Computer Aided Power System Analysis
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Module No # 08
Lecture No # 40
WLS (Contd.)

Welcome friends we have been discussing about this WLS technique so now, let us look at the other aspects of this method. So we are talking about WLS method.

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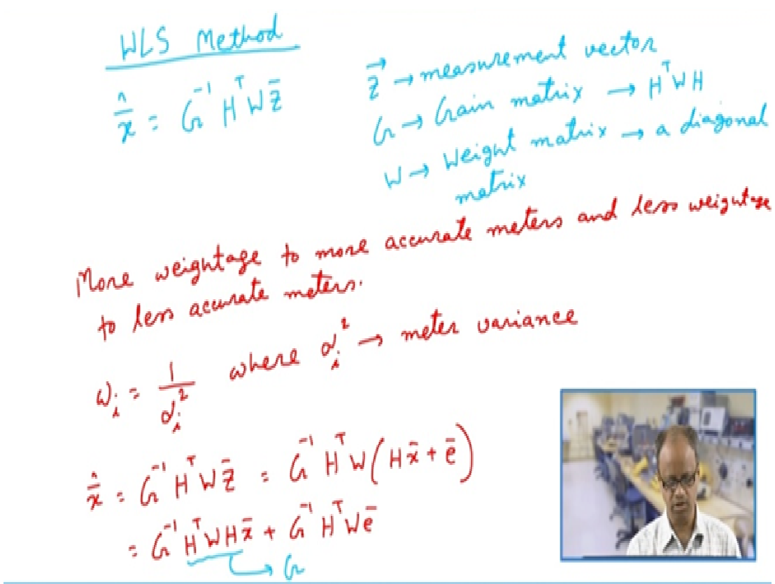

Now in the last lecture, we have derived that is x bar is G inverse H transpose W into Z. This we have already seen where Z is the measurement vector, G is the gain matrix and W is the weight matrix. G2 is given by H transpose W into H and H is the connective vector. Now here, we have said that now this is weight matrix. So now, we did not say anything about that what should be the value of the different weights and weight matrix is it is basically a diagonal matrix having weight values W1, W2 up to Wn at the diagonal entries and all the other off diagonal elements as 0 so this is a weight matrix.

Now, in the last class, we did not at our discuss that how these ways are to be chosen so let us first discuss this. Now, in the last lecture we have a taken example where you have shown that may be that there are 4 may be there are 4 meters which have got different accuracy level so then therefore for our purpose of estimating these states. We should give actually more weightage to the more accurate meters and we should give less weightage to the less accurate meters.

So then our guiding philosophy should be and this is also most nature as our guiding philosophy should be that we should give more weightage to more accurate meters and less weightage to less accurate meters. So that is our guiding principle now the question is how do we give this? Now, we need to remind ourselves that if there is an meter which is very accurate. So then it is a variance of the error which is being supplied by the manufacturer would be also much less. For example, for any meter, whenever you purchase any meter that particular meter as also got its some specified value of variance of errors which is being specified by the manufacturer itself.

So then therefore and this particular variance is nothing but the variance of the errors because as we have already set that these errors are actually basically random quantities so because these errors are essentially random quantities. So then therefore associated with any meter, there is some standard deviation or variance of the errors. And if these errors are much less, so then therefore this variance of the errors will also be much less.

So then therefore, more accurate meters will have much less variance and the less accurate meters should be much more variance so then therefore, if you wish to give more weightage to more accurate meters. So then therefore one very simple process is that we simply take the inverse of the variance as the weight of that particular meter. So then therefore, we choose as wi = 1/ sigma i square where sigma i square is the meter variance.

It is actually the meter variance means that is basically the variance of the errors associated with this meter and this particular value is been supplied by the manufacturer. Because this manufacturer in a meter they do a certain amount of test on it just to test that I mean what is the accuracy level of this meter. So during that accuracy test, they do determine that what is the variance of the errors associated with the meter so we call it that sigma i square is the meter variance.

