Computer Aided Power System Analysis
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Lecture – 04
Bus Admittance Matrix with Mutual Impedance

Welcome to this module of this course, Computer Aided Power System Analysis. Up to the last class, we have discussed about the formation of the bus admittance matrix of any general large power system. But in that formulation, we have not considered the mutual impedance between the lines. So here in this lecture as well as in the next lecture, we would be looking into the fact that how do we take into account the mutual impedance between any 2 lines into account while forming the bus admittance matrix. So what we are trying to do is as actually follows.

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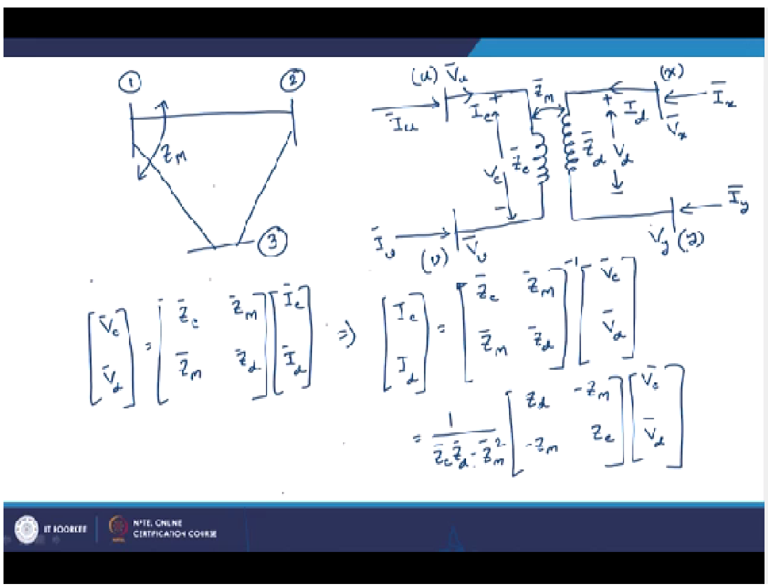

Suppose I do have, so this is let us say bus 1, this is bus 2, this is bus 3 and let us say there is 1 transmission line between all these buses and suppose that these 2 line have got some mutual impedance connecting between these 2 lines. So now what we are trying to do is as follows. Now suppose I do have let us say bus u, bus v, it is a bus x, bus y. So it is bus u, bus v, bus x, bus y. Between u and v, there is an element connected.

Between x and y, also there is an element connected. The current through this element, let us say, Ic. The current through this element is Id. The impedance of this element is Zc. The impedance of this element is Zd and these 2 elements are interconnected or rather they do have a mutual impedance between them as Zm. And let us say that the current which is being injected here Iu, Iv.

These are all complex quantity. This is Ix. This is Iy. So this is the setup. So what we are actually trying to do is, essentially we are trying to formulate or rather find out an equivalent circuit between bus u, bus v, bus x and bus y such that after we form the equivalent circuit connecting all these 4 buses, we would be able to apply the standard rules of forming the bus admittance matrix.

So essentially that is our goal. So to do that, so now let us see we have that and let us say that this voltage across this is Vc. Voltage across this element is, let us say, Vc. Voltage across this is element is Vd. So we can write down VcVd=Zc Zm Zm Zd*Ic Id. So a complex quantity because after all Vc would be ZcIc+Zm*Id and also Vd would be ZmIc+Zd*Id. So therefore, Ic Id would be Zc Zm Zm Zd inverse Vc Vd. Now it would be, so this would be equal to 1/ZcZd-Zm square*Zd-Zm-ZmZc*VcVd.

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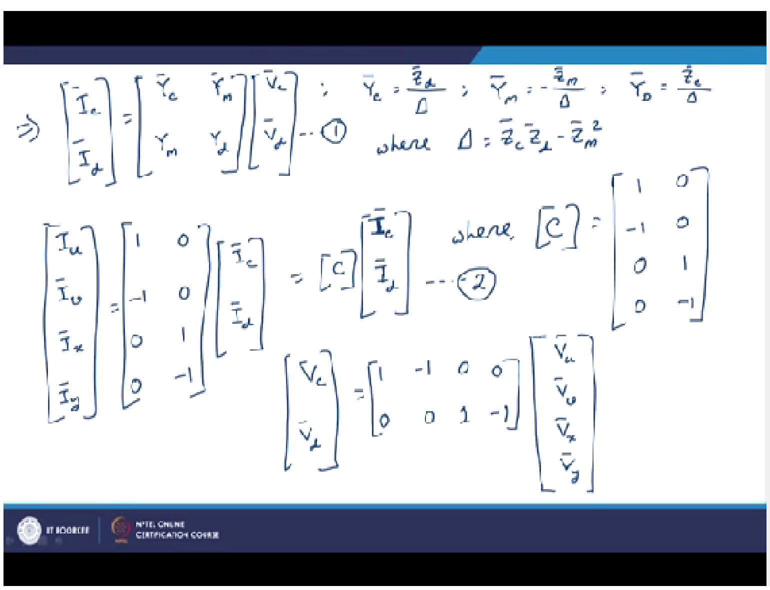

