Computer Aided Power System Analysis
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Module No # 07
Lecture No # 35
Line Outage Sensitivity Factor

Hello welcome to this lecture on the particular course on computer aided power system analysis in the last lecture we have discussed about the generation outage sensibility factor. In this lecture we will start talking about the line outage sensitivity factor. Now to before we discuss about what is mean line outage sensitivity factor ahh rather how to calculate this particular line outage sensitivity factor we should first look into the concept of Thevenin equivalent circuit as observed from any particular bus in a large power system.

So first let us look at the concept of Thevenin equivalent circuit so we have Thevenin so you are talking about Thevenin equivalent circuit.

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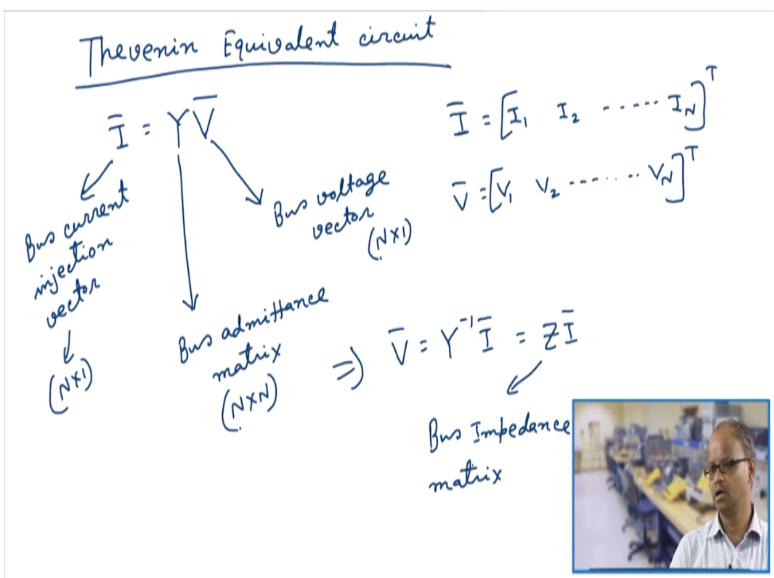

So Thevevin equivalent circuit now we know from our basic power system analysis whatever we have done that we know that I vector I = matrix Y into vector V vector I is the bus current injection vector. Vector V is bus voltage vector and this is bus admittance matrix now if there is n number of buses in my system so then in that case this is N cross 1 this is N cross N and this is N cross 1 and we note that we just recollect that I vector is essentially I1, I2, IN transpose T, V is V1, V2, VN and transpose T.

Here we also have to note that all to this currents I1, I2, IN V1,V2, VN they are all essentially complex quantities but taking a rather doing an abuse of notation we not putting in over power over any of this quantities because we have already put an over power to denote I and V are two vector anyway. So now from this we can write down that V = Y inverse into I or in other words V = ZI where Z is the recall bus impedance matrix. Now suppose due to some reason this vector I is being part out by a vector delta I.

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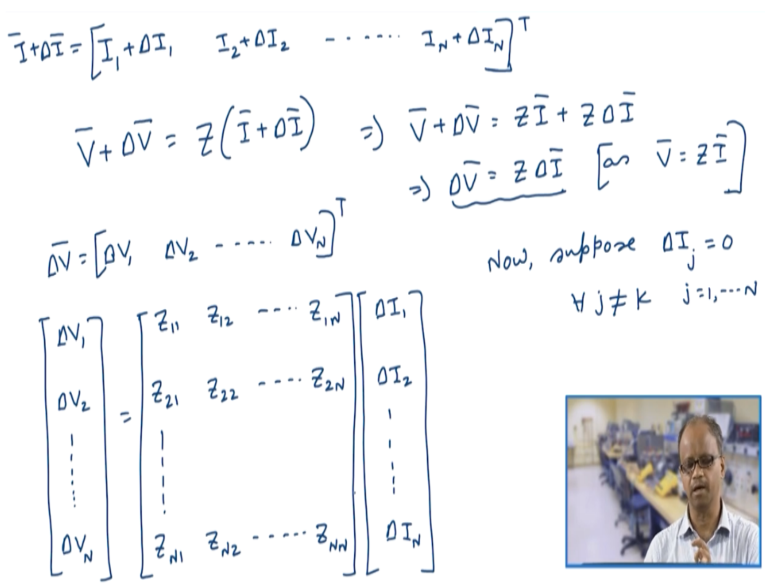

