Computer Aided Power System Analysis
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Lecture – 03
Bus Admittance Matrix

Welcome to this third lecture of this course called computer aided power system analysis, today we would be talking about something very central to the power system analysis which is called actually a bus admittance matrix, so now let us start.

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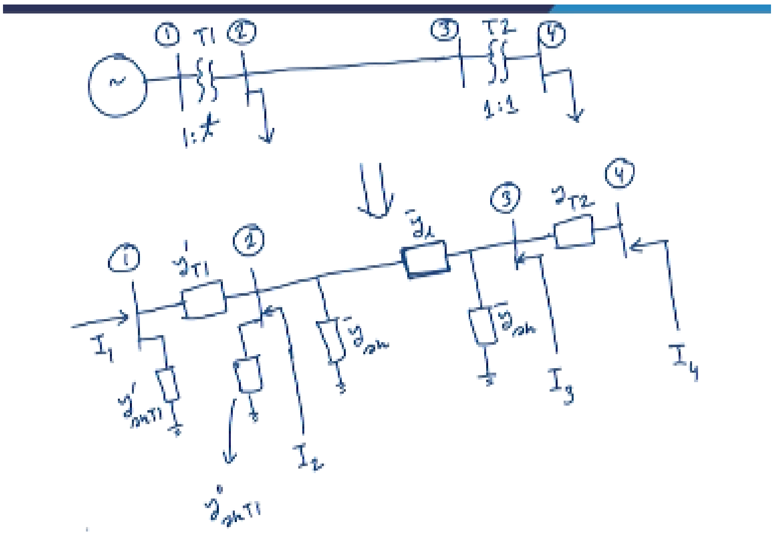

So, what we take suppose, I do have a system, a generator followed by a transformer, followed by a transmission line followed by another transformer and let us say another bus, let us say there is load and let us assume that there is some load here and this is bus 1, this is bus 2, this is bus 3, this is bus 4, now suppose if this is the bus and now, suppose if this is the system so then and also assume that this transformer is 1 is to t.

And this particular transmission line is actually medium link transmission line and this transformer is 1 is to 1, so then therefore according to our earlier discussion, this particulars network can be represented by this equivalent circuit, so this is bus 1, this is source, bus 1 then because this is an off nominal tap changing transformers, so then there will be a pi-model between bus 1 and 2 and we have already seen that what are these parameters of this pi model.

So, this is bus 2, then for this transmission line also there would be another pi model, this is corresponding to this transmission line, so this is the pi model, so this is the pi model and then so we reach bus 3 and then because this is 1 is to 1 transformer, so then it would be only represented by an series impedance, so this is bus 4 and then we have a load. Now, as you have already discussed now for each bus, whether there is a load or a generator or nothing at each bus we can define something called current injection.

So, let us say at bus 1, I do have got a injected current I1, at bus 2 also we can have some injected current I2, at bus 3 also we can have some injected current I3 and at bus 4 also we can have some injected current I4, now here as we have already discussed, I1 would be actually corresponding to the current contributed by the generator, I2 would be nothing but the negative of the load current as we have already seen in the last class.

I3 would be actually 0 because at this bus, there is neither any load connected nor any generator connected in shunt, it would be 0 but for the purpose of our very general analysis, we will still keep this I3, so I mean this is some injected current its value is 0 but still we are defining it by a variable called I3 and of course, I4 is nothing but the, again negative of the load current of the load drawn at bus 4.

Now, let us say that this particular impedance now, this is a series impedance, so for this series impedance we can also calculate this equivalent admittance, so this is let us say y line and this is y shunt, this is y shunt, this is let us say yT2, let us say this is transformer 1, this is transformer T2, this is transformer T1, this is transformer T2, so this is yT2 and this is actually yT1 * T.

So, let us say we say that this is, y dash T1 and this is let us say some other value y, let us say shunt T1, y dash shunt T1 and let us say this quantity is y double dash shunt T1, please note that y dash T1 would be nothing but y * T where actually y is the leakage admittance of this transformer y dash shunt T1 would be nothing but y *; yT * T - 1 and y double dash sh T1 would be T – 1 * y.

So, I mean all these things we have already derived, so then we are not really putting it here, so now this is the equivalent circuit, now with this equivalent circuit we would now like to write down the KCl equation at each and every note. So, now at bus 1, this current I1 would be actually equal to the current here and here.

