

**Computer Aided Power System Analysis**  
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**Module No # 08**  
**Lecture No # 37**  
**Line Outage Sensitivity Factor (Contd.)**

Hello welcome to this lecture on the course of computer aided power system analysis so far we have been discussing this line outage power flow method. Now in the last lecture we have seen that how the change in the current flow over an existing line can be actually computed if some other line is outaged but now as we have already discussed at the starting. When we have actually started discussing about this sensitivity factors what we actually want that what would be the change in real power flow over any line if some line is gone.

So now today we would be looking at that how can we calculate the change in real power flow over the existing line if some other line is outage. So for that let us look at our last expression so this is the last expression we have obtain in the last class. So here  $I_p$  so here basically  $P_{pq}$  is the line which is existing line in this circuit  $m-n$  is the line between bus  $m$  and  $n$  which has been outaged and  $Z_b$  and  $Z_c$  these are all the impedance of the individual lines.

And we have already very much we have actual stressed at this point. And the other elements are nothing but the element of the bus impedance matrix. Now we as we already know that we can calculate the bus impedance in matrix from the knowledge of the bus admittance matrix. So then therefore all this quantities at the right hand side are essentially basically dependent on the line parameter so which are already know so then therefore these particular ratio is actually constant quantity.

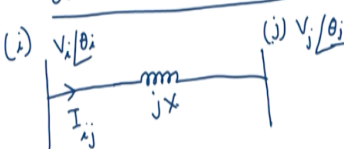
Now here obviously for obvious reason we can see that has all these impedance are essentially a complex quantities so then therefore these ratio would be essentially a complex quantity but then here what you are taking about we are talking about the change in the line power change in the line current flow so then obviously we are probably saying here that what would be the change in the line current flow over the line between buses  $b$  and  $q$  that is essentially the complex current over the line between bus  $p$  and  $q$  due to outage of the line between bus  $m$  and  $n$ .

Now we do is we now apply the assumptions of the DC load flow we have already discussed what is mean by DC load flow. So in that DC load flow we said that we are take the assumption that all the bus voltages are equal to 1.0 per unit all the bus voltage angles are approximately equal to 0 they are not exactly equal to 0 but they are very close equal to 0 for example let us say 1 degree, 2 degree, 4 degree, 5 degree or even let us say -1 degree, -2 degree, - 5 degree etc.

So then therefore they are reasonably close to 0 so then therefore theta i – theta j also reasonably equals to 0 this is an assumptions and we also neglect all resistances of the lines so if I now apply these assumptions then what we get is as follows.

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Under DC-load flow assumptions:



$$P_{ij} = \frac{V_i V_j}{X} \sin(\theta_i - \theta_j)$$


$$\approx \frac{\theta_i - \theta_j}{X}$$

as  $|V_i| = |V_j| = 1$   
and  $\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$

$$I_{ij} = \frac{V_i \angle \theta_i - V_j \angle \theta_j}{jX}$$

$$= \frac{(V_i \cos \theta_i - V_j \cos \theta_j) + j(V_i \sin \theta_i - V_j \sin \theta_j)}{jX}$$

$$\approx \frac{\theta_i - \theta_j}{X} \left[ \begin{array}{l} \text{as } \cos \theta_i \approx 1; \cos \theta_j \approx 1 \\ \sin \theta_i \approx \theta_i; \sin \theta_j \approx \theta_j \end{array} \right]$$



So now what we are trying to do is under DC load flow assumptions now suppose I do have a bus i and bus j so it is bus i bus j so we are now doing all these calculations on this low assumptions so I do have an bus in and bus j and between bus in bus j is the line that because we are neglecting all the resistances so then obviously this line would be represented by out inductance only so let us say this is jx and this is Vi angle let us say theta i and this is Vj angle theta j so we know that Pij is actually Vi Vj / x sin we should use capital X here that is the standard notation we use so we should use capital X actually.

So capital X so theta i – theta j now we have already assume that Vin Vj = 1 and theta i – theta j is almost equal to 0 so as so then therefore it turns out to be theta i – theta j / x as according to assumption Vi = Vj = 1 and sign theta i – theta j almost equal to theta i – theta j this is due to the

due to our assumptions that  $\theta_i - \theta_j$  is almost equal to 0. So under DC load flow equation this is the expression of  $P_{ij}$  now on in this if I now calculate what is the current  $I_{ij}$  so now what is the expression of current?

