Computer Aided Power System Analysis

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Lecture - 42
Error Analysis

Hello friends. Welcome to this lecture on the course of computer aided power system analysis. In the last lecture, we have looked into some examples of this weighted least square estimation technique and from that example, we have said that it is very difficult for us to recognize that which set of estimated values to be accepted.

Because as we already said that in the case of state estimation, what we would be doing that we would be taking the measurements at a regular interval and then with those measurements taken at regular intervals would be also estimating the states of the network or for the system at regular interval. So then therefore just by looking at those values, we would not be able to recognize that which particular estimated values are really good enough for us.

So then we said that we need to do some kind of error analysis or rather we need to do some kind of statistical analysis on these estimated values as well as the on these estimated errors so then we would be able to find out that which set of measured values are really acceptable to us. So then therefore today we would be looking at some aspect of this error analysis.

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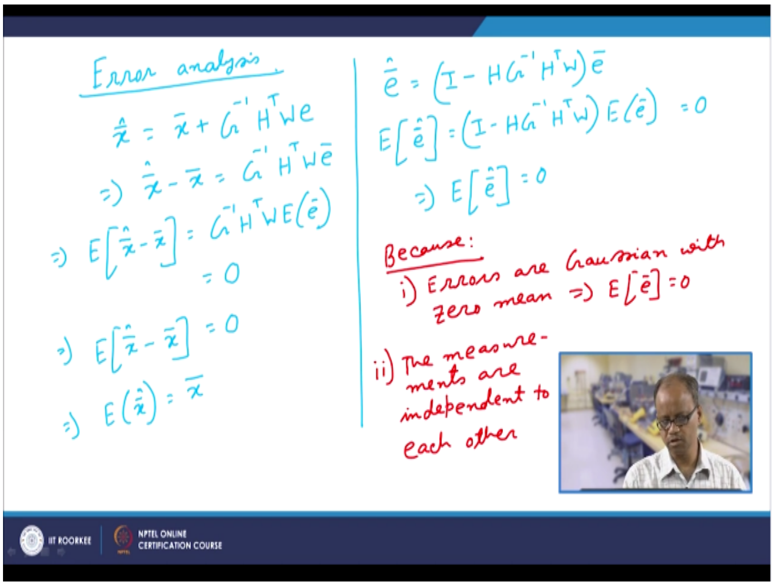

Now we recall that our estimated values x bar hat is given by x bar+G inverse H transpose W*e that we have already seen. So then therefore we said that x bar-x hat bar is=G inverse H transpose W*e, it is the error vector. Now if we take the expectation of x bar-x so what we get is that we get that G inverse H transpose w of expectation of e and that we have said is=0. So then therefore E of x hat bar is=0 so then therefore E of x hat bar is=x.

So this we have already seen, so we have said that this expected values of this estimated values you always converge to the true values. We also have seen that estimated errors is given by identity matrix-H G inverse H transpose W*e that you have already seen and therefore we have said that expected values of the estimated errors would be I-H G inverse H transpose*w*expected error right.

So that would be 0. So on other words we have taken that is equal to 0. Now we have done up to this and why have you done this, because we assume we have our basic assumption which is a true assumption to a large extent because this is due to the fact that errors are Gaussian with zero mean. Zero mean means that expected value of errors would be is =0.

And we have already explained what is mean by this expected value of I mean what is mean by this expected I mean why this particular zero mean is assumed that we have also explained on some earlier class and we also said that this measurements are independent of each other so this issue today we need to look at in more detail measurements are independent to each other.

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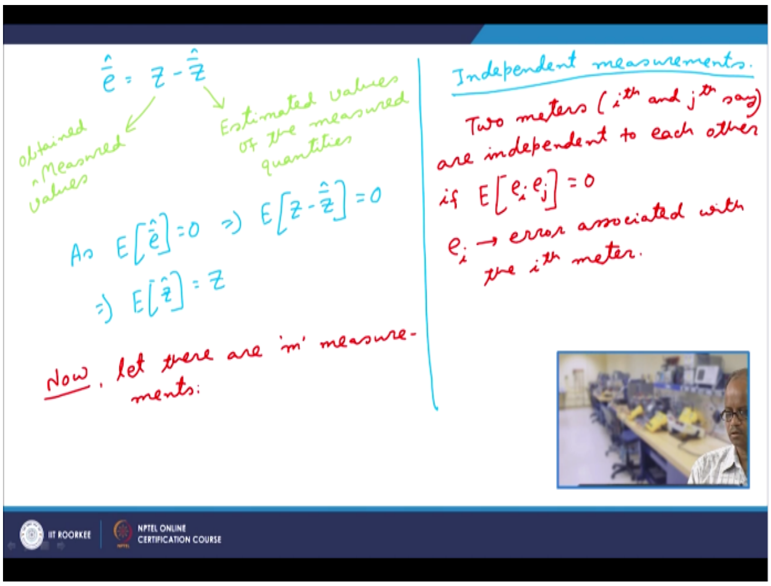

