

Computer Aided Power System Analysis
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Lecture - 25
AC - DC Load Flow (Contd..)

Hello, welcome to this lecture on computer aided power system analysis. In the last lecture we have discussed about the unknown quantities as well as the equations corresponding this AC – DC load flow. And we have seen that in the case of HVDC system there are actually 5 equations which do describe the operation of an HVDC link but then there are 9 unknown quantities involving both the rectifier side as well as the inverter side.

Now we have argued that because there are only 5 equations but 9 unknown quantities so then therefore we need to pre-specify 4 quantities and then subsequently we can solve the remaining 5 quantities utilizing this 5 equations. Now here the question comes that out of this 9 quantities which 4 quantities we actually need to pre-specify and then which 5 quantities we actually need to solve.

Actually as it happens, that out of this 9 quantities we can basically pre-specify any 4 quantities and it is possible to calculate the remaining 5 quantities utilizing this 5 equations. So now let us look at some example.

(Refer Slide Time: 01:53)

Case - i)
 Specified quantities: $\alpha, P_{dr}, \gamma, V_{di}$

Steps

- $$P_{dr} = V_{dr} I_d = V_{dr} \frac{V_{dr} - V_{di}}{R_{dc}}$$

$$\Rightarrow P_{dr} R_{dc} = V_{dr}^2 - V_{dr} V_{di}$$

$$\Rightarrow V_{dr}^2 - V_{dr} V_{di} - P_{dr} R_{dc} = 0$$

$$\Rightarrow V_{dr} \text{ can be calculated.}$$
- Calculate $I_d = \frac{V_{dr} - V_{di}}{R_{dc}}$

To calculate
 α ✓
 P_{di} ✓
 Q_{di} ✓
 6 other quantities

So in this example, so case 1, so let us say some case. We will be actually discussing 2 cases. So in case 1, let us say the specified are specified quantities. These specified quantities are α , P_{dr} , γ , V_{di} . Now you may wonder that we are actually trying to calculate P_{dr} but then here we are actually specifying P_{dr} . Now the answer lies the fact that whenever we are actually specifying any HVDC link, we usually say that this HVDC link is at this particular moment is let us say supplying 1000 MW.

Because whenever we are trying to solve the load flow solution of any HVDC system so in that load flow solution we need to know that what is the amount of power which is being carried by the HVDC link. Until and unless we do not know that what is the power being carried by the HVDC link we really cannot solve for this AC and DC system together.

So then therefore because we already know that what is the amount of power being carried by this HVDC link, so it is quite a practical assumption to know that this value of P_{dr} is actually available to us. So we have got this 4 quantities known. So then utilizing this 4 quantities we now need to calculate the remaining quantities.

That is our final goal is to calculate the all the other quantities in that particular vector, unknown vector that is basically all the remaining 6 quantities plus we also need to

calculate Q_{dr} , P_{di} , Q_{di} and 6 other quantities. So this is our goal. So we need to solve from this 4 quantities and this 5 equations we need to solve all this other quantities. So let us see how to do this.

So we will solve it in steps and these are very simple steps, nothing; so steps. So first step. P_{dr} is known. Now P_{dr} we know that P_{dr} is actually $V_{dr} * I_d$. Now what is I_d ? I_d is actually $V_{dr} * V_{dr} - V_{di}/R_{dc}$. Please note that this R_{dc} is also known. R_{dc} is nothing but the resistance of the HVDC link and here we assume that all the parameters of all the lines that is all the AC lines as well as the HVDC link are known.

So then therefore R_{dc} is known. So then therefore from this, so then we can write down this equation as $P_{dr} * R_{dc} = V_{dr}^2 - V_{dr} * V_{di}$. So we can write down as $V_{dr}^2 - V_{dr} * V_{di} - P_{dr} * R_{dc} = 0$. So this is a quadratic equation involving V_{dr} . Now in this please note that this V_{di} is known. It is already pre-specified. So this V_{di} is known. P_{dr} is already known because it is already pre-specified.

