Computer Aided Power System Analysis
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Module No # 09
Lecture No # 41
WLS Example

Hello friends, welcome to this lecture on computer aided power system analysis. We have been discussing the basic procedure of weighted least square estimation method. We have already discussed that what are the basic techniques of this and we have also looked into the calculation methods. So today in this lecture we will now start looking at 1 simple example to illustrate the calculation procedure with the numerical example.

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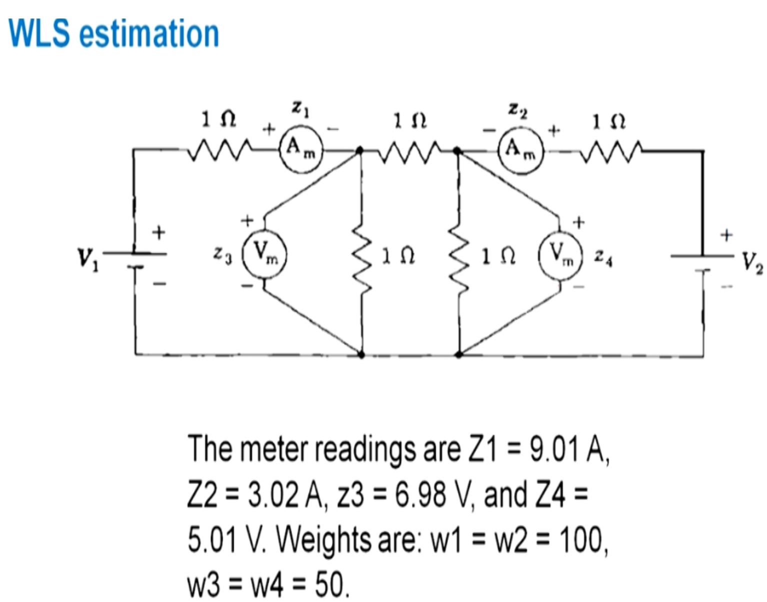

So what you have is let us say that we do have an circuit. And this is circuit so everything here is basically 1 ohm and we have got 2 voltage sources V1 and V2. And we have to put 4 meters there are 2 ammeters Z1 and Z2 so I mean this is what actually first measurement. This is our second measurement and there are also 2 volt meters this is Z3, this is Z4. So essentially what we have trying to measure is that we are simply trying to measure the current flow through this path.

Current flow in this branch voltage across this branch are essentially the voltage at this point because i mean this point is taken as the reference point. And we are also measuring the voltage at this node with respect to measurement. And this meter readings are Z1 is 9.01 ampere, Z2 is 3.02 ampere, Z3 is 6.98 volt and Z4 is 5.01 volt.

Now you see that all these values are not exactly an integer values because as an example here. Probably the actual value is 9 ampere but, then here because of this meter errors this value is being actually read as 9.01 ampere. Similarly probably the true value of this current is actually 3 ampere but here because of this meter error it being registered as 3.02 ampere. Similarly probably the actual or rather the true value at of the voltage at this node is 7 volt but again because of the error it is being read as 6.98 volt.

And similarly probably the actual value or rather the true value of the voltage at this node is 5 volt but again because of the meter error it is being registered as 5 volt. Now, we have also discussed in the last class that we do take the weights as the inverse of the variance so then therefore so then essential accordingly, these 2 ammeters are essentially found to be more accurate.

So then therefore their words are taken be 100. This is also 100, this is also 100 and then these 2 meters are not that accurate, so they are little more inaccurate. So then therefore their words are taken as 50. Now our unknown state vectors are vector or rather essentially or unknown quantities which we try to estimate from this 4 measurement. This is V1 and V2 so then therefore the first thing we have to do is to express these measurements. With respect to the or rather we have to express these measurements in terms of this unknown voltages or rather in terms of this unknown quantities. So here, what we do is so as we said that we do express the unknown quantities.

