Computer Aided Power System Analysis

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Lecture - 28
Example of Gaussian Elimination Method

Hello, welcome to this lecture of this course on computer aided power system analysis. In the last lecture, we have looked into the basic procedure of Gaussian elimination method. We have also discussed very briefly that how to incorporate this Gaussian elimination method in terms of this matrix operation. So in this lecture, we would be looking into this matrix operations in detail followed by one example, one small example. So let us start.

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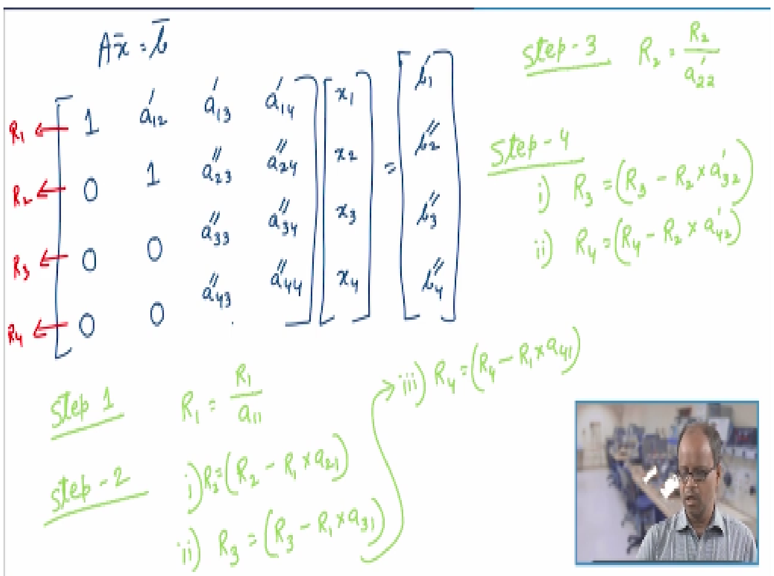

So we have what we have is say again we take an example that A x = b and we are talking about say a 11, a 12, a 13, a 14; a 21, a 22, a 23, a 24; a 31, a 32, a 33, a 34 and a 41, a 42, a 43, a 44 into the unknowns are x 1, x 2, x 3, x 4. And on the right hand side you have got b 1, b 2, b 3, b 4. So this is the case. We denoted that this we have got rows, 4 rows R 1 row 1, row 2, row 3, row 4.

So our steps would be that as we have looked into that the first step 1 would be step 1 that we say that R 1 = R 1/a 11. So essentially you would be dividing this entire row by a 11. So then therefore what will happen? So here we will have 1. So this would be now given to 1. So then therefore this would change to 1. This would change to 1 and this would be changed to a 12/a 11. This would be a 13/a 11. This would be a 14/a 11.

And this would be b 1/a 11. So let us say that this would be changed to a 1 dash, a 13 dash, 14 dash and it would be b 1 dash. So this is after step 1. In step 2 what we do? In step 2 there are 3 parts. Step 2 there are 3 parts actually. Part 1 that we do R 2 – R 1 * a 21. So we say that this is, so we say that R 2 = R 2 minus so new R 2 would be equal to R 2 – R 1 * a 21. So what we do is that we have to multiply this entire row by a 21.

So then here we will get one a 21 and then here of course a 12 dash * a 21, a 13 dash * a 21, 14 dash * a 21 and then here also we will have b 1 dash * a 21 and when I do subtract this entire new row from this so it will go to 0. So it becomes 0. This becomes 0 and it would be a 22 – a 1 dash. It would be a 22 – a 12 dash * a 21 and etc. etc. So let us say this one will also be changed to a 22 dash, a 23 dash, a 24 dash and b 2 dash.

Then in step, then in the next we would say that R 3 = R 3 – R 1 * a 31. And in the third step we write that R 4 = R 4 – R 1 * a 41. So this we will do. So once we do this so by the same logic what we will find that this would be 0. This also would be 0. So then what we will have here that it would be 0. It will also be turn out to be 0 and all these terms will change; a 32 dash, a 33 dash, dash, dash, dash, dash. So this completes step 2.

So what we have done essentially we have essentially made all these terms in column one 0 except this first pivot. Then in step 3 what we do? We do that in step 3 what we do? So now we work with this new R 2. So we do that R 2 = R 2/ a 22 dash. So what we do? Essentially we actually divide this entire row by a 22 dash. So by doing so we make this to be 1, make this to be 1 and this becomes a 23 dash/ a 22 dash.

