Computer Aided Power System Analysis
Prof. Biswarup Das
Department of Electrical Engineering
Indian Institute of Technology-Roorkee

Lecture - 34
Linear Sensitivity Factors (Contd.)

Hello, welcome to this lecture on computer aided power system analysis. In the last lecture we have been discussing about the linear sensitivity factors and in that context we have also discussed in detail about the very basic concept of generation outage sensitivity factor. Today, we would be looking into some another sensitivity factor followed by some other detailed discussion about GOSF.

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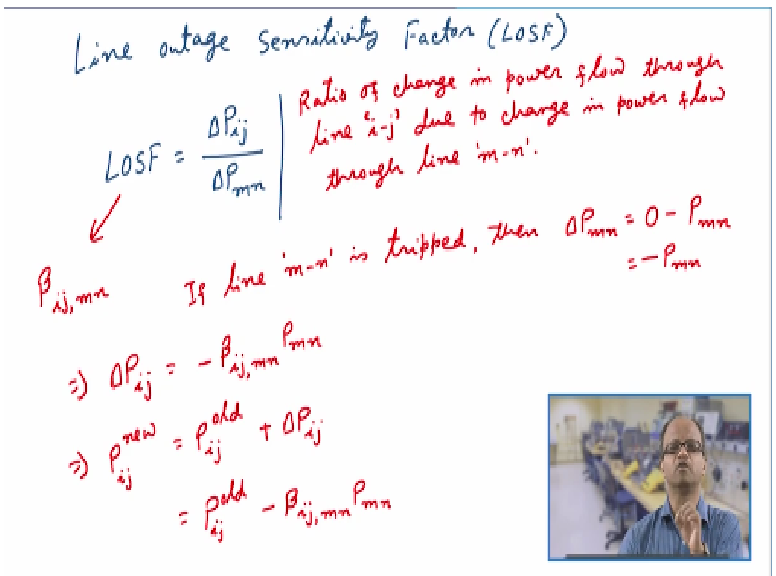

So what we do is today we first look at that line outage, first look at we said that in the last class that there are 2 types of sensitivity factor, one is called generation outage sensitivity factor and another is called line outage sensitivity factor. So today first let us look at line outage sensitivity factor, is called LOSF. Similar to GOSF, LOSF is defined as del P ij/del P mn. So what does it mean?

It basically mean it is ratio of change in power flow through line i - j due to change in power flow through line m – n. So we call it as beta ij, mn right? So ij is the change where it is taking place and m – n is the line in which this change has taken place. So now suppose I have got a line let us say between m and n and if that line is going out of order, that is if this line is getting tripped. So then therefore what would be delta P n?

So if line m – n is tripped say then delta P mn will be when it is tripped so then this, so then basically the power flow through it would be 0 because it is an open circuit and earlier its initial value was P mn. So it is – P mn, right? So this is it. So then therefore when I know this, so then therefore delta P ij would be minus of beta ij, mn * P mn and so then therefore P ij new would be P ij old + delta P ij. So it is P ij old – beta ij, mn * P mn.

So then from this expression if this quantity is known, so then therefore just by utilizing this very simple algebraic expression we can simply calculate what would be the new power flow over line i - j in case line m – n is tripped. Now obviously, if there are let us say 10 lines so then therefore we need to know that what would be this sensitivity factor each pair of line, right?

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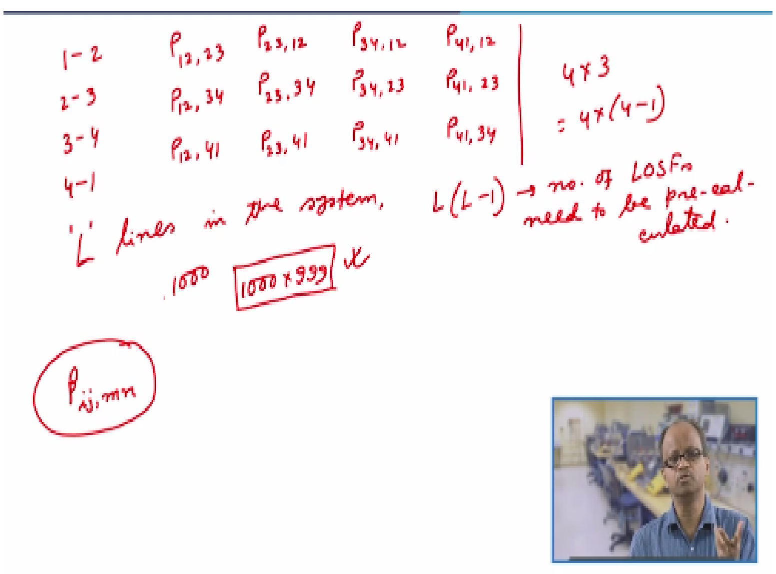