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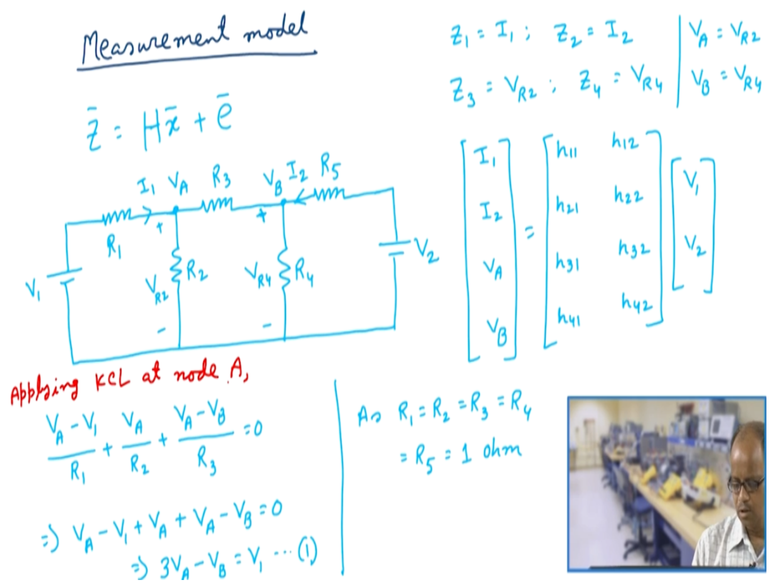

So then, we do have our measurement model is measurement model we have seen is essentially that we have seen that measurement model is Z=Hx + e. So now here, to start our state estimation algorithm, we have to know what is this matrix h. Now this matrix h can be computed from the circuit analysis of the given circuit. So let us again draw it know I mean how it is done.

So then essentially what we have to do? We have to essentially express all these measured values in terms of these 2 unknown voltage V1 and V2. Which can be done only by through appropriate circuit analysis? So now, I mean let us do that so our circuit is something like this and then I think it is so our circuit is like this. So this is V1, this is V2 and this is R1, R2, R3, R4, R5 and here we are measuring I1 so this is my Z1 we are measuring I2 so this my Z2. And we are measuring here VR2 and we are also measuring here V R4, V across R4.

So, in our measurement model Z1 = I1, Z2 = I2. Please note that, here we are talking about dc circuit. So then therefore there is nothing called these absolute magnitude or angle. So this all everything would be just numerical values Z3 is VR2. That is the voltage across this resistance are 2 and Z4 isVR4 so that is what is on measurement. So then essentially you have to find out that what would be the expressions of I1, I2.

Now let us say this point I say VA, let us say this point is B. So let us say this particular voltage is VB. So then therefore, we say that VA, VB. We have to now find out this matrix is essentially h11, h12, h21, h22, h31, h32, h41, h42 into V1, V2. So, we have to now express I1 in terms of V1 and V2. So, we have to now essentially find out these coefficients h11, h22 and all these 8 coefficients from the circuit analysis.

And here of course, we note that V, VA= VR2 and VB= VR4 that we note. So now to do this how to do this so now, we have to do our standard circuit analysis so at this point and at this points we apply this nodal equations. So if I do apply this nodal equations, what I get is that VA-V1 / R1+VA / R2+VA-VB / R3=0 am just applying KCL at this point. Now here, if you note, if you note that all these resistances R1, R2, R3, R4 and R5 are =1 ohm.

So then therefore, because all R as because all resistances are 1 ohm. So then therefore, you can write down that VA-V1+VA+VA-VB=0. So this equation turns out to be 3VA-VB=V1. So, this is equation 1 these we are doing as R1=R2=R3=R4=R5=1 ohm. So, this is our first equation now, we apply KCL at this second point so applying KCL, so this will write that here applying KCL at node A we get this.

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Applying KCL at node B

$$\frac{V_B - V_2}{R_5} + \frac{V_B}{R_4} + \frac{V_B - V_A}{R_3} = 0$$

$$\Rightarrow V_B - V_2 + V_B + V_B - V_A = 0$$

$$\Rightarrow 3V_B - V_A = V_2 \quad \text{---(2)} \quad \Rightarrow 9V_B - 3V_A = 3V_2 \quad \text{---(3)}$$

$$3V_A - V_B = V_1 \quad \text{---(1)}$$

$$(3) + (1) \Rightarrow 9V_B - \cancel{3V_A} + \cancel{3V_A} - V_B = 3V_2 + V_1$$

$$\Rightarrow V_B = \frac{3V_2 + V_1}{8} = 0.125V_1 + 0.375V_2 \quad \text{---(4)}$$

$$\text{From (1)} \quad 3V_A = V_B + V_1 = \frac{3V_2 + V_1}{8} + V_1 = \frac{9V_1 + 3V_2}{8}$$

$$\Rightarrow V_A = \frac{3V_1 + V_2}{8}$$

Now, we applying again now applying KCL at node B. What we get? We get is, if we look at this VB-V2 /R5. So we get VB-V2/R5+VB/R4+VB-VA/R3=0. So therefore, VB-V2+VB+VB-VA=0 or in other words 3VB-VA=V2. So, this is equation 2. So I have got equation 2 is this and we equation is 3VA-VB=V1 that was equation 1. From this, so what we do is from equation 2 I get 9VB-3VA=3V2.