Now before we look into this issue in more detail, we need to recollect one very small thing. We also know that these are expected or rather estimated value of error is actually given by z-z hat. So this is the actual true measurements and this is the estimated measurements. So we again say that this is the true measurement. So these are the true or rather the measured value, we should say it is obtained measured values, should say obtained measured values and this is the estimated values of the measured quantities.

So this is we already seen. Now as expected value of 0 so that means expected value of z-z hat is also 0 or in other words expected value of z hat is=z. Now let us look at the case of this independent in nature. Now what is mean by independent measurements? Independent measurement means now let us look this is an important issue, independent measurements. Now what is mean by independent measurements?

We say that two meters are independent to each other. We say that two meters say ith and jth say are independent to each other if the expected value of ei ej is=0 where ei is the error associated with ith meter. Now let us say that I have got now let there are m measurements. Now we would be doing some generalized discussion m measurements, m can be any value.

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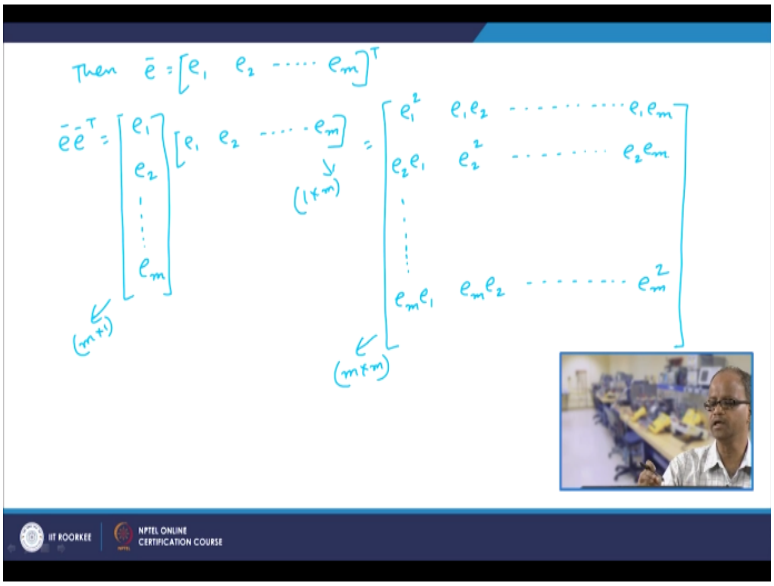

So then vector e would be e1 e2 em transpose, so when therefore e e transpose would be what? It would be e1 e2 up to em and e transpose would be e1 e2 up to em. So then therefore this would be with something like this that is e1 square then e1 e2 then dot dot dot e1 em. Then, you have got e2 e1 e2 square then dot dot dot e2 em. Then, what we have is em e1 em e2 dot dot dot em square.

So this would be the matrix, so this would be an m x m matrix, this is m x 1 and this is 1 x m. So this is an m x m matrix. Now if I do take the expectation operator at both the sides so then what I get?

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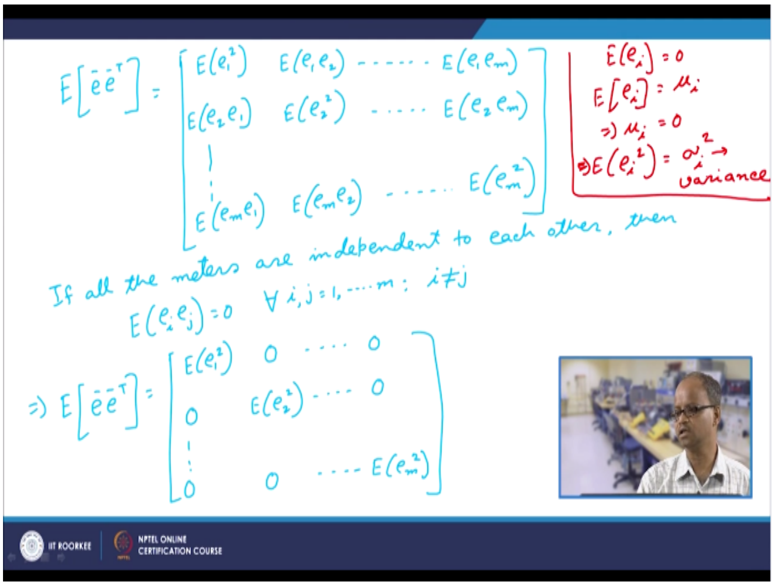

