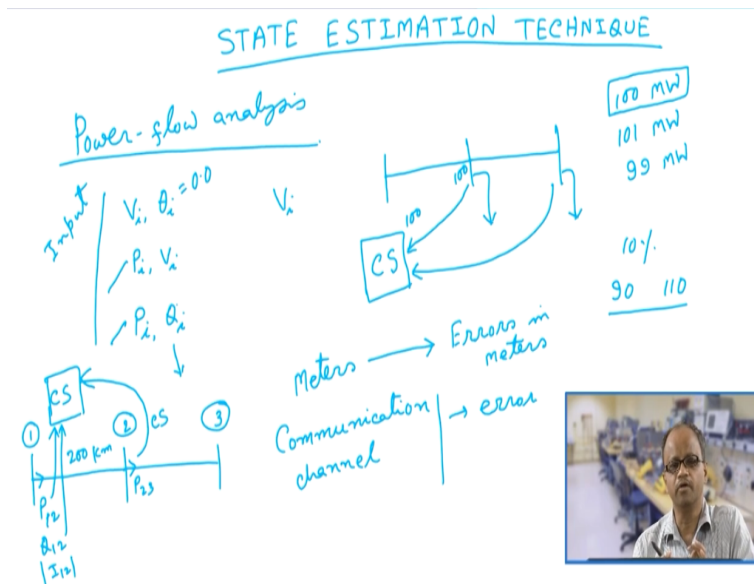


**Computer Aided Power System Analysis**  
**Prof. Biswarup Das**  
**Department of Electrical Engineering**  
**Indian Institute of Technology – Roorkee**

**Module No # 08**  
**Lecture No # 38**  
**State Estimation Technique**

Hello friends welcome to this lecture on the course of computer aided power system analysis in the last lecture we have introduced the basic concept of this state estimation technique and in this lecture we would be exploring more from this particular method.

**(Refer Slide Time: 00:45)**



So let us start so we are discussing state estimation technique so when we have starting discussing about state estimation technique in the last lecture we said that we are it is necessary for us to continuously monitor the system let the power flow or the current magnitude over in the line goes beyond they are corresponding permissible limit now we also said that if we know all the values required for performing the load flow analysis then we could be simply keep on doing this low flow analysis periodically and then simply calculate the power flow as well as the current flow magnitude over all the lines and then take appropriate action whatever it is necessary.

Then we also argued that well sometimes or rather in many occasions it is not very convenient to get all these values of power injection which is required for the load flow analysis instead of this

power injection values which are required for performing the load flow analysis it would be sometimes much easier to get the measured values of the power flow over the existing lines over some existing lines.

Now there is also another aspect of it another aspect is as follows again we look at this power flow analysis again our inputs are  $V_i$   $\theta_i$  for slack  $P_i$   $V_i$  for a  $P_v$  and  $P_i$ ,  $Q_i$  for  $P_q$  now in actual implementation from where I will get this  $P_i$ ,  $P_i$  I mean this  $P_i$  this  $P_i$  and this  $Q_i$  I mean we can always argue that well this  $\theta_i$  will this  $\theta_i$  would be always = 0 so then therefore we need not have to be measure it this  $V_i$  they are already known to us because we probably know that what is this setting of the voltage controller of the generator.

So then therefore if we know that any particular generator is meditate to control its terminal voltage at least 1.01 per unit then in that case we can say well my  $V_i$  is specified magnitude is let us say 1.01 power unit so then therefore we can also argue that yes probably for these two values we really not need any meters but then that we need at least meter for  $P_i$  and  $Q_i$  at the load bus is because of this fact that in any system the loads at any bus will keep on changing so then therefore until and unless we know that each and every in stand up what is the amount of load which is being consumed from any bus right.

We will not be able to know their appropriate exact value and if we do now know there exact value so then therefore our low calculation itself would be erroneous and if we get erroneous value of voltage magnitude and angles of all the buses we will get also get erroneous values of calculated values of power flow over a line and as well as the line current magnitude over and above.