So then therefore just as an example if I got in my system let us say 4 lines and this 4 lines are 1 – 2, 2 – 3, 3 – 4 and 4 – 1, right? So I will have let us say P 12 due to 23, P 12 due to 34, P 12 due to 41. Similarly, I will have P 23 due to 12, P 23 due to 34, P 23 due to 41. Please note that P 12, 23 and P 23, 12 they are not the same. They will not be the same. We will look into this but they will not be same. They are not equal to each other, right?

And similarly we will have P 34, 12; P 34, 23; P 34, 41 and lastly P 41, 12; P 41, 23; P 41, 34. So we have all these combinations. So what would be these number of combinations. So now this combination, this is 12. So it is basically 4 * 3. It is obvious that 4 * 4 – 1. So for each of this line, so then therefore if I have got M, so then therefore if I have got let us say L lines in the system.

So then therefore total number of LOSF, I mean this particular sensitivity factors which need to be calculated is L (L – 1) number of LOSFs need to be calculated. Need to be I would say that pre-calculated. So then therefore if I have let us say 1000 lines, so then I have calculate 1000 * 999. So it is quite a large number. We may say, yes this is a quite a large number.

But as we will see in future in this course only in some lecture in the future we will see that these beta coefficients, these beta coefficients, they will basically depend on the line parameters and so then therefore if I know this line parameters and they have got a and essentially each and every of this coefficient has got one expression, very simple expression to calculate.

So then therefore it is extremely easy to code that particular expression. So then therefore calculating these many coefficients will actually take no time by using the today's computation facility. So although this number is pretty high but then basically calculating these coefficients take absolutely no time because these coefficients only depend on the line parameters.

And as this line parameters are known calculation of this parameter I mean calculation of this LOSFs are also very easy. Now today after this brief discussion about LOSF we will now go into, we now look into the detail that how to calculate GOSF. So far we have been only calculating saying that if we know this particular values of GOSF and if we know these values of LOSF then we can calculate this new values of power and etc.

But now we have to now really see that how this GOSF are to be calculated and what form they do take and until and unless we do this we will not be able to appreciate this advantage of this algorithm.

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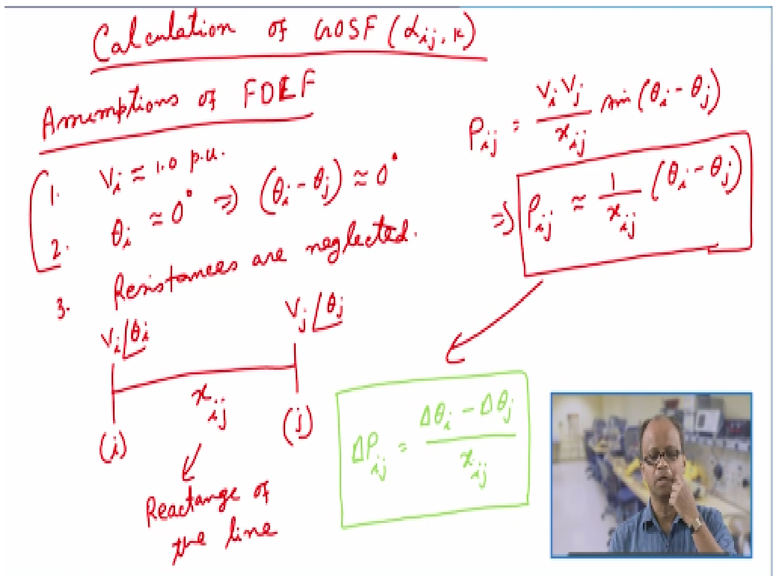

So let us look into the calculation of L ij, k. For this we have to take some, we need to take now please recollect the assumptions of FDLF. What was the assumptions of FDLF? One assumption is that V i is almost = 1.0 per unit and another assumptions was essentially that all theta i is 0 degree. So then therefore theta i – theta j is also 0 degree, right? of course they are not 0. Now here we take another assumption.