So that means the injected current at bus 1 is I1 is being part out from I1 to delta I1. Similarly injected current at bus 2 is being part out from I2 to delta I2 and so on and hence so forth and injected current at bus N is being part of from IN to delta IN right. So then therefore so then if this is part out so then we can say that this is nothing but I + delta I vector why this particular delta I vector is obvious that this delta I vector is nothing but delta I1, delta I2, delta IN transpose.

Now because this I vector is being part out from its original value to I + delta I so then therefore as a result this V vector will also be part out from its original value to V+ delta V so then therefore we can write down that V + delta V = Z into I + delta I. Please note that this V is change from V to V + delta V because of this change of I from I to I + delta I. So from this we know that V + delta V = Z into I + Z into delta I or in other words delta V = Z into delta I as V = ZI.

So then therefore I can write down of course here in that of course here that delta V vector would be is nothing but delta V1, delta V2, delta VN transpose so then now we can write down this relationship as delta V1, delta V2 to delta VN in that would be equal to here it is delta I1, delta I2 to delta IN. Please again note that all this quantities are complex quantities and similarly Z11, Z12 to Z1N, Z21, Z22 to Z2N and so on and hence so forth ZN1, ZN2 to ZNN.

Please note that this elements Z11, Z12 up to ZNN etc all this are also complex quantities but just to ease of notation we are not putting any bar or over any of them. Now suppose that they current injection is changed only at a particular bus K so that means now suppose that for a delta Ij = 0 for all j not equal to k and j of course j = 1 to N and it has got some value at bus k which is equal to delta Ik.

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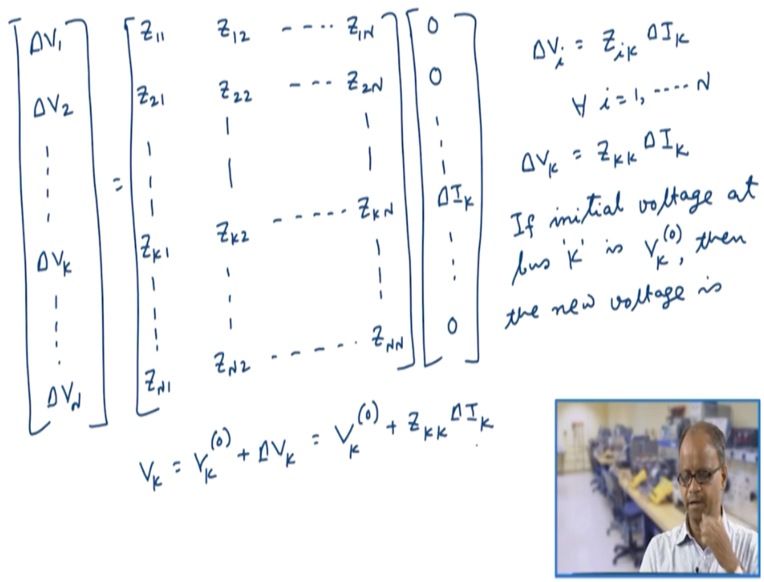

So then therefore in that case what we will happen so we can write down that delta V1, delta V2 to delta Vk to delta VN so it would be Z11, Z12 to Z1N, Z21, Z22 to Z2N, Zk1, Zk2 to ZkN and 1 to ZNN and it would be everywhere it would be 0, 0 dot delta Ik and 0. So then from here we can write down that delta Vi = Zik delta ik that is very straight forward delta Vi would be Zik into delta Ik for all i = 1 to N.

So then therefore delta Vk would be Zkk into delta Ik now so this is the change in the voltage at bus k due to the change in the injection current at bus k so then therefore if the initial voltage at bus k is Vk say 0 then the new voltage is so then this new voltage would be equal to Vk would be equal to Vk naught + delta Vk that would be equal to Vk naught + Zkk delta Ik. So now we have got this equation that Vk = Vk naught + Zkk delta Ik so we have got the equation as.