We must remind again we must note that this meter variance is nothing but the variance of the errors associated with this meter. Now here we can see that if for an accurate meter this particular value of sigma i square is nothing is very low so then therefore omega it would be high and vice-versa and for a less accurate meters this value of sigma i square would be much more. So then therefore wi be much less so now here, let us look at the other aspects of it.

From here, we have x hat = G inverse H transpose Wz so we can write it as G inverse H transpose W and we have already seen z is Hx + some e. So it is G inverse H transpose w Hx + G inverse H transpose We.

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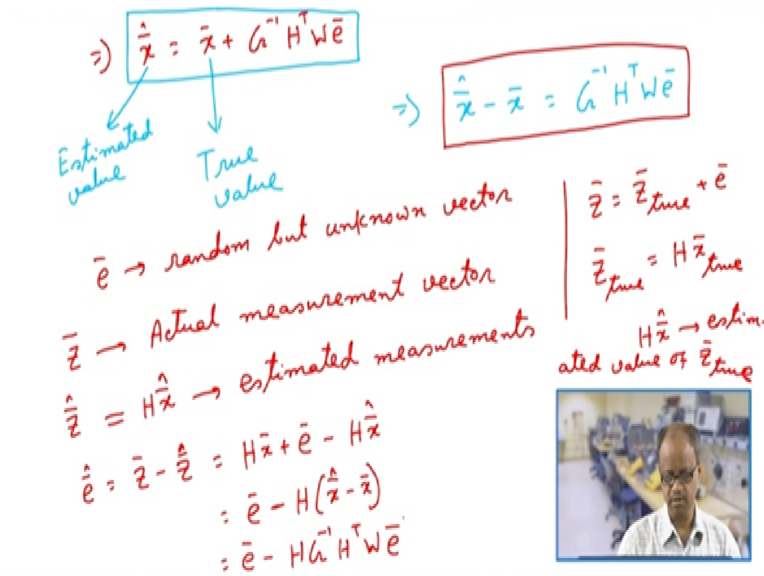

So then therefore x hat would be x + G inverse H transpose W into e because this quantity is nothing but GH transpose WG is G. So then G inverse G is identity matrix. So the next hat is equal to x bar + e. So then therefore we can see here that this estimated value of the states would be different from the true values of the x. We must say that this is estimated value and this is true value.

So here you can see that this estimated value of this state would be different from the true value because of this presence of error. Had there been no error? Had there been this error would be 0? So then obviously x hat would be a which is also intuitively very correct. Because if there is no error in the meter. So then therefore whatever measurement would be recorded by this meter they would be absolutely ideal values and so then therefore whatever estimation so we do carry out using these ideal values. They would also be ideal values of the true vectors ideal values of the state vector.

But because of the presence of error this estimated values are not equals to the true value. So now therefore we can write down from here that x hat - x is = G inverse H transpose w into e. So this is also very important result we would be using this important result some point of time. Now we have to now understand one thing that this vector is actually an random but unknown vector.

It is an error vector is a random vector but it is an unknown vector. Why we call it is an unknown vector because even for a meter, for example, if a manufacturer specifies that a particular meter has got an error of let us say + - 2% that does not mean that whenever in a measurement would be taken by the meter that error would be either +2% or -2% that error could be any value between - 2% to + 2% and it can take any value and these values are absolutely random.

So then therefore at any instant of time we just do not know what value it has taken. It value is absolutely random we do not know what value it has taken. We can only be sure that if this meter has not gone out of order by some means then the error associated with this measurement would be within + - 2%. But its actual value would be never known. So it is an unknown vector. So now the question is that if it is an unknown but random vector do we have any option of let see estimating their values?

For example, here, we do not know what is the true values but here we are also able to estimate the values of the state vector. So similarly as this error vector is also a random but unknown vector. Is there any option for us to estimate their values? Now the best what we can do is not to best what we can do is that we have already got this particular vector that is the actual measurement vector so this is actual measurement vector.

