Computer Aided Power System Analysis
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Module No # 08
Lecture No # 39
Weight Least Square (WLS) Method

Hello friends welcome to this lecture on computer aided power system analysis in the last lecture we have introduced the concept of a state estimation and we have also started looking at the measurement model of its simple electric circuit. So we today we will continue with that so what we did till last lecture is so we are essentially talking about weighted least square method.

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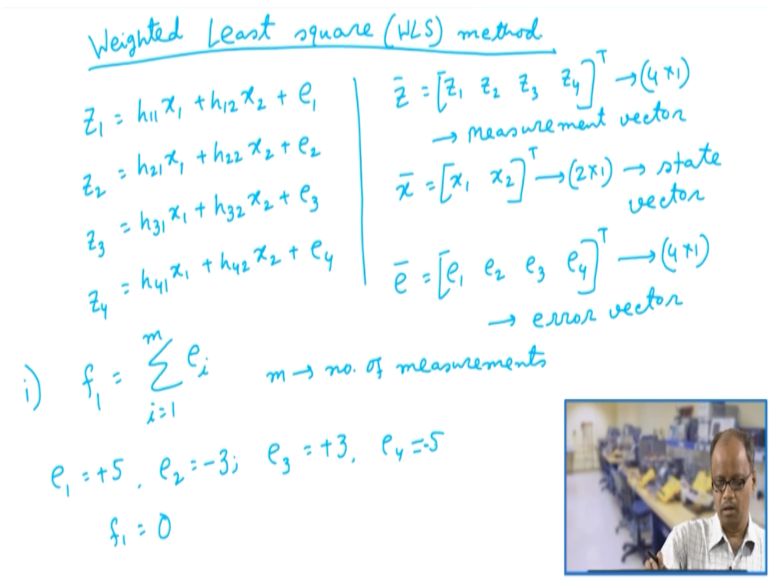

Now weight is weighted least square method we still did not explain so we should be explaining this to do in today's lecture. Now what we have done in the last class that we have considered a simple electric circuit and in that simple electric circuit we have taken one electric source and one current source and circuit we have taken one voltage source and one current source and there was several resistances connected in various combinations and we said that the voltage source and current source they are unknown.

So we have treated them as the unknown quantities or rather this gate variables which we wanted to know and for that we have also put a 4 meters some them are ammeters some them are volt meters and then we said that this measurement they can be expressed as a function of this state variables + some errors so what we did is what we said that let this measurements we said now here we said that Z1 that h11 x1 + h12 x2 and we said that it will also have some error which is unknown error but which is random in nature.

So h22 x2 + e2, Z3 is h31x1 + h32 h2 + e3 and Z4 h41x1+h42x2+e4 now here this Z vector Z1, Z2, Z3, Z4 transpose so this is 4 cross 1 vector this we call as the measurement vector where this quantities Z1, Z2, Z3, Z4 are nothing but the measured values x vector x is x1, x2 transpose so this is 2 cross 1 vector we called it as state vector. State vector is nothing but the vector a comprising of the quantities which are to be determined and we also denote error vector e as e1, e2, e3, e4 transpose again this is a 4 cross 1 vector so we called it as an error vector.

Now as we said earlier that our goal is to find out the values or rather the estimate the values of x1x2 such that the effect of this errors are minimized. So then therefore we need to derive some objective function or some kind of index which will essentially compute the net effect of all the errors now there can be several indexes in this one such index would be that we take that this index f we call it this index f we can say that would be taking the case where we are simply = 1 to m here m is the number of measurements.

So then here in this we can say that f1 that i mean one such measure could be that what would be doing that would be simply add all these errors and then basically what we are saying here is that we would be simply trying to minimize the sum total or rather the algebraic sum total of this errors. So our goal probably could be that we would like to estimate this value of x1 and x2 such that the algebraic some total of this error could be minimized.

Now this looks elegant but the problem is that it has got a very basic problem that these values e1, e2, e3, e4 they because they are essentially random in nature so then therefore they can be either positive or negative so then therefore if we take just their algebraic sum so it may happen that they will cancel each other automatically for example. Suppose for example say that e1 e is let us say some unit value let us say 5 +5 and this is the error.

