Computer Aided Power System Analysis

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Lecture - 27
Gaussian Elimination Method

Hello, welcome to this lecture on computer aided power system analysis. In the last lecture we have discussed about the sparsity of the Jacobian matrix and then we have also introduced the basic concept of the Gaussian elimination method. So now let us look at the details of the Gaussian elimination method. So we will first take an example and then we will try to generalize this method.

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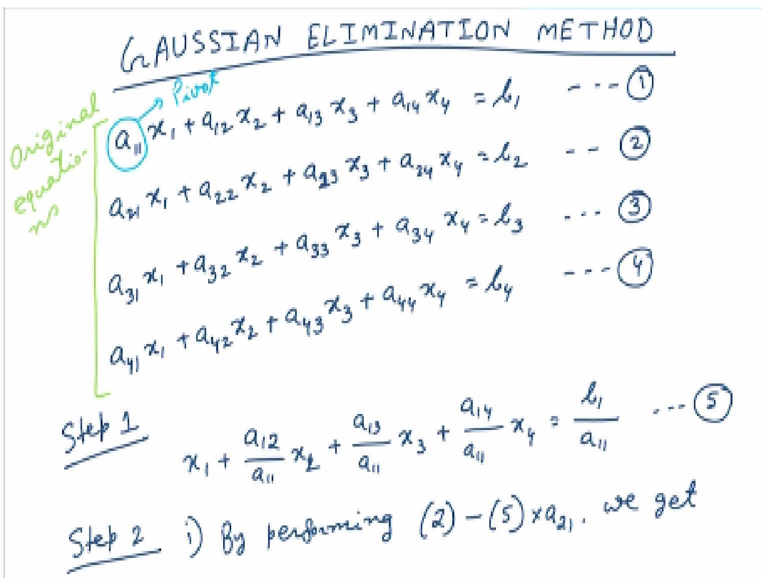

So let us look at that Gaussian elimination method. Now as we have discussed in the last class, so let us look, let us take a small example of a 4 * 4 matrix. So we had got this equation a 11 x 1 + a 12 x 2 + a 13 x 3 + a 14 x 4 = b 1; a 21 x 1 + a 22 x 2 + a 23 x 3 + a 24 x 4 = b 2; a 31 x 1 + a 32 x 2 + a 33 x 3 + a 34 x 4 = b 3 and lastly a 41 x 1 + a 42 x 2 + a 43 x 3 + a 44 x 4 = b 4. So let us say that this is equation 1, 2, 3, 4.

So what we do in the first step, in this first step, step 1 so we say that these are the original equations. These are the original equations. So in the first step what we do is we do divide this equation 1 by a 11. So the first we simply take it as a pivot element we say that a 11 is a pivot element. So we say that a 11 is a pivot element. So we take this as a pivot element and then divide entire equation 1 by this pivot element.

So when I do this we do get that x 1 + a 12/a 11 x 2 + a 13/a 11 x 3 + a 14/a 11 x 4 = b 1/a 11. So let us say that this is equation 5. Then what we do is we now multiply equation 5 by a 21, a 31, and a 41 one by one and then subtract the resulting equations from equation 2, 3, and 4 respectively. So let us say step 2 and step 2 will have part 1. Part 1 is that, what we do, that we first multiply equation 5 by a 21 and then subtract the resulting equation from equation 2.

So then therefore by performing we write equation 2 – equation 5 * a 21. We get, what we get? Equation 2 – 5 * a 21.

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$$\left(a_{21}-a_{21}\right)x_1+\left(a_{22}-\frac{a_{12}}{a_{11}}a_{21}\right)x_2+\left(a_{23}-\frac{a_{13}}{a_{11}}a_{21}\right)x_3+\left(a_{24}-\frac{a_{14}}{a_{11}}a_{21}\right)x_4=\left(b_2-\frac{b_1}{a_{11}}a_{21}\right)$$