This become a 24 dash/a 22 dash. So these values would be changed. So let us say that it becomes a 22 double dash. This becomes a 24 double dash. And this also becomes b 2 dash/a 22 dash. So let us say this becomes b 2 double dash. After that what we do in step

4, in step 4 we have 2 steps. In step 1 we do that R 3, so now you would be working with this. So R 3 = R 3 – R 2 * a 32 dash. So essentially what we are doing?

Now because we have got one unity here so we have to multiply this entire row by a 32 dash. So once we multiply this entire row by a 32 dash so then therefore here we will get one a 32 dash and here of course a 23 double dash * a 23 dash and etc. etc. It will be a 24 double dash * a 3 etc. etc. Then when I essentially subtract this new row from this we get here 1, 0 and this term becomes a 33 dash – a 23 double dash * a 32 dash.

So then therefore this value will also change. So it will become a 33 double dash some value. This will also change from this a 34 dash – a 24 double dash * a 32 dash. So it will also change to a 34 double dash and this one will also change from b 3 dash to b 3 dash – b 2 double dash * a 32 dash. So this also becomes b 3 double dash. And here also we then in step 2 in the part 2 of so please from here this is, so this is part of.

And this R 4 = R 4 – R 2 * a 42 dash. So this also another step we have to do or rather another part we have to do. So once I do that by the same logic this becomes 0. These values get changed. So they all become a 34 double dash and a 44 double dash and b 4 double dash.

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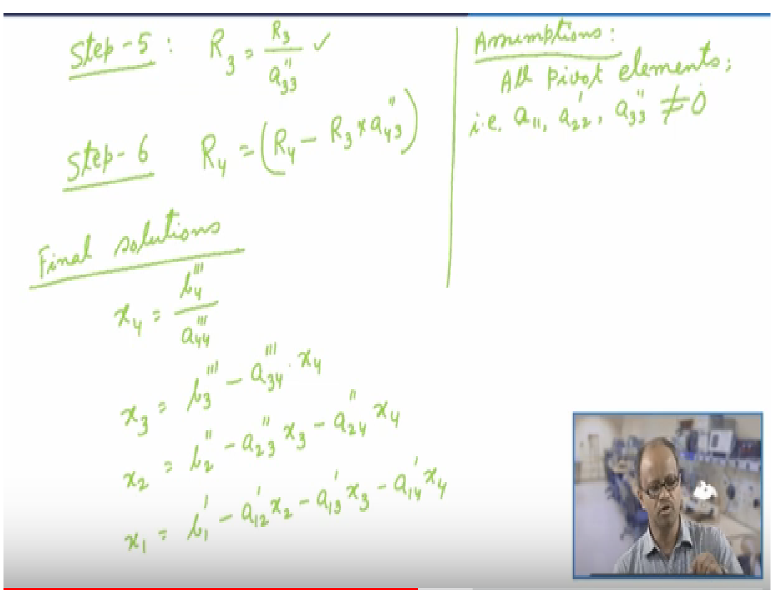

So in step 5 what we do is, so step 5 what we do is we do so now we work with this and we simply divide this entire row by a 33 double dash. So then what we do is that R 3 we do R 3 = R 3/a 33 double dash. So once we do that what we get, once we divide everything so what we get is this becomes 1. So this becomes unity and this becomes a 3 double dash/a 33 double dash. So it is changed.

So let us say that it has become a 34 triple dash. And this becomes b 3 double dash/a 33 double dash. So this becomes b 3 double dash. And the last stage, step 6 what we do is, step 6 what we do is we do R 4 = R 4 – R 3 * a 43 double dash. So once we do that so then what we get? So this becomes again 0 and this becomes and this also changed because it will be a 44 double dash minus a 34 triple dash * a 43 double dash.

So then therefore this value will also change. So it will be also a 44 triple dash and it will also change because it will be changed to b 4 double dash – b 3 triple dash * a 43 double dash. So this will also become b 4 triple dash. So once we do that so then our final solution would be, so then the final solutions would be. So we start with the bottom. So we start with the bottom ones; x 4 would be given by from here b 4 triple dash/a 44 triple dash.