Let us say e2 is let us say -3 some value e3 is let us say +3 and e4 is let us say -5 so then therefore f1 would be = 0 e1+ e2 + e3 + e4, e4 would be 0 but the point is individually these errors are very high. So then therefore although they are net effect that is the net algebraic sum would be 0 but individually because this errors are very very high so then therefore individually this measurements are extremely erroneous.

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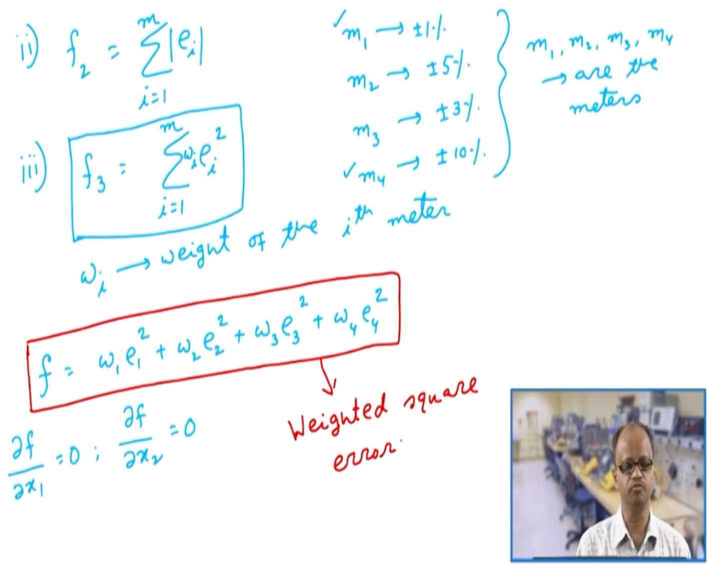

So then therefore this measure cannot be used second measure could be now we said now here in this earlier case that particular problem arose because of this fact that we are taking the algebraic sum now instead of that suppose we take this the magnitude of them. So then in that case obviously this problem of automatic cancellation of the errors will not be there but the problem is that this particular function is not really very much differentiable in an elegant manner.

So then therefore this particular measure or rather this particular objective function can also not be used because this quantity is not a really very much differentiable or rather it is quiet difficult to work with this particular objective function when you have to do some kind of differentiation. So then therefore what we do is what we use is something called so to do this we use something called ei square.

So when we do use ei square so then therefore again this particular objective function also does not have the problem of automatic cancellation of the errors and also this is also very much easily differentiable. Now here in this case we have given equal weightage to each and every measurement but depending on the accuracy of the meter. For example m1 could be let us say 1% error that is basically this m1 and m2 could be let us say can have let us say +-5% error it can have let us say +-1% error.

m3 could be have some something called +-3% error and m4 could be very erroneous let us say +-10% error and this m1, m2, m3 and m4 are the meters. So then therefore if we use this particular objective function or rather this particular measure what we are doing is we are actually giving equal weightage to a very accurate meter that is m1 or a very bad meter that is m4 and also to the other intermediate meter which is not very fare.

So then therefore what we do that to give adequate weightage to the accuracy of the different meter we do use some weight so we do some weight so we call it wiei square where wi is the weight of the so it is weight of the ith measure ith meter so weight of the ith meter. Now obvious question comes that what should be the value of the weight well now issue we will come little I mean little later but right now let us understand let us now accept this fact that for our estimation purpose we do use at this particular measure or this particular objective function is the most appropriate because it has got a very beautiful differentiable property and it also does not allow automatic cancellation of the error.

So now in our case so then therefore in this present case if 3 would be actually w1e1 square + w2e2 square + w3e3 square + w4e4 square. Now as we said our objective is in fact now here I mean instead of saying f3 we can simply write it as f so this now so now our measure is called f so this is the objective function f or rather the measure. So now as we said that we would like to estimate our state such that the net effect of the error are is actually minimized and we have developed some quantifiable measure that is given by this particular expression.