R_{dc} is nothing but the HVDC link. R_{dc} is nothing but the resistance of the HVDC link which is also known. So then in this quadratic equation only unknown is which is V_{dr} . So now because everything else is known so then we can calculate V_{dr} . So V_{dr} can be calculated. Now here there is one little question. The little question is that it is a quadratic equation. So then therefore we will get 2 values of V_{dr} .

Which value of V_{dr} we should take? We should only accept that value of V_{dr} which is greater than V_{di} because after all the current can only flow from rectifier side to inverter side. But it cannot flow from inverter side to the rectifier side. So then therefore out of this 2 values of V_{dr} we should only choose the value which is greater than V_{di} . So so we choose the value which is greater than V_{di} .

Then step 2, once we know V_{dr} then we calculate $I_d = V_{dr} - V_{di}/R_{dc}$. So here V_{dr} we have just now calculated in step 1. V_{di} is already known. R_{dc} is known. So I_d is calculated. Now what we do? So now we then we go to step 3.

(Refer Slide Time: 07:35)

$$3. \quad V_{dor} = \frac{V_{dr} + \frac{3}{\pi} X_{cr} N_r I_d}{\cos \alpha} \quad \left| \quad \text{as } V_{dr} = V_{dor} \cos \alpha - \frac{3}{\pi} X_{cr} N_r I_d \right.$$

$\Rightarrow V_{dor}$ is known (calculated)

$$4. \quad V_{dr} = V_{dor} \cos \phi_r \quad \Rightarrow \quad \cos \phi_r = \frac{V_{dr}}{V_{dor}} \rightarrow \text{known}$$

$$5. \quad Q_{dr} = P_{dr} \tan \phi_r \rightarrow \text{calculated.}$$

$$6. \quad V_{dor} = \frac{3\sqrt{2}}{\pi} n_r a_r E_{tr} \rightarrow a_r \text{ is calculated.}$$

$$7. \quad P_{di} = V_{di} I_d \rightarrow P_{di} \text{ is calculated}$$

In step 3 we write that $V_{dor} = V_{dr} + \frac{3}{\pi} X_{cr} N_r I_d$ divided by $\cos \alpha$. Now from where we have got this equation? We have got this equation from as from please recollect that there is one equation that $V_{dr} = V_{dor} \cos \alpha - \frac{3}{\pi} X_{cr} N_r I_d$. So then therefore if we just rewrite this equation we get this expression of V_{dor} . Now in this expression what happens? In this expression what are quantities known?

α is known. So α is known. I_d we have just now calculated in the last step. V_{dr} we have calculated in the first step. X_{cr} and N_r they are all known quantities. For example we have already said in the last lecture that for a 12 – pulse HVDC link $N_r = 2$ and X_{cr} is nothing but the transformer leakage reactant. So then therefore in this expression everything is known. So then therefore V_{dor} is known.

So therefore V_{dor} is known or calculated, right? So V_{dor} is calculated. In step 4 what we known, what we calculate? We have already seen that we have got an equation which says that $V_{dr} = V_{dor} \cos \phi_r$. we have already seen this equation in the last lecture and so then therefore $\cos \phi_r$ is V_{dr}/V_{dor} . Now in this V_{dr} is known. V_{dr} we have already calculated in the first step. V_{dor} we have just now calculated in the last step.

So then therefore $\cos \phi_r$ is known. So then this is the third quantity we have calculated. So now here you see there are 6 other quantities. So then this is one quantity we have calculated. This is another quantity we have calculated. This is the third quantity we have calculated, right? So out of the 6 quantities we have already calculated 3 quantities. Now once we know $\cos \phi_r$ so then we calculate $Q_{dr} = P_{dr} \tan \phi_r$.

ϕ_r we have just now calculated. Q_{dr} and this P_{dr} is already pre-specified. So then therefore I can calculate Q_{dr} . So then it is calculated. So it is calculated. So here also Q_{dr} is calculated. Now in step 6, step 6 we write that V_{dor} is equal to, $V_{dor} = 3 \sqrt{2/\pi} N_r a_r E_{tr}$. Now here in this expression V_{dor} we have just now calculated. V_{dor} we have just now calculated in step 3. N_r is known.