So it would be b 4 triple dash/a 44 triple dash. Then what would happen to x 3? x 3 would be basically now from this equation x 3 + a 34 triple dash * x 4 = b 3 triple dash. So it would be b 3 triple dash – a 34 triple dash * x 4 and x 4 would be from here. So it is – a 34 triple dash * x 4. Similarly, x 2 would be b 2 double dash – a 23 double dash * x 3 – a 24 double dash * x 4. Please look at this equations.

So x 2 would be b 2 double dash – this into this minus this into this. All these are known and x 3, x 4 we have just now calculated from here so it known. And x 1 would be essentially from this equation b 1 dash – a 12 dash x 2 – a 13 dash x 3 – a 14 dash x 4. So that is how it would go. Now here we have assumed that well we did not face any kind of problem with the pivoting.

Pivoting means that whenever that is basically whenever we have tried to divide for example here, here and here, right? These values a 11, a 22 dash and a 33 double dash they are all nonzero. So here the assumptions we have taken that assumptions where in our calculation here that assumptions where that all pivot elements that is a 11, a 22 dash and a 33 double dash are not equal to 0 because if that is 0 so then this divisions do not exist.

So in that case we will not be able to get the modified row. However, if there is any problem that if this for example if a 22 dash becomes 0 here for example after this elimination, so then what we will do? So we will simply swap this with this right and then we will start working with this. So then therefore whenever we will face that some pivot element or let us say one of two pivot element has become 0 or rather the pivot element which we wish to work on now has become 0 we will simply swap this row with the row just below it.

If it is I mean if the corresponding element in that same column is actually nonzero. So in that case we will simply make that particular element to be the pivot element and then continue. Now for this simple 4 * 4 system this is pretty simple. Obviously now we can very easily understand that if we do extend it for any N * N system this methodology is extremely just absolutely the same.

So this can be easily extended to any general N * N matrix relation. Now let us look at some, at one very simple example, simple numerical example. So now let us look at this example, one simple numerical example. So we have got, again we have taken one 4 * 4 matrix.

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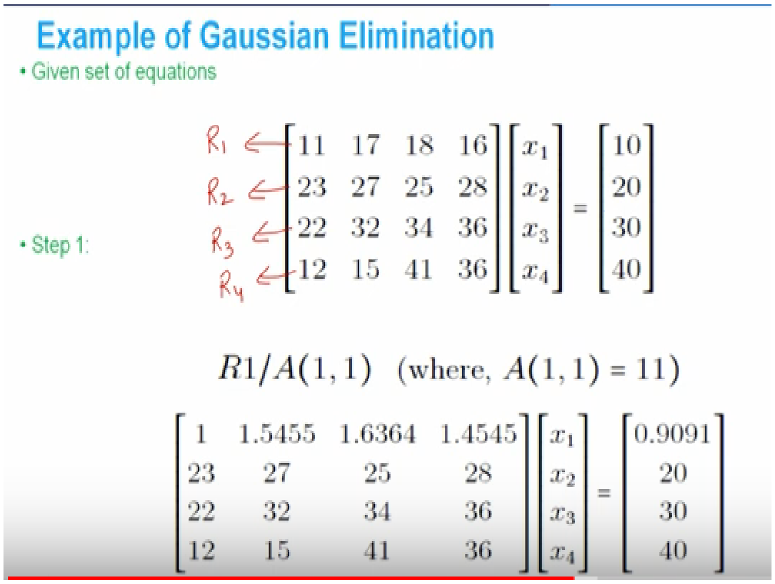

And there are 4 unknowns and these are known. So b 1, b 2, b 3, b4. So in step 1 what we do? Now again this is R 1, this is R 2, this is R 3, this is R 4. So this is R 1. Again it is R 1, this is R 2, this is R 3, this is R 4. So then in the first case we are dividing R 1/a 11. So that is this as we take it pivot so where A 1,1 is 11. So we are taking this as pivot. So we are dividing everything. So when we are dividing everything we are getting this. Everything is changed. So this is changed. These 3 rows are remaining unchanged.