I mean when we are calculating of GOSF that all this lines are completely reactive. That is the resistances are neglected. So these are the 3 assumptions. Now with this assumption now suppose I do have a line between bus i and bus j so this is bus i, this is bus j. This has got an voltage V i angle theta i. This has got an voltage V j angle theta j. and this because this resistance are neglected, this has got an reactance x ij.

This is the reactance. This is the reactance of the line. So with this we know that P ij is well known expression V i V j/x ij sin (theta i – theta j). So then therefore I can write down now if I apply these two assumptions, if I apply these two assumptions, so then therefore I can write down P ij = V i and V j both are 1. So 1/x ij because both are 1 and this is theta i – theta j because sin theta i – theta j is almost = 0.

So then therefore sin theta i – theta j is almost = theta i – theta j numerically. So we have taken this. So we have taken these two assumptions. So we get this expression. So then from here I can write down delta P ij is actually delta theta i – delta theta j/ x ij. So then therefore from this expression if I know the value of delta theta i and delta theta j I would be able to calculate the value of delta P ij. Now, how to calculate this value of delta P ij.

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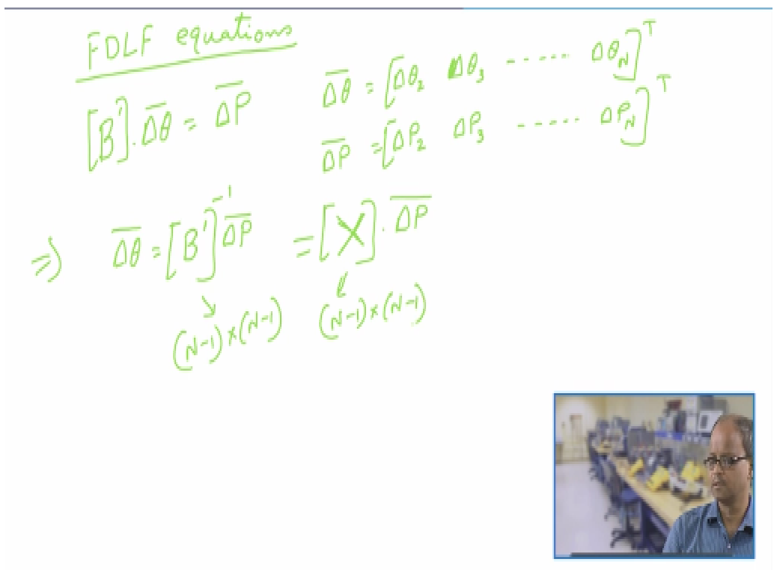

For this FDLF, if we look at FDLF equations what is this first FDLF equation? We have got this FDLF equation B dash * delta theta vector = delta P, right? B dash * delta theta vector = delta P. B dash is a matrix, we already know that this is a constant matrix. Now what is delta theta vector? Delta theta vector it is a vector, it is a vector and B dash is a matrix, it is a matrix.

So delta theta vector is delta theta 2, delta theta 3…delta theta N transpose and delta P vector is delta P 2, delta P 3, delta P N transpose. So from here I can write down that delta

P vector is nothing but inverse of this B dash matrix * delta theta vector. We do call it capital X matrix. It is a capital X matrix * delta theta vector. So we call it capital X matrix * delta theta matrix.

Now because B dash is a (N * N), so it will also be an (N * N). So then therefore it should be delta theta vector = B dash matrix * delta P matrix.