Now this actual measurement vector would be actually the true value and the error, right. So this actual measurement vector we know that this is basically nothing but the true value and the error. Now here, we do not know what is the true value but the best estimate of the true value available with us is nothing but the estimated value. So then therefore we can only estimate that what should be the true value of the measurement as so I can define another quantity of Z hat that = H into x hat and what is this? It is actually the estimated measurements.

Now what is exact when we write that it is estimated measurements what actually does it mean? As we say that Z is actually Z true + some error. This error I do not know Z true also I do not know and this Z true is basically we already said that Z true is nothing but H into x true. Now we also do not have these values of H x we also do not have these values of H true but we have just now estimated this particular x true and this estimated value we have is the x hat. So x hat is the closest value available to us, as for us x true is concern so x hat is nothing but the closest value of x true. So then therefore the closest value of Z true would be also H into x hat.

So this we can say that this is the closest value of the Z true and this closest value of the Z true we are calling as estimated measurements, right. So this is basically the closest value of the z true and these we are calling as estimated measurement. So then therefore, my estimated error would be Z-Z hat, right so Z- Z hat So here basically here now essentially we are simply replacing Z true as H into H hat and H into H hat is nothing but estimated measurement which is essentially nothing but the estimated value of z true, right.

So we can also write it is, so this is estimated value of Z true. So then therefore my estimated error would be Z - Z hat. So this is H into x + e - H into x hat, so in therefore e - H into x hat - x. So then therefore it is x hat - x. So now, if I now replace this expression here so what we get is e - HG inverse H transpose w into e, right.

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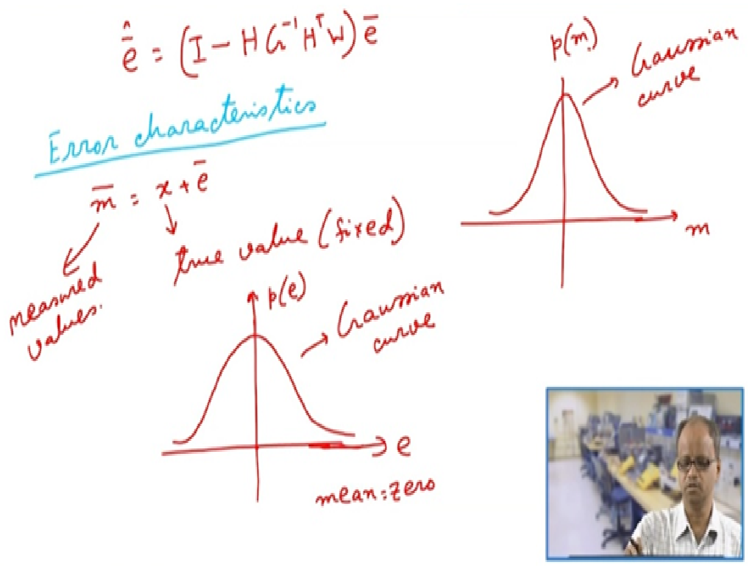

So then therefore I can say that e hat estimated value of error is identity matrix - H into G inverse H transpose into w into e. So this is a constant matrix because a H is a constant matrix, G is a everything constant matrix. So this is a constant matrix so then estimated error is also related with the actual error by this. Of course, we do not know this, right. Now the question is that how good or how bad measurements are these.

So now we have got these values that so now basically this is the true value or other this is essentially the difference between the estimated state and the true state and this is the expression of the estimated error. Now to understand that how good or how bad is this estimated error or how good or rather how close is the estimated state to the true state to understand these values we need to look into in more detail about the errors. So we need to now talk about the error characteristics.

So now if it is an error characteristics, now error means that is you are talking about the measurement error. Now for our in fact it has been already experimentally seen that if we take a measurement by an healthy meter. Healthy means that which has not gone out of order, if I take measurements by a meter which has not gone out of order repeatedly let say 1000 times, 10,000 times, million times.