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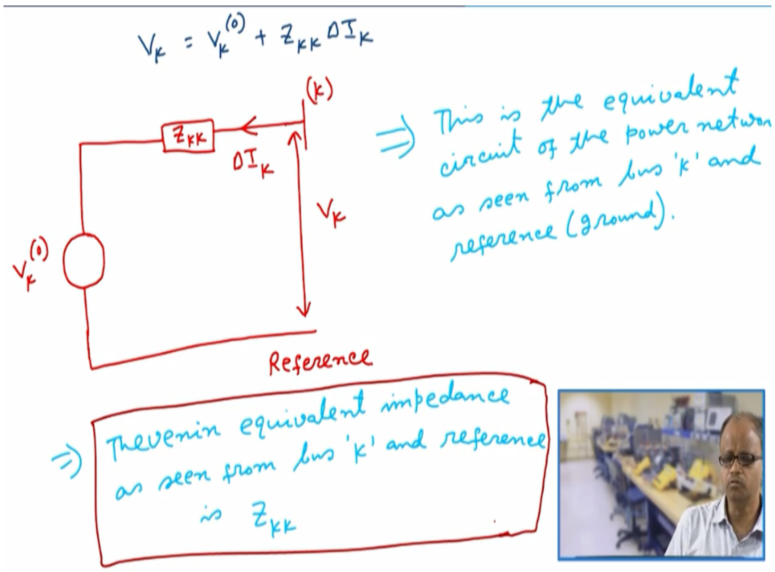

So we have got the final equation as Vk = Vk naught + Zkk to delta Ik now the question is that how do we represent this particular equation as a equivalent circuit. So now we can represent this particular equation as an equivalent circuit and this would look like something like this so we have got some source some impedance then this is bus k this is the reference and this is Vk this is Vk naught this is Zkk and the current injector is delta right.

So then therefore this particular equivalent circuit very easily very conveniently represent this particular equation. So Vk = this drop Zkk into delta Ik + this drop so Vk = this drop + this one that is fine so then therefore this is the equivalent circuit of the entire power grid as perceived from bus k and ground. So we can write down that this is the equivalent circuit so this is the equivalent circuit of the power network as seen from bus k and reference.

So reference is nothing but ground so it is ground now if I wish to find out that what is the Thevenin equivalent impedance of the entire grid as observed from bus k and ground. So then from our very basic understanding of the network theory we know that whenever we are trying to calculate the Thevenin equivalent impedance what you have to do we have to simply short circuit all the voltage sources and simply open circuit all the current sources and then simply calculate the equivalent impedance as perceived from the given two terminals.

Now in this case there is no open now in this case there is no current source there is only voltage source that is this we cannot so then therefore if we wish to calculate this equivalent Thevenin impedance so then what you have to do some can be shot it once we shot it so therefore the equivalence impedance between this and this as perceived from bus k would be nothing but Zkk so then therefore you can write down when therefore Thevenin equivalent impedance as seen from bus k and reference is Zkk.

So this is the very interesting result so this is a very interesting concept okay now the question is well we have found out the Thevenin equivalent impedance of the entire power grid as perceived from bus k and ground but whatever would be Thevenin equivalent impedance of the entire network as perceived from between two buses let us the bus m and bus n so let us look at that. So now we are now trying to find out that what would be the Thevenin equivalent impedance of the power grid as perceived from bus m and bus n.

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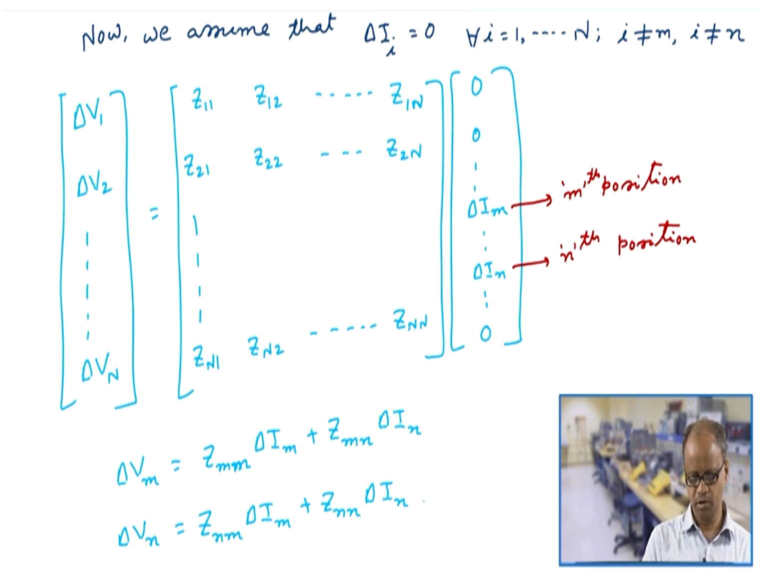

