

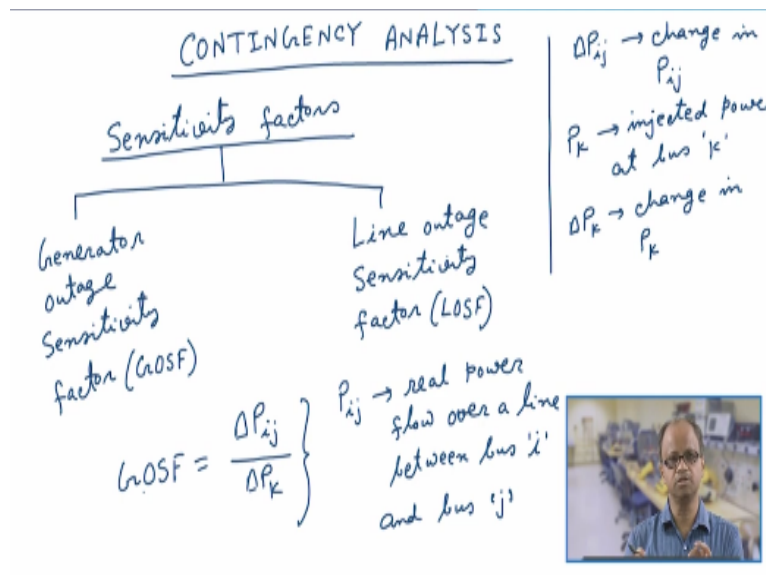
Computer Aided Power System Analysis
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Lecture - 33
Linear Sensitivity Factors

Welcome to this lecture on the course of computer aided power system analysis. In the last lecture we have discussed about the concept of contingency analysis and in that lecture we have argued that for the purpose of contingency analysis it is not required to have a very accurate value. Rather we are only interested that whether the system would be operating in a secured zone or not in the case of any contingency that is the outage of either the generator or a transmission line.

So then we that because of this fact that we really do not have to compute the full AC power flow analysis to analyze the effect of any single contingency but rather we can possibly take some simplifying assumptions and do some kind of rough calculation to arrive to our conclusion whether the system would be operating in a secured zone or not after a contingency has occurred. So then essentially what we have done is, so we are talking about contingency analysis.

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Now today we will be looking into the basic concept that how do we go about it. So contingency analysis. So we said that we really do not have to do any kind of a complete AC load flow analysis but we have to do some sort of approximate calculation. So these approximate calculations we carry out by using something called sensitivity factors. So we use something called sensitivity factors. Now what are the sensitivity factors?

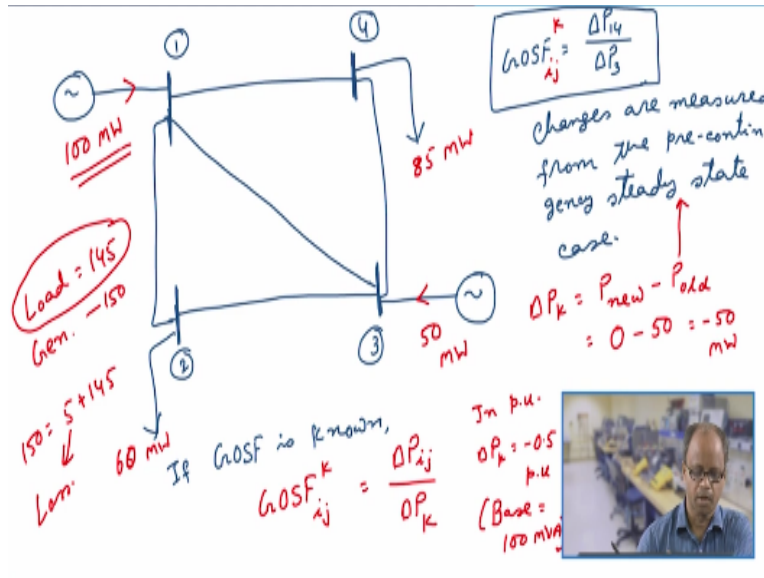
This sensitivity factors, now there are actually two kind of sensitivity factor which is called generator outage sensitivity factor. This is GOSF, generator outage sensitivity factor. And the second one is line outage sensitivity factor. So we have got two types of sensitivity factor or rather we use two types of sensitivity factor which is called generator outage sensitivity factor and another is called line outage sensitivity factor.