Or in other words, so we can write IcId=Yc Ym Ym Yd Vc Vd where Yc is given by Zd by this, delta Ym is given by -Zm/delta and Yd is given by Zc/delta where delta=ZcZd-Zm square. Now from this circuit, we can write down that IuIvIxIy=, if I try to represent it all this injected current

IuIvIxIy in terms of these branch currents, so we get some matrix now. What this matrix, clearly here Iu would be equal to Ic and Iv would be equal to -Ic.

Similarly, Ix would be equal to Id and Iy would be equal to -Id. So then therefore, Iu would be equal to Ic, so it is 0 1 0. Iv is -Ic, so it is -1 0. Ix=Id, so it is 0 1. And Iy=-Id, it is 0 -1. So let us say this is matrix C, some matrix C*IcId. Where matrix C is, this matrix 1 0 -1 0 0 1 0 -1. Also Vc=Vu-Vv and Vd=Vx-Vy. So then therefore, if I write down VcVd=, you have something called here Vu Vc Vx Vy.

So it would be something like this. So Vc=Vu-Vv. So it would be 1-1 and others would be 0 and Vd=Vx-Vy, so it would be 0 0 1 -1. So we can clearly see that this matrix is nothing but transpose of this matrix C. So this row, sorry this column becomes this row and this column becomes this row.

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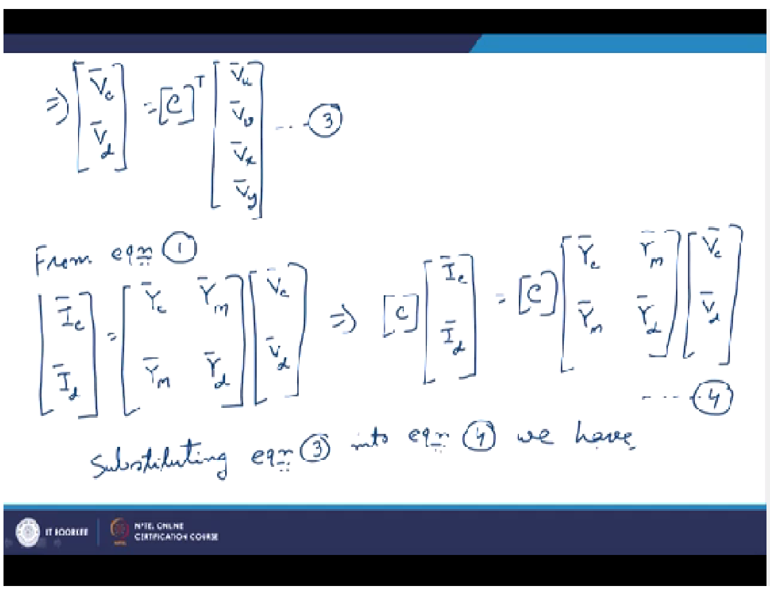

So then therefore, we can write down that VcVd=matrix C transpose VuVvVxVy. Now we have IcId= this, so now we would be, let us say that this is equation 1. We should set equation 1. We should say that this is equation 2 and we should say that this is equation 3, suppose. From equation 1, IcId=YcYmYmYd VcVd. Now I premultiply everything by the matrix C. So I premultiply everything by matrix C, so then I get premultiplying everything by matrix C, I get C*IcId=matrix C*YcYm.

Now in this equation, we will now put equation 3. So this is, let us say this equation 4. So substituting equation 3 into equation 4, we have.