Pivot

$$\Rightarrow a_{22}^{(1)}x_2+a_{23}^{(1)}x_3+a_{24}^{(1)}x_4=b_2^{(1)} \quad \cdots (6)$$

$$\text{ii)}\quad (3)-(5)\times a_{31} \Rightarrow$$

$$\left(a_{31}-a_{31}\right)x_1+\left(a_{32}-\frac{a_{12}}{a_{11}}a_{31}\right)x_2+\left(a_{33}-\frac{a_{13}}{a_{11}}a_{31}\right)x_3+\left(a_{34}-\frac{a_{14}}{a_{11}}a_{31}\right)x_4=\left(b_3-\frac{b_1}{a_{11}}a_{31}\right)$$

$$\Rightarrow a_{32}^{(1)}x_2+a_{33}^{(1)}x_3+a_{34}^{(1)}x_4=b_3^{(1)} \quad \cdots (7)$$

$$\text{iii)}\quad (4)-(5)\times a_{41} \Rightarrow$$

So what, equation 2 is a 21 minus again a 21 x 1 + a 22 minus we multiply a 1, we multiply everything by a 21. So it is a 12/a 11 * a 21. So a 12/a 11 * a 21 x 2 + a 32 minus it would be a 11, it would be a 21 and it would be a 13 x 3 and lastly plus it should be a 24 – a 14/a 11 a 21 * x 4 = b 2 – b 1/a 11 * a 21. So this we can write down as this is 0, x 1 is gone. So let us say this is a 22.

Let us say this is the first updated, this is the changed value a 22 square bracket, within bracket 1 means that is the first modified value x 2 plus we say that it is a 23 (1) x 3 plus we say that a 24 (1) x 4 is equal to let us say this is the first modified value. This is equation 6. Then in step 2 what we do? We do equation 3 – equation 5 * a 31. So what we get? We get (a 31 – a 31) x 1.

So we are basically eliminating x 1 from all this equations. Then we get a 32 – a 12/a 11 * a 31 x 2 + a 33 – a 13/a 11 * a 31 * x 3 + a 34 – a 14/a 11 * a 31 * x 4 = b 3 – b 1/a 11 * a 31. So this is what we will get. So we multiply everything by a 31 and then subtract this from this. So what we get? So we get from here x 1 is again gone. So it is a 32 the first updated value of x 2 + a 33 first updated value of a 33 x 3 + a 34 that is the first updated value x 4 = b 3 this is the first updated value. Let us say this is equation 7.

Then in the third, let us say in the third what we do? We do equation 4 – equation 5 * a 41. So what we get?

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$$(a_{41}-a_{41})x_1 + \left(a_{42} - \frac{a_{12}}{a_{11}}a_{41}\right)x_2 + \left(a_{43} - \frac{a_{13}}{a_{11}}a_{41}\right)x_3 + \left(a_{44} - \frac{a_{14}}{a_{11}}a_{41}\right)x_4 = \left(b_4 - \frac{b_1}{a_{11}}a_{41}\right)$$

$$\Rightarrow a_{42}^{(1)}x_2 + a_{43}^{(1)}x_3 + a_{44}^{(1)}x_4 = b_4^{(1)} \quad -(8)$$

Step-3

Devide (6) by $a_{22}^{(1)}$

$$\Rightarrow x_2 + \frac{a_{23}^{(1)}}{a_{22}^{(1)}}x_3 + \frac{a_{24}^{(1)}}{a_{22}^{(1)}}x_4 = \frac{b_2^{(1)}}{a_{22}^{(1)}} \quad \ldots (9)$$

Step-4

i) (7) − (9) × $a_{32}^{(1)}$

So what we get? We get (a 41 – a 41) * x 1 + a 42 – a 12/a 11 * a 41 * x 2 + a 43 – a 13/a 11 * a 41 * x3 + a 44 – a 14/a 11 * a 41 * x 4 = b 4 – b 1 a 11 * a 41. So a 14 a 41 a 13 a 11. So this is it. So this can be written as x 1 is gone. So a 42 (1) x 2 + a 43 (1) first updated value x 3 + a 44 (1) x 4 = b 4 (1). So this is 8. So then what are the equations we have? So we have got these equations.

So now we have got equation 7. So now what we have got? We have basically from the initial 4 equations we have now basically eliminated x 1. So then we have got resultant 3 equations and in this 3 equation there are only 3 unknowns x 2, x 3, x 4. And you have got 6, 7, and 8. Now what we will do is, so we have got 6, 7, and 8. Now what we will do is, we will now take a 21 as pivot. So we will now take this as pivot, this one.