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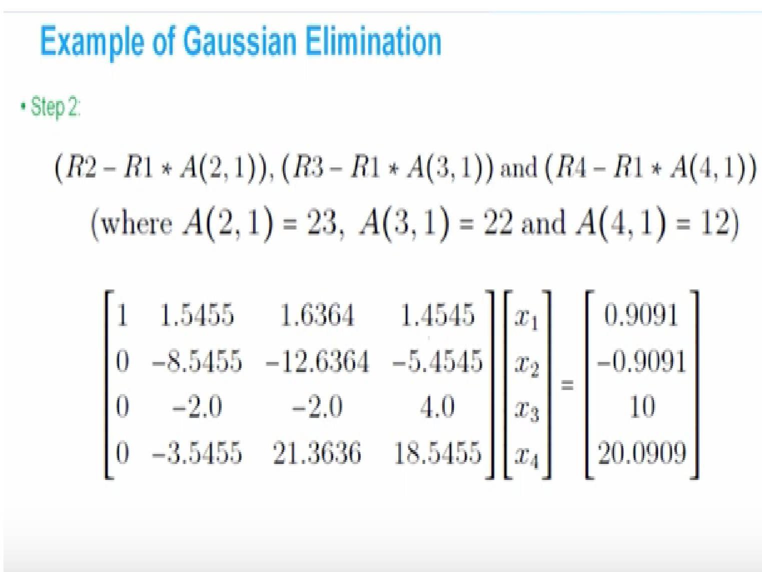

In step 2 we have got 3 parts. First is that R 2 – R 1 * A(2, 1). So then therefore what we do in the first case that this row minus this row into 23. So what we will do now we will now simply multiply this changed first row with basically 23. So when you multiply this changed first row with 23 so this element becomes 23 and this becomes something else. And when we do subtract them, so then this becomes 0 and these elements change.

So then therefore these elements change and similarly also these element change. And same what we do is that after that we take this one. That is we multiply this row with 22. So then therefore we get 122 here and then we simply subtract this row from this row. So we get another 0 and these values also get changed and the last stage we do multiply this row with 12. So we get 12 here and also these values change and then we simply subtract this changed row from this row.

So here we get 10 and this values becomes something else. So once we get these values, so I mean this is basically after this, after step 2. So in step 3 what we will do is that we will simply divide this entire row, this entire second row by -8.5455. So then when I divide this entire second row by -8.5455 what we will get, that I will get simply one unit here and these values will also change. And that is what we are exactly doing.

So we are dividing this entire second row by – 8.5455. So we get some value. So here you see all these are negative. So then therefore when I divide all this negative quantity by another negative quantity this becomes positive. So this value is changed and now what we do is and you see and this 3 and next two are remaining same. So this row sorry so this third row and this fourth row they remain as such same.

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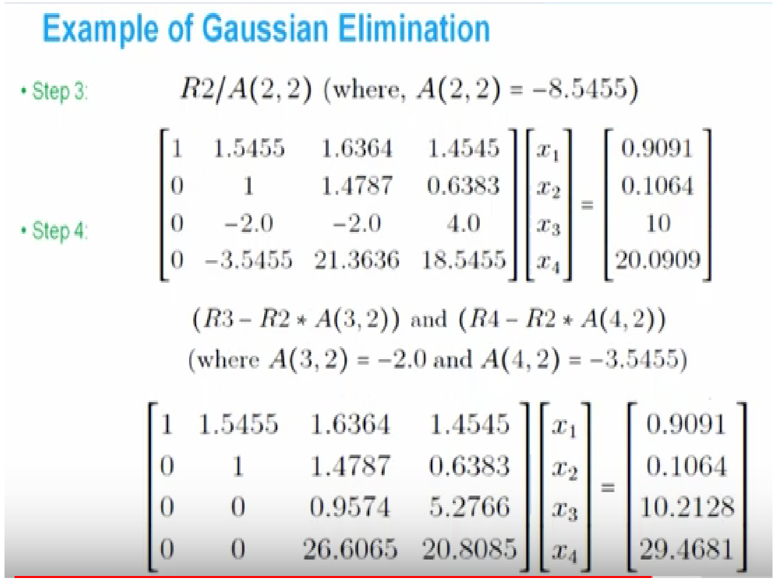

Then in step 4 what we do? That we now multiply this row by – 2 and then subtract the product form this row. So this becomes 0. And so this becomes something else. So this becomes 0 and this element changes. This element changes and this element also changes. Please note that because this is 10 and this is – 2 so then obviously this will be more than 10. Similarly, we do now multiply this row by – 3.5455 here.