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At Bus 1

$$\bar{I}_1 = \bar{V}_1 \bar{y}'_{shT1} + \bar{y}'_{T1}(\bar{V}_1 - \bar{V}_2)$$
$$= (y'_{shT1} + y'_{T1})V_1 - y'_{T1}V_2 \quad \cdots (1)$$

At Bus 2

$$\bar{I}_2 = \bar{V}_2 \bar{y}''_{shT1} + \bar{y}'_{T1}(\bar{V}_2 - \bar{V}_1) + \bar{y}_{sh}\bar{V}_2 + \bar{y}_l(\bar{V}_2 - \bar{V}_3) \quad \cdots (2)$$
$$= -y'_{T1}V_1 + (y''_{shT1} + y'_{T1} + \bar{y}_{sh} + \bar{y}_l)\bar{V}_2 - \bar{y}_l V_3$$

At Bus 3

$$\bar{I}_3 = \bar{y}_{sh}\bar{V}_3 + \bar{y}_l(\bar{V}_3 - \bar{V}_2) + \bar{y}_{T2}(\bar{V}_3 - \bar{V}_4)$$
$$= -\bar{y}_l\bar{V}_2 + (\bar{y}_{sh} + \bar{y}_l + \bar{y}_{T2})\bar{V}_3 - \bar{y}_{T2}\bar{V}_4 \quad \cdots (3)$$

Then, therefore we write this equation at bus 1; I1 would be equal to, we write V1 * y dash y sh T1 * V1 * y dash sh T1 that is this current + y dash T1 * V1 – V2 + y dash T1 * V1 – V2, please note that all these are actually complex quantities, so we must take care to put over bar over them, so there are over bar, there are over bar, so all these are complex quantity, so if we do take so, y dash sh T1 + y dash T1 * V1 - y dash T1 * V2, so this is equation 1.

At bus 2, the injected current at bus 2 that is this would be equal to the this current, this current + this current + this current, now this current is equal to again, this current + this current, so then therefore this injected current I2 would be equal to this current + this current + this current + this current, so let us write down, so then therefore it would be I2 would be V2 y sh T1 double dash, this is; this current + this current is y dash T1 * V2 – V1.

So, this is y dash T1 * V2 – V1, all these are complex quantity plus now, this current; this current is nothing but y sh * V2, so y sh * V2 + the last component would be this current and this current is nothing but yl * V2 – V3, so it is yl * V2 – V3, so if we collect the terms we get y dash T1 V1 + y double dash y sh T1 + y dash T1 + y shunt + yl V2 - yl V3, so this is the second equation.

Similarly, at bus 3, at bus 3 this injected current if I apply KCl at this point, this injected current would be equal to this current + this current and this current is again = this current + this current so then, therefore this injected current I3 would be = this current + this current + this current, so we need to take into account all these 3 terms, so let us do that. So, the first is V3 * y shunt that is this current.

So, I3 y shunt * V3 + then this current, this current would be yl * V3 – V2, so it is yl * V3 – V2 + the last term would be this current and this current is nothing but yT * V3 – V4, so yT2 * V3 – V4, so collect all the terms together, so this is –yl V2 + y sh + yl + y T2 * V3 – yT2 * V4, so this is equation 3.

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At Bus 4

$$\bar{I}_4 = \bar{y}_{T2}(\bar{V}_4 - \bar{V}_3) = -\bar{y}_{T2}\bar{V}_3 + \bar{y}_{T2}\bar{V}_4 \quad \cdots (4)$$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \end{bmatrix} = \begin{bmatrix} \bar{y}'_{shT1}+\bar{y}'_{T1} & -\bar{y}'_{T1} & 0 & 0 \\ -\bar{y}'_{T1} & \bar{y}'_{shT1}+\bar{y}'_{T1}+\bar{y}_{sh}+\bar{y}_l & -\bar{y}_l & 0 \\ 0 & -\bar{y}_l & \bar{y}_{sh}+\bar{y}_l+\bar{y}_{T2} & -\bar{y}_{T2} \\ 0 & 0 & -\bar{y}_{T2} & \bar{y}_{T2} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$$

Injection current vector

Symmetric, complex

Bus admittance matrix

Bus voltage vector

Lastly at bus 4, we would write the equation of I4, now this injected current I4 would be nothing but equal to this current, so it is very simple, so it is yT2 * V4 – V3, so it is yT2 * V4 – V3, so this = - yT2 * V3 + yT2 * V4, so this is equation 4. Now, what we will do is; we will collect all these 4 equations together and write them in terms of a matrix, so if I do that what I do is; I get I1, I2, I3, I4 = some matrix which we will now fill up one by one, V1, V2, V3, V4.