Now if we know E_{tr} , we would be able to calculate a_r . Now from where E_{tr} will come? Now please again recollect our discussion regarding the basic procedure of the sequential AC – DC load flow. What we do in the case of sequential AC – DC load flow? We first assume the flat start. Please note that essentially it is an AC grid and in this AC grid between 2 buses we have got one HVDC link connected.

And we are trying to solve for everything. So then when we start our sequential load flow, what we do we first take the flat start. So then therefore the moment we do take the flat start so then therefore this value of E_{tr} is already known. Initially, this value of E_{tr} is known. So then therefore initially when we are starting our calculation this E_{tr} is known. So once this E_{tr} is known so then therefore a_r is also known.

So then I should actually, so therefore a_r is calculated. So out of this 6 quantities so this is the 4th quantity we are calculating. The rest two are, so then we have still have to calculate 4 more quantities. Which 4 more quantities? Two quantities from here and this P_{di} and Q_{di} . Step 7, we calculate P_{di} as $V_{di} * I_d$. Now, in our specification, V_{di} is already known. So then therefore so P_{di} is calculated, right? So P_{di} is calculated.

So we, because V_{di} is known and P_{di} is calculated, so P_{di} is calculated. So here this P_{di} is also calculated. Then what we do?

(Refer Slide Time: 14:27)

8. $V_{doi} = \frac{V_{di} + \frac{3}{\pi} X_{ci} N_i I_d}{\cos \gamma} \rightarrow V_{doi}$ is calculated

9. $\cos \phi_i = \frac{V_{di}}{V_{doi}} \rightarrow \cos \phi_i$ is calculated

10. $Q_{di} = P_{di} \tan \phi_i \rightarrow Q_{di}$ is calculated.

11. $V_{doi} = \frac{3\sqrt{2}}{\pi} N_i a_i E_{xi} \rightarrow a_i$ is calculated

Therefore, P_{di} , Q_{di} , P_{di} , Q_{di} are known. Utilising these values, we carry out one iteration of AC load flow and therefore, updated values of E_{xi} and E_{xi} are obtained.

Then we again write down the equation $V_{doi} = V_{di} + \frac{3}{\pi} X_{ci} N_i I_d / \cos \gamma$. Now here in this equation V_{di} is already pre-specified. γ is pre-specified. I_d we have calculated. N_i is known. N_i is number of 6 – pulse converters connected in series at the inverter side. N_i is known. And X_{ci} is nothing but the transformer leakage reactance at the inverter side. So then therefore V_{doi} is also calculated.

So then therefore V_{doi} is calculated. So once we know the V_{doi} , so then after that what we do is we calculate $\cos \phi_i = V_{di} / V_{doi}$. So then therefore $\cos \phi_i$ is calculated. So this is the fifth quantity we are calculating, fifth quantity we are calculating, right? This is the fifth quantity we are calculating because here V_{di} is known. This is actually pre-specified and V_{doi} we have just now calculated.

Once we know $\cos \phi_i$ so then in the next step I can calculate $Q_{di} = P_{di} \tan \phi_i$. So then therefore Q_{di} is calculated. So in this Q_{di} is also calculated. And out of the 6 quantities we have already calculated 5 quantities. Which one is remaining? Remaining is a_i . that is the transformer tap ratio at the inverter side. So to calculate the transformer tap

ratio what we do is, we write that V_{doi} is equal to, we write that $V_{doi} = 3 \sqrt{2/\pi} N_i a_i E_{ti}$.

So here V_{doi} we have just now calculated from step 8. N_i which is known. N_i is nothing but the number of 6 – pulse converters in series at the inverter side. E_{ti} again, when we start our iteration we first take this, we first take this flat start. So then therefore E_{ti} magnitude is also known. So then therefore when E_{ti} is known everything else is known. So then a_i is calculated. So a_i is calculated.