Now if I take the expectation operator at both the side so what I get, I get something like very interesting, so I take expectation operator and expectation of a matrix is nothing but now is essentially the elements of the expected value of a matrix is nothing but the expected value of the individual element. So then it would be simply expected value of e1 square then expected value of e1 e2 and then expected value of e1 em.

Then, similarly it would be expected value of e2 e1, expected value of e2 square and similarly expected value of e2 em. Then, it would be expected value of em e1, it would be expected value of em e2 and expected value of em square. This will be the case. Now we have said that all the meters are independent of each other. So then therefore if all the meters are independent to each other, then what we have from this basic definition?

Expected value of ei ej is=0 that is for all i, j is=1 to m within either that i is not equal to j so in plain English what does it mean that all these off diagonals would be 0. So e1 e2 would be 0, e2 e1 would be 0, so then all off diagonals would be 0, so then therefore expected value of would be essentially expected value of e1 square then 0 all are 0 then 0 expected value of e2 square then it is 0 then all are 0 and this is expected value of em square.

So it would be a diagonal matrix. Now what is mean by expected value of e1 square? If I do a simple analysis, all expected values of ei is=0 so then therefore if I say that ei if I say that expected value of ei if I denote as mu i so then therefore mu i is=0. So then if this mean is 0 so then therefore ei square is nothing but the standard deviation or rather this variance. It is nothing but the variance right.

So if this mu i is=0, so then therefore this expected value of ei square is nothing but the variance.

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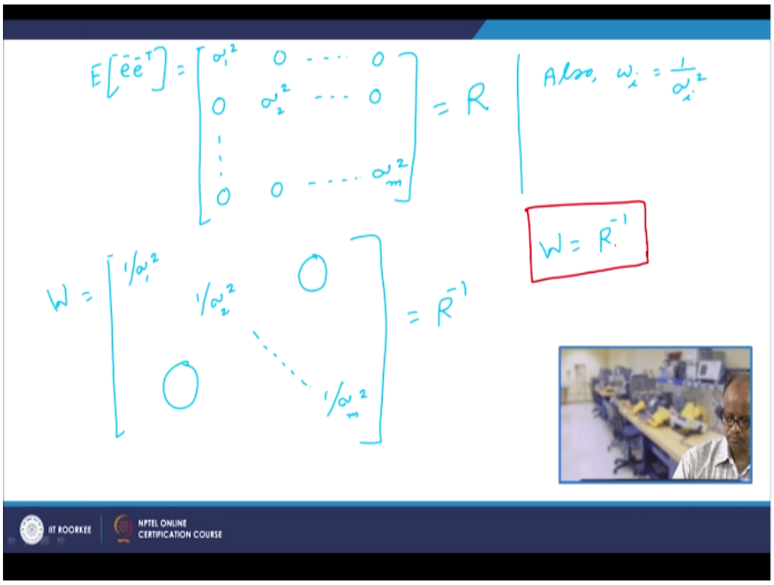

So then therefore what happens is expected value of e e transpose is nothing but variance 0 0 0 0 0 0 sigma m square right. We call it matrix R, so this is the matrix R which is the basically the, it is basically diagonal matrix in which all diagonal elements are nothing but the variance of individual meter and also we said omega i that is the 1/sigma i square.

So then therefore my weight matrix is nothing but 1/omega sorry sigma 1 square 1/sigma 2 square dot dot dot 1/sigma m square and all off diagonal terms are 0. So then this and this is essentially R inverse. So it is a very beautiful relation that my weight matrix is nothing but the inverse of this. So weight matrix is nothing but the inverse of the variance matrix of all the meters. Of course, this is only true when the meters are assumed to be independent to each other.

Yes, usually these are true because usually these are true but not always but here we are for the sake of simplification we are assuming that these meters are independent to each other. So then therefore w is=R inverse. Now we do something very interesting.