Current flow this is of course  $V_i \angle \theta_i - V_j \angle \theta_j / jx$  so it is  $V_i \cos \theta_i - V_j \cos \theta_j + j V_i \sin \theta_i - V_j \sin \theta_j / jx$ . Now here again we apply that  $V_i = V_j = 1$  so  $V_i = V_j = 1$  now  $\theta_i$  and  $\theta_j$  is almost equal to 0 therefore  $\cos \theta_i = \cos \theta_j$  also = 1 then therefore this term vanishes and here this  $j$  cancels out and  $V_i$  this is 1 and  $\sin \theta_i$  is almost equal to  $\theta_i$  and  $\sin \theta_j$  is almost equal to  $\theta_j$ .

So then therefore this also trans out to be  $\theta_i - \theta_j / x$  here what we are doing here is basically as  $\cos \theta_i$  is almost equal to 1 and  $\sin \theta_i$  is almost  $\theta_i$  and  $\cos \theta_j$  is almost equal to 1  $\cos \theta_j - \sin \theta_j$  they are equal to 0 and  $\sin \theta_j$  is almost equal to  $\theta_j$  so this should be  $i$ . So then therefore what we can see now so from this expression and this expression we can see that under DC load assumptions the expression of  $P_{ij}$  and  $I_{ij}$  are just the same.


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Under DC-load flow assumption, expressions of  $P_{ij}$  and  $I_{ij}$  are the same.

Under DC-load assumption

$$\frac{\Delta P_{pq}}{P_{mn}^{(0)}} = \frac{x_{pq}}{x_{mn}} \left[ \frac{(x_{pn} - x_{pm}) - (x_{qn} - x_{qm})}{x_{Th,mn} - x_{pq}} \right] \rightarrow \text{LOSF}$$

$x \rightarrow$  reactances of individual line  
 $\checkmark \rightarrow$  Element of the  $[B]^{-1}$  matrix

$$P_{pq}^{(new)} = P_{pq}^{(0)} + \Delta P_{pq}$$


So you note that we arrived that under DC load flow assumption expression of  $P_{ij}$  and  $I_{ij}$  are the same. So then therefore now if I now apply DC load flow assumptions in this expression so then what will happen all this  $z$  that is all this impedance is should be replaced by they are corresponding reactance's  $\Delta P_{pq}$  will be replaced by  $\Delta P_{pq}$  and  $I_{mn}^{(0)}$  could be replaced by

$P_{mn0}$  because the expression of  $I_{pq}$  and expression of  $P_{pq}$  is the same expression of  $I_{mn}$  would be equal to the expression of  $P_{pn}$ .

And here under the DC load flow assumptions we would be actually be neglecting the resistances so then therefore everything would be replaced by their corresponding imaginary values. So then therefore we can write down under the DC load flow assumptions and which we are writing so then therefore we say that under DC load flow assumption we can write  $\Delta P_{pq} / P_{mn0}$  would be equal to  $X_b / X_c$  into  $Z_{pn} - Z_{pm}$ .

So it is it would be  $X_{pn} - X_{pm} - X_{qn} - X_{qm}$  and this is  $X_{\theta m}$  Thevenin -  $X_b$  so is in the  $X$  again we note that these this and this they are reactance's of individual line and this are element of the let say  $B$  dash we have already seen it  $B$  dash inverse matrix. So now here we have got 1 complete expression so where it is only dependent upon the line parameter. So this expression is called this expression is  $B$  this expression inside this box it is the line outage sensitivity factor for a single continents.

So we have obtained that what is the so again we can see that this line outage sensitivity factor can be computed very easily if we just know the line parameters of the system so then therefore this outage factors can be computed  $(\cdot)$  (13:13) and they can be stored in the computer memory so then therefore whenever any line is outaged or whenever we would like to simulate that what would be effect of this outage of this line on the other existing lines.

So we can simply take that what is the value of the existing power flow over the line which is to be outaged and then simply multiply that existing power flow with this corresponding sensitivity factor to arrive at the change in the power flow over any other line. So obviously from here if I know this  $\Delta P_{pq}$  so then therefore we know that this  $P_{q}$  so then from here also you can say that  $P_{pq}$  new would be of course it would be  $P_{pq}$  old +  $\Delta P_{pq}$ .

So then therefore once we calculate this value by utilizing this LOSF after that we can very easily calculate what would be the value of new  $P_{pq}$ . Now let us see that what would be that what is the effectiveness of this method I mean does this method gives lot of error or what is the error. So for that now let us look at IEEE 14 bus system we have already shared that what is the data on this.