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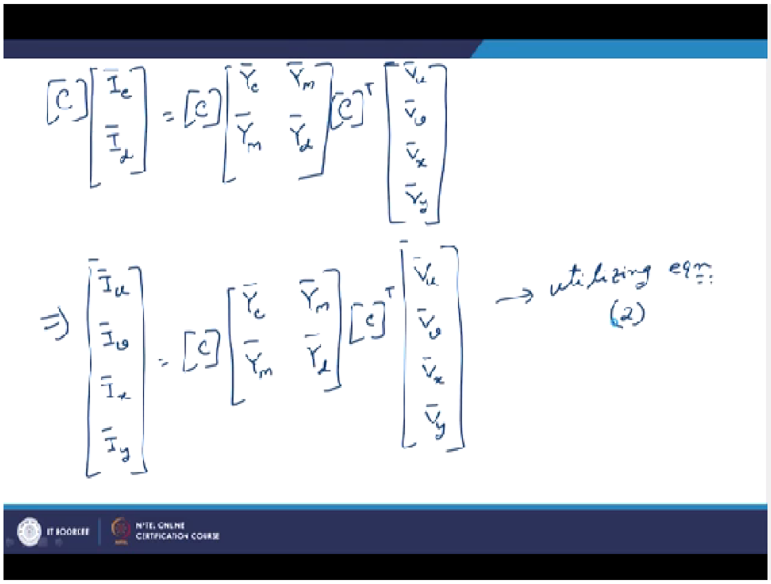

That C*IcId=matrix C, and then it is VcVd and VcVd is C transpose this, C transpose VuVvVxVy. Now C*IcId, now from C*IcId is nothing but IuIvIxIy. So if I utilize equation 2 here, so then therefore, we have IuIvIxIy=matrix C YcYmYmYd C transpose Vu, all are complex quantities, VuVvVxVy. So this is utilizing equation 2. Now we have to look at that what is this matrix? C*YcYmYmYd C transpose.

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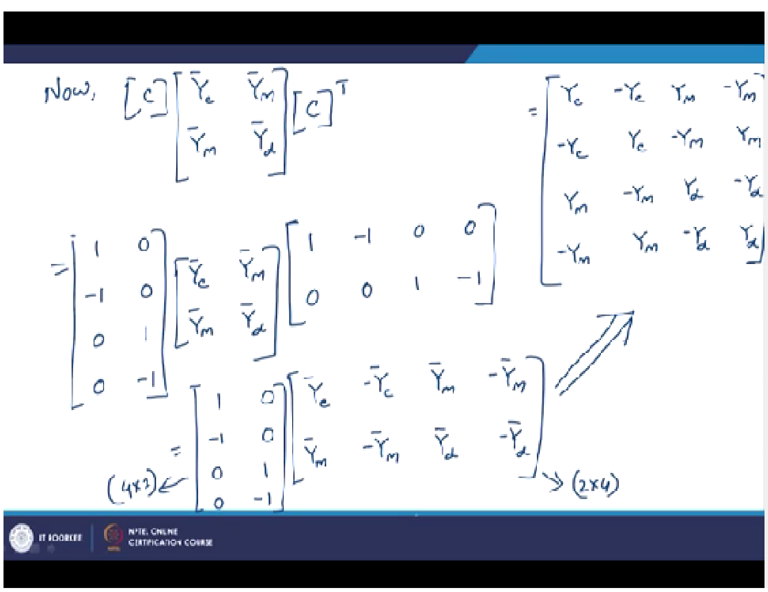

So now let us look at. Now C*YcYmYmYd C transpose. That would be equal to? C matrix is 1 0 -1 0, this matrix. So this is 1 0 -1 0 0 1 0 -1, this is YcYmYmYd. And C transpose matrix C is, this is just 1 -1 0 0 0 0 1 -1. So what I would get? So I would get here from here 1 0 -1 0 0 1 0 -1 and this would be Yc -Yc Ym -Ym, it would be Ym -Ym Yd -Yd. So this is a 4*2 matrix and this is a 2*4 matrix.

So then therefore, there product would be a 4*4 matrix and this product would be, so it is Yc -Yc Ym -Ym. This is -Yc Yc -Ym Ym. This plus is not required. Then what we have? We have Ym -Ym Yd -Yd. Then we have -Ym Ym -Yd Yd. So this is the matrix. So this matrix is this. So then therefore, what we have?