So then therefore we need to put meters here now if I put a meter and even if I assume that well I do have this communication back bone to transfer this measured values to some control center here so we say that this is control center so we do have some communication channel from here to here and from here to here to transfer this measure values to this control center and there are two issues.

First issue is that we are putting meters and second issue is this communication channel now what are these issues? So two issues had arise one issue from due to the meters now when you

say that we are taking this measurement from the meters we need to remember that no meter in this world is absolutely accurate each and every meter has got some inherent errors so then therefore in the actual accurate value.

Actual value let us say 100 megawatt at some bus and if this meter has got let us say 1% error just 1% error so then this meter probably some point of time it will be measuring 101 megawatt or may be at some other point of time it would be measuring 99 megawatt of power. So then therefore because of this error inherent in the meter it will not ever measure this exact ideal values which is equal to 100 megawatts rather it would be measuring some values which is not true value.

So then therefore this control center will when it will get the measured values this control center will actually get the values not exactly 100 but then we get 101 or 99 but even then we can argue well this errors are not very much for all engineering purpose these values are acceptable so then therefore even if we perform our load flow analysis with this small erroneous values still we will get our the calculated values of real power flow as well as line current magnitude within an acceptable error.

So then therefore this calculated values of real power flow as well as the line current magnitude will give us an reasonable amount of confidence about the current health of the system that is agreed. But now suppose this meter for any reasons suddenly started malfunctioning it can because after all the meters already put and it is not possible for anybody and it is not possible for anybody to monitor this meter on continuous basis.

Now suppose this meter has got some how due to some circuitry problem inside it has started malfunctioning. So then therefore now this error instead of becoming one person now this error let us say becomes suddenly start it is now operating within 10% so then therefore if this actual values is 100 and instead of sending 100 it is now sending either 90 or may be 110 these values are unacceptably large.

These values contain unacceptable level of errors so if we calculate the load flow analysis with this erroneous values so obviously we are going to get it very erroneous estimate of the system. So then therefore when you are talking about the issues is meters these this issue is essentially

errors in meter so this is the basic issues. Now what is the issue regarding this communication channel? This communication channel were assuming that this communication channel as got no error has such now.

So then therefore if the values sent by the meter let us say a 100 so this is communication channel also sending a value of 100 but it may sometime happen that due to some error in the communication channel of course now a days with the advancement of the communication technologies this chance of this error but even that it may happen that sometime this communication channel fails so then therefore instead of getting this values 100 we may get some completely garbage values or sometimes we even may not get any value.

So then therefore communication channel also has got some error so then therefore whenever we are trying to measure any values let say injected power values by using meters these values which are ultimately being transmitted to the control center may content will large amount of error. Now the question is of course this very natural question is in the last lecture we said that well instead of again we are taking this 3 bus system so this is the slack bus and we said that we are measuring here  $P_{12}$  this is bus 1 this is bus 2 and this is bus 3,  $P_{12}$ ,  $Q_{12}$  and let us say  $I_{12}$  magnitude.

And let us say here you are measuring  $P_{23}$  let us say so I have got 4 unknown so I will get 4 equations I can solve now the question is again the same question come. Now here suppose my control center is here some control center is somewhere here now here also we have put 3 meters 1, 2, 3 and also we have put another meter here so then here also you have put 4 meters now because you have put 4 meter now here also because you have put meters here so this meter also can have error.

And although this meters are located at our control center again it is not possible for anybody to continuously monitor this meter whether they have started malfunctioning or not. So then therefore this meter also may contain error again this meter also may contain error now again because we said that this is 200 kilometers so again from here to here also we have to transfer this value from this location to this locations so this is the communication channel and this communication channel also may contain error.

And although these meters are located at the local station itself but even then there has to be some communication media some link something which will also transfer these values to here from here to here and this here to here and similarly from here to here so there also can be some error. So then therefore what we can see is and that whether I do power flow analysis that I mean whenever we are taking some measurement either for purpose of the power flow analysis or you are taking some other measurement to a eliminates some of the difficulties as associated with measuring the injected quantities of current buses.