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### Line outage analysis in IEEE-14 bus system

Line no.	Pline (ori)	For outage of line no. 18				For outage of line no. 2			
		LSOF (2)	$P_{18}$	Pline (cal)	Pline (ACLF)	LSOF (6)	$P_2$	Pline (cal)	Pline (ACLF)
1	1.6262	-0.0363	-0.0777	1.6290205	1.6309	1.0113	0.7464	2.381034	2.4451
2	0.7464	0.0351	-0.0777	0.7436727	0.742	—	0.7464	—	—
3	0.8655	-0.0294	-0.0777	0.8677844	0.869	0.1721	0.7464	0.993955	0.9987
4	0.5314	-0.0682	-0.0777	0.5366991	0.5364	0.3597	0.7464	0.799880	0.7986
5	0.3662	0.0574	-0.0777	0.36174	0.362	-0.4845	0.7464	0.727830	0.7256
6	-0.358	-0.0363	-0.0777	-0.355179	-0.3545	0.1716	0.7464	-0.22991	-0.2352
7	-0.7068	0.5224	-0.0777	-0.747390	-0.7442	0.4944	0.7464	-0.3377	-0.3481
8	0.2689	-0.4868	-0.0777	0.3067244	0.3014	-0.0258	0.7464	0.288157	0.2815
9	0.1063	-0.1923	-0.0777	0.1212417	0.1192	0.0101	0.7464	0.113838	0.1113
10	0.2893	0.6604	-0.0777	0.2379869	0.2433	-0.0288	0.7464	0.267803	0.272
11	0.1156	0.9961	-0.0777	0.038203	0.0351	-0.0166	0.7464	0.103209	0.1046
12	0.0852	-0.0857	-0.0777	0.0918589	0.0925	-0.0024	0.7464	0.083408	0.0842
13	0.2005	-0.325	-0.0777	0.2257525	0.2277	-0.0086	0.7464	0.19408	0.1952
14	0.0025	-0.0125	-0.0777	0.0034713	0.0024	0.0048	0.7464	0.006082	0.0024
15	0.2664	-0.4743	-0.0777	0.3032531	0.2991	0.021	0.7464	0.282074	0.2791
16	0.0123	-1.0039	-0.0777	0.090303	0.0904	0.0168	0.7464	0.024839	0.0234
17	0.0655	0.4085	-0.0777	0.0337596	0.0329	0.011	0.7464	0.073710	0.0719
18	-0.0777	—	-0.0777	—	—	0.0167	0.7464	-0.06523	-0.0666
19	0.0233	-0.1021	-0.0777	0.0312332	0.0305	-0.0025	0.7464	0.02143	0.0224
20	0.0857	-0.4102	-0.0777	0.1175725	0.1192	-0.011	0.7464	0.077489	0.0794

Now what we are now doing now we are now here showing the results corresponding to the outage of two lines one is line number 18 and one is line number 2. What is line number 2 and 18 would be available in the circuit diagram as well as the from the data. Now this is the original power flow in this P line ori is the original power flow all this lines are line number 1 to 20 and this is the corresponding LSOF actually should be LO line outage sensitivity factor it should be LOSF.

And this are the corresponding values now P18 now we are here talking about that this outage of line number 18 and from this column we can see that line number 18 has got an pre outage power level =  $-0.0777$  so this is the value. So when I multiply this with this and then add with the original so now after that what we do is simply now multiply this into this so then we get this delta Pq1 and then we add it with the to the original value after that we get this new calculated values of the lines new calculated values of the power flow over the existing lines because of the outage of line number 18.

Here if we just note that here we have shown it dash with because this particular line is outage so then obviously there is no sense to calculate the new power flow over this line so you have not calculated so now this is the estimated value of new power flow over the existing lines obtained by the method of line outage sensitivity factor and then complete AC power flow analysis has been carried out in which line number 18 has been removed from the data file.

So then therefore one complete load to analysis has been carried out in which there are all 14 buses all loads are everything is fine I mean all generation pattern all loads everything is there only thing is that now instead of having 20 lines in the data file now we will we are only having 19 lines where this particular line number 18 has been removed from the data file and from that and so then after doing this load flow we have got this new power flow over the lines right.

So after we do the load flow by removing line number 18 from the data file we get this power flow over the lines. So we can see that these values are reasonably close to each where each and every close where all these values are reasonably close to each other. So then therefore there is not really much of any or rather these errors are acceptable errors for the power present engineering it is acceptable.