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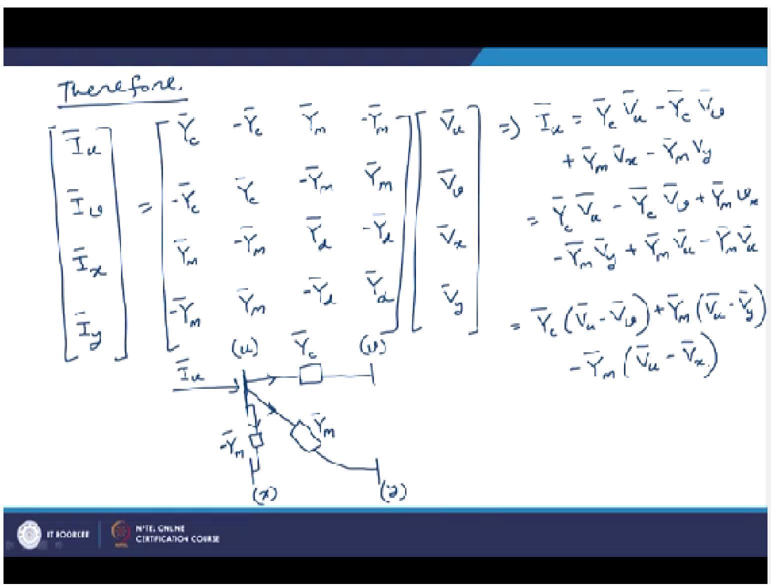

Therefore, we have IuIvIxIy=, this big matrix 4*4 and it would be Vu Vv Vx Vy. So this would be all complex quantity, this would be all complex quantity. So Yc -Yc Ym -Ym. So it is Yc -Yc Ym -Ym. -Yc Yc, so it is negative, -Yc Yc -Ym Ym. Then Ym -Ym Yd -Yd, so Ym -Ym Yd -Yd. This is -Ym Ym. This is just the negative of it. -Ym Ym -Yd Yd. So what we got?

We got a bus, you know admittance matrix connecting the injected currents. So what we have got? So you see here what I have got? Iu Iv Ix and Iy, these are nothing but the injected current at bus u bus v bus x and bus y respectively. And these voltage is Vu, sorry I should have mentioned either, this is voltage Vv. This is Vx and this voltage is Vy. So I have got this voltages Vy Vv Vx and Vy.

So what we have got? We have essentially connected this injected, 4 injected currents, Iu Iv Ix and Iy. With these 4 bus voltages, Vu Vv Vx and Vy. So then therefore, so it may appear that we have done our job but the problem is now, so then therefore, this is nothing but the bus injected current. This is the bus voltage vector and this is the bus admittance matrix as far as these 4 buses are concerned.

Now it may appear that we have done our job. But the point is, we really cannot utilize this, because this particular bus admittance matrix cannot be directly incorporated into our normal standard bus admittance matrix. So then therefore, we need to do something else. So what we will do is, so now, we will now utilize these equations and we will go through this. Now here, there are, so Ic Id Vc Vd.

So now let us write down these equations. So now these equations are, so now we have to write down the equations, are Iu=YcVu-YcVv+YmVx-YmVy, these are. To this, we simply add YmVu and then also subtract YmVu, so what we get is YcVu-YcVv+YmVx-YmVy+YmVu and -YmVu. So we have simply added and subtracted YmVu from these expressions. So we got these.

So from here, what I get? I take Yc as Vu-Vv from these 2 terms. Then we take YmVu, this minus this -Vy and then -YmVu-Vx. So this is the expression of Iu. So then if this is the expression of Iu, so then what we can have that for this, so now we have got I v x y. So this is bus u, this is bus v, this is bux x, this is bus y. And we have got injected current Iu and now, so now we can see that this injected current Iu is actually constitutes of these 3 terms.

So if we can represent these 3 terms by means of some equivalent impedance or admittance, so then therefore, we can express this first row of this equation or rather first row of this particular matrix relation by an equivalent circuit. So now let us see how do we do this? So from this, so I have got Iu. So now if I put Yc here and then I put Vu-Vy, so this is, I put here Ym, this is Ym. And between these 2, I put another admittance -Ym.

So now what we have? This current which is entering this bus is constituting of 3 currents, this current+this current+this current. This current is obviously Vu-Vv*this admittance. This current is obviously Vu-Vy*this admittance. And this current is actually Vu-Vx*this Y and this is precisely this particular relation shows this current is nothing but Yc*Vu-Vv. This current is actually nothing but Ym*Vu-Vy.

And this current, this third current is nothing but equal to -Ym*Vu-Vx. So then therefore, what we have done that we have represented the first row of this matrix equation by an equivalent circuit. So then similarly we can also represent the, all the other rows of this equation by some other equivalent circuit and then we have to actually add those 4 equivalent circuits together and we have to arrive at the compact equivalent circuit representing this particular network. So this we will do in the next lecture. Thank you.