So towards that goal we assume that now we assume that delta I = 0 for i = 1 to capital M and i is not equal to m and i is not equal to n. So therefore what we have simply trying to say that only at bus M and bus N there are some power turbation in the injected current at all the other buses there is no power turbation in the injected currents.

So then in that case what we have is we can write down the delta V1, delta V2 up to delta VN and then Z11, Z12 to Z1N, Z21, Z22 to Z2N this is ZN1, ZN2 to ZNN and we have 0, 0 something called delta Im and let us say then delta In and then all are 0. So this is the mth position and this is the nth position and position so then from this what we can write down about delta Vm and delta Vn so we can write down about delta Vm and delta Vn so we can write down as delta Vm = Zmm delta Im + Zmn into delta In.

And similarly we can write down delta Vn = Znm into delta Im + Znn into delta In so these are the power turbation due to the so then these are the power turbation of the voltage due to the power turbation in the injected currents at bus m and n.

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$$V_m = V_m^{(0)} + \Delta V_m = V_m^{(0)} + Z_{mm}\Delta I_m + Z_{mn}\Delta I_n$$

$$V_n = V_n^{(0)} + \Delta V_n = V_n^{(0)} + Z_{nm}\Delta I_m + Z_{nn}\Delta I_n$$

$$V_m = V_m^{(0)} + (Z_{mm} - Z_{mn})\Delta I_m + Z_{mn}(\Delta I_m + \Delta I_n)$$

$$V_n = V_n^{(0)} + Z_{nm}(\Delta I_m + \Delta I_n) + (Z_{nn} - Z_{nm})\Delta I_n$$

So then therefore we can write down Vm final Vm is as usual Vm0 + delta Vm so then Vm0 is the initial voltage + Zmm into delta Im + Zmn into delta In. Similarly Vn = Vn0 that is initial voltage of bus n + delta Vn that would be equal to Vn0 + Znm delta Im + Znn delta In. So we have got these two equations now we will do some simple algebraic manipulations of these two equations as follows right that Vm = from here Vm0 + Zmn – Zmn into delta Im.

So what I have done we have done I am simply subtracting Zmn into delta Im now because I am subtracting Zmn into delta Im so then I must also add it and if I add it so then what I will get is so I get Zmn into delta Im + delta In this is very simple what i have done I have simply subtracted Zmn into delta In and I have added Zmn into delta Im so these two cancel out and the original system remains same.

Similarly Vn would be equal to Vn naught + here what we do here is first Znn – Znm into delta Im so what I have done we have simply subtracted Zmn into delta Im so that therefore I have to add this Zmn into delta In to make this ahh I mean (()) (24:13) system so then therefore if we do that so we have got Znn into delta Im + delta In so this is the equation. So now we have got these two equations of course this equation comes from this.

Now how do we represent this two equations can we represent this two equations so in an equivalent circuit so we would now like to represent this equations in equivalent circuit so when we represent this equations in an equivalent circuit what we get is so we.