So, let us partition this matrix just for the you know and then let us say this, this and this, so for I4 it is, - yT2 V3 + yT2 V4, this is 0, this is 0, for I3 it is – yl V2, so minus yl V2 + y sh + yl + yT2, y sh + yl + yT2, so it is y sh + yl + y T2 – yT2 V4 - y T2 V4 and this is 0. At bus 1, it is y sh dash T1 and y dash T1, so this is y dash T1 + y dash T1 and this is - y dash T1, so this is - y dash T1, this is 0, this is 0.

And for bus I2 y - y dash T1 - y dash, sorry, y dash T1 + y double dash sh T1 y dash T1 y double dash sh T1 + all these are complex quantities, please note all these are complex quantities, y double dash sh T1 + y dash T1 + y sh + y dash T1 + y sh + yl, this is - yl and this is 0, so this is the matrix. Now, if we look at this matrix there are certain things, which is quite obvious.

If we look at the diagonal terms, we find that this diagonal terms are nothing but the sum total of the admittances connected at a particular bus for example, at bus 1, in this figure 2 admittances are connected y dash sh T1 and y dash T1 and the diagonal term corresponding to bus 1 is nothing but the sum total of this and this similarly, at bus 2, there are 4 admittances connected that is y dash T1 y double dash sh T1 y sh and yl and the diagonal term corresponding to bus 2 is nothing but the sum total of this + this + this + this.

Similarly, at bus 3, there are 3 admittances connected which are nothing but yl y sh and y T2 and again at bus 3, we can see that the diagonal terms is nothing but the sum total of this 3 admittances and at bus 4, there is only one admittance connected directly that is yT2 and at bus 4 that this diagonal term is nothing but yT2, there are also few zero elements for example, between bus 1 and 3 and bus 1 and 4.

And now, if we look at this diagram, we can see that there is no direct connection between bus 1 and 3, neither between bus 1 and 4, so then therefore similarly, if we look at bus 4, at bus 4 there is no direct connection between bus 4 and 2; bus 4 and 2 they are not directly connected only bus 4 and 3, they are directly connected, so then therefore we can see that whenever there is no direct connection between two buses, this corresponding off diagonal element is 0.

And the non-zero off diagonal element are nothing but the negative of the admittance connected between any two buses for example, between bus 1 and 2, there are only one admittance is connected that is y dash T1 and so then therefore between bus 1 and 2, so this element which is so then this element which is corresponding to this first row and second column is nothing but the negative of this admittance.

Similarly, this element which is corresponding to second row and first column that is also equal to this that means, it is also nothing but the negative of this admittance connected between bus 1 and 2, similar observations whole also true for the all the other elements, all the other non-zero off-diagonal elements, so then therefore we can see that this matrix is essentially, it is a symmetric matrix, it is a complex matrix.

Complex matrix means that all the terms or rather all the nonzero terms are complex quantities and there are also few zero elements of this matrix, now for our analysis we do denote this vector I1, I2, I3, I4 as injection current vector, this vector is called bus voltage vector and this matrix is called bus admittance matrix. Now, here in this small example, the size of this bus admittance matrix is 4/4 and in our system there are also 4 buses.

So, then therefore we can see that the size of this bus admittance matrix is nothing but equal to the number of buses in the system, so then therefore if there are N buses in the system, so then therefore the size of this bus admittance matrix should be = N cross N.

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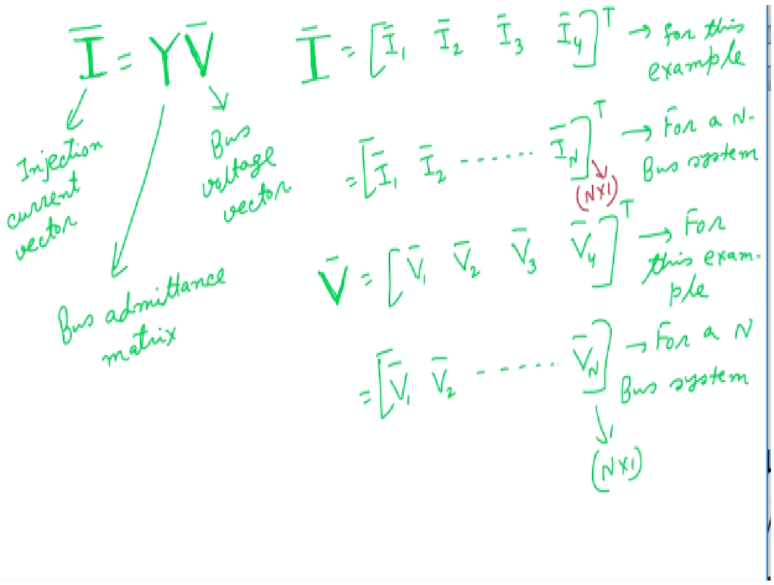