But inevitably these measured quantities will be associated with two types of error one error is due to the meter error and another error is due to the communication error so now so then therefore the question comes now in the case of power flow analysis whenever we have done power flow analysis what we have assume we have assume that whatever input values are available to us these are the input values.

So whatever input values are available to us these values are ideal absolutely accurate without containing any error. But now our problem becomes little more difficult and complicated because in the case of when you have done power flow analysis we have not considered the possibility of any error in any of the measurement because we have just now argued it is almost always true that to get these values of this  $P_i$  and this  $Q_i$  we have to put some we have to some meters.

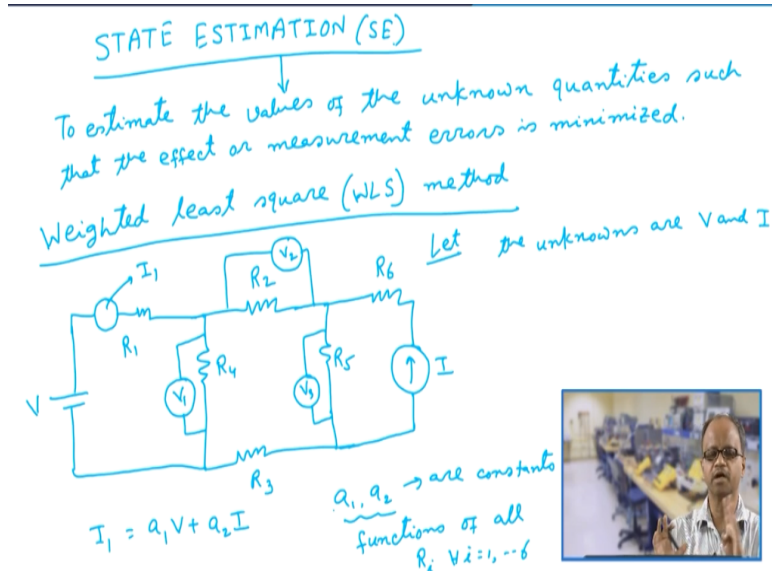
So now our problem is becomes more complicated in the sense now we have to consider the possibility of errors in our measured quantities. So whether we are utilizing measured quantities for the purpose of power flow analysis or for any other purpose so then we have to consider the possibilities that we have got some associated error in the measured quantities and we have to now because they are now errors in the meter quantities errors in the measured quantities so then therefore we will not be able to calculate the ideal values of our unknown quantities.

We have to only estimate we can only estimate their values that is so then therefore the state estimation technique is basically concerns about estimating the unknown quantities in the presence of measurement error or when we say that when we use the term measurement error where actually taking into account the fact that this error may come due to the inherent error of the meters or may crop up due to the error in the communication channel or both.

So now our problem is that how to calculate or estimate the unknown quantities in my system in the presence of metering errors so that is the essence of the state estimation error. So when the essence of state estimation method is estimating the values of unknown quantities in the presence of metering error. So we are saying that trying to estimate the unknown quantities because of this fact that because I have got input quantities which are metered quantities which are measured quantities I have got inherent error.

So therefore my measured quantities which are nothing but the input to my algorithm are not accurate are not ideal. So then therefore my output of the algorithm whatever that is so then therefore my calculated quantities will also not be ideal right. So then therefore our effort or our goal is now to calculate this values or rather the estimate these values such that the effect of this measured errors are minimum so the whole process of state estimation we call it state estimation SEE

**(Refer Slide Time: 16:00)**



State estimation we call it SE so the whole purpose of SEE is to estimate the values of unknown quantities such that the effect of the errors are minimized so the goal is to estimate again I repeat we are utilizing we are using the estimate naught calculation because are solved because when we say solved we assume that my input quantities are absolutely ideal and my so then therefore my output quantities will also have their most accurate values or rather the ideal values.