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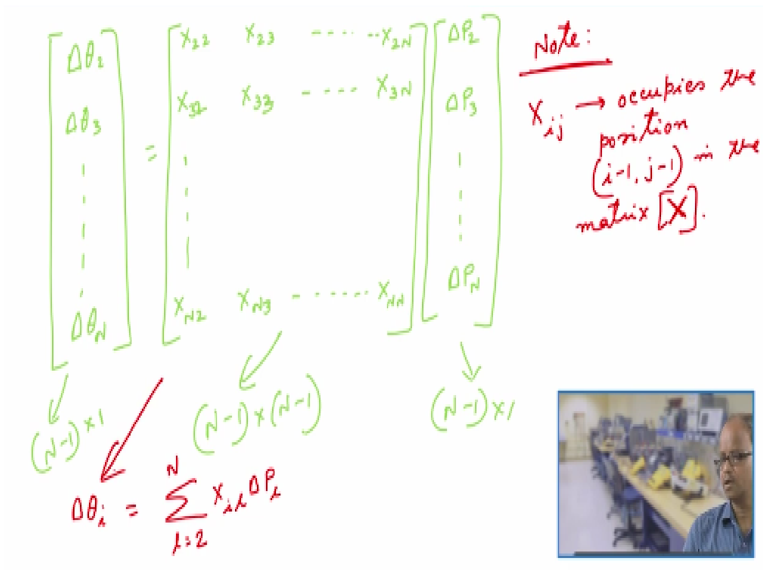

So then therefore if we just expand this. So can write down that this is X 22, X 23. Now here we are abusing our you know, abusing our notations but we will just this is X 32, X 33 and X N2, X N3, X NN. These Xs are element. Now here when we are talking about X 22, X 22 is actually occupying the place 11. X 23 is occupying the place 12. X 33 is occupying the place 22. Similarly, X NN is occupying the place (N – 1), (N – 1).

Please note that this is an (N – 1) * vector sorry (N – 1) * vector I made a mistake. This is actually, this is (N – 1) * (N – 1). This is also (N – 1) * (N – 1). So this is (N – 1) * 1 vector. This is also (N – 1) * 1 vector. So this is (N – 1) * (N – 1) vector, right? And although it is the position 11 but we are writing it X 22 just to denote that this element connects the change I mean connects the angle at bus 2 with the power injected at bus 2.

This element connects angle at bus 2 with the power injected at bus 3. And this element connects angle at bus 3 with the power injection bus 2 and so on so forth. So then therefore please note a very important note it is to avoid any sense of confusion. Notice that X ij occupies the position (i – 1, j – 1) in the matrix X. This is very important.

So then therefore from this expression delta theta i would be given by summation k = say 2 to N X ik delta P k alright? It will be X ik delta P k. So this would be known.

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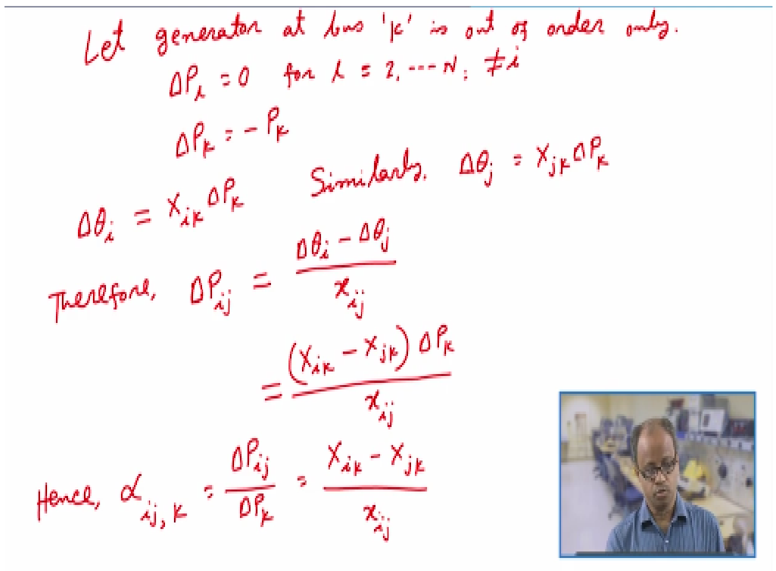

Now suppose, let generator that because we are taking this k so let us change this to be easier for to remember, easier to remember. Let us say index is l, so i l delta P i from this. Now let the generator at bus k is out of order only. So then therefore what happens? At all the other buses delta P k is 0 for k = say 2 to N but not equal to i. So all the other buses it is 0 again.

Delta P l = 0 for all l = 2 to N not equal to i and delta P k = - P k that we know. So then therefore in this column there would be only one quantity which would be nonzero, other would be zero. So right now we would be only taking this expression delta P k. So then therefore from this, so from this expression and from this we can write down that delta theta i would be = X ik * delta P k.