And then, if we plot their measured values we will find that these plots basically follow Gaussian. So then this particular measured values follow a Gaussian. And because these measured values are nothing but the true value and plus error now we know that this so now what we are doing is essentially what we are doing is that we are taking measurements of a quantity by a certain meter of a fixed quantity by a certain meter many time.

Let say 1 million times or let say or 10,000 times so then what we have got is actually we are taking let say measurement values m is x true + e, x is the true value. So this is fixed value this is constant value which I do not know this is the fixed value. These are the measured values now if we take the measurement of a fixed value by a meter say 10,000 times or million times it will follow a something called a Gaussian graph or normal graph something called a Gaussian graph.

So we know what is meant by Gaussian curve. So it is a Gaussian curve, right. And it will have a 0 mean so it would be quite symmetric about this particular vertical axis. So it is the quite symmetric, so this is the value, so let say this is the some value m and this is the probability of m, right. Now these measurements follow a Gaussian curve now because this is a true value so then therefore error is nothing but measured values minus some true value.

So because these measured values also now because these measured values follow a Gaussian curve. So then therefore these errors will also follow a Gaussian curve because error is nothing but this measured value - a fixed value. So then therefore error will also a follow a Gaussian curve. So then therefore error, so errors will also follow a Gaussian curve. So errors will also follow some Gaussian curve.

So we say that it is probability of error and this is error and this is also a Gaussian curve, right. And this error will also have a mean 0 so it will also have a mean 0 mean = 0. What is mean is equal to 0? So that means this is always 0. This mean is equal to 0 why that should be 0 because, see if for any particular meter which has not gone out of order.

If I take the measurement with this meter, let say as earlier times. So then what will find that this particular measurements values are almost symmetric with each other and if I take the average, they will come almost equal to 0 because these values are random. So then therefore if we take these measurements for a billion times, so then almost equal values of positive and negative values will actually exist.

So then therefore they will ultimately cancel each other. So then therefore their mean is 0 and because this particular measurement quantities has got their mean is 0. So then therefore error will also have 0 mean. So first characteristics is that errors are 0 mean, second characteristics as is that these errors are independent with each other. What is meant by independent with each other? Independent means that if I take, suppose for example, we are taking the measurement of 2 quantities by using 2 meter and if these meters are independent to each with us each other.

So that means the error according with meter 1 will not be influencing the error occurring with meter 2 and vice-versa. So then therefore the error according with meter 1 and meter 2 that would be independent of each other that means there is there would be no kind of correlation between the errors of these 2 meters. So then we say that these errors or other these errors of the meters are independent with each other and they are also have 0 mean.

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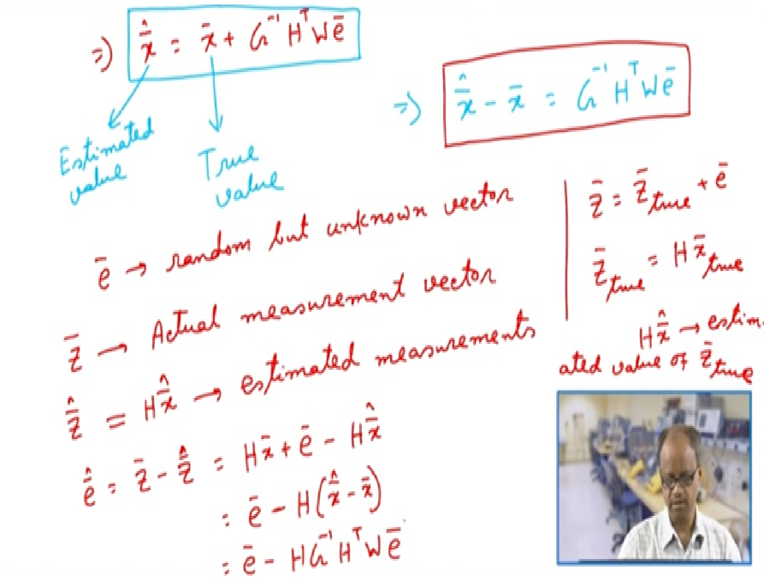

So then therefore our error characteristics is first is that they have got 0 mean. And these are independent from each other so you just write independent. Now what is meant by mathematically 0 means that expectation of the error is 0. So if I say that Ith meter is has got 0 mean should not write e bar actually should write E of ei 0. What is ei? ei is the error of the I x meter and ei is the expected value of ei.