But here because our input quantities are having errors so then therefore my output quantities will also have errors so then therefore I never can calculate their ideal values. So I can only estimate their values such that the effect of error is minimal so estimate the values of the unknown quantities such that the effect of measurement errors is minimized so that is the basic goal of state estimation technique.

Now the most popular method for estimate the values of unknown quantities such that effect of the measurement errors is minimized is called is weighted least square method so we call it WLS method. So we are now going to discuss about weighted least square method now here we need to remind ourselves that our goal is to do state estimation of an electric power grid that is our final goal which we will do but to introduce the basic theory and the concepts of weighted least square estimation method we will first take every simple circuit DC circuit.

So let us take DC circuit something like this and this drawing an arbitrary circuit honestly is nothing much something like this. So there is a current source there is voltage source so that is  $V$  this is  $R_1$  this is  $R_2$  this is  $R_3$  this is  $R_4$  this is  $R_5$  this  $R_6$  this is  $v$  and this is  $I$ . So this was the voltage source so this is a current source. Now suppose let the unknowns are  $V$  and  $I$  so I do not know what is values of this  $V$  and  $I$ .

We do not know what is the value of  $I$ , I want to calculate their values so now for that what I do I let us say I am simply taking the you know I am taking some measurements so I put some ammeter here say I put some ammeter here so I measured this values  $I_1$  and let us say I put one volt meter across this let us say so this values voltage and let us say I put another volt meter across it let us say this is  $V_2$  let us say I do put another volt meter across it and this taking arbitrarily.

Let us say I am also putting another volt meter across it let us say  $V_3$  and so and hence so forth so I have put 4 meters and then I am taking this measurement so I have put four meters. So I know this measured values and from these I would like to estimate the value of  $V$  and  $I$ . Now from our basic circuit analysis because after all this is a simple linear circuit all these are resistance and this is voltage source this is current source or whatever it is.

And then what we can do that if I do this analysis so then what I can what I will find is that if I write the expressing of  $I_1$ .  $I_1$  would be some constant let us say  $a_1$  into  $V$  + some other constant into  $a_2$  into  $I$  and where  $A_1$  and  $A_2$  are constants some constants and these are actually all functions of these are all function of all  $R_i$  where for  $i = 1$  to  $6$  so then therefore this  $a_1$  so then  $a_1$  and  $a_1$  would be some expression involving  $R_1, R_2, R_3, R_4, R_5, R_6$  similarly  $a_2$  also would be some expression involving  $R_1, R_2, R_3, R_4, R_5$  vice versa right. I mean if we do just our simple circuit analysis as we do in our first year so we will be find out this expression.

**(Refer Slide Time: 22:19)**

$$\begin{aligned}
 m_1 \leftarrow I_1 &= a_1 V + a_2 I \\
 m_2 \leftarrow V_1 &= a_3 V + a_4 I \\
 m_3 \leftarrow V_2 &= a_5 V + a_6 I \\
 m_4 \leftarrow V_3 &= a_7 V + a_8 I
 \end{aligned}$$


$a_i; i=1, \dots, 8$  are all constants and functions of  $R_1, R_2, \dots, R_6$ .

$a_1 = h_{11}; a_2 = h_{12}$   
 $a_3 = h_{21}; a_4 = h_{22}$   
 $a_5 = h_{31}; a_6 = h_{32}$   
 $a_7 = h_{41}; a_8 = h_{42}$

$$\begin{aligned}
 m_1 &= h_{11} x_1 + h_{12} x_2 \\
 m_2 &= h_{21} x_1 + h_{22} x_2 \\
 m_3 &= h_{31} x_1 + h_{32} x_2 \\
 m_4 &= h_{41} x_1 + h_{42} x_2
 \end{aligned}$$

$V = 10 = x_1$   
 $I = 5 = x_2$   
 $h_{11} = 5$   
 $h_{12} = 2$

Ideal value of  $m_1 = 50 + 10 = 60$



So similarly what I write is so let us say that  $I_1$  let us say  $a_1$  so let us say  $V$  and let us say  $a_2$   $I$  similarly let us say  $V_1$  would be something  $a_3$  into  $V$  +  $a_4$  into  $I$  similarly  $V_2$  would be let us say  $a_5$  into  $V$  +  $a_6$  into  $I$  similarly  $V_3$  would be  $a_7$  into  $V$  +  $a_8$  into  $I$ . So here we again note that all  $a_i, i = 1$  to  $8$  are all constants and functions of  $R_1, R_2$  to  $R_6$  right. Now what we have done we have actually taken 4 measurement so let us say this is  $M_1$  we call it  $M_1$  so this is measurement 1 so this is measurement 2 this is measurement 3 this is measurement 4.