So this is the 6th quantity we have solved. So then therefore by following this very simple algebraic steps we are being able to calculate all the 6 remaining quantities as well as the quantities which are of our interest. So now what happens? After we finish this calculation, so then therefore after we finish this calculation, after finishing this calculation, so therefore P_{dr} , Q_{dr} , P_{di} , Q_{di} are known.

Once they are known so then after that what we do? So utilizing this equivalent injections we now solve one iteration of the AC load flow, just one iteration. We do not solve this AC load flow completely. We just do the one iteration of this AC load flow. After we do this one iteration of this AC load flow so then what we get? So then, so we write, so utilizing this values we carry out one iteration of AC load flow.

And thus, and once we carry out one iteration of AC load flow so then what we get after we carry out the one iteration of AC load flow? We get the updated values of E_{tr} and E_{ti} . And therefore updated values of E_{tr} and E_{ti} are obtained. So once we get this updated value of E_{tr} and E_{ti} so after that we again recalculate, we again follow steps 1 – 11.

So after we get this updated value of E_{tr} , E_{ti} so then therefore we again follow the steps 1 – 11, update the values of P_{dr} , Q_{dr} , P_{di} , Q_{di} , do another iteration of AC load flow then we get another updated values of E_{tr} and E_{ti} . Then again simply follow the steps 1 -11. Then get updated values of P_{dr} , Q_{dr} , P_{di} , Q_{di} . Then again solve another iteration. Get updated value of E_{tr} , E_{ti} and then come back.

So we keep on doing this iteration till this AC load flow converges in the usual sets. So that is how this particular algorithm actually works. So we can see here that although these equations are completely nonlinear because out of this 5 equations, 4 equations involve cosine term. So then therefore 4 equations are completely nonlinear. But then we really do not have to invoke any kind of complicated Newton – Raphson technique or etc.

For solving this 5 equations we can simply solve for the remaining unknown quantities which are left after our pre-specified quantities we can simply solve this remaining unknown quantities by simple algebraic manipulation of this 5 equations. So then therefore these calculations are really very computationally light.

Because they are computationally light so then therefore the overall efficiency of the AC – DC load flow is not really hampered much, not at all hampered. It has very little enhancement of this code. So this is the basic philosophy. Now we have just now only seen only one case when we have got specified quantities α , P , δ , γ and V , δ . Let us take another case and then let us see that how these calculations.

Now here let us say that instead of P , δ and V , δ we just let us say reverse them. We just see that V , δ and P , δ is known and α and γ . So then in case 2, so now let us consider case 2.

(Refer Slide Time: 23:23)

Case - ii) $\alpha, \gamma, P_{di}, V_{dr}$ are specified.

Steps

i) $P_{di} = V_{di} I_d = V_{di} \frac{V_{dr} - V_{di}}{R_{dc}}$

$\Rightarrow R_{dc} P_{di} = V_{di} V_{dr} - V_{di}^2 \Rightarrow V_{di}^2 - V_{di} V_{dr} + R_{dc} P_{di} = 0$

ii) $I_d = \frac{V_{dr} - V_{di}}{R_{dc}} \rightarrow I_d$ is calculated

V_{di} is calculated

So in case 2, just it is another example, in case 2 alpha, gamma, P di and V dr are specified. So if alpha, gamma, P di and V dr are specified. So in the earlier case we said that alpha, gamma, P dr and V di, now here we are seeing alpha, gamma, P di and V dr. So then how do we calculate this? So the steps are, steps are very simple. This goes in a very you know this almost in the same manner.

So we say that P di we know that $P_{di} = V_{di} * I_d$. So this is nothing but $V_{di} * V_{dr} - V_{di} * R_{dc}$. So we can write down that $R_{dc} P_{di} = V_{di} V_{dr} - V_{di}^2$. So then therefore it is $V_{di}^2 - V_{di} V_{dr} + R_{dc} * P_{di} = 0$. Again this is a quadratic equation involving V di. In this quadratic equation which quantities are known? The quantities known are V dr which is specified. P di which is specified. R dc which is known.