Because from in this expression if I keep on expanding it, right l = 2 to N only delta P k would be nonzero but all the other delta P l would be 0. So then delta theta i would be X ik * delta P k. Similarly, delta theta j would be = X ij * delta P k. So then therefore delta P ij = delta theta i – delta theta j/small x ij. So this is = X ik – X i it would be X jk delta P k/X ij. Hence alpha ij, k, what is alpha ij k GOSF.

That is the change in, so that is = delta P ij/delta P k would be = X ik – X jk/x ij.

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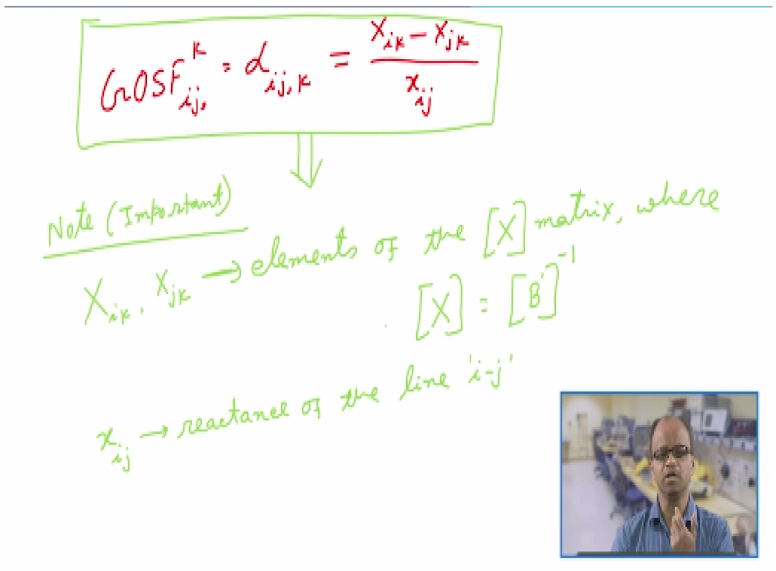

So we write again this, so therefore we have got GOSF ij k that is we denote as alpha ij, k that is = X ik – X jk/x ij. So this is the expression. If we look at this expression, now here in this expression this one, two things very important to note, very important to note. We have to note, very important, I would say it is important, I would like to mark that this is very important note.

Capital X ik and Capital jk they are the elements of the X matrix where X matrix is nothing but inverse of the B dash matrix, very important. And small x ij, please note this is capital X ij and this is small x ij. This is the reactants, individual reactants of the line ij. This is very important to note. One should not be confused that this is also the reactants. This is the elements of the matrix A. These two are elements of the matrix A.

Now here we need to note something, again. B dash matrix is a constant matrix. That we have already seen when we have discussed about FDLF. Now because B dash matrix is a constant matrix, so then therefore its inverse is also a constant matrix. So then X matrix is a constant matrix. So this all these elements are known. Because all these elements are known so then therefore X ik and X jk are known and X ij which is nothing but the reactants of the line, this is also known.

We further note that this B dash matrix is actually formed by the knowledge of the line parameters. We have already also seen it when we have discussed about FDLF. So then therefore this B dash matrix is also formed by the or rather or basically from the knowledge of the line parameter. So then therefore this capital X matrix is ultimately formed by the knowledge of the line parameters and X ij is obviously the line parameter.

So then therefore, this alpha ij, k is a constant quantity for every combination of ij and k it is a constant quantity and this constant quantity is only dependent on the line parameters. It is not dependent on any operating condition. So therefore for any given system, the moment we know the line parameters we can simply pre-calculate these values by the simple expression.

And because this expression is so simple even if these values of ij and k are very large, very large but then because this expression is very simple it is very easy to quote and so therefore it will take hardly any time for the modern day computers to pre-calculate this particular sensitivity factors and to store them. So then what we will do?

We will simply first pre-calculate the sensitivity factors and store them and after that whenever there is an outage of any generator we will simply recalculate the new power flow over all the lines by the expressions what we have already seen in the last lecture. So this is the basic philosophy. Now let us look, now the question comes into mind that what is the effectiveness of this?

That is if I do calculate my new power flow over the lines, obviously because these are dependent on some assumptions, especially of the FDLF for example we are taking the assumption that my voltages are all 1 per unit. We are taking the assumption that my angles are all almost equal to 0. We are also basically I mean we are also neglecting the line resistances, right? So then therefore it involves some approximation.