Now what do they mean? Actually what do they mean? So GOSF is defined as now what is GOSF? GOSF is basically defined as first let me write down the expression and then we will define what are these quantity. It is defined as $\frac{\Delta P_{ij}}{\Delta P_k}$. Right now it is looking very cryptic but let us explain. P_{ij} is the real power flow over a line between bus i and bus j . So this is P_{ij} and what is ΔP_{ij} ?

And ΔP_{ij} is of course then change in P_{ij} . Similarly, P_k is the injected power at bus k and of course ΔP_k is change in P_k . So then therefore what we are essentially saying? We are essentially saying that GOSF denotes that by how much or rather basically GOSF is nothing but the ratio of change in power flow over a line between bus i and j due to the change in real power injection at bus k .

So this is the, this is basically the definition of GOSF, generator outage sensitivity factor. So what it is exactly means. So let us again take some very simplifying example.

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Let us take some very simple example. So this is bus 1, bus 2, this, this, this thick lines are buses; nothing but that is the junction points; bus 3, bus 4 and let us say there is also another line between them. How does it matter and let us say there is one generator connected at bus 1 and let us say another generator is connected in bus 3 and there are loads, there are loads, there are loads etc.

So now for example if I take a line between let us say bus let us say 1 and 3 or let us say bus 1 and 4. So then therefore GOSF would be and let us say we are talking about the change in real power injection at bus 3. So that means GOSF for this case would be $\Delta P_{14} / \Delta P_3$. ΔP_{14} is nothing but how much the by how much the amount of real power flow has changed over this line between bus 1 and bus 4 due to some change in injected real power at bus 3.

Now the question is from which this change would be measured. Basically, I mean whenever we are talking about any change so then obviously that change would have to be measured from some reference value. This reference value is the essential I mean this particular reference value is essentially nothing but the pre-contingency steady state. So then therefore these changes are measured from the pre-contingency steady state case.

So now what we are actually saying? Now let us go into little deeper. Suppose I know this value of GOSF by some means, suppose. Suppose that GOSF is known. So if GOSF is known by some value I mean whatever, I mean by some means I mean we will see that how this particular GOSF is to be calculated. So if GOSF is known so then therefore if I know that what is the change in P_3 then we can immediately calculate that what would be the change in P_{14} .

Similarly, we will also have to define GOSF for each and every combination. For example for our case we can simply say that it is let us say this is GOSF and let us say ij due to k . So we can say that this GOSF is basically ij due to k . So then therefore when I say that GOSF with a subscript ij and with a superscript k right it only says that this is $\frac{\Delta P_{ij}}{\Delta P_k}$.

So then therefore for each combination of this generator and the line we will have different values of GOSF, right? And if this GOSF are known so then therefore if there is a change in the real power injection here so then therefore immediately we can calculate what would be the new amount of or rather what would be the change in the real power flow over a line between bus i and bus j due to some change in injected power at bus k .

Now suppose, just take an example. Now suppose that this particular generator has been generating 100 MW and let us say this particular generator has been generating 50 MW suppose in the steady state. So then therefore now if this particular generator goes out so then therefore what will happen, so then therefore this particular generator would not give any amount of power.

So then therefore ΔP_k in fact any ΔP_k is actually given by $P_{\text{new}} - P_{\text{old}}$. When we say that P_{old} this is nothing but this initial steady state value. So then therefore if this particular generator goes out so then in this case what would be the value of ΔP_k ? It would be 0 because when it is going out it is not giving any value of power and earlier it was 50. So then this ΔP_k would be -50 MW right.

And if our base is let us say 100 MVA so then therefore in per unit delta P k would be = - 0.5 per unit for base = 100 MVA, correct?