Now, in more concise term we do write it as bus injection current vectors I, I am just trying to put them in a big you know slightly, it is a vector which is called Y; capital Y, this capital Y matrix is nothing but the bus admittance matrix * V; V is the bus voltage vector, so then here I stands for I1, I2, I3, I4 in this example and it would be equal to I1, remember all these complex quantities, I2 dot dot dot IN transpose for a N bus system.

So, this I is again, this is the injection current vector, this V is bus voltage vector and this Y is bus admittance matrix, so I mean what would be V, this vector V for this case would be again in the same line, V1, V2, V3, V4 transpose for this example and in general, it would be V1, V2 dot dot dot VN for a N bus system. So, then therefore for a general N bus system, the size of this vector would be N cross 1.

And for an N bus general system, the size of this vector also would be N cross 1 and what about this matrix Y?

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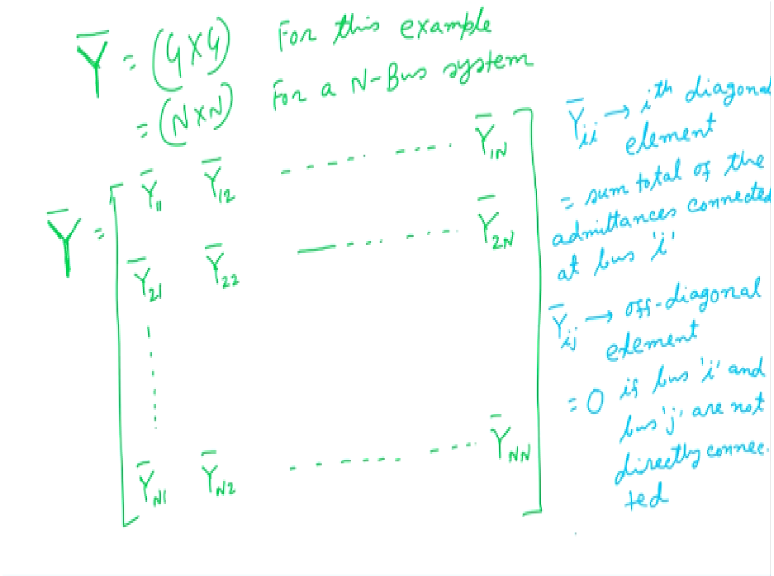

It is a 4 cross 4 matrix for this example and this would be an N cross N matrix for a N bus system, now this matrix Y is represented like this, it is Y11 Y12 dot dot dot Y1N, all these Y's are nothing but the element of this matrix, Y21 Y22 Y2N and YN1 YN2 YNN, now so in general what we have observed, so we can write down them in a more Yii that is the ith diagonal element is nothing but sum total of the admittances connected at bus I.

Yij that is the off diagonal element that would be 0, if bus i and bus j are not directly connected.

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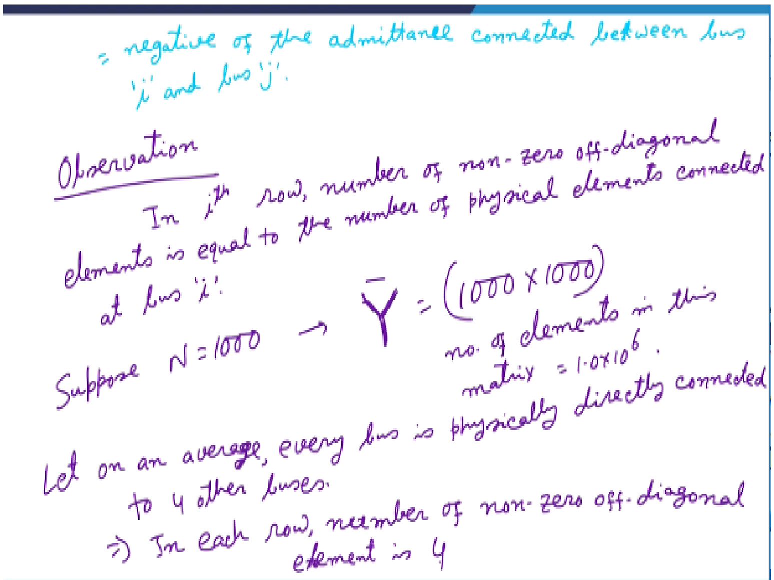