So the question comes that if we calculate our new values of line power flow by accepting this approximations, obviously those approximate values would have some error. How do they compare if I do the same calculation by running a load flow. So let us see that.

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Line data for IEEE-14 bus system

Branch no.	From bus	To bus	R (p.u)	X (p.u)	B/2 (p.u)	Tx. Tap
1	1	2	0.0194	0.0592	0.0528	0
2	1	5	0.054	0.223	0.0492	0
3	2	3	0.047	0.1979	0.0438	0
4	2	4	0.0581	0.1763	0.0374	0
5	2	5	0.0569	0.1738	0.0339	0
6	3	4	0.067	0.171	0.0346	0
7	4	5	0.0134	0.0421	0.0128	0
8	4	7	0	0.209	0	1
9	4	9	0	0.5562	0	1
10	5	6	0	0.2522	0	1
11	6	11	0.095	0.1989	0	0
12	6	12	0.1229	0.2557	0	0
13	6	13	0.0661	0.1302	0	0
14	7	8	0	0.1762	0	1
15	7	9	0	0.011	0	1
16	9	10	0.0318	0.0845	0	0
17	9	14	0.127	0.2703	0	0
18	10	11	0.082	0.192	0	0
19	12	13	0.2209	0.1999	0	0
20	13	14	0.1709	0.3479	0	0

So what we have here is, we have here is, we are considering here a IEEE – 14 bus system and so this is the line data and so basically this is the bus data. So there are 14 buses. There are 3 generators at bus 1, bus 2 and bus 6. And these are the generator patterns, load patterns, everything etc. is given. And then here there are also 20 lines. So I have got this 1 to 20 lines.

And here also some of the lines are also for example these lines are nothing but transformers; this, this, this are transformers. These also are transformers because we see it has got only X. There is no resistance. There is no shunt charging capacitance. So these

5 are essentially transformers. Others are lines. And these are transformer. That also can be seen by this entries that I mean transformer tap is 1, 1, 1, 1 or 1, 1, 1. Others are 0, 0 means that I mean these are basically lines. Now if we do this analysis of GOSF. So we get this.

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Generator outage analysis in IEEE-14 bus system

		For outage of generator at bus 2				For outage of generator at bus 6			
Line no.	Pline (ori)	GSOF (2)	ΔP_2	Pline (cal)	Pline (ACLF)	GSOF (6)	ΔP_6	Pline (cal)	Pline (ACLF)
1	1.6262	-0.9709	-0.183	1.803875	1.7891	-0.7628	-0.112	1.711634	1.7203
2	0.7464	-0.1935	-0.183	0.781811	0.7776	-0.4344	-0.112	0.795053	0.7753
3	0.8655	0.0089	-0.183	0.863871	0.8606	-0.1568	-0.112	0.883062	0.8888
4	0.5314	0.0556	-0.183	0.521225	0.5207	-0.2928	-0.112	0.564194	0.5678
5	0.3662	0.0821	-0.183	0.351176	0.3511	-0.2977	-0.112	0.399542	0.395
6	-0.358	0.047	-0.183	-0.3666	-0.3622	-0.1203	-0.112	-0.34453	-0.3357
7	-0.7068	0.1064	-0.183	-0.72627	-0.7217	-0.0033	-0.112	-0.70643	-0.7189
8	0.2689	-0.0013	-0.183	0.269138	0.2691	-0.2562	-0.112	0.297594	0.3173
9	0.1063	-0.0004	-0.183	0.106373	0.1064	-0.1009	-0.112	0.117601	0.1255
10	0.2893	-0.0078	-0.183	0.290727	0.2887	-0.6729	-0.112	0.364665	0.3309
11	0.1156	-0.0059	-0.183	0.11668	0.1154	0.2459	-0.112	0.088059	0.0748
12	0.0852	-0.0007	-0.183	0.085328	0.0851	0.0277	-0.112	0.082098	0.0778
13	0.2005	-0.0031	-0.183	0.201067	0.2003	0.1312	-0.112	0.185806	0.1782
14	0.0025	-0.0065	-0.183	0.00369	0.0025	-0.0204	-0.112	0.004785	0.0022
15	0.2664	0.0052	-0.183	0.265448	0.2667	-0.2358	-0.112	0.29281	0.3151
16	0.0123	0.0055	-0.183	0.011294	0.0124	-0.2287	-0.112	0.037914	0.0512
17	0.0655	0.0037	-0.183	0.064823	0.0657	-0.1548	-0.112	0.082838	0.0944
18	-0.0777	0.0057	-0.183	-0.07874	-0.0776	-0.2368	-0.112	-0.05118	-0.039
19	0.0233	-0.0012	-0.183	0.02352	0.0232	0.05	-0.112	0.0177	0.016
20	0.0857	-0.0038	-0.183	0.086395	0.0854	0.1574	-0.112	0.068071	0.0567