So then therefore this is a quadratic equation. We can solve for this quadratic equation. By the standard technique we get 2 values of V di. We take that value of V di which is less than V dr. So we here in this case we have to take that value of V di which is less than V dr. So once we get this value of V di so then what we do is then we calculate $I_d = V_{dr} - V_{di} / R_{dc}$. So then from here we write that V di is calculated.

Then here also I d is calculated. Once we know I d and V di, after that what we do is if we just look at it after that we can simply follow the step 3, step 4, step 5, step 6, step 7

up to step 11 as it is. Then we can calculate all this other quantities. So now what are the major steps, so then basic algorithm of this AC – DC load flow. So then how do we simply write down the basic algorithm for.

(Refer Slide Time: 26:58)

- Schematic (overall) algorithm for AC-DC load flow
- i) Assume flat start for the AC system.
 - ii) Solve the DC equations to calculate P_{dr} , Q_{dr} , P_{di} , Q_{di}
 - iii) Run one iteration of AC load flow
 - iv) Go to step (ii) till convergence is achieved.

So let us just a schematic algorithm or rather I should say overall algorithm for AC – DC load flow. So what are the overall algorithms? So what that algorithm is that first we take, assume flat start for the AC systems. So we assume flat start for the AC system. So then therefore E_{tr} and E_{ti} is known. Then we write that solve the DC equations to calculate P_{dr} , Q_{dr} , P_{di} , Q_{di} . Then what we do?

We run or rather run or execute one iteration of AC load flow. So when we run one iteration of AC load flow then we get the updated values of E_{tr} and E_{ti} and then again we solve the DC quantity etc. etc. So then therefore what we say in the next that go to step 2 till convergence is achieved. So this is the very basic algorithm for AC – DC load flow. So that is how, so we can see that there is no change in the AC load flow and etc.

So we have already explained all these things. So now let us look at some very simple example.

(Refer Slide Time: 29:49)

Example of AC-DC load flow in 5 bus system

- One bipolar HVDC link is connected between bus 4 and 5 (rectifier at bus 4 and inverter at bus 5)
- Base voltage of the AC system is 132 kV

$$R_d = 10.0 \Omega;$$

$$N_r = N_i = 2; \frac{3}{\pi} X_{cr} = \frac{3}{\pi} X_{ci} = 6.0 \Omega.$$

So in this example, so in this example again we take this 5 bus system. So one bipolar HVDC link is now what we have been saying that in this 5 bus system we have got one bipolar HVDC link is connected between bus 4 and bus 5, rectifier at bus 4 and inverter at bus 5. Basically, the base voltage of the AC system is 132 KV. One thing we must actually mention here that the calculations at the DC side we usually do with the actual values.

Calculations at the DC side we actually do with the actual values. So now these are the various quantities. R_d is nothing but the DC link resistance. $N_r = N_i = 2$ because we are considering 12 – pulse converter LCC. And X_{cr} X_{ci} , it is actually giving $3/\pi$ into this. I mean we are taking X_{cr} and X_{ci} , X_r it should be X_r and X_i which is nothing but basically transformer leakage reactants at the rectifier, transformer leakage reactants at the inverter. This is = 6. That is $3/\pi$ is 6.

(Refer Slide Time: 30:55)

Example of AC-DC load flow in 5 bus system

Combination-1

$$\alpha = 5^\circ, P_{dr} = 100 \text{ MW}; \gamma = 18^\circ, V_{di} = 250 \text{ kV}$$

$$V_{dr} = 253.938 \text{ kV}; i_d = 393.8 \text{ Amp.}; Q_{dr} = 16.276 \text{ MVAR};$$

$$P_{di} = 98.45 \text{ MW}; Q_{di} = 35.024$$

$$P_4 = -1.15 - 1.0 = -2.15 \text{ p.u.}, Q_4 = -0.6 - 0.16276 = -0.76276 \text{ p.u.},$$

$$P_5 = -0.85 + 0.9845 = 0.1345$$

$$Q_5 = -0.4 - 0.35024 = -0.75024 \text{ p.u.}$$

Now the first combination what I have said, that in the combination alpha we have assumed 5 degree. P dr 100 MW, alpha 18 degree, gamma 18 degree. That is V di this. Now because everything is given in you know megawatt and MVR and etc. So we calculate everything in actual values. I mean we do not do the calculation in the DC side in the per unit values. We do the calculation at the DC side in the actual values.