And would be equal to negative of the admittance connected between bus i and bus j, so now here there is one very interesting observations we have to make, the interesting observation is that how many off diagonal elements are there in each row of this bus admittance matrix, let us look at that. So, the question is how many off diagonal elements are there in each row of this bus admittance matrix?

So to understand that first let us look at the diagram for example, bus 1 is connected to bus 2 only directly, so then therefore in this diagram, so therefore in this diagram, I am sorry, so therefore in this matrix, there is only one non-zero off diagonal element in row 1, bus 2 is directly connected between bus 1 and bus 3 and we observed that corresponding to row 2, there are 2 off diagonal nonzero elements.

Bus 3 is connected directly between bus 2 and bus 4, we again observed that corresponding to row 3, there are 2 non- zero off diagonal elements and bus 4 is directly connected only with bus 3 and we again observe that there is only one non-zero off diagonal element at row 4; fourth row, so then therefore we can conclude that in each row, so we write that in each row observation; in each row number of nonzero off-diagonal elements = the number of physical elements connected at bus.

So, you should write actually; we should write more secondly in ith row, so it is connected bus I, now there is a very interesting thing suppose, N = 1000 that is we are considering again 1000 bus system, so then therefore the size of the bus admittance matrix would be 1000 cross 1000, so then total number of elements in this matrix is 1.0 * 10 to the power 6 and also let on an average, every bus is physically, directly connected to say 4 other buses.

If that is the case so then in that case, what would be the number of non-zero off-diagonal elements in each row, it would be 4, so then therefore in each row, number of nonzero element is 4, so then therefore, what is the total number of nonzero element in this matrix?

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Total number of non-zero element in the matrix

$= 4 \times 1000 + 1000 = 5000$

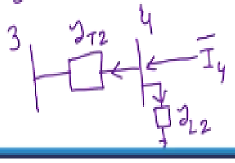
$\Rightarrow$ Total number of zero elements $= 1000000 - 5000$

$= 995000$

$\Rightarrow$ Sparsity

$\%$ sparsity $= \dfrac{995000}{1000000} \times 100 = 99.5\%$

Bus admittance matrix is a highly sparse matrix.



$$\bar{I}_4 = \bar{y}_{T2}(\bar{V}_4 - \bar{V}_3) + \bar{y}_{L2}\bar{V}_4$$
$$= -y_{T2}\bar{V}_3 + (\bar{y}_{T2} + y_{L2})\bar{V}_4$$

So, then the total number of nonzero element in this matrix is; total number of nonzero element in the matrix = 4 * 1000; 1000 is nothing but the number of row + there are also 1000 diagonal element in each row, so then therefore there will be also 1000 more nonzero element, please note that these 4000 are essentially due to the off-diagonals nonzero elements and these 1000 are nothing but diagonal elements which are obviously nonzero.

So, it comes out to be only 5000, so then the total number of nonzero element in the matrix is 5000, so then therefore total number of elements or rather zero elements we call is nothing but 2, 3, 4, 5, 6 – 5000, so it = 000 and it is 599; 99500, now we define something called sparsity of the matrix. Sparsity means that what is the relative percentage of the elements which has 0, in this case what is the percentage sparsity?

Percentage sparsity = 995000 divided by 1000000 * 100, so it will come out to be 99.5%, so then therefore any bus admittance matrix is an highly sparse matrix, so then therefore our conclusion is that bus admittance matrix in general is a highly sparse matrix, so this is the one of the most important observation of this bus admittance matrix, so then what we do in actual calculation, we actually do something such that we really do not have to work with this zero elements.

Hopefully, we would be able to cover some aspects of this in this course in future sometimes, now there are 1 or 2 very minor points of this, here in this case as we have already discussed, here in this case all these 2 loads are considered to be constant PQ loads because as we have already discussed in the earlier lectures that we do take constant PQ loads because constant PQ loads do give us the most pessimistic results.