So then therefore what we want to do that we want to estimate this states x1 and x2 such that these quantity minimized because this particular quantities essentially reflecting the net effect of the errors on the system. So now because we wish to minimize this such that minimize their effect so then therefore it follows from simple differential calculus that del f1x1 should be 0 and del f/ del x2 also should be 0 right.

So we are basically trying to minimize this function such that with respect to x1 and x2 that means that we are simply trying to estimate it x1, x2 such that this particular effect of the errors is minimized.

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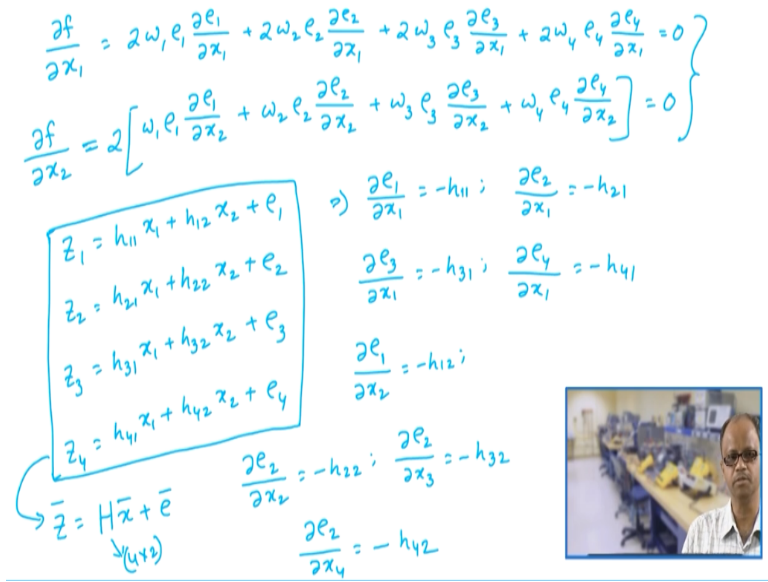

So then therefore if I now do this analysis so now what do I get? I get that del f del x1 is basically 2w1e1 del e1/del x1 + 2 w2e2 del e2 / del x1+ 2w3e3 del e3 / del x1 + 2 w4e4 del e4/ del x1 this should be 0. Similarly del f del x2 would be 2w1e1 del e1/del x2 + w2e2 del e2 / del x2+ w3 e3 del e3 / del x2 + w4 del e4 / del x2 = 0. Now if I take this two equations together and write them in a matrix form so then what do I get? So I get that so I get del e1 so we get something like this.

Now before doing this we have to do something else now what is del e1 del x1? Now we have got Z11 = h11 x1 + h12 x2 + e1, Z2 = h21+ x1+h22 x2 + e2, Z3 = h31 x1+h32 x2 +e3, Z4 = h41 x1 + H42 x2 + e4. So then therefore if I do del e1 del x1 what I will get. I will get –h11 and similarly del e2 del x1 would be – h21 del e3 del x1 would be – h31 del e4 del x1 would be – h41 del e1 sorry del e1 / del x2 again – h12 del e2 / del x2 would be – h22 del e2 / del x3 would be – h32 and del e2 / del x4 = - h42. So then therefore if I now substitute this so then what equation I get now if I now substitute all this in this so we get an equation that del e1x1 = h11 del e2x1 is h21, h31, h41.

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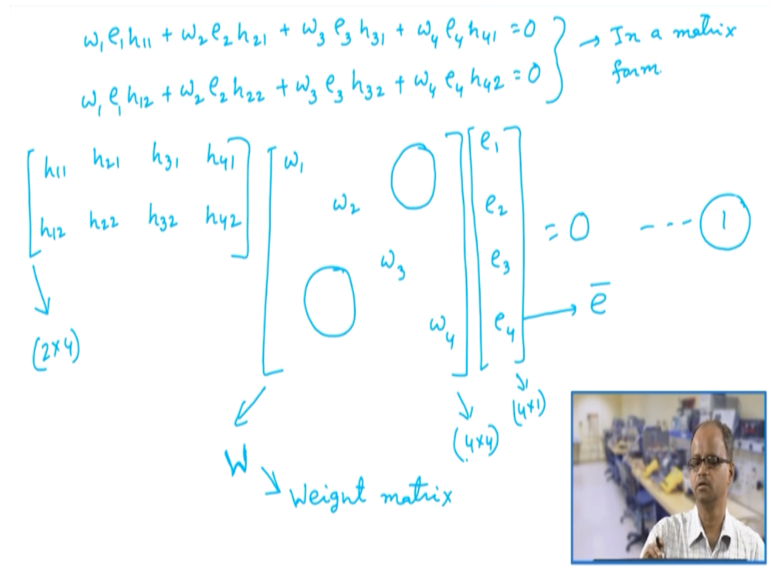