So then this method give us a reasonably good amount of or rather reasonably accurate estimation. Similarly when you are similarly also another experiment has been done where line number 2 has been outaged so we can see that for line number 2 the original power flow is 0.7464 so this is the 0.7464. So this is the original power flow and this is the LOS again this is this LOSF now please note that when we are changing the line to be outage so then obviously the corresponding LOSF values of also would be changed.

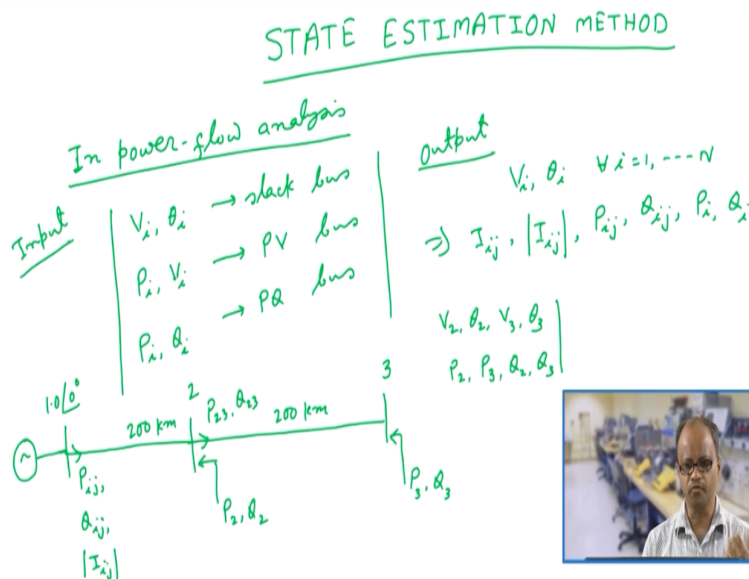
So then therefore this LOSF value would not be equal to this LOSF values so we have got this LOSF values so we have got this original power flow so then from that we calculate the what is the change in the power flow over an existing lines so we get this calculated values obtained by the method available LOSF and then we have also done the load flow analysis of this system by now removing line number 2 but now when we are removing line number 2 we have actually also reinstated line number 18 in the data file.

So here also in this case we have down the AC load flow for 19 lines are or rather done the AC load flow of this entire system having only 19 lines in this system so now we have got two values that this is the most accurate values at the last column this is the most accurate value and at the last but one column this is value of the power flow obtained by using this LOSF approach again we can see that these two quantities are quiet close to each other.

So then therefore this method also gives us a reasonably good estimate of the new power flow over all the existing lines so when a particular line is outage. So from this analysis so we can see that this line outage so then basically by this line outage sensitivity factors as well as this generator outage sensitivity factors they to gives us a reasonable amount or rather reasonable confidence in the estimated values because we have done through experiment we have already shown that the computer simulation that the values of obtained by utilizing this methods that this out is sensibility factors they are pretty close to the values which are obtained by utilizing actual AC power flow method.

So this completes our discussion of the contingency analysis so now we would like to discuss now we now like to start discussion about our state estimation method. So now we start our discussion about so now we start a new method or rather new topic state estimation technique.

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Now what is state estimation method and why it is necessary so this introduction we would like to discuss in this lecture to appreciate the necessity of this state estimation method let us we can revise it basic concept of so power flow analysis what we do the power flow analysis what we do we have certain inputs and we do calculate certain outputs. So now what are the inputs are  $V_i$ ,  $\theta_i$  at this slack bus and then  $P_i, V_i$  at the PV bus and  $P_i, Q_i$  at the PQ bus right so these are the input quantities and what are the outputs?

Outputs are essentially  $V_i \theta_i$  at all buses for all  $i = 1$  to  $N$  and from that we can calculate  $I_{ij}$  then magnitude of  $I_{ij}$  and then  $P_{ij}$  that is the line power flow  $Q_{ij}$  and anything and everything injected power at all the other buses in fact these are already given any injected reactive power at the buses where it is not given that means we can calculate anything and everything whatever I would like to know about the system.

So once we know these values that is this magnitude and angle at each and every bus with the help of we can essentially calculate anything and everything about the system. Now suppose we want to monitor this system because every time because essentially on a power system the loading as well as the generation level in the system always changes periodically at least some loading level changes periodically and to respond to that late loading levels so then obviously this generator generation pattern also changes.