So let us say this is equation 3. So 3+1 I am doing that is, 9VB-3 VA+3VA-VB=3V2+V1. So then therefore, this and this got cut 8VB = 3V2+V1/8. So then therefore, this is sorry and there is little mistake so then therefore, so I can simply VB= this. So that I can say that it is 0.125 V1+3/8 would be 0.375 V2 so this is the first equation. So this is equation 4 now, because I have got this VB so now, we have to now calculate VA so from equation 1. So 3VA=VB+V1. So VB is 3V2+V1 / 8+V1. That means 9V1+3V 2 / 8 or in other words VA=3V1+V2 / 8.

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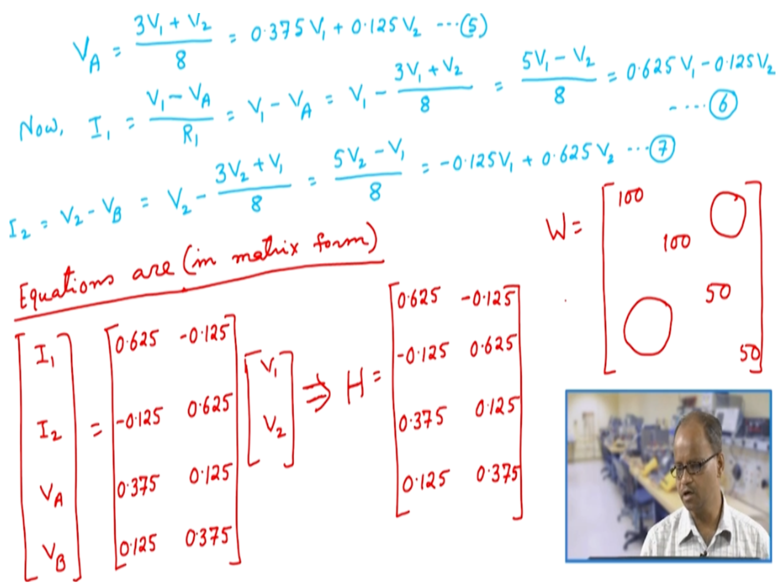

VA=3 V1+V2 so 3V1+V2 / 8 so it is 0.375 V1+0.125 V2 so this is equation 5. So, I have got these 2 voltages V and V B so after that, we have to calculate what is I1. So now I1 is nothing but now V1-VA/R1. So, this is V1-VA so this is V1-VA is 3V1+V2/8. So, it is 5 V1-V2/8. So it will be 0.625 V1-0.125 V2 so this is equation 6.

And similarly I2 is V2-VB and V2 is and VB is what is VB? VB is 3 V2+V1/8 so 3V2+V 1 / 8. So, it is 5 V2-V1/8 so it is -0.125 V1+0.625 V2. So, have got all the expressions of this measurement values so now, equations are so then therefore equations are now let us look at that what was our equations are? I1, I2, V1, V2, so I1, I2, VA, VB. So then therefore, so then in matrix form so have got I1, I2, VA, VB.

There are something we will fill up this blanks and this would be V1, V2. And so now, first thing I 1 is 0.625 so this is 0.625 V1, -0.125 V2, -0.125 V2, I2 is -0.125 V1 and +0.625 5 V2 so +0.625 VA,VA is 0.375 V 1, 0.375 V1, +0.125, +0.125 and VB. VB is 0.125 and 0.375 so, it is 0.125, 0.375 so, this is the measurement model. So these are all measured quantities and this is the unknown x vector.

So then therefore, this is h matrix. So then therefore, we can simply say that my h matrix is given by so. This is my h matrix and this h matrix is given by 0.625, -0.125, -0.125, 0.625, 0.375, 0.125, 0.125, 0.375. So, this is my h matrix so then therefore we can see that is h matrix always evaluated on the basis of the circuit analysis. So then therefore, h matrix is nothing but the relation between the true value of the measured quantities and the true values of the unknown quantities. And that can only be obtained by the circuit analysis of the system. So this is my h matrix so now with this h matrix we will do our calculation.