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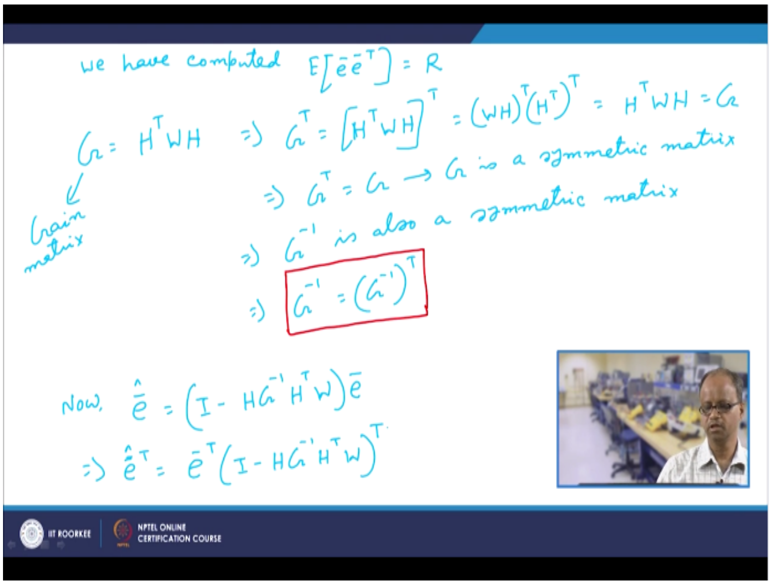

Now so far we have done we have computed expectation of e e transpose and that we have seen that this is nothing R. Now we also have the estimated errors. So now let us look at that what would be the expected value of this estimated error. Now before I look that I mean before we actually discuss it let us first understand that what is the nature of this G matrix. G matrix is that is the gain matrix is given by H transpose WH, so this is the gain matrix.

H matrix we have already seen and please note that this particular W matrix is nothing but a diagonal matrix. Now if I take G transpose what I get H transpose WH whole of transpose so we get WH transpose*H transpose of transpose. So therefore H transpose WH. So then therefore this is G so then therefore G transpose is=G so G is a symmetric matrix. Because G is a symmetric matrix, so then therefore G inverse is also a symmetric matrix.

Because inverse of a symmetric matrix would be also a symmetric matrix, so then therefore G inverse would be is=G inverse T. So this is a very important relation which we will utilize. Now what is we have already seen that expected value of errors is given by I-HG inverse H transpose W*e. So then therefore what is e transpose? It would be e transpose*I-HG inverse H transpose W*T right. So this would be the case.

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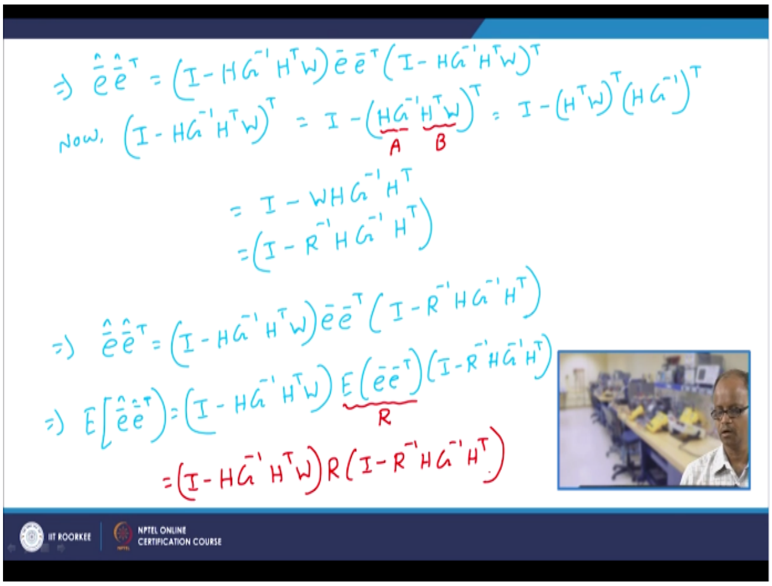

So then therefore we would now like to find out so then therefore what is e e transpose, so it is I-HG inverse H transpose W, so what is this HG inverse H transpose W*e and e transpose is e hat sorry e transpose*I-HG inverse H transpose W*T. Now what we have? So now so this is the expression and we have to do some simple maths on it. Now let us look that now what is HG inverse H transpose WT.