So we get that w1e1 h11 + w2 e2 h21 + w3e3h31 + w4e4h41 = 0 and similarly if is substitute del e1 / del x2 and etc I mean these 4 into this we get that w1e1 h12 + w2e2 h22 + w3e3 h32 + w4e4 h42 = 0. So this is what we get now we need to put them in a matrix form so put them in matrix form if I wish to put them in matrix form so then what do I get so first to write is simple is that what we write is W1, W2, W3, W4 so this is a diagonal matrix this is all 0.

So this is 4 cross 4 matrix then we have here e1, e2, e3, e4 so if I now this is also 4 cross 1 matrix so now if I multiply this I will get w1e1, w2e2, w3e3 and w4e4. So this is 4 cross 1 so there is an then therefore that i will get is h11, h21, h31, h41 and h12, h22, h32, h42 = 0 and this is a 2 cross 4 matrix. So then therefore if I do everything so I will get this equation obviously this is this and this is w1e1 + w so this and this would be w1e1, w2e2, w3e3, w4e4.

So the first row would be h11 into w1 h21 w2e2 h31 w3e3 + h41 w4e4 and similarly the second we will get it. So now this matrix we denote as the capital w matrix this is an capital w matrix so this is the weight matrix we call it has weight matrix this is weight matrix and this is a vector we had already seen that this we denote as a vector e now in this expression if I do take this expression can be written as vector z = sum matrix h into vector x + vector e.

Where this matrix h is h11, h12, h21, h22, h31, h32 h41, h42 so this would be of 4 cross 2 matrix so if I take so if I write this 4 equations in a matrix form I get that Z = sum matrix h into x bar +

e bar x bar is the state vector e bar is the error vector z is the measurement vector and h is the matrix connecting matrix and this is an 4 cross 2 matrix.

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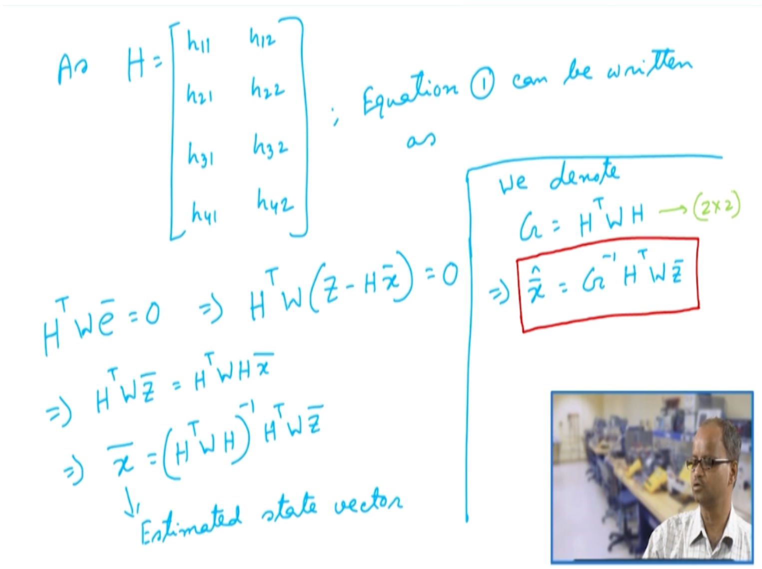

And h would be as h is h11, h12, h21, h22, h31, h32 and h41, h42 so then therefore if I say that that this is equation one so then equation one can be written as H transpose W into e = 0 right. Now from that e is equal to nothing but Z – Hx so then therefore now H transpose W and e is Z – Hx = 0. So then therefore I can write down that H transpose WZ, Z is the measurement vector = H transpose W Hx so then therefore I derive that x = H transpose WH inverse into H transpose WZ.