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WLS estimation (contd..)

$$\mathbf{H} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$$

$$\mathbf{H}^T\mathbf{W} = \begin{bmatrix} 0.625 & -0.125 & 0.375 & 0.125 \\ -0.125 & 0.625 & 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 100 & \cdot & \cdot & \cdot \\ \cdot & 100 & \cdot & \cdot \\ \cdot & \cdot & 50 & \cdot \\ \cdot & \cdot & \cdot & 50 \end{bmatrix}$$

$$= \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix}$$

So now this is our H matrix so our H matric is we have already seen 0.625, -0.125, -0.125, 0.625, 0.375, 0.125 and 0.375 and 0.125 and 0.375. And our W matrix is nothing but our W matrix that is weight matrix is nothing but, we will decide that it is a diagonal matrix. So, it is a 4 cross 4 matrix. So it is 100, 100, 50 and 50 and these out diagonal quantities are 0. These out diagonal quantities are 0. So then, so this is my W matrix so 100, 100, 50, 50.

So then therefore, we have got this H matrix so then we do H transposing into W. H transpose so then this is the H transpose into W so we get this. So H transpose W is given by this after I do this H transpose W. Then we calculate this G matrix as this is the game matrix. G matrix is nothing but the gain matrix so it is H transpose W into H. So this is H transpose W we have already got H 62.50 and minus so this quantity is H transpose W. So this is H transpose W and into H.

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WLS estimation (contd..)

$$G = H^T W H = \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$$

$$= \begin{bmatrix} 48.4375 & -10.9375 \\ -10.9375 & 48.4375 \end{bmatrix}$$

So then, it is this so it a 2 cross 2 matrix now we have already argued and we have already shown theoretically in that. If there are I mean if there are n number of values which are to be estimated, if there are n number of values which are to be estimated. So that, in that case the dimension of the gain matrix should be allows n cross n respective of the number of the measurement used.

Now, in this case, in this particular circuit n= 2 because, our unknown quantities are V1 and V2. So then therefore n=2 and because n=2 so then therefore, our g matrix would be 2 cross 2 matrix. So then it is a this 2 cross 2 matrix. One thing we need to understand that it is a symmetric matrix now how do I prove that this g matrix is symmetric matrix? We will show it but then we can show it very easily that I mean G= H transpose into W into H. And if I do take it transpose, we will ultimately get back this. So then therefore, G is a symmetric matrix and it is indeed a symmetric matrix we can see here, so G is a symmetric matrix.

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WLS estimation (contd..)

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{z}$$

$$= \begin{bmatrix} 48.4375 & -10.9375 \\ -10.9375 & 48.4375 \end{bmatrix}^{-1} \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \mathbf{z}$$

$$= \begin{bmatrix} 0.0218 & 0.0049 \\ 0.0049 & 0.0218 \end{bmatrix} \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1.2982 & 0.0351 & 0.4386 & 0.2281 \\ 0.0351 & 1.2982 & 0.2281 & 0.4386 \end{bmatrix} \begin{bmatrix} 9.01\text{ A} \\ 3.02\text{ A} \\ 6.98\text{ V} \\ 5.01\text{ V} \end{bmatrix} = \begin{bmatrix} 16.0072\text{ V} \\ 8.0261\text{ V} \end{bmatrix}$$

So then, now our estimated values x1 hat and x2 hat, we have said that always we do always we denote our estimated values with a hat symbol. So it would be G inverse H transpose into W into Z. So then I mean this is the G inverse so this is G that inverse H transpose W we have already calculated so this is H transpose W and this Z. So this G inverse is this then it is H transpose W and Z is a vector.

It is basically nothing but the measurement vector and that is nothing but Z1, Z2, Z3, Z4. So this into this now this is a 2 cross 2 matrix. And this is a 2 cross 4 matrix. So then therefore, this whole quantity would be 2 cross 4 matrix so then I have got it 2 cross 4 matrix. So this into this is nothing but 2 cross 4 matrix and this 2 cross 4 matrix I got. So this into this and then we substitute the values of Z1, Z2, Z3, Z4.

So we please recall that our values are 9.01, 3.02, 6.98 and 5.01 so then, if I do this I get this estimated values so these are the estimated values. From this estimated values we can possibly in far that well then probably the true values of this V1 and V2. It would be somewhere probably, this I mean V1 would be 16 and V2 would be 8 but well.

