is independent, this particular meters are independent.

But then this is because this depends on all measurements, so then therefore as z hat depends now again as z hat depends on all measurements e hat also depends on all measurements and that because e hat depends on all measurements so then therefore any value of ei hat says is actually contributed by all meters and as a result all estimated errors are actually contributed by all meters and then therefore none of the estimated errors is actually independent of each other, they are all dependent on each other.

So this is the basic reason. We have started with our goal that we wish to essentially minimize the function some function f and so we actually tried to minimize the function that it is ei square wi. So this is what we want to minimize. So this was our original objective function. Now after we do all our analysis and after we obtain our estimated states and after that also we obtained our estimated errors and etc.

You would be definitely would be like to know well after we have done all these exercise what is the value of f, that is although we done everything to minimize this value of f but we also would like to know what is the numerical value of f, I mean is this value 0, 1, 2, 3, 5, 500 or what. Now the question is theoretically this value of f is actually dependent on the true error of the meters which is not known to us.

So then therefore the only option for us to calculate the value of f is to substitute here the estimated value of the errors because we do not know that what is the actual or the true error so then therefore because we wish to calculate the value of f we really cannot calculate the value of true value of f.

But rather we can only calculate the value of f with the value of the estimated errors and so then therefore because we would be able to calculate the value of f with the value of the estimated error so then therefore we can only estimate the value of the objective function. So then therefore we say that we only are able to calculate, estimate the value of the objective function and that would be given by ei hat square*wi, so this is the estimated value of the objective function.

So we write that this is the estimated value of the objective function. So this is the estimated value of the objective function. Now if this is the estimated value of the objective function, now what is its nature, would this value would be a completely deterministic value or would be a completely random value? So that is the question. Now we here need to understand that this value of the errors are all actually random.

So then therefore the value of the estimated states which would be calculating at each calculation we also cannot predict so then therefore they would also be random in nature. If this value of the estimated states will be also random in nature, when you say random that means we would not be able to really predict that what would be their value, so in a sense they would be random in nature.

So then therefore when we calculate the value of estimated value of the measurements utilizing those estimated value of states so then therefore this vector z hat would be also random in nature that means that we also cannot predict their value. So then therefore e hat vector that is the estimated error vector will also be random in nature so then as a result f hat will also be random.

So then therefore I mean when you say it is random it means that if we do the state estimation of a given circuit repeatedly say after 5 minutes and then we get these values of f hat so then these values would not be deterministic at all that means that we really cannot predict them by any particular formula or equation or anything that they actually random in nature and because they would be essentially random in nature so then therefore it is quite probable that they would probably have some kind of probability distribution function.

So then therefore because f hat is a random value that means if we calculate the value of f hat for a large number of state estimation exercises, they will not follow any particular equation but they would be random in nature so as a result they would definitely follow some certain probability distribution function.

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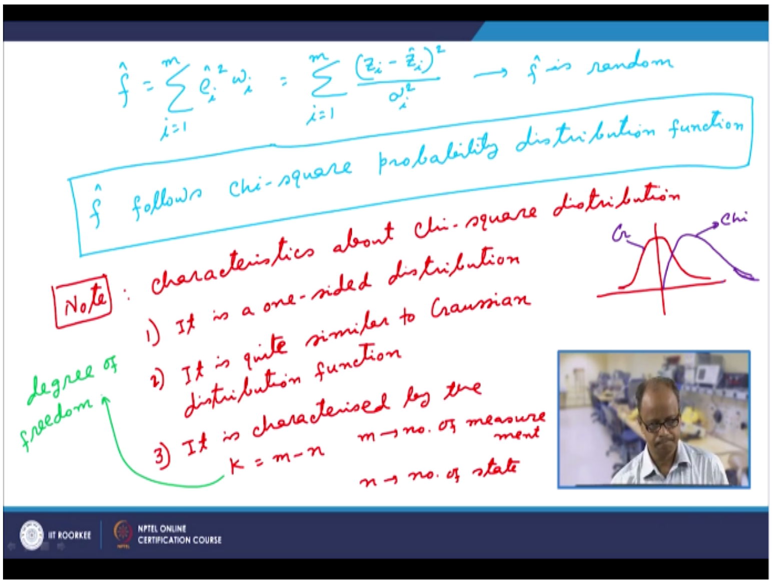