So this is the estimated value of the estimated vector now we denote G that is we denote that G we call it gain matrix is HWWH so then therefore estimated state vector we can hat so this is an x bar hat stands for the estimated state vector so we write it as G inverse H transpose WZ. So this is the final expression of the estimated value now why do we daily call it a weighted least square method.

Now let us look at this what we have done we have actually our measure is this is our measure and what is this measure? We are first squaring the errors and then we are multiplying this squared errors by their vect and then we are minimizing this resulting function. So then therefore this is actually this function is nothing but this function basically give the weighted square error. So this function is nothing but it gives is the weighted square error right.

Because this function gives weighted square error and we are trying to find out this estimated state by minimizing this particular weighted square error. So that means we are simply trying to minimize or rather we are simply trying to make this particular function or rather this weighted square error as the least values so then therefore this is called weighted least square method. So what we are essentially trying to we are trying to find out the estimated vectors by such that the weighted square error is the least. So then therefore and that precisely why it is called the weighted least square method.

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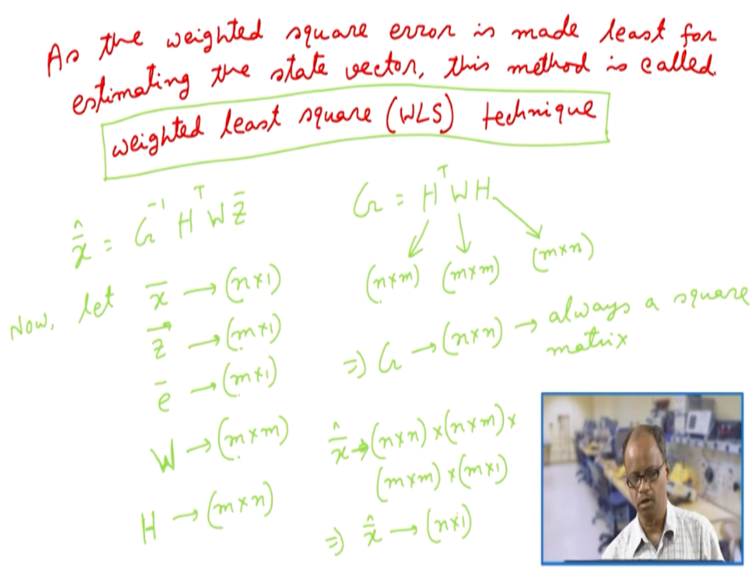

So we write that this has the weighted square error is made least for estimating the state vector, this vector is called weighted least square WLS. So now here x again let us look at that it is G inverse H transpose WZ now let us look at the general case G is given by H transpose WH. Now let my in a most general case x that is my unknown vector is an n cross 1 that is there at total n number of quantities to the estimated Z is an m cross 1 that is there at total number of measurement is m.

So then therefore e also would be m cross 1 and W would be that is weight matrix would be an m cross m matrix. So then therefore what would be H, H would be an m cross n vector m cross n matrix (()) (30:08) common. So then therefore what would be G this is H transpose this is m cross m W is m cross m and this is H is m cross n so then therefore what would be G so then therefore G would be m cross m cross n.

So would be m cross n so it would be always square matrix so here in this case what would be G in our case what would be G in our case G would be basically 2 cross 2. So in our case G would be here G is 2 cross 2 where G is 2 cross 2 so then therefore what would be x vector dimension it is n cross n H transposes n cross m that is H transpose W is m cross m and Z is m cross 1 so then it ultimately x is n cross 1.

So n cross n, n cross m, m cross n, n cross m it is n cross 1 what it should be so therefore we have obtained the expression of the estimated values of the state vectors by minimizing the weighted squared error. So essentially we finish today now in the next lecture we will continuing to see the other aspect of this particular method thank you.