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WLS estimation (contd..)

$$\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \\ \hat{z}_4 \end{bmatrix} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 16.0072 \text{ V} \\ 8.0261 \text{ V} \end{bmatrix} = \begin{bmatrix} 9.00123 \text{ A} \\ 3.01544 \text{ A} \\ 7.00596 \text{ V} \\ 5.01070 \text{ V} \end{bmatrix}$$

$$\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} 9.01 \\ 3.02 \\ 6.98 \\ 5.01 \end{bmatrix} - \begin{bmatrix} 9.00123 \\ 3.01544 \\ 7.00596 \\ 5.01070 \end{bmatrix} = \begin{bmatrix} 0.00877 \text{ A} \\ 0.00456 \text{ A} \\ -0.02596 \text{ V} \\ -0.00070 \text{ V} \end{bmatrix}$$

Now let us look at that what is the estimated measured value? What are that? What is the true value of the measurement? So now, what happens that if this is the estimated value of the x. So then therefore, I can also estimate now we are taking that as if that this estimated values are we are assuming as if that the estimated values of x and true values of x. So then therefore, if I substitute this true values of x into expression Z= Hx. So then I get the estimated values of Z.

So now this is the H matrix and this estimated values of x so that H into so this as is giving me this particular this is the Z hat matrix so this is the estimated value. So then therefore, error estimated errors are the values obtain by the meters actual value obtain by the meters that is 9.01, 3.02, 6.98 and 5.01 - the estimated values of the measurements. So this is this so you can see that is estimated errors are pretty small.

So we can possibly think is our estimation is reasonably correct. Now here, the point is that here this calculation only has been done for a particular set of measurements. Now please note that, if in this circuit even we take this measurements repeatedly, that is if I take this set of measurements right now and then if I take another set of measurements by utilizing the same meters and the same circuits after let us say 10 minutes. We may get simply some other values of the measurement values now suppose for example we take another set of measurements and then for Z4 that is that VB we get 4.40 instead of 5.01.

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WLS estimation (contd..)

- If Z4 = 4.40 V instead of 5.01 V (other measurements remain same)

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 1.2982 & 0.0351 & 0.4386 & 0.2281 \\ 0.0351 & 1.2982 & 0.2281 & 0.4386 \end{bmatrix} \begin{bmatrix} 9.01\ \text{A} \\ 3.02\ \text{A} \\ 6.98\ \text{V} \\ 4.40\ \text{V} \end{bmatrix}$$

$$= \begin{bmatrix} 15.86807\ \text{V} \\ 7.75860\ \text{V} \end{bmatrix}$$

Suppose let us say this is 4.40 instead of 5.01, this is something happens to the meter. So it is now giving me the reading of 4.40. Now until and I do have some other means to know that whether this meter has gone out of order or not. So therefore, you have to accept this values 4.40. As the actual I mean you have to accept this value as the correct value given by the meter. So if you use 4.40 and if we again do this same analysis you see so then I get this value. So, if you can see that earlier my estimated values are 16 and almost =16.00 and 8.0 but now my values are 15.86 and 7.75. So, these are now considerably changed and also if I do this estimated now compute estimated errors.

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$$\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} 9.01 \\ 3.02 \\ 6.98 \\ 4.40 \end{bmatrix} - \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 15.86807\ \text{V} \\ 7.75860\ \text{V} \end{bmatrix} = \begin{bmatrix} 0.06228\ \text{A} \\ 0.15439\ \text{A} \\ 0.05965\ \text{V} \\ -0.49298\ \text{V} \end{bmatrix}$$

Estimated errors are now much more increased. Now from this 2 sets of measured values our question is that which 1 should I accept? Is that any method for us to accept that? Well, this is the true one or let us say or let us say this particular set of values is true one. How do I decide upon this as such just by looking at this measurement values we do not have any idea that which 1 is should be more correct.

So then therefore, we have to do some kind of statistical analysis on these obtained values. Some kind of statistical analysis on this obtained values and then based on those statistical analysis. We have to decide that out of let us say these 2 which 1 should be the acceptable. In fact here, we have taken only 2 sets of measurement. But please note that, when we are doing this doing this exercise of state estimation.

This particular exercise of state estimation is actually kind of continuous process. That is, we do take the set of measurements after let us say 5 minutes or after let us say 10 minutes or whatever it is. So it keep on taking set of measurements at a regular interval and then at that same interval we also keep on estimating the states of the system that means we are simply trying to monitor that, what is happening to the system.

So then therefore, until and unless we do develop some statistical method and do apply that statistical method on a particular on any particular set of estimated values. We would be unable to decide whether these estimated values are really useful or not. So then therefore these questions are very important questions. In the scope of this state estimation analysis and we will start addressing this issue from the next lecture thank you.