And my unknowns are we are saying it is  $V$  and  $I$  so for our analysis we say that my unknowns  $I$  always denote as state variables  $X_1$  and  $X_2$ . So here  $X_1$  and  $X_2$  so then therefore I can say so now if I now rewrite this equations in the common parlance so what I will write is that  $M_1 = h_{11} x_1 + h_{12} x_2$  where of course I am writing  $a_1 = h_{11}$   $a_2 = h_{12}$  similarly  $m_2 = I$  can write  $h_{21} x_1 + h_{22} x_2$  obviously  $a_3 = h_{21}$ ,  $a_4 = h_{22}$  similarly I can write down  $M_3 = h_{31} x_1 + h_{32} x_2$   $M_4 = h_{41} x_1 + h_{42} x_2$ .



Of course  $h_{31}$ ,  $h_{32}$ ,  $h_{41}$ ,  $h_{42}$  we can identify I mean let us write it for this set of completion if I have  $h_{31}$  where just denoting it for the purpose of more of general analysis so  $a_6 = h_{32}$  and  $a_7$  is denoted as  $h_{41}$  and  $a_8$  is  $h_{42}$ . Now what happens these values  $M_1$ ,  $M_2$ ,  $M_3$ ,  $M_4$  these are the measured values now these quantities  $h_{11}x_1 + h_{12}x_2$  is the ideal values that if this value is let us say I mean if this value let us say if  $V$  is let us say 10 volt and if let us say  $I = 5$  ampere.

So I know what is  $a_1$  and I know what is  $h_{11}$  I know what is  $h_{12}$  so if this value is let us say so now if I put  $X_1 = 10$  this is  $10 = x_1$  this is  $10 = 5 = x_2$ . So now if I put  $10 = x_1$  and  $5 = x_2$  and  $h_{11}$ ,  $h_{12}$  everything is known so then these expression will give me the ideal value this is the ideal value. But what this meter this ammeter will measure because of this inherent error this meter  $m_1$  or  $m$  meter will measure these ideal value + or - some error so these meter because of this error it will not report to this ideal value this meter will actually report some error along with the ideal value.

So then therefore this ideal values let us say for example let us say  $h_{11}$  is let us say 5 and let us say  $h_{12}$  is let us say 2 so then when here the ideal value of  $m_1$  is ideal value of  $m_1$  is  $m_1$  would be  $h_{11}$  into  $x_1$  that is  $50 + 2$  into  $5$  that is  $10$  that is  $60$  so ideal value also  $m_1$  is also  $60$  but this is ammeter so then this as got it is inherent error. Now the point is now we also need to remind ourselves so on very basic fact of I mean that if I take the measurement of this same quantity in a circuit by the same error repeatedly then the value what we get from that meter is not the same every time.

**(Refer Slide Time: 27:59)**

$m_1 \leftarrow I_1 = a_1 V + a_2 I$   
 $m_2 \leftarrow V_1 = a_3 V + a_4 I$   
 $m_3 \leftarrow V_2 = a_5 V + a_6 I$   
 $m_4 \leftarrow V_3 = a_7 V + a_8 I$


$a_i; i=1, \dots, 8$  are all constants and functions of  $R_1, R_2, \dots, R_8$ .