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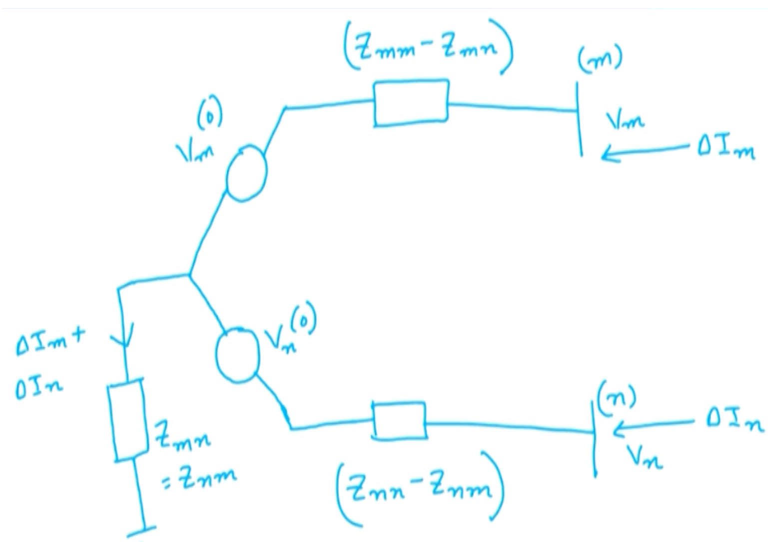

So what we do is so we say that it is an bus m this is bus n so this is bus m this is bus n and we have here so you have the voltages is Vm here this voltages is Vn injected current here is delta Im injected current here is delta In and this current goes from here this current goes from here they meet together so then here the total current is delta Im + delta In and this impedance is Zmn = Znm.

Please note that because why was matrix is a symmetric so then therefore the inverse of it so that is nothing but the bias impedance matrix will also be a symmetric matrix so as a result Zmn would be is equal to Znm this would be Vm0 this would be Vn0 this would be Zmm – Zmn and this would be Znn – Znm. So this would be the equivalent circuit so from this equivalent circuit what we get that Vm = delta Im into Zmm – Zmn last Vm naught so let us see so Vm = Vm naught + delta m into Zmm - Zmn + Zmn into delta Im into delta In.

And Vn = Vn naught + Znm delta Im + this so then this circuit faithfully represents these two equations so then this is nothing the equivalent as perceived from bus m. So this is the equivalent circuit as perceived from bus m and n. Now from here i can find out practically everything now if I want to find out the equivalent impedance as perceived from bus m to ground so what I get so first thing what we have to do you have to make this two sources short circuited so once you do this short circuit and then if I wish to find out that what is the equivalent Thevenin impedance as perceived from bus m and ground.

So then what it would be this would be Zmm – Zmn + Zm Zmn so this and this cancels out so Zmm remains so then therefore from this the equivalent Thevenin impedance between bus m and n sorry bus m and ground would be = Zmm which we have already seen. Similarly as observed from bus m to ground we can find out that the Thevenin equivalent impedance would be Zmn – Znm + Znm so then this particular Znm cancels out and so then therefore the Thevenin equivalent impedance as perceived from bus n and ground would be = Znn which is absolutely correct as we have already observed.

Now what would be the Thevenin equivalent impedance as perceived from bus m to n so then for that what we have to do you have to simply make our journey from bus m and then we have simply finish our journey at bus n and then we have to simply add and then we have simply see that I mean what are the impedance we get so then when we start our journey at bus m and then finish our journey at bus n we find that this impedance and this impedance are in series so then therefore the Thevening equivalent impedance as perceived from so Thevenin equivalent impedance as seen between bus m and n is this is denoted as Z Thevenin mn that is given by Zmm – Zmn + Znn – Znm.

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$$\Rightarrow \boxed{Z_{Th,mn} = Z_{mm} + Z_{nn} - 2Z_{mn}} \quad \boxed{\text{as } Z_{mn} = Z_{nm}}$$

So then therefore as Zmn = Znm so then therefore Z Thevenin between bus m and n is given by Zmm + Znn – 2Zmn so this is a very powerful and interesting result so then therefore you can find out that once you know the bus admittance matrix of a system then we will simply take the inverse we will find out the bus impedance matrix and once we point out the bus impedance matrix from that bus impedance matrix elements we can readily calculate the Thevenin equivalent impedance of the entire power grid as perceived from bus m and n.

Of course here we have we have actually written down this particular expression due to this fact that Zmn = Znm right and this is due to this fact that my bus impedance matrix is basically a symmetric matrix so in this lecture we have looked into the Thevenin equivalent impedance of the entire power network first we have looked into the Thevenin equivalent impedance as perceived from any particular bus to ground and then we have looked into Thevenin equivalent impedance as perceived between any two given buses. So in the next lecture we would be continuing this particular discussion thank you.