So, then therefore you do take the constant PQ loads but now suppose, instead of these loads being constant PQ loads suppose, if this loads are let us say constant impedance load, so then how this bus admittance matrix would be modified? The answer is simple, if this is constant impedance load for example, so then we will still assume that there is some current which is being injected to it.

And that current would be equal to this current + the current through this admittance and then therefore, following these equations what we have already derived here would be simply writing down the equation for this injected current equal to this current + this current and this current is nothing but essentially V4 – V3 * y T2 and this current would be if there is some admittance connected here.

So and then therefore this current would be yl2, so then therefore what will happen; in the expression of this particular current apart from this also, this admittance would be added, I mean just as an example, for example just to illustrate this point suppose, there is a bus, let us say this bus 4 in our example and this is bus 3 and in our earlier example between bus 4 and bus 3, there is an impedance admittance connected yT2, so this is yT2; yT2.

And this load is now on constant impedance load having an impedance yl2; admittance yl2, I mean this is not a constant PQ load, earlier in this case because it was in constant PQ load, we have simply assumed that this current drawn by this load is equal to the negative of the injected current or rather we said that this injected current at bus 4 is equal to the negative of the actual current drawn by this load.

But now this load is on constant impedance load or rather constant admittance load, so then we have to see how this is to be modelled, so in this case also, we now define the injected current like this, please note in the earlier case, in this previous case what we did; we have assumed that this current is being contributed by this load, this current is being actually contributed by this load.

But now what we are saying that this injected current is not being contributed by some load rather we are assuming that there is some fictitious injected current which is now being injected in it, of course if we do not consider this, so then therefore this I4 would be 0, of course they would be 0 but then if we do take it as an variable, so then in that case what will happen that this I4 would be equal to nothing but the current through this plus current through this, so then I4 would be = yT2 * V4 – V3 +; this current is yl2 * V4.

So, it turns out to be – yT2 * V3 + yT2 + yl2 * V4, now if we do compared this expression of I4, with the expression of I4 which we have already derived, what we find here; we simply find that this contribution of this constant impedance load or rather the contribution of the constant admittance load is being added to the term corresponding to V4, so in this matrix what will happen?

This particular off- diagonal term which is corresponding to fourth row and third column, this will not change but this diagonal term corresponding to fourth row would be changed from yT2 to yT2 + yl2, so that therefore if there is an constant impedance load connected at any bus that particular admittance would be absorbed in the diagonal term and the current in that admittance and essentially the load current in through that admittance term would not, I mean would be basically would not be considered explicitly, right.

Instead of that we will assume that there is an fictitious injected current I4, so then therefore if there is an constant impedance load or constant admittance load, so that particular impedance or admittances would be absorbed in the diagonal terms of the bus admittance matrix.

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$$\bar{I}_i = \bar{Y}_{i1}\bar{V}_1 + \bar{Y}_{i2}\bar{V}_2 + \cdots + \bar{Y}_{iN}\bar{V}_N$$

$$= \sum_{k=1}^{N} \bar{Y}_{ik}\bar{V}_k$$

$$\boxed{\bar{I}_i = \sum \bar{Y}_{ik}\bar{V}_k}$$

Now, before we conclude there is only one small thing we need to do now, from this expression I = Y * V, we can write down that we can write down the expression of ith injected; we can write down the expression of the injected current at bus I as Yi1 * V1 + Yi2 * V2 + dot dot dot YiN * VN, so it would be sum total of Yik * Vk; k varies from 1 to N, so the expression of injected bus injected current is given by Yik Vk, so this is actually the starting point of the analysis of a power system.

So, in this lecture what we have done; we have looked into the basic concept of the bus admittance matrix and we have seen that how this bus admittance matrix is formed with a small example and from that small example, we have observed that we have actually deduced that how any general large power system network can be represented very, very conveniently with in terms of this bus admittance matrix.

And all the terms of this bus admittance matrix are also defined especially that if there is no element connected between any 2 bus, so corresponding to that 2 elements, this off diagonal element would be 0 and this diagonal element should be nothing but the sum total of all the physical elements connected to bus I and the non-zero off-diagonal elements should be equal to the negative of the physical admittance connected between 2 buses.

Furthermore, this is a symmetric matrix, this is a square matrix and this is also a sparse matrix, so with this small introduction of bus admittance matrix, we stop today, we will continue with this discussion in the next lectures, thank you.