Again first thing, here it was pivot, we call it pivot. Now in the next phase, we will not take this one as pivot. So when we take this one as pivot so then what we do? We divide this entire equation by a 22 (1). So when we do that, so in step 3 what we get? We divide equation 6 by a 22 (1). So what we get? We get is x 2 + a 23 (1)/a 22 (1) x 3 + a 24 (1)/a 22 (1) x 4 = b 2 (1)/a 22 (1). So now we call it, this is equation 9.

In step 4 what we do is we now multiply this equation 9 with a 32 (1) and then subtract the resultant equation from equation 7. So then what we do is we do in the first stage, we do (7) – (9) * a 32 (1). So we got 7 – 9 * a 32 (1). So what we get?

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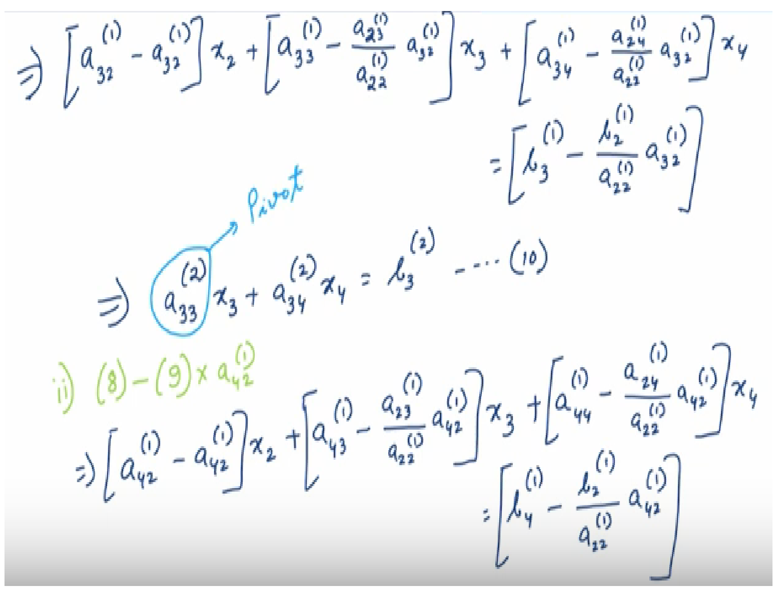

So we get is a 33 (1) minus it would be a 23 (1)/a 22 * a 32 (1). So it will be a 23 (1)/a 22 (1) * a 32 (1). This into x 3 + a 34 (1) – a 23 (1)/a 22 (1) sorry it would be a 24 (1) a 22 * a 32 (1) * x 4. That would be b 3 (1) – b 2 (1)/a 22 (1) * a 32 (1). So we get this. So from here what we get? x 2 is eliminated. So from this we now get a 33 second updated value x 3 + a 34 second updated value x 4 = b 3 second updated value.

So this let us say this is equation 10. Then in step 2, then in part 2 of this step what we do is we do equation 8 – 9 * a 42 (1). So we do equation 8 – equation 9 * a 42 (1). So we get a 42 (1) – a 42 (1). This is x 2 plus a 43 (1) – a 23 (1)/a 22 (1) * a 42 (1). This plus a 44 (1) – a 24 (1) a 22 (1) * a 42 (1). That is = x 4 = b 4 (1) – b 2 (1)/a 22 (1) * a 42 (1), right? So we get this.