So then here we will also get one – 3.5455. So once we get this – 3.5455 then what will happen? So then after that we will simply subtract this row from this row so we will get another 0. Then this value changes, this value changes and this value also changes. So you see after this two steps we get these two and this two to be unity and all this rows, all this elements to be 0.

So in the last two stage, in the last in the next stage what we will do? We will simply divide this entire row by 0.9574. So once we divide this entire row by 0.9574 we get one unity and we get something here we get something here.

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Example of Gaussian Elimination

- Step 5: $R3/A(3,3)$ (where $A(3,3) = 0.9574$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & 1 & 1.4787 & 0.6383 \\ 0 & 0 & 1 & 5.5114 \\ 0 & 0 & 26.6065 & 20.8085 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 0.1064 \\ 10.6672 \\ 29.4681 \end{bmatrix}$$

- Step 6: $(R4 - R3 * A(4,3))$ (where $A(4,3) = 26.6065$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & 1 & 1.4787 & 0.6383 \\ 0 & 0 & 1 & 5.5114 \\ 0 & 0 & 0 & -125.83 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 0.1064 \\ 10.6672 \\ -254.3484 \end{bmatrix}$$

So we get unity, something here, something here because we have 5.27/0.9574. So it will be more than 5.27. So we get some value. So we get also some value. And then after that we simply multiply this row by 26.6065 and then subtract the resulting row from this row. So we get here 0, right and then we get here something. So it will be almost – 130 or something. So we get something. And then we get also here something.

So now what we have got? So now after that our final solution would be x 4 would be this divided by this.

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Example of Gaussian Elimination

- Final Solution: $x_4 = 254.3484/125.83 = 2.0214$

$$x_3 = 10.6672 - 5.5114 \times 2.0214 = -0.4735.$$

$$x_2 = 0.1064 + 1.4787 \times 0.4735 - 0.6383 \times 2.0214 = -0.4837.$$

$$x_1 = 0.9091 + 1.5455 \times 0.4837 + 1.6364 \times 0.4735 - 1.4545 \times 2.0214 = -0.5086.$$

So the final solution x 4 would be this divided by this. All are minus so then this minus has been omitted. X 3 would be this minus 5.5114 * x 4. So it is exactly 10.6672 – 5.5114 * x 4 x 4 is this. And x 2 would be 0.107 and x 2 would be 0.1064 minus this into this minus this into this. So now here what happens that because this is negative so it is 0.1074 – 1.4787 – 1.4787 * x 3. Now x 3 is actually – 0.4735.

So then it is minus into, so minus of minus it becomes plus. So it is 1.4787 * 0.4735 – 0.6383 * x 4 and x 4 is 2.0214. So when I plug this and do this calculation we get -0.4837 and x 1 would be that 0.9091 minus this into x 2 minus 1 point this into x 3 minus this into x 4. So because x 2 also now negative so this becomes plus. X 3 is also negative, so this becomes plus but x 4 is positive. So this becomes minus.

So once we do this calculation we get this value. So these are the final solution. So this so then by this method one can easily solve the any set of N * M matrices or rather any set of N * N linear equations involving n unknowns. One condition here should be that this N linear equations involving n unknowns should be independent in nature. That is if they are not independent in nature so then therefore this resulting matrix A matrix would be basically a singular matrix.

So then therefore if the singular matrix, so then now so then therefore even if we do carry on this Gaussian elimination method on the singular matrix, ultimately at some point of time we will face struggle and we will not be able to calculate the final answers. So then therefore one has to be very sure or rather one has to be very careful that these equations which we are taking, these equations are independent in nature so that this resulting coefficient matrix that is matrix A is actually nonsingular in nature.

Although this Gaussian elimination method has got some I mean it is a very simple method and also it is very easy to code. But then it has got some basic problem. Now let us look at some problems of Gaussian elimination. So there is some limitation in Gaussian elimination method and this limitation are basically nothing but this fill-in problem.

And this fill-in problems is nothing but that when we are trying to do Gaussian elimination method and if this original A matrix if there are some 0 values in the process of Gaussian elimination method those 0 values becomes ultimately nonzero. So then if this values becomes nonzero so then therefore we are not really being able to exploit the advantage of sparsity, right?

So then therefore we need to little more careful in carrying out this operations of this Gaussian elimination method. So this aspect we will continue in the next lecture. Thank you.