So then therefore because this loading level keeps on changing on as well as to track of that loading level as this particular generation level also keep searching. So then therefore the power flow over all the lines keeps in changing now when it is keeping on changing so when this power flows are keeping on changing so therefore there may be chance at some point of time the power flow over any line or rather the we can say that this magnitude of current flow over any line that may cross the limit. Now you that crosses the limit so then obviously we have to take some corrective action.

Please note that here in this case we are not even talking about any kind of contingency we are not talking about any kind of outage condition we are simply talking about this steady state condition where no element has been outaged all the elements are in action they are already in place but because of the change in loading as well as generation pattern the line power flow as well as the current flow over the existing lines they can keep on changing.

So then therefore there can be chance that at some of time the power flow or current over any existing line that can go beyond its maximum limit. So if it goes beyond the maximum limit so then that particular line was in danger and if that condition persist for a long time so then this line may be permanently damaged so then therefore it will incur huge cost (25:55). So to prevent that possibility we need to monitor the system on a continuous basis.



Now obviously we at the first glance we can say that well if I know all these quantities so then therefore every time or rather on a let us say continuous basis we can do this power flow analysis and then once we do this power flow analysis on this we can calculate all these quantities so then obviously should be able to keep track of the system continuously. But then unfortunately for doing this power flow analysis all these quantities are absolutely required if any single quantity here is missing this power flow analysis cannot be done that is the first part.

Second problem is that we are here talking about that this injected power at all these buses for example injected power at all this  $P_v$  bus as well as the  $P_q$  bus and the voltage magnitude at the  $P_v$  bus now what happens that if I wish to do this calculation. These calculations would be done at some central computer station at some particular location. So then therefore to that location all this required data need to be given.

Now it may or may not be possible to install meters at all these points to always keep on measuring that what is the injected power at each and every bus. Suppose I do have a simple 3 bus system and I wish to do a load flow analysis so I have a generator so this is my slack bus so here let us say I know it and this is a let us say this is the load bus so this is bus 2 this is bus 3 so this is I have got  $P_3$   $Q_3$  in fact they are direction to be reversed.

So I have got  $B_3$  and  $Q_3$  here also I have got  $P_2$  and  $Q_2$  so then therefore if I wish to do load flow analysis of this system I need the values of  $P_3Q_3$ ,  $P_2Q_2$  and once I know this I would be able to do this analysis. But possibly instead of now possibly instead of measuring all these quantities in probably too much is here as far as this analysis is concerned especially if my let us say men's server station or men control center is located somewhere here near the generator possibly should be easier for us to measure the power flow over the line measure the reactive power flow over the line and measure the magnitude of current flow over the line.

So in that case what will happen if my control center is located here so I do not have to really rely on these values which will have to be communicated to me rather I would get these values at a local station. Of course here we are simply taking a very elementary system just to point out the fact that some times or in let us say several occasions it is much easier to measure the line power flow as well as the current magnitude over the line.

Now basically so now actually what happens when we do a load flow analysis now here in this system is essentially there are 4 unknowns  $V_2$ ,  $\theta_2$ ,  $V_3$ ,  $\theta_3$  so then they are therefore in the so then therefore to solve for this 4 unknowns we need 4 equations rather 4 quantities. So in the normal load flow analysis our known quantities are  $P_2$ ,  $P_3$ ,  $Q_2$ ,  $Q_3$  so I have got 4 equations 4 unknown we solve.

Now if I wish to solve for this 4 unknown it is not always necessary that I should have only these 4 known quantities I can have also some other quantities which are expressible in terms of this unknown quantities and this quantities and let us say out of this 4 equations 3 equations would be I would be getting from here right and then again also then possibly suppose for example that if I have the means of which means that I can communicate some data from this center to this center say that this distance is 200 kilometer let us say this distance is also 200 kilometer.

So then therefore I would like to say for example in this case that well I do have the means to communicate some data from this center to this center but unfortunately my communication network from bus 3 to bus 1 is not still existing I cannot really communicate that data is that long distance so then therefore if I do not have that so then therefore I cannot really put a meter over here then simply keep on sending what is the value of  $P_{23}$  and  $Q_{32}$  to do this calculation.

Rather what I can do is that I can possibly measure this values let us say  $P_{23}$  or  $Q_{23}$  or whatever so then therefore what I get 4 quantities from here let us say anyone quantities of from here and then I can possibly try to solve for it that is the one problem. So then therefore sometimes instead of these I can getting these injected values. Sometimes or rather in many occasion so it is much more convenient to get some other measured values for the purpose for of analyzing the system. So we stop here today now and in the lecture we will also continue with this basic concept of this state extension method thank you.