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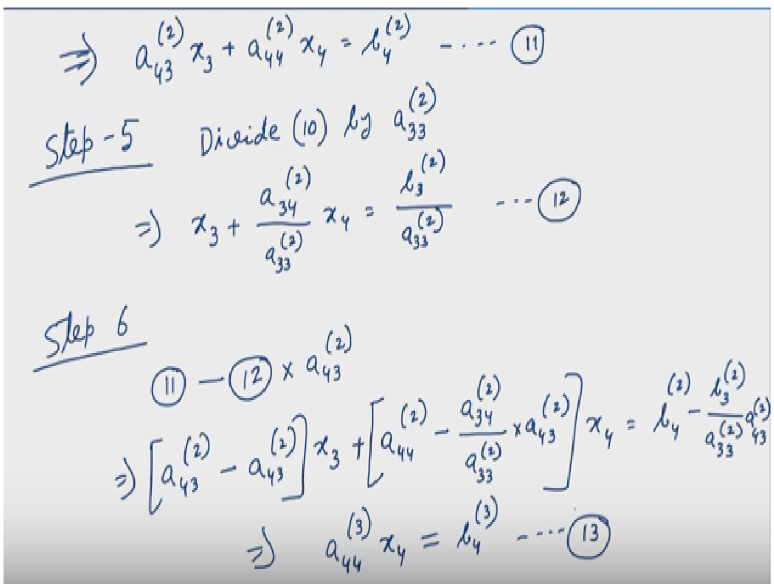

So once we get this, so then this we can write down as x 2 is gone. Once x 2 is gone so we can write down as that this is a 43 (2) x 3 + a 44 (2) x 4 = b 4 (2) right? So a 43 x 3 (2). So this is equation 11. So I have got equation 11. So now what we have got? We now from the 3 equations 6, 7, and 8 we have now eliminated x 2 and by eliminating x 2 we have now reduced it to 2 equations involving only x 3 and x 4.

So now what we will do is, we will simply take this as pivot. So we will simply take this as pivot and then what we will do is so then once we take this pivot so then we write in step this is step 5. So in step 5 so what we do is we divide equation 10 by a 33 (2). Divide equation 10 by a 33 (2). So by this what we get? We get is x 3 + a 34 (2)/a 33 (2) * x 4 = b 3 (2)/a 33 (2).

That is what I get. a 34 (2)/a 33 (2) * x 4 = b 3 (2)/a 33 (2), right? So this becomes equation 11, this is equation 12 and then in step 6 what we do is we do equation 11 – equation 12 * a 43 (2). So what we get? We get [a 43 (2) – a 43 (2)] * x 3 + [a 44 (2) – a 34 (2)/a 33 (2) * a 43 (2)]. This is x 4. That would be equal to b 4 (2) – b 3 (2) * a 33 (2) * a 43 (2). So from this equation x 3 is gone.

So this we can say that this is a 44 third updated value * x 4 = let us say b 4 (3). So this is equation 13. So now equation 13 only involves x 4. This is known, this is known. So then therefore we now so now how do we solve.

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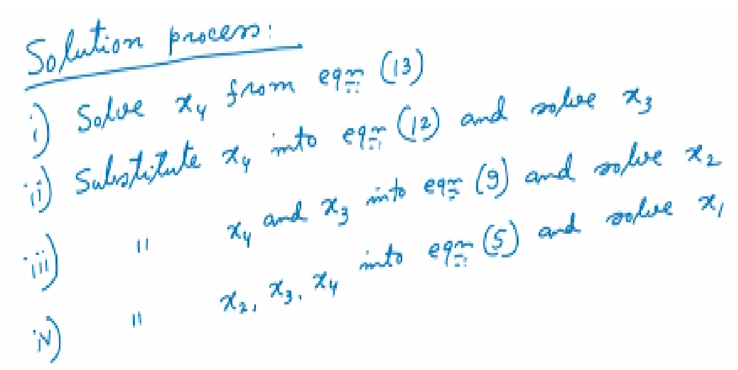

So then the solution process, now we will start from backward. So what is solution process/ Solve x 4 from equation 13. Then what is that? Once we get x 4 from here we will plug this x 4 into equation 12 and then solve x 3 from equation 12. Then what we do? Substitute x 4 into equation 12 and solve x 3. Then what we do? Substitute x 4 and x3 into equation, which equation we should now use?

So now x 4 and x 3 are known so now after that we can simply substitute x 4 and x 3 in equation 9 and then solve for x 2, into equation 9 and solve x 2. And then the last one is that we are simply so x 2, x 3, x 4 into equation 5 and solve x 1. So that is how, so you see we are being able to solve all this quantity x 1, x 2, x 3, x 4 just by simple algebraic manipulation without involving any matrix inversion.