So when we do these calculation with actual value by following the steps we get V dr is equal to this value, I d is equal to this, Q dr is this, P di, Q di everything. So all these values are known. Once we know these values so now we calculate the equivalent injection at P 4. Now please note in the 5 bus system originally at 5 bus system there was already a load connected 115 MW. So in per unit it is already -1.15.

On top of that P dr is 100 MW so that is equal to -1. So then therefore now the net power injected is this plus this. That is minus. So now net load at bus 4 is actually 215 MW. So that translates into 2.15 per unit. At bus 4 the basically the reactive load which was already connected was 60 MVR. So this is -0.6. And here we have got and also this is absorbing 16.276 MVR. So in per unit it is 0.16276.

So then therefore net reactive power load here is -0.6 * point this plus this. So it is, so net reactive power load at bus 4 has also increased. So it is -0.7626. So this plus this. At

the inverter side, at bus 5 the original AC load connected was 85 MW. So then therefore it has, so basically corresponding to that load, my injected real power is - 0.85. But the inverter is injecting a value of P_{di} , injecting, it is injecting.

It is giving into this AC bus a value of 98.45 MW. So then therefore net injected power would be is equal to this minus because this is load plus this. This is the generation as far as bus 5 is concerned. This is generation, so then net real power injected is this plus this, so 0.1345 per unit. And at the rectifier side, sorry and at the inverter side Q_{di} is 35.024 MVR. Please note that both inverter and rectifier they do absorb reactive power from the AC grid.

So then therefore as far as this AC grid is concerned this inverter is also acting as a load as basically, is also acting as a reactive power load. So then therefore this is the reactive power load due to this Q_{di} and this is the earlier AC reactive power load which was connected, which is actually 40 MVR which is already connected. So the net reactive power load at bus 5 is increased. So then these are the new quantities.

So then please note that in the DC side we are doing the calculations using the actual values but when we are converting them into the AC side we are converting them in the per unit values and please note that this base MVA is same in everywhere, at the AC side and the DC side base MVA is same. So once we get this equivalent values of, net values of P_4 , Q_4 , P_5 , Q_5 we do run one iteration and then update the values of E_{tr} and E_{ti} and then again recalculate and recollect and then again keep on doing this. And once we do this then so then this whole algorithm converges in total iteration of 5 iteration.

(Refer Slide Time: 35:11)

Example of AC-DC load flow in 5 bus system

Bus no.	Without generator Q limit			
	$ V $ (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1.0	0	0.68984	0.46301
2	1.0	-0.63995	0.5	-0.17235
3	1.0	-4.91128	1.0	1.54134
4	0.82813	-17.48682	-2.15	-0.76277
5	0.91332	-3.89028	0.13449	-0.75025
Total iteration = 5				

$$a_r = 0.8714$$

$$a_i = 0.8149.$$

And these are the results and if you compare this results you will find that these values have actually gone down significantly because of the fact that at bus 4 and bus 5 this rectifier and inverter converter they are actually acting as reactive power load. So as if there is an extra reactive power load. So then therefore their voltages has gone down and the net P inj and etc. that also changed.

In this case, net P inj has got now -0.25 and here it also increased and this net Q inj etc. have increased. And these are the final values of a r and a i. So that is how this AC – DC load flow is executed. So this is the end of our AC – DC load flow. So from the next lecture onwards we would be looking into the other aspects of this particular course. Thank you.