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If all $GOSF_{ij}^k$ are known

New power flow over the line: $\Delta P_{ij} = (GOSF_{ij}^k) \Delta P_k$

$P_{ij}^{new} = P_{ij}^{old} + \Delta P_{ij}$

Let: M - Generators in the system

1 -> Slack

M-1 -> PV bus.

Assumption: Lost generation is taken by the slack bus.

Now suppose we know if all GOSF ij k are known. We will see that how this GOSF k are known I mean or rather how to actually calculate this particular sensitive factors. So then therefore if all this GOSF case are known so then therefore by this we can simply calculate delta P ij = GOSF ij k * delta P k, right? That we can simply calculate. So just this multiplied by this. This is already known.

This we have just now calculated as we have shown. So then therefore this is immediately known. So then therefore what would be the new power flow over, so the new power flow over the line that is P ij new that would be = P ij old + delta P ij. Now this P ij old is already known, correct? So this particular P ij old is already known. This is nothing but the pre-contingency steady state value. So this is already known.

We have just now calculated this quantity from here. So we will simply add these two quantities and we will get the estimate of that what would be the new power flow over the line. So then therefore here what we are doing? So we are simply not using the normal AC load flow. We are simply utilizing some sensitivity factors or rather this particular sensitivity factor GOSF ij k.

Obviously, the question comes into mind that how to calculate them. We will discuss it in some later stage in this course. Now here what happens is the following. When I say now for any generator now suppose I have got let us say M generators in a system, M generators in the system. Let, we know that one has to be slack and the rest $M - 1$ would be the let us say PV bus or rather the generator bus, that we know.

Now the point is if I say that one generator has gone out of order. So then therefore this particular generator is not really giving any power. So then therefore there will be total deficit between generation and the load. So then therefore with all probability this particular system will go into insecure zone of operation anyway if we do not compensate for this loss of generator.

For example if in this system for example if in this system let us say this load is 85 MW and let us say this load is let us say 65 MW sorry 60 MW. So then total load is 145 and total generation is 150. So 150 is equal to let us say $5 + 145$. Here we are just assuming that let us say this 5 is lost just as an example. Now suppose if this generator goes out so then therefore it is not giving 15 MW.

So then therefore in this case what will happen? I have got total load is still 145 but by now my generation is only 100 MW. So then obviously this generator would not be able to supply all the loads. So as a result voltage will automatically come down. So whatever we do we will just not be able to operate the system in the secured zone. It will simply shut down after some time.

Because this, because the amount of generation is not equal to the load. So then therefore in that case what we have to do essentially if we have to prevent the complete shutdown of the system, we have to essentially restore to load shedding which we do not want to do obviously. I mean restoring to load shedding should be our least priority.

So then therefore whenever we are saying that some generator is being out so therefore we must also say that well there is some mechanism to compensate for this loss of generation. Now in this case what we do? So when we are talking about this one, so in this expression it is assumed that the lost generation is totally taken by the slack bus. That is the assumption.

That is we assume that this lost generation is totally taken by the slack bus. So here the assumption is that lost generation is taken by the slack bus. We take this assumption because we say that our slack bus is the highest rated bus or rather basically our slack generator is essentially the highest rating generator so then it should be able to take the extra amount of generation.

But then obviously our slack generator is not a generator which has got infinite capacity. So if it does not have infinite capacity then it may happen that in case of outage of a big generator then and if our slack generator has to take the or rather has to compensate for the loss of that entire generation by that particular big generator it may happen that this particular slack generator would be completely out of its rating.

So this assumption although it is a very convenient assumption but this assumption is not really very feasible because there is a change that in this process, this particular slack generation would not be able to supply the entire power because of the simple fact that it may simply exceed that this particular requirement of supplying the total real power now may exceed its rating.

So if that happens, so then obviously this particular slack generator will also not be able to supply the entire generation. So then in that case also we have to restore to load shedding.

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The lost generation is compensated by the other remaining generators combinedly.