So now we write f hat is actually ei hat square wi so we can write it as now what is this i is=1 to m so this is i is=1 to m so it is zi-zi hat square/omega is sigma i square and this is random and f hat is actually random in nature is random. Because it is random so then therefore it must follow, it should follow some certain probability distribution.

Now if we do this exercise for a very large number of times and we plot and we calculate the probability distribution we find I mean it has been found out that f hat follows a chi-square, it is called chi-square or chi-square, chi-square probability distribution function, f hat follows so this is a very important extremely important fact we need to remember this that f hat follow something called chi-square probability distribution function.

So now the question is what is this chi-square probability distribution function. So let us look at that. So chi-square probability distribution function is given by this.

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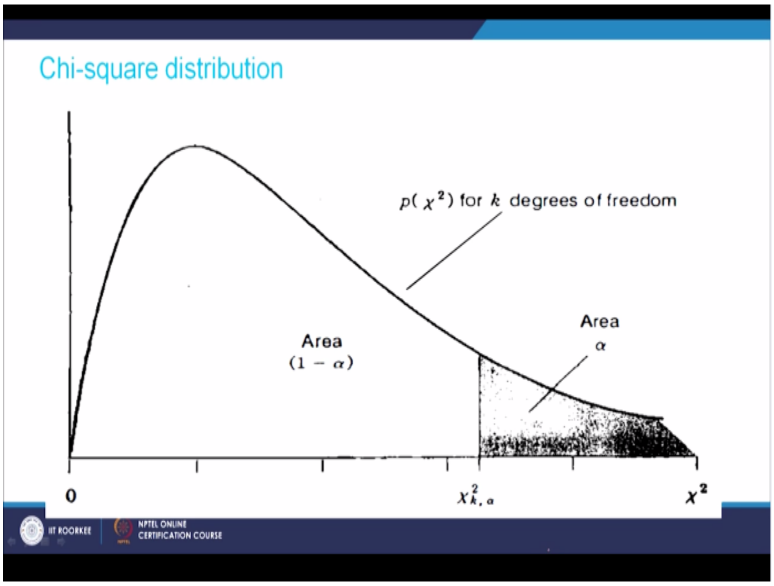

So this is the chi-square probability distribution function, so what are the characteristics of the chi-square probability distribution function. There are several things you can observe. First of all, it is one-sided distribution function; it does not go at the up I mean negative side so this is a one-sided distribution function. So in contrast with the Gaussian distribution function please note that in contrast with the Gaussian distribution function.

So we first we note so we take some note here so note so there are some characteristics note some characteristics, characteristics about we need to note chi-square distribution. First is that it is a one-sided distribution that is the first observation we make, it is not double-sided. What you mean by double-sided? For example, if you look at the Gaussian distribution function, it is something like this.

So it is basically a double-sided distribution function but in contrast chi-square distribution is actually something like this, so chi-square distribution is actually something like this. So this is basically Gaussian, this is Gaussian, this is chi-square so you see in contrast to this Gaussian distribution function, chi-square distribution function is a one-sided function.

Second observation we make is that it is actually quite similar to the Gaussian distribution function, so it is quite similar to although it is one-sided but it is quite similar to Gaussian distribution function. Third thing we must mention it here, third thing is it is characterized by a quantity called a k it is called the order and this k is given my m-n. What is m? m is number of measurement and n is number of states.

So it is actually characterized by something some quantity called k which is basically it is nothing but m-n. So what does it mean? It means that see this particular I mean this particular k is called actually degrees of freedom. So this k is called is basically it is a degrees of freedom. So this k is called degree of freedom. What does it mean? That means for different value of k, we will have different chi-square distribution curve.