$a_1 = h_{11}; a_2 = h_{12}$   
 $a_3 = h_{21}; a_4 = h_{22}$   
 $a_5 = h_{31}; a_6 = h_{32}$   
 $a_7 = h_{41}; a_8 = h_{42}$

$m_1 = h_{11} x_1 + h_{12} x_2 + e_1$   
 $m_2 = h_{21} x_1 + h_{22} x_2 + e_2$   
 $m_3 = h_{31} x_1 + h_{32} x_2 + e_3$   
 $m_4 = h_{41} x_1 + h_{42} x_2 + e_4$

Ideal value of  $m_1 = 50 + 10 = 60$

$V = 10 = x_1$   
 $I = 5 = x_2$   
 $h_{11} = 5$   
 $h_{12} = 2$



That means what I am trying to say is that suppose I have a very simple circuit say I have got a very simple circuit this we have already this kind of experiment let us suppose I have put an ammeter here this is an ammeter and this is an resistance and this is a very simple circuit there is nothing in it V, R. So I have put a resistors so let us say this is 100 volt and let us say this is 10 volt so 10 ohm if I put an ammeter here so then if I now measure this current repeatedly but this ammeter I will not always get 10 I will get obvious sometime 10.1 probably 9.92 sometimes may be 10.05 sometimes may be 9.938 whatever it is but probably know never I will get exactly 10m.

Why this I am getting these values instead of 10 see this is the ideal value but we are never getting ideal value we are getting some erroneous value that erroneous value we are getting because of the inherent of this inherent error of this and this inherent error is actually random in nature. So if I say that this error is let us say if I say that this meter has got an total error is let us say  $\pm 1\%$  that does not mean that this meter will always give  $+1\%$  or  $-1\%$  error this 1 is say that it is  $\pm 1\%$  this is actually the maximum value.

But in practice whenever it will measure some quantity it will give some error which is random in nature within this  $\pm 1$  range. So then therefore what is happening is that this measured value will have some so then ideal value + some error here so and this errors are actually random in nature so then therefore we know our lab experiments when we do take some measurement by it utilizing some measure we usually what we do we take the repeated measurement of the same quantity and then we take the average why do we take the average?

Because the errors in errors are in are basically random in nature so then therefore sometimes the sometime this errors are positives and sometime this errors are negative so then therefore if I to take the average of some repeated measurement this errors will cancel out each other so then therefore we would be so then therefore this average value would be closer to this ideal value but the point to be noted here is that this any meter  $m$  will give the ideal value will say  $x_{true} + \text{some error}$  and this error in some random in nature.

So then therefore here in this case we can say now basically this is the true value that unfortunately I do not know what is this value but what this meter will give that this quantities  $+ \text{some error } e_1$  similarly  $m_2$  will also give some error  $e_2$   $m_3$  will also give some error  $e_3$ ,  $m_4$  will also give some error  $e_4$ . So this is the measurement model so what we have got? We have got now essentially for this case we are taking a very simple case to develop the theory.

So what we have got? We have got a very simple circuit and we are got 2 unknown quantities which you call them as  $x_1$  and  $x_2$ . To estimate this unknown quantities we are utilizing 4 meters and we are now writing down the expression of the measured quantities measured by this 4 meters so when we write the expression we are actually writing the expression of the ideal quantity ideal value which this meters will record if everything had been ideal but in actual life we note that this meters will actually give us a measured value which is equal to the ideal value  $+ \text{or} - \text{some error} + \text{or error}$ .

So here when you see  $e_1, e_2, e_3, e_4$  this  $e_1, e_2, e_3, e_4$  they can take either positive value or negative value which you do not know this all random in nature. So then therefore the measured value  $m_1$  would be this true value  $+ \text{error}$  measured value  $m_2$  would be the true value  $+ \text{error}$  measured value out /  $m_3$  is the true value  $+ \text{error}$  measured value of  $m_4$  would be true value  $+ \text{error}$ .

So now our task is to estimate  $x_1$  and  $x_2$  from this measured quantities  $m_1, m_2, m_3, m_4$  such that the effect of this random errors  $e_1, e_2, e_3, e_4$  would be minimized so this will continue from the next lecture thank you.