This is I transpose, I transpose is I so-HG inverse H transpose WT right so let us say this is A. Now if I say that it is A, if I say that is A and if I say that is B so it is AB transpose, it is B transpose A transpose. So then therefore it would be and therefore I-H transpose WT*HG inverse. So then therefore so which would be equal to I-W is a diagonal matrix so it is WH then it is G inverse*H transpose and W is I-R inverse HG inverse H transpose right.

So this is the case. So then therefore I can say so this is the case. So then therefore I can say that this into this is actually I-HG inverse H transpose W*e*e transpose*I-R inverse HG inverse H transpose. If I take the expected value of both the sides, what I get? If I take the expected value at both the sides, I get is I-HG inverse H transpose W. Then, I get because this is a constant matrix, then I get expected value*I-R inverse HG inverse H transpose.

This quantity we have just now shown that this quantity is R, so then therefore I-HG inverse H transpose W*R*I-R inverse HG inverse H transpose.

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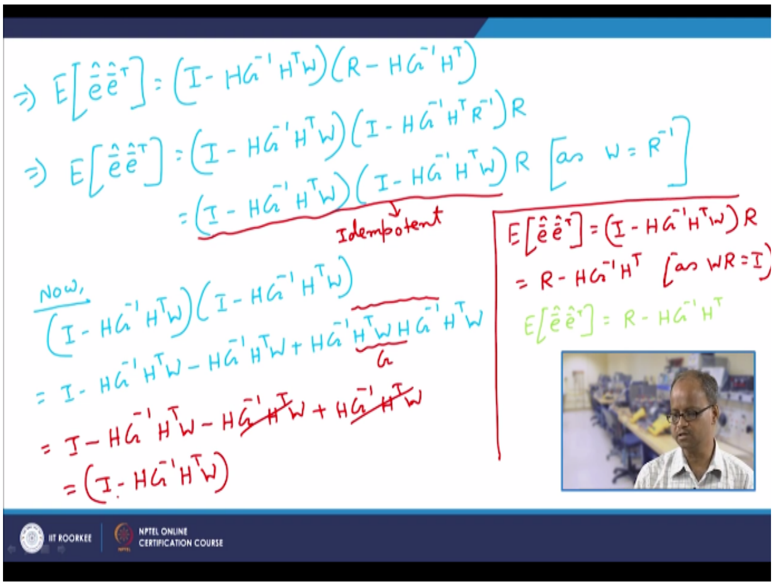

So then therefore what I have got, I have got is expected value of e hat and e hat transpose is=I-HG inverse H transpose W and now what I do, I multiply take this product of R with this so R*I is R-R*R inverse is identity so then this goes HG inverse H transpose, so this is HG inverse H transpose right. So then therefore expected value is now I-HG inverse H transpose W. Now I take R common from both the sides, so I-HG inverse H transpose R inverse*R.

So then therefore it is I-HG inverse H transpose W*I-HG inverse H transpose W*R as we have just now proved that W is=R inverse. So then therefore now if I do multiply everything so what I get? Now what is this quantity? This is an idempotent matrix. Now I-HG inverse H transpose W, now what is this quantity I-HG inverse H transpose W that is=I-HG inverse H transpose W this into this-this into this HG inverse H transpose W+HG inverse H transpose W that is this*this in the same order HG inverse H transpose W.

So I have just put this and this in the same order. Now it is very interesting thing to look at that this quantity is nothing but G. So then therefore G*G inverse is=identity so then therefore it becomes I-HG inverse H transpose W-HG inverse H transpose W+HG inverse, this is G*G inverse is I so this quantity so then this term cancels out, so this terms cancels out H transpose W.

So this and this cancels out so this becomes I-HG inverse H transpose W. So then therefore we have so then what we have expected value of e hat transpose is actually I-HG inverse H transpose W*R. Now because this into this is the same matrix, you see this matrix into this matrix is actually same matrix so this is called an idempotent matrix. So this is=R-HG inverse H transpose as W*R is=identity.

So this is a very important relation, so what we ultimately found out that we have very beautiful relation that expected value of this error is=R-HG inverse H transpose. So then basically expected value of this estimated errors are not exactly equal to R but it actually differs from the original matrix R. So then therefore this matrix plays a major role in our statistical analysis and these issues we would be looking at in more detail in the subsequent lectures. Thank you.