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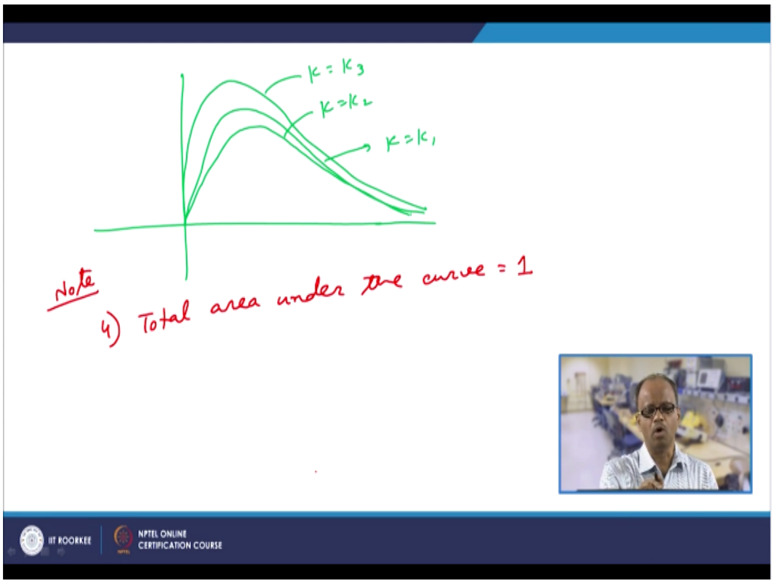

So then therefore suppose for a particular value of k I do have, this is my chi-square distribution curve, so let us say that this is for k is=k1 and for let us say some other value of k is=k2 we will have something like this, so k is=k2 and for some other value of k is=k3 we will have something like this. So for different value of degree of freedom we will have different graph or rather different probability distribution.

So here it is basically shown this graph for this particular probability distribution for a particular degree of freedom which is k. Now obviously the total area under the curve would be 1 so this also we must note so the fourth note would be that the total area under the curve, note so it is the fourth, total area under the curve is=1 because after all it is a probability distribution function, so then therefore total area under the curve would be 1.

Now if this is total area under the curve is 1 so now here as it is shown this is actually split into two parts, one has got an area is=alpha and so then therefore the rest of them is and then therefore this is basically the value of 1-alpha. Now what this alpha and 1-alpha denote? This value of area alpha says that I mean if this value of alpha is let us say 0.01 suppose if this value of alpha is=0.01 so then therefore this value would be 0.99.

So it says that there is a probability of 0.01 such that the calculated value of f hat or rather the estimated value of f or rather the calculated value of f hat will cross the threshold value of this. I mean this is called the threshold value so then essentially there is something called a table, so then therefore for a particular degree of freedom k and for a particular value of alpha there is a certain threshold value.

So then when we say that I have got an alpha so then it simply shows that or rather it simply says that there is 1% chance or rather the probability of f hat to be over or rather to be above this particular value or rather to be greater than this particular threshold value is 0.01 or other words we can also say that the probability of the value of f hat to be less than this threshold value is 0.99 or rather there is a 99% chance that the probability of f hat would be less than this particular threshold value.

So now the question is how do we get this particular threshold value and how do I get this particular value of alpha? So now let us look at that so then essentially there is a table.