5 generators.


$$\gamma_k = \frac{S_k}{\sum_{i=1, i \neq k}^M S_i}$$

100 ← 3X } M=5
 2 }
 4 } OP_k
 5 }
 K=3

$$\gamma_1 = \frac{S_1}{S_1 + S_2 + S_4 + S_5} = \frac{S_1}{\sum_{i=1, i \neq 3}^5 S_i}$$

$\gamma_2 \rightarrow \gamma_2 \cdot OP_k$
 $\gamma_4 \rightarrow \gamma_4 \cdot OP_k$
 $\gamma_5 \rightarrow \gamma_5 \cdot OP_k$

These are constant



So to prevent that what we actually do is essentially what is being done is that this lost generation, the lost generation is distributed, lost generation is compensated by the other remaining generator combinedly. So what does it mean? Suppose for example in my system let us say there are 5 generators and let us say and this 5 generators are 1, 2, 3, 4, 5 and this generator is gone and let us say this generator has been supplying 100 MW earlier.

So then therefore this 100 MW lost generation would now have to be compensated by all this 4 remaining generators. Now the question is how this 100 MW would be distributed among this 4 remaining generators. One simple solution is that simply divide them equally.

But then that is also not a very feasible thing. Because if out of this if anyone is very small generator so even taking care of extra 25 MW maybe may also turn out to be quite problematic for that small generator. So what we do is we simply distribute this lost generation to be taken up by this remaining generators in proportion to their MVA rating.

So what we define is, we define some quantity called gamma k, gamma l which is given by actually what we do is that we do simply distribute them according to their I mean according to the amount of real power which they have been generating. So then what we

do is that or rather we also sometimes do it through their MVA. So then here what we can do is that $S_i / \sum_{i=1}^M S_i$ not equal to 1, right? Sorry it is not equal to 1 sorry.

It would be, so then therefore what is meant by this? So I have got 5 generators 1, 2, 3, 4, 5. Generator 3 has gone out. So then therefore my k is 3. So my k is 3; k is nothing but the generator which is experiencing outage. So k is 3. So then we calculate γ_1 as $S_1 / (S_1 + S_2 + S_4 + S_5)$. So it is nothing but $S_1 / \sum_{i=1}^5 S_i$ not equal to 3, right? So here in this case for example $M = 5$.

So then similarly we also calculate γ_2 , γ_3 not γ_3 γ_4 and γ_5 . So for this we would be calculating γ_1 , γ_2 , γ_4 and γ_5 . So once we get this γ_1 , γ_2 , γ_4 and γ_5 so then now because all these quantities are known these are nothing but the MVA ratings. Now because all these quantities are known so then therefore these quantities are all constant.

They are all constant. So γ_1 , γ_2 , so these are constant quantities. These are constant. So then therefore if these are constant and now the change is actually ΔP_k right? So if this change is ΔP_k so then therefore what would be the share of this ΔP_k ? For bus 1, it would be this share would be nothing but $\gamma_1 * \Delta P_k$. For bus 2 it would be $\gamma_2 * \Delta P_k$.

Bus 4 it would be $\gamma_4 * \Delta P_k$ and for bus 5 it would be $\gamma_5 * \Delta P_k$, right? so then what happens now? So earlier what we have done? Earlier we have only considered that there is a change in the injected power only at bus k that is at the bus at which this generator is going out of operation. But now because of this the injected power at all the other buses are also changing.

So then therefore when we would be calculating the new power flow over the lines we also have to take into account the change in the injected power at bus 1, bus 2, bus 4, and bus 5.