So what we have done is, we have considered the case that this generator at bus 2 has got outage.

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Bus data for IEEE-14 bus system

Bus no.	Type	$\|V\|$ (p.u)	θ (deg)	P_G (MW)	Q_G (MVAR)	P_L (MW)	Q_L (MVAR)	Q_{MIN} (MVAR)	Q_{MAX} (MVAR)	G_{sh} (p.u)	B_{sh} (p.u)
1	1	1.06	0	0	0	0	0	0	0	0	0
2	2	1.045	0	18.3	5.857	0	0	-500	500	0	0
3	3	1	0	0	0	119	8.762	0	0	0.0002	0.502
4	3	1	0	0	0	47.79	3.9	0	500	0	0
5	3	1	0	0	0	7.599	1.599	0	0	0	0
6	2	1.07	0	11.2	44.2	0	0	-500	500	0	0
7	3	1	0	0	0	0	0	0	0	0	0
8	3	1	0	0	0	0	12.9	0	0	0.0023	0.1325
9	3	1	0	0	0	29.499	16.599	0	0	0	0.0633
10	3	1	0	0	0	9	5.799	0	0	0	0
11	3	1	0	0	0	3.501	1.8	0	500	0	0
12	3	1	0	0	0	6.099	1.599	0	0	0	0
13	3	1	0	0	0	13.5	5.799	0	0	0	0
14	3	1	0	0	0	14.901	5.001	0	0	0	0
Base MVA = 100											

So here what we have within this bus in this system we have got 3 generators; bus 1, 2 and 6. Out of this as we have seen, bus 1 we have taken as the slack bus, right? And bus 2 and 6 are the PV buses. So we have considered both the cases one by one. So the first case, we have considered that this generator bus 2 has been out. So when this generator bus 2 are out and by following this calculation we have calculated these GOSF values.

And these are the original power flow over all the lines. Remember, I mean lines means lines plus transformers. Then we have calculated delta P 2. Now this bus 2 has got a generation of 18.3 MW. So if it is out, so then therefore delta P 2 would be – 0.183. So this is exactly it is it will be – 0.1 and 3. So we have got this, we have got this, we have got original value.

So then we calculate this P line by using that expression what we have discussed. Here we have assumed. So here we have assumed that all these generators which have been lost due to the outage of bus 2 had been taken care of by the generator at bus 1 which is nothing but the slack bus. So this is the, this column shows the power flow which is obtained by this approximate method.

And this is the power flow which we obtained by full AC load flow. When we do this full AC load flow, what we do? From our database, from this data we simply remove this bus. We simply remove this. We simply make all of them 0, sorry we simply remove this. We say that we have got only 2 generators 1 and 6, right? this 6 also remains the same. So then automatically this entire extra generation will be taken by bus 1.

So we do this complete AC load flow and if we look at this, these entries they are quite close to each other. They are quite close to each other. So then therefore although there is approximate method but this error involved in them is not much. Similarly, if we look at the case at outage of generator bus 6, again if you look at these two columns, these two are quite close to each other.

So then therefore what we can see here that this approach gives me very quickly the new values of line flows in the event of an outage of any generator without needing the full

AC load flow and these new calculated values are also reasonably close the ideal values. This ideal values are nothing but the values obtained with full AC load flow.

So because these values are reasonably close with those ideal value, so then therefore this is a quite powerful method where I can analyze the effect of contingency on or rather we can analyze the effect of generator contingency on any system without resulting to AC load flow very quickly. So today we will stop here. From the next class onwards or rather from the next lecture onwards we will be looking into the other aspects of this contingency analysis. Thank you.