And this expected values of is all known to everybody. So I am not going to detail in this. So this expected value is nothing but the statistical mean of the any random quantity. So that therefore if all these errors are have got 0. So then therefore if all these errors have got 0 means so then therefore we have E of e1 = 0, E of e2 = 0 and dot e of em will also equal to be 0. So then therefore we can write down that expected value of e1, e2 dot em is also = 0 over and other words expected value of error vector will be equal to 0, right. Now if I utilize this, so expected value of this error vector is if I utilize this into this quantity, so what do I get?

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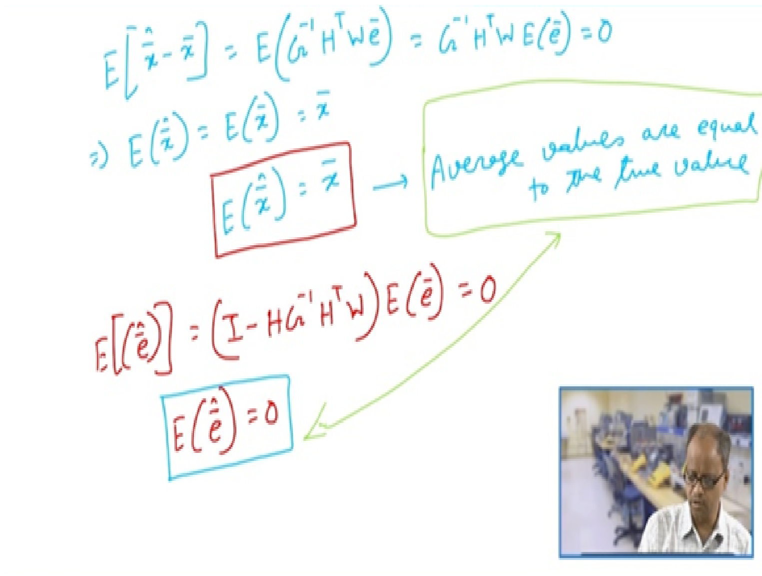

So now what we are trying to do is that now we are trying to do is expected value of x hat - x that would be expected value of G inverse H transpose We, because there is a constant matrix. So it is G inverse H transpose W expected value of e = 0 over in other words expected value of this estimated value would be equal to the true value e expected and because it is a true value. So it would be a constant value. So then therefore if the meters have got 0 error, so then therefore the average value of the estimated quantity would be equal to the true values.

So this is very important result where saying that this average value so that means if I take this estimate for a long time or rather if this take this estimation for a large number of times and then if these meters are have not gone bad. So then therefore their mean value would be equal to the true value and that is precisely why in your laboratory experiments so you always take the more than one measurements with any meter and then we need to take that average.

Secondly, what is the expected value of the estimated error an expected value of estimated error is again is I - HG inverse H transpose W into expected value of error that is = 0. So that means the expected value of the estimated error are also so are also = 0. What in other words mean that if we take the measurements for a long for a very large number of times their error would be cancelling each other, right. So this is the average values are equal to the true value.

This is a very important result equal to the true value and that is the essentially the central idea of this least square method. So if this for an well behaved meter where this meter have not gone bad the average values of the estimated values will always came out to be the true value and the estimated values of error are also 0 that means these errors will ultimately will cancel each other for a large number of measurements. And that why precisely the average value should be equal to the true value. So these and these they are actually interconnected with each other. So we a stop here today you would be looking into the other aspects of this method in the next lectures. Thank You.