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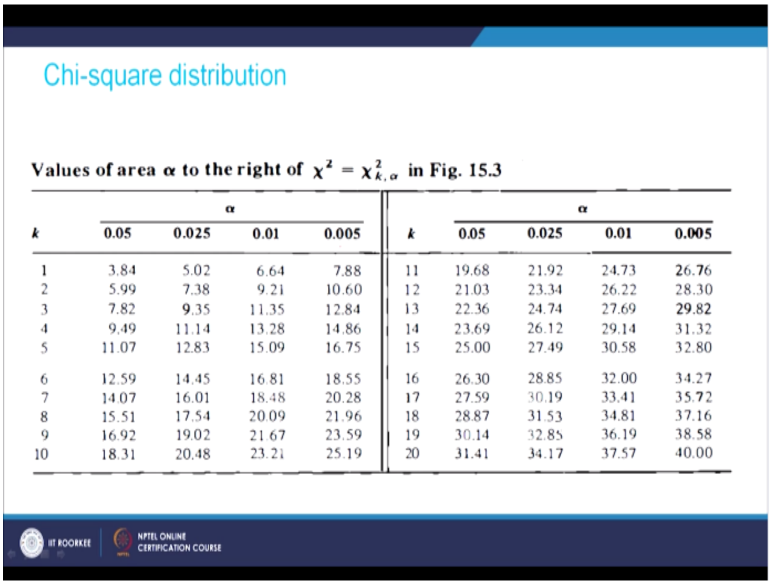

Values of area α to the right of $\chi^2 = \chi^2_{k,\alpha}$ in Fig. 15.3

k	α 0.05	0.025	0.01	0.005	k	α 0.05	0.025	0.01	0.005
1	3.84	5.02	6.64	7.88	11	19.68	21.92	24.73	26.76
2	5.99	7.38	9.21	10.60	12	21.03	23.34	26.22	28.30
3	7.82	9.35	11.35	12.84	13	22.36	24.74	27.69	29.82
4	9.49	11.14	13.28	14.86	14	23.69	26.12	29.14	31.32
5	11.07	12.83	15.09	16.75	15	25.00	27.49	30.58	32.80
6	12.59	14.45	16.81	18.55	16	26.30	28.85	32.00	34.27
7	14.07	16.01	18.48	20.28	17	27.59	30.19	33.41	35.72
8	15.51	17.54	20.09	21.96	18	28.87	31.53	34.81	37.16
9	16.92	19.02	21.67	23.59	19	30.14	32.85	36.19	38.58
10	18.31	20.48	23.21	25.19	20	31.41	34.17	37.57	40.00

So this table is like this. So this table is actually like this and this table shows that there is a value of k. So this is nothing but the freedom I mean these values of k denote nothing but the degree of freedom and this is the value of alpha and these are nothing but the probability that the value of f hat would cross would be this.

So now for example if my degree of freedom is let us say 2 and if my value of alpha is 0.01 so that means this is the corresponding threshold value, this is 9.21. So then therefore what does it mean? It means that when I calculate the value of f hat, there is only 1% chance that the value of f hat would cross the value this particular value 9.21. Similarly, there is only 0.5% chance or rather the probability of f hat to be greater than 10.6 is only 0.005 or in other words the probability of f hat to be less than 10.6 would be is actually 0.995.

Similarly, for any other value of k, for any other value of k and for any particular value of alpha, we do have different value of threshold quantities. So this is the value of k, this is the value of k and we do have this particular threshold values. Now utilizing these threshold values and these particular threshold value or rather this particular chi-square distribution is actually very useful to us. Now why and how this is useful?

At the start of our discussion, we said that the meters whatever we are using for state estimation purpose, they will certainly have errors. Now it may happen that some meters have developed some problem inside it, so then therefore the error with this meter has got exceptionally large, unacceptably large but we do not know it that which one has got. So then therefore if we still keep on doing our state estimation with that particular faulty meters so then whatever estimated values will get would be all very wrong.

So then therefore we need to devise a method, we need to have a method such that we can find out that which particular meter has gone bad or rather we say that we need to find out that which particular measurement value has gone bad, bad means that it is now containing very unacceptable value of error. So then therefore, we need to identify that which measurement data has bad or in other words in the parlance of state estimation we say that we need to devise a method for bad data detection.

That means we need to detect that which particular measurement data has gone bad, so we need to do something called bad data detection such that we can eliminate that particular meter which has gone bad and so as to make our estimated values acceptable. So in the next lecture, we would be looking into this aspect. Thank you.