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Therefore new power flow over line between bus 'i' and bus 'j' would be,

$$P_{ij}^{new} = P_{ij}^{old} + (\Delta P_k) \times (\alpha_{ij,k}) - \sum_{\substack{\lambda=1 \\ \neq k}}^M (\alpha_{ij,\lambda}) (\gamma_\lambda) (\Delta P_k)$$


$GOSF_{ij}^k = \alpha_{ij,k}$
 $\Delta P_k = 0 - P_k = -P_k$

$\gamma_1 = 0.1$
 $\gamma_2 = 0.2$
 $\gamma_4 = 0.3$
 $\gamma_5 = 0.2$

$\Delta P_1 = 0.1 \times 50 = 5$
 $\Delta P_2 = 0.2 \times 50 = 10$

$\Delta P_k = -0.5 \text{ p.u.} = -50 \text{ p.u.}$

$P_{ij}^{new} = P_{ij}^{old} - P_k (\alpha_{ij,k}) + \sum_{\substack{\lambda=1 \\ \neq k}}^M (\alpha_{ij,\lambda}) \gamma_\lambda \Delta P_k$



So then therefore in this case so then therefore new power flow over the line therefore new power flow over the line between bus i and bus j would be P_{ij}^{new} . That would be = $P_{ij}^{old} + \Delta P_k$ into now here just for the sake of you know just for the sake of convenient notation let $GOSF_{ij}$ be denoted as $\alpha_{ij,k}$ suppose. $\alpha_{ij,k}$. So change in ij due to k . So it would be into $\alpha_{ij,k}$ minus, now why it would be minus?

We will explain that why it will be minus, summation let us say $\sum_{l=1}^N$ not equal to k $\alpha_{ij,l} * \gamma_l * \Delta P_k$, that is quite, so this is the case. It is looking to be quite you know intimidating but it is not. First thing first, up to this point we have already explained. Why this minus? This minus would be for the simple reason when bus k is going out of order then that real power would have to be compensated by them.

Now we have already seen that ΔP_k for example in our case would be let us say -0.5 right per unit. Let us say -50 per unit. Just in our example suppose γ_1 is let us say 0.1 , γ_2 is let us say 0.3 , γ_4 is let us say 0.3 and γ_5 is let us say 0.2 suppose. So then therefore what would be ΔP_1 ? It would be $0.1 * 50 = 5$ and we are taking here $0.1 * 50$ because it has to now inject more. It has to inject more.

Because in this case if I have got 5 generators for example, if I have got 5 generators and out of this 5 generator, generator 3 is going out of order. So then to compensate for that bus 1 or rather basically the generator at bus 1 we have to now inject extra amount of power right? So then it would be injecting extra amount of power and that extra amount of power would be injected more. So then therefore it would be positive.

So then therefore here in this case it was going actually out of order and it is actually giving more. So in this case it was actually going, so then therefore it is actually going out of order and in this case it is giving more so then therefore this would be opposite in sign. For example here also if we just similarly ΔP_2 also would be let us say $0.2 * 50 = 10$ and let us say so on and so forth, right?

So then therefore if we apply this, so if we apply this right? So by this we would be able to calculate this new power flow over the line between bus ij for the outage of the generator at bus A. For this we need to know this quantity, for this we need to know this quantity and for this we need to know this quantity. This quantity we have already seen that this is constant quantity. We will be looking into these quantities at some later stage.

And we will also see that these quantities are also constant quantities so then therefore because all these quantities are constant quantity so then therefore in this case if there is an outage of any generator we really do not have to do any kind of AC power flow. We have to simply do some simple possible algebraic operation, multiply and add.

Now before closing this discussion, here suppose for example if we just substitute the value of ΔP_k here for example so if we substitute the value of ΔP_k here so what we will get? P_{ij} new would be P_{ij} old, ΔP_k we have already seen that it is actually the negative of the earlier generation. So it is basically $-\Delta P_k * \alpha_{ijk}$ right and this is here minus and this is also here minus.

So then therefore this would be plus because I have got here one minus and basically this I mean $\Delta P_k = -P_k$. We have already seen in the just a little earlier that ΔP_k is

actually $0 - P_k$. that is $= - P_k$. So then therefore this minus and this minus plus; $l = 1$ not equal to k up to M delta alpha ij l gamma $l P_k$. So then therefore by this expression if I know what is the amount of generation of the generator which is going out of order right.

And if we know all the sensitivity values, this, this and this it is extremely easy to calculate the new power flow over a line without resorting to the complete AC load flow. So we stop here today. In the next lecture we would be continuing this particular topic. Thank you.