So this step can be very easily extended for n equation system. Now let us just look at that how to implement it in terms of matrix operation. So let us look at very quickly, it will take not much of a time.

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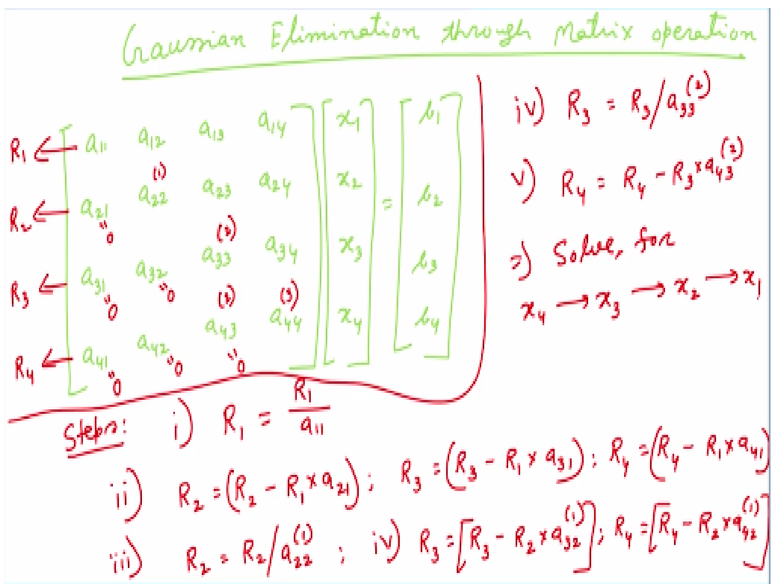

So let us look at that Gaussian elimination through matrix operation. So now what we have got? For example this one if I say that this is so this my matrix is a 1, a 2, a 3, a 4; a 21, a 22, a 23. So these are the equations. So now for matrix operation purpose what we do is we do denote then that this is R 1 row R 1. This is row 1, this is row 2, this is row 3, this is row 4.

So how do we I mean translate this matrix operation I mean how do we really translate all these operations by means of matrix operation? So in the steps what we can do is that in steps 1 is that R 1 this row is actually being R 1/a 11, right? Then we do in 2, what we do? We replace R 2 as = R 2 - R 1 * a 21; R 3 as R 3 – R 1 * a 31 and R 4 as R 4 – R 1 * a 41. So we get this new rows. In step 3 what we now do?

So now once we do this all these terms would be 0. That is basically this would be 0, this would be 0, this would be 0 and these would be 111 111 111. So then we can simply say that this would be 0. This becomes 0, this becomes 0, this becomes 0. So this all become 111. So then after that what we do? That we do R 2 = R 2/a 22 (1). So because you see this will become a 22 (1).

And then in step 4 what we do? In step 4 we do that R 3 = [R 3 – R 2 * a 32 (1)] and R 4 = [R 4 – R 2 * a 42 (1)]. This is what we will do. So after we do this so then what will happen? This will become 0, this will become 0 and this becomes a 33 (2). So after that what we will do? We do in step 4, so from here we are going here. So this is step 4. So in step 4 what we do?

With step 4 we do that R 3 = R 3/a 33 (2) and then in step 5 we do R 4 = R 4 – R 3 * a 43 (2). So this also becomes a 43 (2). So once we do this, this becomes 0, this becomes a 44 (3). So then, then in the last step solve for first x 4 followed by x 3 followed by x 2 followed by x 1. So this is how it will proceed by means of matrix operation. As you can see that these operations are very easy to follow.

So this can be very easily extended for a case where there are n equations and n unknowns. Now here only point needs to be noted that in case in the process of elimination if suppose any term becomes 0. For example let us say a 22 (1) becomes 0 so then in that case we really cannot this. So then in that case, in that step we really cannot take a 22 (1) as the pivot element because if we try to divide this by this pivot element so it will become infinity.

So then whenever any pivot element becomes 0 we do not take that as the pivot element but we go to the next equation. For example this one we take this and then we do take this as the pivot element and then proceed. So this is basically the basic technique of the

Gaussian elimination method. In the next lecture we would be looking at one simple numerical example. Thank you.